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# Pricing in Information-Centric Network Interconnection

Tuan-Minh Pham\*, Serge Fdida\*, Panayotis Antoniadis†

\*LIP6 Laboratory, UPMC Sorbonne Universités, France

†Information Technology and Electrical Engineering Department, ETH Zurich, Switzerland

**Abstract**—We propose a pricing model to study the economic incentives for caching and sharing content in the interconnection of Information-Centric Networks (ICNs). While collaborative caching in ICNs is a key feature to success in improving network performance and reducing delivery costs in content distribution, the current pricing strategies in the Internet are not incentive-compatible with ICN interconnection. In this paper, we clarify the issue by considering the existing value and money flows in today’s Content Delivery Networks (CDNs) and studying their possible evolution in a future ICN interconnection scenario. We model and analyze the interactions in price and caching investments among transit ISPs, access ISPs and content providers in ICN interconnection. Under the assumptions of our economic model, it is proven that ICN interconnection is feasible from an economic point of view and a stable state can be reached. Our numerical results show the case where an access ISP is profitable from caching investment. Interestingly, we observe that in the marketplace described by our model there are more opportunities for competition in ICN interconnection thanks to caching.

**Index Terms**—ICN interconnection, economic incentives, collaborative caching, non-cooperative games, network pricing.

## I. INTRODUCTION

Information-Centric Networking is an approach for the future Internet architecture that deals with the explosion of the supply and demand for content in the Internet [1]. The principal ideas in ICNs are the following: 1) content is located by name instead of by location, and 2) every ICN node can cache and serve the requested content. Therefore, caching and sharing content plays an important role in ICN interconnection. More specifically, each ICN would require the cooperation in caching of other ICNs in order to provide a global high-performance network in content delivery. However, with the contemporary pricing policies in the current Internet, all ICNs in an upstream direction, where content is sent from a customer ISP to a provider ISP, have no incentives to cache and share content since this would reduce their income [2]. This means that without modifications of the existing Internet market the potential of caching collaboration between ICNs will be limited, resulting in poor expected performance of content delivery at a global scale.

Our paper is motivated by the need to define an alternative pricing model that provides suitable economic incentives for caching and sharing content in ICN interconnection. For example, an ISP provider in ICNs may wish to fetch content cached in the networks of one of its customer ISP [3]. Then, in this scenario, the customer ISPs should benefit from the content cached in their networks, or the ISP provider should

pay the customer ISP for the content it provides. A pricing model for ICNs needs to include this fundamental requirement for caching incentives. Much work has been devoted for the study of alternative pricing models for ISP interconnection in the Internet [4]–[8]. However, to the best of our knowledge, there has not been any attempt thus far to study a pricing model capturing the incentives to invest in caching in the current or future Internet architectures.

In a non-cooperative context, an ISP wants to maximize its benefits by following an appropriate strategy in caching investments and pricing. Hence, it is natural to ask the following questions: What is the impact of transit ISPs’ prices? What is the impact of content providers’ prices? What is the impact of caching investments of an access ISP? Can an ISP profit from caching investments? We aim to answer these questions in order to better understand the interdependencies in pricing and caching investments between the main entities in the ICN interconnection under our proposed pricing model. Finally, a decision of any ISP would have an impact on the strategies of the others while each ISP optimizes its decisions in an individual manner. As a consequence, a core attribute of a candidate pricing model is whether it can lead the system to an equilibrium and under which assumptions.

The major contributions of this paper are as follows:

- We clarify the economic incentives in caching and sharing content in ICN interconnection. We analyze the pricing policies in existing network economic models including traditional CDNs and visions of federated CDNs, and explore their adaptation for the case of ICN interconnection.
- We formulate a new pricing model that provides economic incentives for caching and sharing content in ICN interconnection. We analytically prove the existence of equilibrium in a competitive context between two access ISPs, which means that there exists a stable state where the access ISPs will tend not to change their prices. So, our model both provides economic incentives for caching and sharing content, and ensures the existence of an equilibrium for keeping the economy stable and achieving economic growth. To our knowledge this is the first paper that describes a solution to the problem of economic incentives in collaborative caching in ICN interconnection.
- We model and analyze the interactions in price and caching investments among transit ISPs, access ISPs and content providers in ICN interconnection. First, we find

that in a non-cooperative context, compared to another access ISP, an access ISP can gain more utility if it invests more in caching under certain conditions. Second, at equilibrium, we observe that even if the price of an access ISP decreases when its caching investment increases, its utility increases if the transit ISP's price is high. Another interesting result is that in the case of a single transit ISP, this transit ISP cannot increase its price as in a monopoly, due to the impact of caching investments by access ISPs. This means that the economics of ICN interconnection could be better regulated because of the existence of incentives for caching.

The rest of this paper is organized as follows. Section II reviews the related work. The issue of economic incentives in ICN interconnection is described in Section III and our solution is proposed in Section IV. Section V presents the analytical results of caching, pricing and utility. Section VI presents the numerical analysis of caching, pricing and utility. Conclusions are stated in Section VII.

## II. RELATED WORK

Several architectures for content routing in ICNs have been proposed (i.e. CCN [9], DONA [10], PSIRP [11], 4Ward-NetInf [12], Breadcrumbs [13]). All of them are based on minimizing some measures of the route. However, in inter-domain routing between ICNs, it is important to consider economic incentives for caching and sharing content due to the fact that ISPs will cache and share content in order to maximize their utility. In [2], [3], the authors concluded that policies and caching incentives in ICN interconnection are not compatible with the current Internet architecture. However, there have been no systematic studies which propose possible solutions to the problems described. This is the main focus of our work.

