Statistical analysis on the uncertainty of asteroid ephemerides

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STATISTICAL ANALYSIS ON THE UNCERTAINTY OF ASTEROID EPHEMERIDES

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Abstract. The large number of asteroids allows a statistical analysis especially for their orbital uncertainty. It presents a particular interest for Near-Earth asteroids in order to estimate their close approach from Earth and eventually their risk of collision. Using ASTORB and MPCORB databases, we analyse the different uncertainty parameters (CEU, U) and highlight relations between the uncertainty parameter and the characteristics of the asteroid (orbital arc, absolute magnitude, ...).

Keywords: Asteroids, Ephemerides, Dynamics, Databases

1 Introduction

Since February 2010, the number of discovered asteroids exceeds 500 000. The large number of asteroids allows a statistical analysis particularly about their orbital uncertainty. There are currently four main databases for asteroids orbital elements: Minor Planet Center, Lowell Observatory, JPL and Pisa University. We specifically deal with two of these databases: ASTORB (Bowell 2011) and MPCORB (Minor Planet Center 2011a). As from April 26, 2011, a total of 550 293 asteroids are compiled in ASTORB and 550 468 in MPCORB. Both databases provide similar parameters such as name/number of the asteroid, osculating elements, magnitude and information on observations (number, length of the orbital arc) and specific information about the uncertainty (Desmars et al. 2011).

2 Uncertainty Parameters

The two databases provide parameters about ephemerides uncertainty. Only one for MPCORB which is the U parameter and five for ASTORB which are the CEU, CEU rate, PEU and the two greatest PEU.

The U parameter provided by MPCORB is a integer value between 0 (very small uncertainty) and 9 (extremely large uncertainty). It is determined by computing first the RUNOFF parameter:

$$RUNOFF = (dT * e + \frac{10}{P}dP)\frac{k_0}{P} * 3600 * 3 \tag{2.1}$$

where dT is the uncertainty in the perihelion time (in days), e is the eccentricity, P is the orbital period (in years), dP is the uncertainty in the orbital period (in days) and k_0 is the Gaussian constant (in degrees). RUNOFF can be considered as the in-orbit longitude runoff in arcseconds per decade. Finally, U is deduced from RUNOFF by:

$$U = \mathrm{INT}\left(\frac{\ln(RUNOFF)}{C_0}\right) + 1 \tag{2.2}$$

where $C_0 = ln(648000)/9$. The quality of the orbit can be quickly determined. But it would have be preferable to provide the RUNOFF parameter which represents a difference in longitude (a physical value).

The ASTORB database provides five parameters related to ephemeris uncertainty:

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- **Current Ephemeris Uncertainty** (CEU): absolute value of the current 1-ephemeris uncertainty expressed in arcsec. The date of CEU is also provided.
- **Rate of change of CEU** (noted CEU rate) given in arcsec/day
- **Peak ephemeris Uncertainty** (PEU): the next peak of ephemeris uncertainty from the date of CEU. The date of PEU is also given.
- **Greatest PEU**: the first parameter is the greatest PEU in 10 years after the date of CEU and the second one is the greatest PEU in 10 years after the date of next PEU.

3 Comparison of the parameters

The U parameter can be compared to the CEU and its rate of change. Fig. 1 shows a correlation between CEU, rate of change of CEU and U parameter. Asteroids with a low U have also a low CEU and CEU rate.

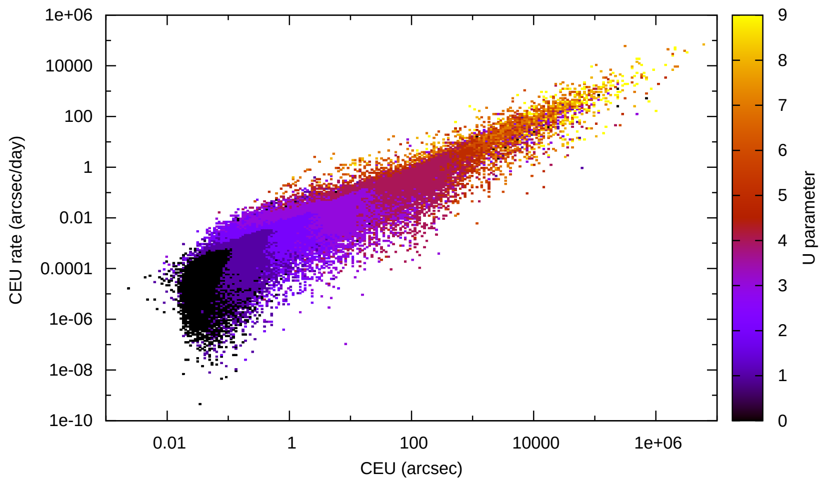

Fig. 1. CEU, CEU rate and U parameter for asteroids in common to the ASTORB and MPCORB databases.

All the parameters seem to be equivalent. The measure of the uncertainty can be provided by only one of these parameters. Nevertheless, one of these parameters could sometimes be missing. Another problem is that some asteroids have a bad parameter U ($U > 6$) and a good CEU ($CEU < 1$ arcsec) and conversely.

To clarify the situation, we have computed our own uncertainties for 4 groups of specific objects. These four groups have been defined considering good and bad U or CEU and two objects in each group have been selected. For each object, we have estimated the accuracy of the ephemerides using two statistical methods of non linear extrapolation.

