Investigation on the accuracy of equivalent electric models for mushrooms strutures

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Abstract— **Planar distributed periodic structures more commonly known as mushroom structures which present a negative refractive index (NRI) in a very narrow frequency band, are usually studied using equivalent electric models under quasi-static approximation. The dispersion equations for different configurations can be established using Bloch theorem for 2D transmission lines (TL). This paper proposes a critical study concerning the validation of results obtained from these utilised dispersion equations through comparing them to those obtained using a full-wave method. An extraction method has been proposed to simulate the frequency dependent of each distributed element. These results show a frequency dependence different from that obtained from effective elements calculated using conventional analytical formulas for the effective material parameters of the unit cell.**

Keywords- Mushroom; left-handed material; equivalent electric models.

I. Introduction

Recent works on artificial magnetic conductors [1],[2] have shown that planar periodic structures as mushroom structures represented in Fig 1, exhibit a High Impedance Surface (HIS) behaviour in a very narrow frequency band when they are illuminated by a plane wave. This property allows the elimination of surface waves as well as the selection of an electromagnetic wave defined by its polarization and its working frequency. So the mushroom structures are often classified as a kind of two-dimensional Electromagnetic band gap (EBG) material. But in most cases, they are classified as metamaterials since a backward-wave propagation phenomenon can be produced under surface excitation conditions. In quasi-static approximation, i.e. when the unit cell size is much smaller than the guided wavelength, a mushroom structure can be treated as a homogeneous medium and represented by an equivalent electric circuit models. According to [1],[2], an effective sheet impedance model composed of a simple equivalent LC circuit can be used for representing a plane wave illumination case. However, for a surface excitation case, an equivalent electric circuit model of composite elements Right/Left-Handed (CRLH) must be applied [3],[4]. Nevertheless, the backward-wave phenomenon leading to negative refractive index (NRI) appears only in a very narrow frequency band. In other words, the dispersion diagrams of mushroom unit cell must be precise in the NRI region, namely when the slope turns negative in a dispersion characteristics representation. For this purpose, several methods based on the 2-D Bloch analysis of TLs were given in [4],[5] and [6] that can determine the equation of dispersion of the structure.

This paper begins by a brief review on the two dispersion equations that are often applied. Next, we present a study concerning the validation of results obtained from these utilised dispersion equations through comparing them to those obtained using rigorous electromagnetic simulations. Lastly, an extraction method using [ABCD] transfer matrix will be presented to determine which of the distributed elements of a unit cell concord with analytic excepted values.

II. 2D Dispersion Equations from Transmission Line Approach

The propagation characteristics of a two-dimensional (2D) periodic electrical network of a mushroom structures can be studied in quasi-static approximation by combining the 2D TL approach with the Bloch theorem [4],[5],[6]. So the unit cell of the structure can be modeled by a 2D-network of four electrical ports as shown in Fig. 2. Where a, w, g, h denote the lattice period, the patch with, the gap width between two adjacent patches and the dielectric thickness respectively.

Each branch port is represented by its [ABCD] transfer matrix and the application of Bloch theorem gives both input and output voltage and current of the unit cell. Applying the boundary conditions with the Kirchhoff's laws at intersection nodes leads to the following reciprocal linear matrix system:

$$[M]\begin{bmatrix}V_x\\I_x\\V_y\\I_y\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \quad (1)$$

where [M] is a 2D hybrid matrix and V_i and I_i, are the voltage and current , respectively.

The two dispersion equations introduced by [4], [5] and [8], use the known linear matrix system principle. Only in [8] the system has been diagonalized.

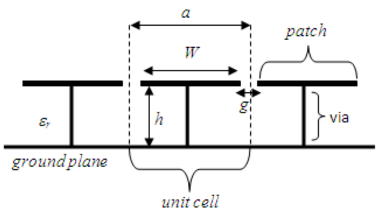

Figure 1. Side view of mushroom structure unit cell.

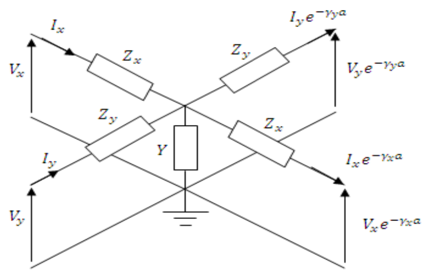

Figure 2. Unit cell of a general 2D periodic TL network.