The optimization problems of economic utilities of ISPs have been considered in the Internet [4]–[8], [14]. For example, Shakkottai and Srikant investigated the economic relationships of ISPs when the traffic is generated by the end-to-end demand between clients and web servers connected to ISPs [4]. Valancius et. al. showed that ISPs in the Internet transit market can achieve a near-optimal profit by using tiered pricing strategies with three or four tiers [5]. A recent study by Altman et. al. addressed the non-neutrality problem of the Internet market in an economic model including an ISP, a content provider, end-users and advertisers. They showed that the ISP and the content provider benefit from the side payment from the content provider to the ISP [14].

However, these, and other solutions in the literature for the case of ISP interconnection cannot be applied to the case of ICN interconnection due to the fundamental differences between the ICN and the current Internet architectures. Our work seeks to address the issue of economic incentives in ICN interconnection by formulating and analyzing a new pricing model in a competitive context, where all players aim to maximize their own profits. The key difference of our model compared to previous studies on economic analysis

in the Internet is that we take into account the key features of ICNs including such as the role of caching and serving content between networks. Our first results provide insights on the provision of incentives for caching in the context of ISP interconnection, a key requirement for a successful implementation of the ICN architecture in the future Internet.

## III. ECONOMIC INCENTIVES IN ICN INTERCONNECTION

The explosion of the supply and demand for content has given rise to the necessity of efficient global content delivery in the Internet. CDNs and ICNs are designed for this objective by sharing an idea of replicating popular content to multiple nodes in the networks. An important difference between them is that CDNs build up the end-to-end content delivery in the Internet at the application level, while ICNs are a proposal for an alternative approach to the core architecture of computer networks. Specifically, in ICNs, a consumer queries content by sending an interest packet containing the content name over its available connectivity. A router speaking ICN forwards the interest packet toward potential sources based on the content name. Any node receiving the interest packet and having the content can respond with a data packet [9]. While ICNs have the potential for high performance and low cost in content delivery, they are still in the design phase and they need to address important incentive issues. In the following, we describe how pricing policies work in the economic models of CDNs that are the current solution for fast global content delivery. We then answer the question whether the contemporary pricing policies could work for the case of ICN interconnection.

### A. Pricing Policies in Existing Network Economic Models

A growing trend in content distribution in the Internet is the use of CDNs by content providers in order to deliver content to end-users with the required performance level. For example, Akamai handles 20% of the total Web traffic today [15]. We will consider the charging policies in CDNs in relation to its architecture. A CDN operator builds a large distributed system of servers which are hosted in ISPs at the edges of the Internet. Content from a content provider who is a customer of CDNs is replicated in several CDN nodes at different locations in the Internet. A content request is served by a CDN node that provides the best performance for content delivery to the user instead of satisfying the request from the original source. The economic relationships in CDNs are shown in Fig. 1a. In the current CDN business model, a content provider pays a CDN provider for distributing content to its customers. If the content provider connects to the CDN through an ISP, the content provider pays the ISP for network access. A customer pays the content provider for content and a part of the delivery cost, and he pays his ISP for network access. The CDN provider pays the ISPs where its servers are located.

The ISPs who own the last mile have begun to launch their own CDNs in order to reduce the traffic amount on the backbone ISPs [16]. These CDNs join together to exchange content and create a federated model called a Federated CDN. We will refer to the former model of CDNs as traditional

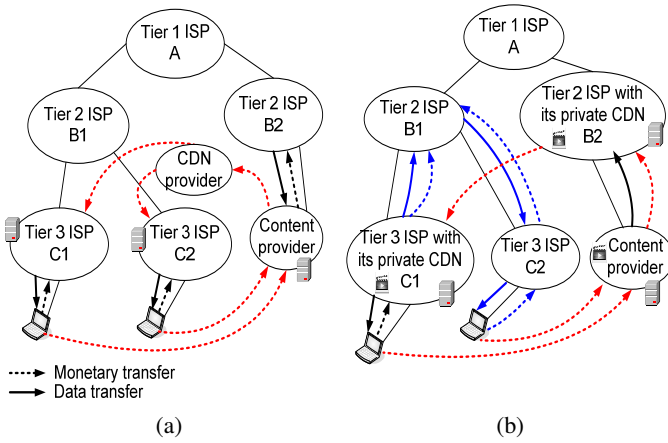


Fig. 1: The economic relationships in CDNs: (a) Traditional CDNs, (b) Federated CDNs

CDNs. In the process of developing a Federated CDN, some CDN providers (e.g. Akamai and Edgecast) launched a set of licensed CDN products which allow ISPs to create their own CDN by combining licensed softwares from CDN providers with their hardware infrastructure [17], [18]. Because ISPs own the network infrastructure over which content is delivered, their CDNs have advantages over traditional CDNs. A main difference in the operation of a Federated CDN is that while a CDN operator in a traditional CDN controls its own caching and sharing system of CDN nodes in the Internet, all ISPs in a Federated CDN have to agree to cooperate in sharing their caching systems.

Due to the difference in operation of a Federated CDN, in the economic relationships, the ISP who plays the role of a CDN provider must have agreements with other ISPs in the federation for exchanging content and accounting [19]. A content provider pays an ISP who provides CDN services for distributing content to its customers. Fig. 1b shows the economic relationships in a Federated CDN including CDN  $B2$  and CDN  $C1$ . In the figure, suppose that for performance reasons a content request from ISP  $C2$  is satisfied by CDN  $C1$ , CDN  $B2$  must set up an agreement with CDN  $C1$  in order that CDN  $C1$  shares content with ISP  $C2$ . In this case, CDN  $C1$  pays ISP  $B1$  when satisfying a content request from ISP  $C2$ . CDN  $B2$  must pay CDN  $C1$  because CDN  $B2$  is paid by the content provider for distributing its content and CDN  $C1$  is not paid by the content provider for that same purpose.