These methods are based on Monte Carlo process in order to create clones of initial parameters from the nominal orbit, representing the region of possible motion. The first one, Monte Carlo using the Covariance Matrix (MCCM), consists in adding a random noise on each initial condition using the covariance matrix. The second one, Bootstrap Resampling (BR) consists in a random resampling of the observations (Desmars et al. 2009). For this test, we have computed 1000 clones of each representative asteroid and then calculated the standard deviation at the date of CEU, on angular separation, which represents the angular deviation of an orbit to the nominal one.

Table 1 gives the comparison between U parameter, CEU and standard deviation provided by clones of nominal orbit with two different methods. For these representative asteroids, the CEU is often close to the standard deviation computed whereas U parameter is misestimated for at least four representative asteroids.

Table 1. Comparison of U, CEU and standard deviation in discordant cases as obtained by clones of nominal orbit with two methods.

	U (mpcorb)	CEU (") (astorb)	σ_{MCCM} (") (our work)	σ_{BR} (") (our work)
2002GM5	9	56 000	55 297	46 934
2006LA	9	100 000	66 185	70 395
2000RH60	5	0.24	0.144	0.171
2003FY6	6	0.76	0.226	0.313
2007WD5	0	1 200	6 812	13 393
1995WZ13	0	5 400	0.122	1.751
4321 Zero	0	0.036	0.019	0.034
31824 Elatus	0	0.24	0.150	0.129

Moreover, the U parameter provides a number which is not related to a physical value (a distance or an angle).

In light of Table 1 and Fig. 1, the CEU seems to be a good parameter in order to estimate the accuracy and predictability of an asteroid orbit as CEU is quickly computable, precise and providing a physical value (an angle). Nevertheless, for some extreme cases (when only few observations are available or for Earth-approaching asteroids), the CEU can be misestimated (Minor Planet Center 2011b). This parameter is not perfect and could be improved for poor observed asteroids.

4 Relations between CEU and orbital arc

The decreasing of the CEU seems obviously related to the increasing of the orbital arc. In this context, we have tried to find a relation between this two parameters (Fig. 2). In this figure, four groups of asteroids can be identified using their orbital arc:

- arc $<$ 10 days
- 10 days $\leq$ arc $<$ 250 days
- 250 days $\leq$ arc $<$ 8000 days
- 8000 days $\leq$ arc

For each group, linear regression can be computed and we have an empirical relation between the value of CEU and the length of orbital arc:

$$\log(CEU) = a \log(arc) + b \tag{4.1}$$

where a and b are given in Fig. 2 for each groups.

If the orbital arc is smaller than 10 days, the CEU is very large and does not much improve when the length of arc becomes greater. Between 10 and 250 days, the CEU is clearly improved if the length of arc increases. For asteroids with an orbital arc between 250 and 8 000 days, the CEU is smaller than 100 arcsec and is still much improved when the length of arc becomes greater. If the orbital arc is greater than 8 000 days, then the CEU is not much improved and reaches its typical minimum value (about 0.1-0.2 arcsec).

5 Conclusion

The best parameter of uncertainty provided by asteroid databases seems to be CEU because it is quickly computable, precise and related to a physical value. An empirical relation between CEU and the orbital arc of the asteroid has also been highlighted.

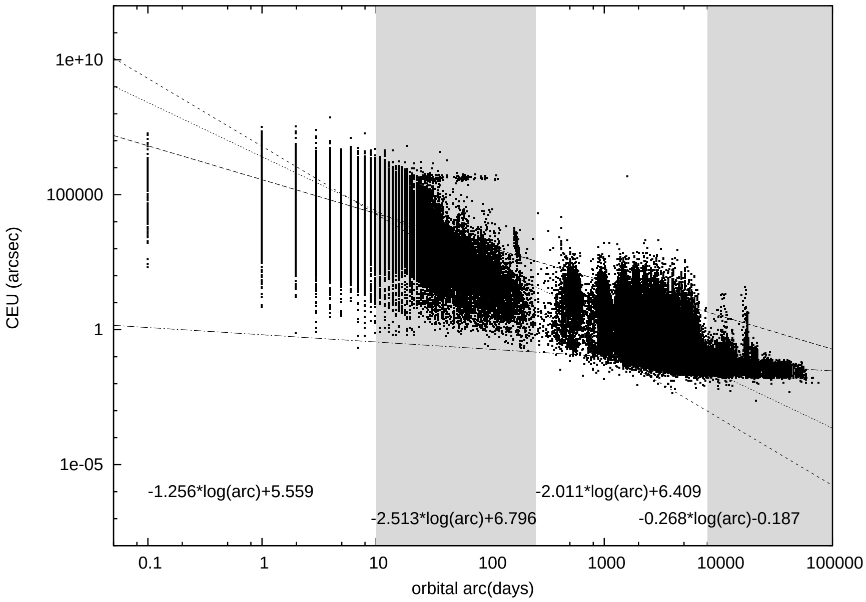

Fig. 2. Relations between the length of orbital arc and CEU.

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References

Bowell, E. 2011, The asteroid orbital elements database, `ftp://ftp.lowell.edu/pub/elgb/astorb.html`

Desmars, J., Arlot, S., Arlot, J., Lainey, V., & Vienne, A. 2009, A&A, 499, 321

Desmars, J., Bancelin, D., Hestroffer, D., & Thuillot, W. 2011, Asteroid Ephemerides, Report ESA/22885/09/F/MOS/100409-R

Minor Planet Center. 2011a, MPCORB database, `http://minorplanetcenter.net/iau/MPCORB.html`

Minor Planet Center. 2011b, Uncertainty parameter U, `http://www.minorplanetcenter.net/iau/info/UValue.html`