A. First model : LC model

In the LC model of the unit cell, introduced by [4] and used in [5], the admittance Y represents either the inductance of a current loop as given by (2) in [1] or the inductance of a metallic via by (3) in [7].

$$L = \mu_o \mu_r h \qquad (2)$$

$$L = \frac{\mu_o h}{2\pi}\left(log\left(\frac{2h}{r}\right) + k\right) \qquad (3)$$

where k is a dimensional constant and r the radius of the metal via. If h>>r then k=-1.

In both [4] and [5], the capacitance C represents the gap between two adjacent patches expressed by their analytical expressions given by (4-a) or (4-b) according to [1] and [2], respectively. Here we have chosen the (4-b) expression.

$$C = W\epsilon_o \frac{(1+\epsilon_r)}{\pi} acosh\left(\frac{a}{g}\right) \qquad (4\text{-a})$$

$$C = a\epsilon_o \frac{(1+\epsilon_r)}{\pi} log\left(\frac{2a}{g\pi}\right) \qquad (4\text{-b})$$

Where ϵ_r and μ_r denote the relative permittivity and permeability. No magnetic media will be considered here, i.e $\mu_r = 1$.

The non trivial solution of the matrix system [M] can be resolved either by reducing the system [4] or directly [6]. The dispersion equations for an isotropic lossless media using both ways are given by:

$$\sin^2\left(\frac{\beta_x a}{2}\right) + \sin^2\left(\frac{\beta_y a}{2}\right) - \frac{1}{2}A_1.B_1 = 0 \qquad (6)$$

$$\sin^2\left(\frac{\beta_x a}{2}\right) + \sin^2\left(\frac{\beta_y a}{2}\right) + \frac{1}{2}(A_2 + B_2 + C_2) + 1 = 0 \qquad (7)$$

where,

$$A_1 = \left[2\sin\left(\frac{\beta a}{2}\right) - jz_1 cos\left(\frac{\beta a}{2}\right)\right],\ B_1 = \left[2\sin\left(\frac{\beta a}{2}\right) - j\frac{1}{2z_2}cos\left(\frac{\beta a}{2}\right)\right]$$

$$A_2 = \left(2 + \frac{z_3}{2z_2}\right)\cos(\beta a), B_2 = j\left(2z_3 + \frac{1}{2z_2}\right)\sin(\beta a), C_2 = -\frac{z_3}{2z_2}$$

When expanding (6) one obtains:

$$\frac{1}{2}(A_2 + B_2 + C_2) + 1 = -\frac{1}{2}A_1.B_1 \qquad (8)$$

with: $z_1 = \frac{1}{jZ_0 C\omega}$, $z_2 = \frac{j\omega L}{Z_0}$, $z_3 = z_1/2$, $z_0 = \frac{\eta_0}{\sqrt{\varepsilon_r}}\frac{h}{a}$

Where Z_0, $\beta_{x,y}$ and, $\beta = \frac{\omega}{c}\sqrt{\varepsilon_r}$ denote the impedance characteristic, the phase constants along x and y directions and the propagation constant, respectively. For an isotropic lossy media, the dispersion equation given by (7) takes the form given in (9) in which the complex propagation constants $\gamma_{x,y} = \alpha_{x,y} + j\beta_{x,y}$ are taking account. The $\alpha_{x,y}$ parameters denote attenuation constants in x and y directions.

$$\sinh^2\left(\frac{\gamma_x a}{2}\right) + \sinh^2\left(\frac{\gamma_y a}{2}\right) - \frac{1}{2}(A_3 + B_3 + C_3) + 1 = 0 \qquad (9)$$

where,

$$A_3 = \left(2 + \frac{z_3}{2z_2}\right)\cosh(\gamma a), B_3 = \left(2z_3 + \frac{1}{2z_2}\right)\sinh(\gamma a), C_3 = \frac{z_3}{2z_2}$$