#### B. Applicability of Existing Pricing Policies to ICN Interconnection

Since there is no cooperation between the caching systems of different traditional CDNs, their pricing policies are not compatible with ICN interconnection. Although Federated CDNs and ICNs are completely different architectures, we can relatively compare Federated CDNs with the interconnection of ICNs by considering the following situation. If all ISPs in the whole Internet built their own CDNs, all these CDNs joined together, and the content was cached and shared

between these CDNs at network levels, the resulting system would be similar to the interconnection of ICNs with respect to caching and sharing content. Despite some similarities between Federated CDNs and ICNs, the pricing policies that have been proposed for Federated CDNs cannot directly apply to ICNs because of economic incentives and architecture issues.

First, the content providers charging policy towards end-users in Federated CDNs and in ICNs should be different. In Federated CDNs, content providers are responsible for delivering content to end-users. Therefore, content providers charge end-users for both the content and distribution costs. In ICNs, any ISP can cache and share content. Any ISP can be a content source for a user if it holds that content in its cache. Because a user can retrieve content from any ISP storing it, the original content source (i.e. content provider) should not be responsible for delivering it. Hence, the content provider cannot charge the user for the distribution cost.

Second, in an upstream direction, the way in which an ISP charges its customer ISPs in Federated CDNs is incompatible with ICNs unless every ISP has agreements with all other ISPs in the Internet. For example, in Fig. 2a, if ICN  $B2$  does not have an agreement with ICN  $A$ ,  $A$  has no incentive to cache content that  $B2$  wants to distribute since it reduces  $A$ 's profit. ICN  $C1$  also has no incentive to share content with ICN  $C2$  if  $B2$  does not pay  $C1$  because  $C1$  is charged by provider ISP  $B1$ . Similarly, ICN  $B1$  and ICN  $C2$  are not motivated to share content with other ICNs when an interest packet is sent from its provider ISP. If  $B2$  does not have an agreement with the other ISPs, the other ISPs have no incentives for caching and sharing content. This results in breaking the principle of content exchange between ICNs. It is unlikely for every ISP to have agreements with all other ISPs in the Internet. Therefore, rather than a complicated mechanism for establishing and managing a complex collection of agreements between all ICNs, we need a new pricing principle that motivates all ICNs to cache and share content. The next section introduces our proposed solution.

## IV. A PRICING MODEL FOR ICN INTERCONNECTION

### A. Pricing Principles

Our pricing model for ICN interconnection is inspired from an example of package delivery. For example, Bob buys an item from the website of a firm. He pays the firm only for the item, not for the transport costs. He requests local store  $A$  to deliver the item to him. If  $A$  has the item in its store,  $A$  will directly transport it to Bob. If  $A$  does not have the item, it can request other stores to deliver the item to  $A$ 's store. For example,  $A$  requests store  $B$ .  $A$  pays  $B$  for only the cost of the transit incurred if  $A$  had its own transport, otherwise  $A$  pays  $B$  for the cost of both the transport and transit incurred. The payment made between  $A$  and  $B$  is similar to the idea of store pickup that many retailers offer.

In our proposed pricing model, a content provider pays an ISP to which it connects for network access and not for distributing content to customers. Hence, while in CDNs

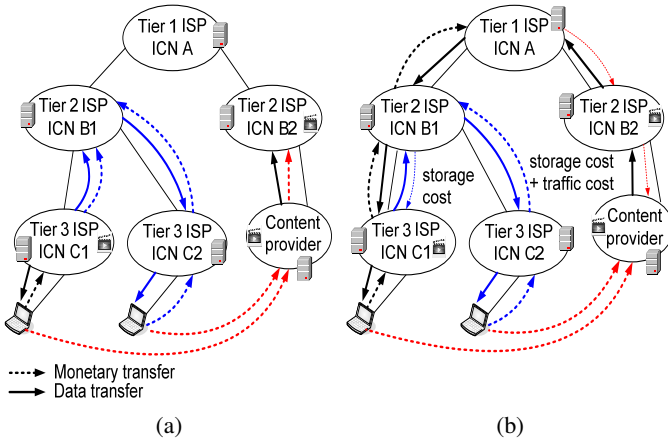


Fig. 2: Pricing in ICNs: (a) An issue of pricing in ICNs, (b) The proposed pricing model in ICNs

customers pay content providers for both content and a part of the delivery cost, in ICNs using our model, they pay content providers only for content, not for the delivery cost. We assume that the customers can enjoy content only if they have bought it from content providers. For example, they need an access right to watch a movie. A customer pays its ISP for delivering content to its device. If the ISP does not have content in its cache, it can ask other ISPs to deliver content to it. If ISP  $A$  asks ISP  $B$  for a content,  $A$  pays  $B$  only for storage cost if  $A$  owns the infrastructure for delivering the content, otherwise  $A$  pays for both traffic and storage cost. In other words, in order to devise pricing models suitable for ICNs, the role of transporting content has to be separated from the role of providing content.

Fig. 2b depicts our proposed pricing model. First, a customer of ICN  $C1$  requests a content item. When the content is delivered to the customer, it is cached in ICN  $C1$ . Then, when a customer of ICN  $C2$  requests the same content, ICN  $C2$  requests the content from the cache of ICN  $C1$  instead of requesting the content from the transit ISP. In this case, ICN  $B1$  pays ICN  $C1$  solely for storage because ICN  $B1$  uses its own infrastructure for transporting the content. ISP  $C2$  has to pay ISP  $B1$  for both traffic and storage.

Our pricing principles provide economic incentives in caching and sharing content in ICN interconnection. In ICN interconnection, using our model, an ICN benefits from services that it provides, and it is motivated to cache and share content without the requirement of establishing an agreement with other ICNs except the ICNs connected directly to it. For example, in Fig. 2b, ICN  $A$  has incentive to cache content because  $A$  must pay  $B2$  for storage cost when it retrieves content from  $B2$ . ICN  $C1$  wants to share content with  $C2$  because it benefits from satisfying an interest packet sent from its provider ISP  $B1$ .