B. Second model : Composite Right/Left-Handed (CRLH) model

In this CRLH model of the unit cell, the admittance Y represents a shunt capacitance C_R in parallel with an inductance L_L, and the impedances $Z_{x,y}$, represents an inductance $L_R/2$ in series with a capacitance $2C_L$, where L_L, C_R, C_L and L_R represent, respectively, the inductance of a metallic via given by (3), the interaction capacitance between the patch and the ground plane given by (10), the gap between two adjacent patches expressed by (4-a) or (4-b) and the inductance of the patch L_R for which no expression has been clearly set in previous work but we found that expression (2) gives a good accuracy .

$$C_R = \frac{\epsilon_o \epsilon_r W^2}{h} \qquad (10)$$

A specific diagonal transformation of the [M] matrix can be employed to set the following dispersion equation [8], [9] for an isotropic media ($Z_x = Z_y = Z/2$):

$$\sin^2\left(\frac{\beta_x d}{2}\right) + \sin^2\left(\frac{\beta_y d}{2}\right) + \frac{ZY}{4} = 0 \qquad (11)$$

where Z and Y, are the impedance Z and admittance Y per unit length, respectively defined by the well-known following relations :

$$Z = j\omega a L_R\left(1 - \frac{1}{\omega^2 C_L L_R}\right) \qquad (12)$$

$$Y = j\omega a C_R\left(1 - \frac{1}{\omega^2 C_R L_L}\right) \qquad (13)$$

Inserting (12) and (13) in (11) yields the following dispersion relation $\vartheta(k)$:

$$\sin^2\left(\frac{\beta_x d}{2}\right) + \sin^2\left(\frac{\beta_y d}{2}\right) = \frac{1}{4}\vartheta^2\left(1 - \frac{{\vartheta_\epsilon}^2}{\vartheta^2}\right)\left(1 - \frac{{\vartheta_\mu}^2}{\vartheta^2}\right) \qquad (14)$$

where [9], $\vartheta^2 = a^2\omega^2\epsilon_o\mu_o$, ${\vartheta_\epsilon}^2 = a^2\frac{\epsilon_o\mu_o}{L_L C_R}$, ${\vartheta_\mu}^2 = a^2\frac{\epsilon_o\mu_o}{L_R C_L}$

The solutions of this dispersion relation can be rewritten in the form $k(\vartheta)$:

$$\vartheta_{\pm}^2 = \frac{1}{2}\left(F \pm \sqrt{F^2 - \left(2\vartheta_{\epsilon}^2\vartheta_{\mu}^2\right)}\right) \tag{15}$$

where , $F=\vartheta_{\epsilon}^2+\vartheta_{\mu}^2 + 4sin^2\left(\frac{\beta_x d}{2}\right) + 4sin^2\left(\frac{\beta_y d}{2}\right)$

The dispersion diagram can be plotted in the well-known first Brillouin zone: $\beta_x d: 0 \rightarrow \pi,\ \beta_y d = 0$ for the ΓX direction, $\beta_y d: 0 \rightarrow \pi,\ \beta_x d = \pi$ for XM direction and $\beta_x d = \beta_y d:\ \pi \rightarrow 0$ for MΓ direction.

In preserving the same specific transformation of the [M] matrix, we found that the dispersion equation for an anisotropic media ($Z_x \neq Z_y$) is given by:

$$\mathrm{Z_x} \sin^2\left(\frac{\beta_x d}{2}\right) + \mathrm{Z_y} \sin^2\left(\frac{\beta_y d}{2}\right) = -\frac{\mathrm{Z_x Z_y}}{2}\mathrm{Y} \tag{16-a}$$

Or: $$\frac{\mathrm{Z_x}}{\mathrm{Y}}\cos(\beta_x d) + \frac{\mathrm{Z_y}}{\mathrm{Y}}\cos(\beta_y d) - \frac{\mathrm{Z_x}}{\mathrm{Y}} - \left(\mathrm{Z_x} + \frac{1}{\mathrm{Y}}\right)\mathrm{Z_y} = 0 \tag{16-b}$$

This dispersion equation can be solved analytically using rigorous mathematical and analytical software. Otherwise the roots can be researched numerically.

III. Accuracy of Dispersion Equations

To set the accuracy of the dispersion equations previously introduced, we compare them to dispersion diagrams obtained using the rigorous electromagnetic eigenmode solver provided in CST Microwave Studio software. Results of three different mushrooms unit cell cases are represented in Fig. 3, Fig. 4 and Fig. 5, respectively.