### B. The Economics of ICN Interconnection

The dynamics of economics of ICN interconnection depend on two important factors: pricing and caching. In this section,

we describe the model of ICN interconnection in which we consider both the caching and pricing parameters, and formally define the problem of ICN interconnection in a competitive context between access ISPs.

The model involves four types of roles: transit ISPs that provide wide-area transport for access networks, access ISPs that connect end-users to the content network, content providers (CPs) that provide their content to end-users, and end-users that consume content. We will refer to the access ISPs and transit ISPs as just ISPs when we want to refer to both of them. In the model, we focus only on content traffic since the traffic of interest packets is negligible compared to the traffic of content packets. We denote the set of access ISPs by  $\mathbb{N}_a$ , and the set of ISPs and CPs by  $\mathbb{N}_m$ . We assume that each access ISP connects to one transit ISP. We define  $H(A)$  to be the transit ISP who is the network service provider for access ISP  $A$ .

For convenience, we summarize all of our notation in Table I.

In the model, each element can have different pricing strategies. The pricing is based on usage, i.e., a price per gigabyte or per a satisfied request. The pricing parameters are defined as follows:

- Each ISP sets a network price for transporting content by using its infrastructure and a caching price for providing content from its cache. For ISP  $K$ , we denote the network price by  $p_K^{(n)}$ , the storage price by  $p_K^{(s)}$ , the total price by  $p_K$ , and the price strategy space by  $P_K = \{p_K = p_K^{(n)} + p_K^{(s)} : p_K^{(n)}, p_K^{(s)} \in [0, p_{max}]\}$  where  $p_{max}$  is the highest possible price. The relationship between network cost and storage cost is represented by the parameter  $\beta_K \in [0, 1]$ ,  $p_K^{(s)} = \beta_K p_K^{(n)}$ .
- Each CP sets a content price that users have to pay for consuming content, and a caching price for providing content from its cache. For CP  $O$ , we denote the content price by  $p_O^{(c)}$ , the storage price by  $p_O^{(s)}$ , the total price by  $p_O$ , and the price strategy space by  $P_O = \{p_O = p_O^{(c)} + p_O^{(s)} : p_O^{(c)}, p_O^{(s)} \in [0, p_{max}]\}$ . The relationship between content cost and storage cost of the content provider is expressed by the parameter  $\beta_O \in [0, 1]$ ,  $p_O^{(s)} = \beta_O p_O^{(c)}$ .

Each ISP supporting ICNs can adopt a different caching strategy. A content request from a customer of an access ISP can be satisfied from the cache of this access ISP or another access ISP or a transit ISP. We denote the total content demand from the customers of access ISP  $A$  by  $\sigma_A$ . For ISP or CP  $M$ ,  $\alpha_{(K,M)}$  denotes the relative content demand of ISP  $K$  that is satisfied from the cache of  $M$ , where  $\alpha_{(K,M)} \in [0, 1]$  and  $\sum_{M \in \mathbb{N}_m} \alpha_{(K,M)} = 1$ . The values of  $\alpha_{(K,M)}$  are dependent on the caching strategies of ISPs. For example, if ISP  $K$  invests in caching capacity and has an effective caching strategy, the value of  $\alpha_{(K,K)}$  will tend to be high.

In a competitive context, an access ISP attempts to maximize its utility. The strategies available to access ISP  $A$  are

the different prices that it might choose in  $P_A$ . The strategy space denoted by  $\mathbb{S}$  is the Cartesian product of the strategy sets of all access ISPs,  $\mathbb{S} = \times P_K$  where  $K \in \mathbb{N}_a$ . Let  $S$  denote an arbitrary member of the strategy set  $\mathbb{S}$ ,  $S = \{p_K \in P_K : K \in \mathbb{N}_a\}$ .  $S$  denotes a combination of strategies of all access ISPs. Each player simultaneously chooses a strategy for maximizing its payoff, and the combination of strategies chosen by the players determines a payoff for each player. Let  $\mathbb{U} = \{U_K : K \in \mathbb{N}_a\}$  be the set of the payoff functions of the players.  $U_K$  denotes player  $K$ 's payoff function.  $U_K(S)$  is the payoff to player  $K$  if the players choose the strategy  $S$ .

The utility function of an ISP or a CP is defined as their profit received from providing his services. ISP  $K$  incurs a marginal cost  $c_{(K,M)}$  when ISP  $K$  gets a content unit from content source  $M \in \mathbb{N}_m$ . To describe the utility function of access ISP  $A$ , we classify three cases from which access ISP  $A$  gains utility. First, it satisfies an interest packet from its cache. The utility that access ISP  $A$  receives in this case is  $\sigma_A \alpha_{(A,A)} (p_A - c_{(A,A)})$ . Second, an interest packet requested from the customers of access ISP  $A$  is satisfied from the cache of any element in the set of access ISPs, transit ISPs and CPs excluding access ISP  $A$ . The content demand satisfied from the cache of element  $M \in \mathbb{N}_m \setminus \{A\}$  is  $\sigma_A \alpha_{(A,M)}$ . The utility that access ISP  $A$  receives when satisfying an interest packet in this case is  $p_A - p_{H(A)} - c_{(A,M)}$ . Third, access ISP  $A$  satisfies an interest packet requested from the customers of other access ISPs. The content demand that is requested from access ISP  $B \in \mathbb{N}_a \setminus \{A\}$  and satisfied from the cache of access ISP  $A$  is  $\sigma_B \alpha_{(B,A)}$ . When satisfying an interest packet in this case, access ISP  $A$  gains the utility  $p_A^{(s)} - c_{(A,A)}$ . The total utility of access ISP  $A$  is the sum of the utility in the three cases:

$$\begin{aligned}
U_A(S) &= \sigma_A \alpha_{(A,A)} (p_A - c_{(A,A)}) \\
&+ \sigma_A \sum_{K \in \mathbb{N}_m \setminus \{A\}} \alpha_{(A,K)} (p_A - p_{H(A)} - c_{(A,K)}) \\
&+ (p_A^{(s)} - c_{(A,A)}) \sum_{K \in \mathbb{N}_a \setminus \{A\}} \sigma_K \alpha_{(K,A)}.
\end{aligned} \tag{1}$$

Given the pricing strategies of transit ISPs and CPs, and the caching strategies of ISPs and CPs, the access ISPs compete to maximize their utilities. We formulate the competitive problem as a normal-form game with the set of players  $\mathbb{N}_a$ , the strategy set  $\mathbb{S}$ , and the set of the payoff functions  $\mathbb{U}$  [20].

In order to answer if the economics of ICN interconnection using the proposed pricing model is consistent with a steady state where no access ISP wants to deviate from his predicted strategy, we need to solve the problem of the existence of a Nash equilibrium for the game. Specifically, the price strategy  $S^* = \{p_K^* \in P_K : K \in \mathbb{N}_a\}$  constitutes an equilibrium if  $S^*$  solves the following optimization problems for all player  $K \in \mathbb{N}_a$ :

$$\max_{p_K \in P_K} U_K(S^* \setminus \{p_K^*\}, p_K). \tag{2}$$

TABLE I: Summary of notations

Notation	Meaning
$\mathbb{N}_a$	The set of access ISPs
$\mathbb{N}_m$	The set of access ISPs, transit ISPs and content providers
$p_K^{(n)}$	The traffic price of ISP $K$
$p_K^{(s)}$	The storage price of ISP or CP $K$
$p_K^{(c)}$	The content price of CP $K$
$P_K$	The price strategy space of ISP $K$ , $P_K = \{p_K = p_K^{(n)} + p_K^{(s)} : p_K^{(n)}, p_K^{(s)} \in [0, p_{max}]\}$
$P_O$	The price strategy space of content provider $O$ , $P_O = \{p_O = p_O^{(c)} + p_O^{(s)} : p_O^{(c)}, p_O^{(s)} \in [0, p_{max}]\}$
$\mathbb{S}$	The game strategy space, $\mathbb{S} = \times P_K$ , where $K \in \mathbb{N}_a$ .
$S$	An arbitrary member of the game strategy set $\mathbb{S}$
$U_K(S)$	The payoff to player $K$ if the players choose the strategy $S$
$\mathbb{U}$	The set of the payoff functions of the players, $\mathbb{U} = \{U_K : K \in \mathbb{N}_a\}$
$\sigma_K$	The total content demand from the customers of access ISP $K$
$\alpha_{(K,M)}$	The relative demand for content from ISP $K$ 's consumers that is satisfied from the cache of $M$ , $M \in \mathbb{N}_m$
$c_{(K,M)}$	The cost that ISP $K$ has to invest in a content unit for getting it from the cache of $M$ , $M \in \mathbb{N}_m$
$\beta_K$	The scaling parameter between network cost and storage cost in the pricing strategy of ISP or content provider $K$
$\rho_K$	The parameter of the user sensitivity effects of the price of ISP or content provider $K$ on the content demand
$a$	The parameter $2a$ expresses the total potential demand of users

## V. ANALYSIS OF CACHING, PRICING AND UTILITY

In this section, we present our analytical results at equilibrium in a context of ICN interconnection where two access ISPs compete for maximizing their utilities. We start by describing the simplified model and computing the utility. We then present our analytical results including the existence of equilibrium, and the impact of the charging costs of the transit ISP as well as the caching investment of an access ISP on equilibrium prices.

### A. A Simplified Model

In order to analytically study the equilibrium of our proposed pricing model for ICN interconnection, we consider a simplified model including one transit ISP, one content provider, and two access networks that compete on prices to attract consumers for maximizing their utilities given their caching strategies (Fig. 3). We denote the two access ISPs by  $A$  and  $B$ , the transit ISP by  $C$ , and the content provider by  $O$ .

In the simplified model, we consider a linear function of content demand of users who connect to access ISPs  $A$  and  $B$ . The linear demand function is [21]:

$$\begin{aligned}
\sigma_A &= a - \rho_A p_A + \rho_B p_B - \rho_O p_O^{(c)}, \\
\sigma_B &= a + \rho_A p_A - \rho_B p_B - \rho_O p_O^{(c)},
\end{aligned}$$

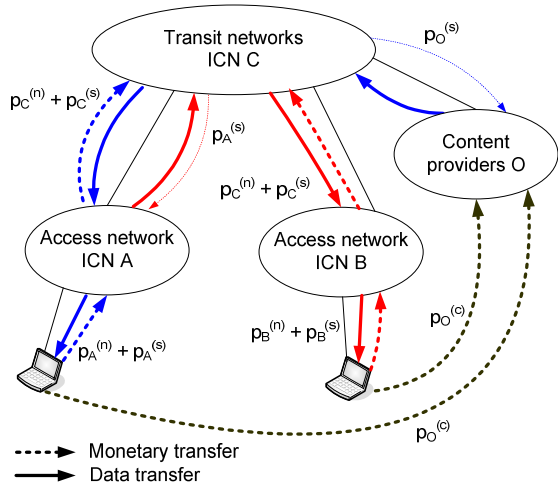


Fig. 3: A simplified model

where  $a$ ,  $\rho_A$ ,  $\rho_B$  and  $\rho_O$  are positive constants. The parameter  $a$  expresses the total potential demand of users. The parameters  $\rho_A$ ,  $\rho_B$  and  $\rho_O$  represent the demand response effects and the user sensitivity effects on the prices. For example, when the price of the CP increases, the demand from users to both ISPs decreases. When the price of ISP  $A$  increases, a part of users will switch from ISP  $A$  to ISP  $B$  and the demand to ISP  $A$  decreases while the demand to ISP  $B$  increases.