Case 1:

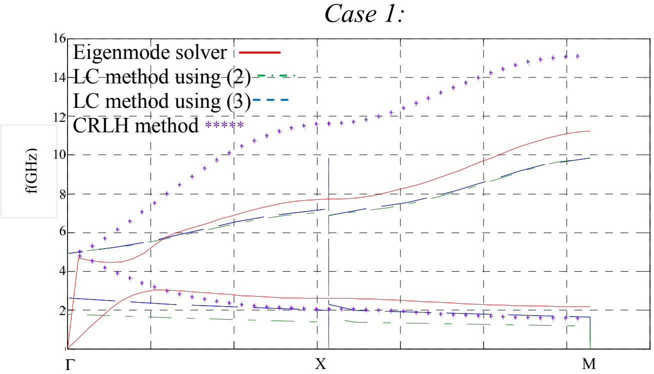

Figure 3. Comparison of dispersion diagram for W=9.6mm, a=10mm, h=3.08mm, r=0.125mm, $\epsilon_r = 2.33$

Case 2:

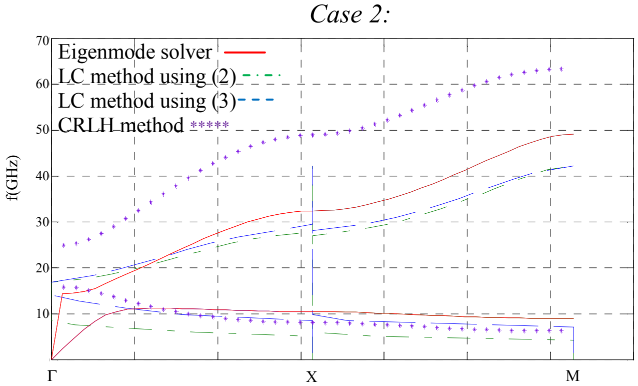

Figure 4. Comparison of dispersion diagram for *W=2.25mm, a=2.4mm, h=1.6mm, r=0.15mm*, $\epsilon_r = 2.2$

Case 3

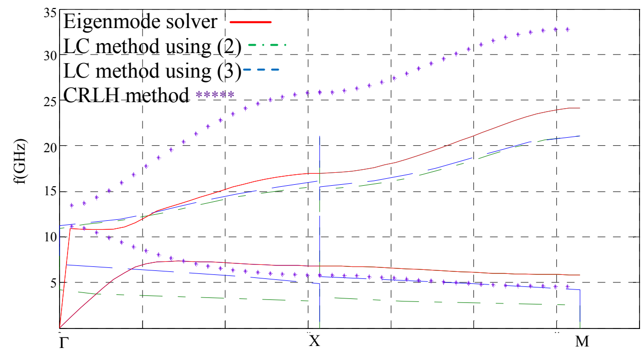

Figure 5. Comparison of dispersion diagram for *W=4.5mm, a=4.8mm, h=1.6mm, r=0.2mm*, $\epsilon_r = 2.2$

We can notice a good concordance of results for both methods for the quasi-static approximation region. As it can be seen, better results can be obtained when using equation (3) rather than (2). At higher frequencies for which the quasi-static approximation cannot be used, only the LC model keeps a correct fit while the CRLH method is no longer valid. To confirm this observation, an extraction method using rigorous electromagnetic simulations of a unit cell of mushrooms structure is proposed to obtain the frequency dependence of each equivalent electrical element and to compare the resulted performance to that when using a CRLH model unit cell. This extraction method has been validated by a canonical case using ADS electric software of Agilent.

IV. Extraction Method

To determine the equivalent electrical elements of a unit cell mushroom as a CRLH model unit cell, three different simulations of [S] parameters are necessary. The first one concerns a unit cell mushroom without the gap. In this case, as shown Fig. 6, just the inductance L_R has to be considered in the Z impedance. Using the conversion of [S] into [Z] impedance, we can extract the inductance L_R with the following expression:

$$L_R = 2\frac{(Z_{11}-Z_{12})}{j\omega} \tag{17}$$

where Z_{11} and Z_{12} denote the impedance elements of [Z] impedance.