We now compute the utility of ISPs and CPs. In order to obtain the utility function of access ISP  $A$ , we apply the formulas in the general model to the simplified model with  $\mathbb{N}_a = \{A, B\}$  and  $\mathbb{N}_m = \{A, B, C, O\}$ . Reversing the roles of  $A$  and  $B$ , we will have the utility function of access ISP  $B$ . Using (1), we obtain the total utility of access ISP  $A$ :

$$\begin{aligned}
 U_A(S) &= \sigma_A \alpha_{(A,A)} (p_A - c_{(A,A)}) \\
 &+ \sigma_B \alpha_{(B,A)} (p_A^{(s)} - c_{(A,A)}) \\
 &+ \sigma_A \sum_{K \in \mathbb{N}_m \setminus \{A\}} \alpha_{(A,K)} (p_A - p_C - c_{(A,K)}).
 \end{aligned} \quad (3)$$

Transit ISP  $C$  gains its utility if access ISPs  $A$  or  $B$  request a content item through ISP  $C$ . The utility function of ISP  $C$  is:

$$\begin{aligned}
 U_C(S) &= \sum_{K \in \mathbb{N}_a} \sigma_K \alpha_{(K,C)} (p_C - c_{(C,C)}) \\
 &+ \sigma_A \sum_{K \in \mathbb{N}_m \setminus \{A,C\}} \alpha_{(A,K)} (p_C - p_K^{(s)} - c_{(C,K)}) \\
 &+ \sigma_B \sum_{K \in \mathbb{N}_m \setminus \{B,C\}} \alpha_{(B,K)} (p_C - p_K^{(s)} - c_{(C,K)}).
 \end{aligned}$$

Content provider  $O$  achieves its utility from the content that the customers consume and from the interest packet that is satisfied from its cache. We note that a content source can be any ISPs or the content provider. The utility function of the

CP is:

$$\begin{aligned}
 U_O(S) &= \sigma_A \alpha_{(A,O)} (p_O^{(s)} - c_{(O,O)}) \\
 &+ \sigma_B \alpha_{(B,O)} (p_O^{(s)} - c_{(O,O)}) \\
 &+ (\sigma_A + \sigma_B) p_O^{(c)}.
 \end{aligned}$$

### B. Analysis

In order to solve the Nash equilibrium in the competitive game between access ISPs, we compute the derivatives of the utility functions of ISPs  $A$  and  $B$ , and solve the following system of equations:

$$\begin{aligned}
 \frac{\partial U_A(p_A, p_B)}{\partial p_A} &= 0, \\
 \frac{\partial U_B(p_A, p_B)}{\partial p_B} &= 0.
 \end{aligned} \quad (4)$$

Propositions 1-3 will give the main analytical results of equilibrium and the impacts of caching and pricing on equilibrium prices.

*Proposition 1:* Given the caching strategies of transit ISPs, access ISPs and CPs, if the demand function of every access ISP is continuous, monotonically decreasing with respect to its price, and monotonically increasing with respect to the prices of other access ISPs, then there exists an equilibrium price.

*Proof:* The second partial derivatives of the utility of access ISP  $A$  with respect to  $p_A$  and  $p_B$  is

$$\frac{\partial^2 U_A}{\partial p_A \partial p_B} = \rho_B - \frac{\rho_B \alpha_{(B,A)} \beta_A}{1 + \beta_A}.$$

Since  $\rho_B > 0$ ,  $\alpha_{(B,A)} \in [0, 1]$ , and  $\beta_A \in [0, 1]$ , for all  $p_A \in P_A$  and  $p_B \in P_B$ , we have  $\partial^2 U_A / \partial p_A \partial p_B \geq 0$ . In addition,  $U_A$  is twice continuously differentiable in  $p_A$  and  $p_B$ . Thus, by Topkis's Characterization Theorem [22], the utility function of access ISP  $A$  has increasing differences in  $(p_A, p_B)$ .

Similarly, the utility function of access ISP  $B$  has increasing differences in  $(p_A, p_B)$ .

Since  $P_A$  and  $P_B$  are compact subsets of  $\mathbb{R}$ ,  $U_A$  and  $U_B$  are continuous in  $p_A$  and  $p_B$ , and  $U_A$  and  $U_B$  have increasing differences in  $(p_A, p_B)$ , the competition between access ISPs is a supermodular game. By Theorem (Milgrom and Roberts) [22], there exists a pure Nash equilibrium, which demonstrates Proposition 1.  $\blacksquare$

Proposition 2 describes the relationship between transit ISP's prices and access ISP's prices at equilibrium. The proof of the proposition will be provided in the Appendix.

*Proposition 2:* Given the caching strategies of transit ISPs, access ISPs and CPs, if the user sensitivity effects of access ISP's prices are similar, at equilibrium, the price of an access ISP is monotonically increasing with respect to the price of the transit ISP for a given price strategy of the content provider, while it is monotonically decreasing with respect to the price of the content provider for a given price strategy of the transit ISP.

Proposition 3 shows the impact of access ISP's caching on its prices at equilibrium.