The second simulation, is represented by the Π-network shown in Fig 7, It concerns two unit cell mushrooms with the gap between two adjacent patches.

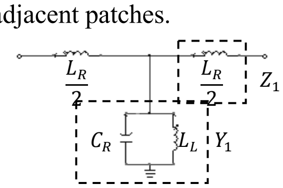

Figure 6. Equivalent circuit of a unit cell mushrooms without gap.

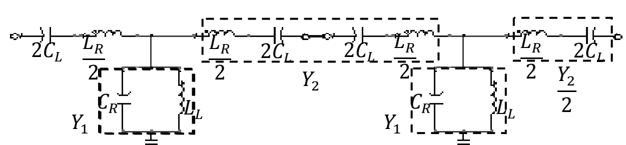

Figure 7. Equivalent circuit of two adjacent unit cell mushrooms.

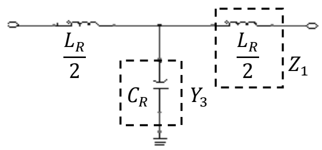

Figure 8. Equivalent circuit of a unit cell mushrooms without via and gap.

So, using the transfer of [S] into [ABCD] matrix the capacitance C_L (which represents the electric coupling between the two adjacent unit cell mushrooms) can be extracted with the following expression:

$$C_L = j\omega\left(\frac{1}{B_{II}} - \frac{1}{j\omega L_R}\right) \tag{18}$$

where $B_{II} = \frac{1}{Y_2}$ denotes the B element of [ABCD] matrix.

The third simulation shown in Fig. 8 concerns a unit cell mushroom without the metallic via and gap.

As shown previously, the [ABCD] matrix is used to extract the capacitance C_L (which represents the electric coupling between the patch and the ground plane) using the following expression

$$C_R = \frac{C_{III}}{j\omega} \tag{19}$$

where $C_{III} = Y_3$ denotes the C element of [ABCD] matrix

Finally the inductance L_L (which represents the metallic via) can be extracted by inserting (11) in the [A B C D] obtained from the first simulation.

$$L_L = \frac{1}{j\omega(C_I - jC_R\omega)} \tag{20}$$

where $C_I = Y_1$ denotes the C element of [ABCD] matrix of the first simulation.

V. ACCURACY OF CRLH MODEL USING EXTRACT METHOD

The extraction method has been employed for the two cases previously exposed in Fig. 3 and Fig. 5. In each of the two cases, a comparison is made between the analytic values of CRLH model calculated from (2) (3), (4-a) and (10) and those of the extraction method at two different frequencies as exposed in tables 1 and 2.

TABLE I. COMPARISON BETWEEN EXTRACTED AND CALCULATED ELEMENT FOR THE CASE 1 AT 2 GHZ AND 6 GHZ

	Calculated elements	Extracted elements at 2GHz and 6GHz
LR	3.9nH	3.46nH – 3.5nH
LL	1.78nH	1.5nH - 2.2nH
CR	0.62pF	0.5pF - 0.91pF
CL	0.36pF	0.2pF – 0.6 pF

TABLE II. COMPARISON BETWEEN EXTRACTED AND CALCULATED ELEMENT FOR THE CASE 3 AT 6 GHZ AND 12 GHZ

	Calculated elements	Extracted elements at 6GHz and 12GHz
LR	2nH	1.8nH - 2.2nH
LL	0.56nH	0.2nH – 0.9nH
CR	0.24pF	0.18pF - 0.57pF
CL	0.14pF	0.2pF - 0.25pF

It can be noted from these results some good concordance at low frequency and a divergence at high frequency especially for C_R and C_L. This divergence is due to the fact that the frequency is not considered in the analytic values of CRLH model. One can also note a good agreement of the value of L_R which prove that the choice of equation (2) was appropriate.

VI. CONCLUSION

An investigation on the accuracy of equivalent electric models of mushroom structures has been performed through the comparison of analytic results to those obtained from the use of rigorous electromagnetic eigenmode solver. An extraction method has been proposed to determinate the frequency dependence of each distributed element of a unit cell mushroom. The results confirm that it is necessary to take frequency into account in the CRLH model.

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