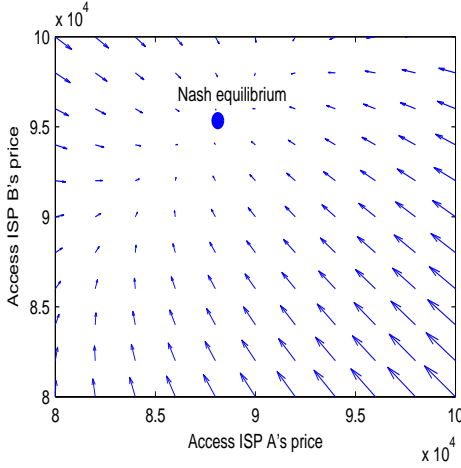


Fig. 4: Convergence to equilibrium

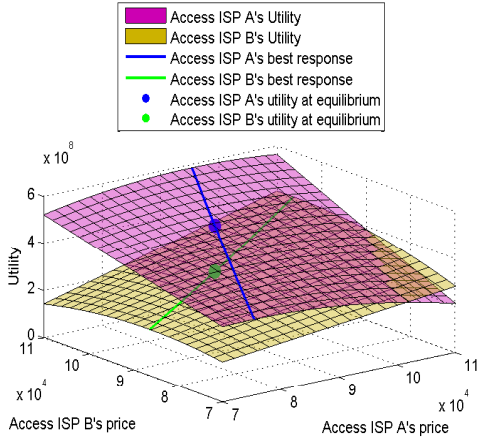


Fig. 5: Utilities of access ISPs at equilibrium

*Proposition 3:* Given the caching strategies of the transit ISP, access ISPs and CPs, at equilibrium, the price of access ISP  $K$  with respect to the investment in caching is

- monotonically increasing if  $c_{(K,K)} - c_{(K,O)} - p_C > 0$ ,
- monotonically decreasing if  $c_{(K,K)} - c_{(K,O)} - p_C < 0$ ,

where  $p_C$  is the price for receiving the content from the transit ISP,  $c_{(K,K)}$  is the cost of satisfying the content from the own cache of access ISP  $K$ , and  $c_{(K,O)}$  is the cost of satisfying the content from the content provider.

*Proof:* Because the role of access ISPs  $A$  and  $B$  is similar, we will prove the proposition for ISP  $A$ . Using the utility functions of  $A$  and  $B$  (3), we find the solution of (4) with respect to  $p_A$  and  $p_B$ . By differentiating the solution of  $p_A$  with respect to  $\alpha_{(A,A)}$ , we obtain

$$\frac{\partial p_A^*}{\partial \alpha_{(A,A)}} = \frac{2(1 + \beta_A)(c_{(A,A)} - c_{(A,O)} - p_C)}{3(1 + \beta_A - \alpha_{(B,A)}\beta_A)}.$$

Because  $\beta_A \in [0, 1]$ , and  $\alpha_{(B,A)} \in [0, 1]$ , the sign of  $\partial p_A^*/\partial \alpha_{(A,A)}$  is similar to the sign of  $c_{(A,A)} - c_{(A,O)} - p_C$ , which demonstrates Proposition 3. ■

Proposition 3 implies that in case the transit ISP charges a high price, the investment of access ISPs in caching will lower the price that consumers have to pay for content at equilibrium. In case the transit ISP charges a low price, the investment of access ISPs in caching does not bring any benefit to their customers.

## VI. NUMERICAL RESULTS

We present some numerical results in order to illustrate our analysis in caching and pricing in ICNs. We demonstrate the following: 1) the existence of equilibrium prices in the proposed pricing model, 2) the impacts of the prices of transit ISPs and content providers on the equilibrium prices of access ISPs, and 3) the effect of caching investment on the equilibrium prices of access ISPs.

We consider an ICN model composed of one content provider  $O$ , one transit ISP  $C$ , and two access ISPs  $A$  and  $B$  who are competitive for maximizing their utilities (or revenue equivalently). In our scenario, the content demand of users that connect to access ISP  $A$  is given by  $\sigma_A = 10000 - 0.1p_A + 0.1p_B - 0.1p_O$ . The content demand of users that connect to access ISP  $B$  is given by  $\sigma_B = 10000 + 0.1p_A - 0.1p_B - 0.1p_O$ . The operational costs of any access ISP for getting a content unit from any content source are the cost parameters  $c = 2$ . The caching investment of an access ISP is represented by the ratio of the number of content requests that the ISP can satisfy from its cache and the one that the ISP has to forward to other ISPs or content providers. The parameters of access ISP  $A$ 's caching investment are given by  $\alpha_{(A,A)} = 0.7$ , and  $\alpha_{(A,B)} = \alpha_{(A,C)} = \alpha_{(A,O)} = 0.1$ . Likewise for access ISP  $B$ ,  $\alpha_{(B,B)} = \alpha_{(B,A)} = \alpha_{(B,C)} = 0.3$ , and  $\alpha_{(B,O)} = 0.1$ . Transit ISP  $C$  sets its price including network cost and storage cost to  $p_C = 60000$ . Content provider sets its price including storage cost and content cost to  $p_O = 45000$ . In a survey of CDN pricing, CDN customers spending \$250,000-\$500,000 per year pay \$0.06 per gigabyte delivered [23]. We assign  $p_C$  to a price number that is scaled to that CDN price. Note that it is the value difference between price setting of elements that affects the numerical results rather than a specific value. The relationship between ISP's network cost and ISP's storage cost, and the one between CP's storage cost and CP's content cost is represented by a scale. For simplicity, they are set equally, and given by  $\beta_A = \beta_B = \beta_C = \beta_O = 0.1$ . In our numerical analysis, all parameters are set to their values in the above setting unless explicitly specified.

First, we illustrate the existence of equilibrium in price competition between two access ISPs in the proposed pricing model for ICNs. Given the caching strategies of the access ISPs, transit ISP and CP, and the prices of the transit ISP and CP, access ISPs  $A$  and  $B$  set their prices simultaneously for maximizing their utilities. Assume access ISPs simultaneously choose a price strategy under a best-response behavior, and the combination of strategies chosen by the access ISPs determines a payoff for each access ISP. Thus, the aggregate trends of the system is appropriately represented by the vector field  $(p_A, p_B) \mapsto (\partial U_A(p_A, p_B)/\partial p_A, \partial U_B(p_A, p_B)/\partial p_B)$ .



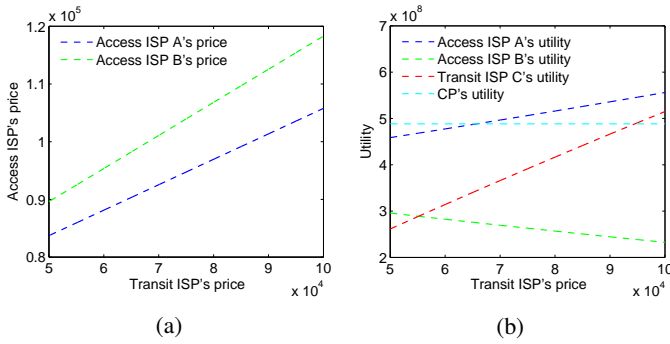


Fig. 6: Impact of transit ISP's prices

Fig. 4 plots the vector field that shows the convergence to equilibrium. The figure demonstrates the existence of a Nash equilibrium point at which no access ISP can profitably deviate given the price of another access ISP. Fig. 5 represents the utilities of access ISPs at the equilibrium prices, and the best response of an access ISP in its price given the price of another access ISP. In the figure, ISP A's utility is higher than ISP B's utility as a result of higher caching investment of ISP A.

Second, we show the impact of the transit ISP's price on the price of access ISPs and the utility of all entities in Fig. 6. We vary the transit ISP's prices for studying their impacts. In Fig. 6(a), the prices of both access ISPs at equilibrium increase if transit ISP's price increases. The results show the fact that the transit ISP's price affects directly the cost of the access ISPs for delivering a content unit. Hence, the access ISPs choose to raise their prices when the transit ISP's price increases. The results conform to Proposition 2. In Fig. 6(b), we see that the utility of access ISP A increases while the utility of access ISP B decreases. In addition, access ISP A's utility is larger than access ISP B's utility although access ISP A's price is less than access ISP B's price. This occurs because the caching investment of access ISP A is larger than the one of access ISP B. We also observe that the transit ISP's utility gets larger when increasing its price. However, we will show that the transit ISP cannot gain a monopoly when we study the impact of caching investment.

Third, we study the impact of the CP's price on the price of access ISPs and the utility of all entities by varying the CP's price while setting the default values to other parameters. The results in Fig. 7(a) confirm the analysis in Proposition 2 that access ISP's prices reduce if the CP chooses to raise its price. It comes from the fact that the CP's price does not have a strong impact on delivering content due to caching at ISPs while it has a direct impact on the content demand from users. Consequently, the access ISPs lower their prices for attracting more users in response to the decrease of content demand due to the increase of the CP's price. In Fig. 7(b), the utilities of both the access ISPs and the transit ISP decrease because the user demand reduces when the CP increases its price. The utility of the CP increases until its price reaches a threshold, and then it slows down because of the reduced demand from the users.

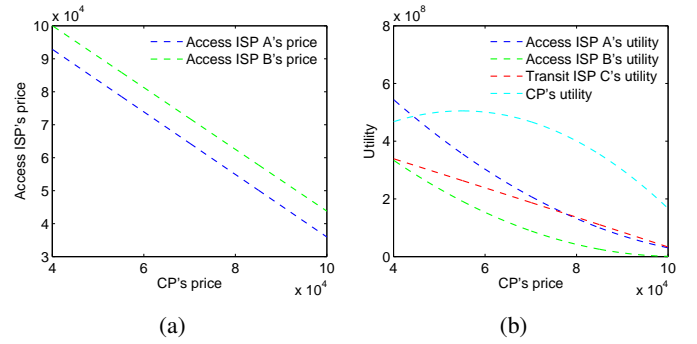


Fig. 7: Impact of content provider's prices

We finally present the impact of caching investment of an ISP on its price at equilibrium. We change the caching investment of access ISP A by varying the ratio of the number of content requests satisfied from its cache and setting the default values to the ratio of the number of content requests satisfied from the caches of the transit ISP and the CP. In Fig. 8(a)-9(a), access ISP A's price at equilibrium decreases when its caching investment increases. It conforms to Proposition 3 because the transit ISP's price is high when compared with the difference between the operational cost of retrieving content from access ISP's cache and the one from CP. In Fig. 8, if transit ISP's price is negligible when compared with access ISP's price, access ISP A's utility does not increase when it invests in caching. This negative result does not occur in practice due to the fact that the transit price is very high. Fig. 9 shows the results of a more practical situation when the difference between transit ISP's price and access ISP's price is reasonable. The results show that even though access ISP A's price at equilibrium decreases, its utility increases when its caching investment increases. The results also show that the utility of the transit ISP decreases when an access ISP invests in caching. When comparing Fig. 6(b) and Fig. 9(b), we see that the transit ISP cannot behave as a monopoly. In other words, the price of the transit ISP could be better regulated in the context of ICN interconnection.

## VII. CONCLUSIONS AND FUTURE WORK

In this paper we have raised the issues of economic incentives for caching and sharing content, and the potential benefits of an ISP from investing in caching in ICN interconnection. We addressed these open questions, which are important for a successful implementation of ICN interconnection, by studying a pricing model that provides economic incentives for caching and sharing content in ICN interconnection. We proved the existence of equilibrium in a competitive ICN interconnection market under our proposed pricing model. This result is significant because it implies that a stable solution with suitable economic incentives in collaborative caching is feasible in the ICN interconnection paradigm. Our work also contributes to a better understanding of the interdependencies between pricing, caching investments and the utility of the transit ISPs, access ISPs, and content providers under the

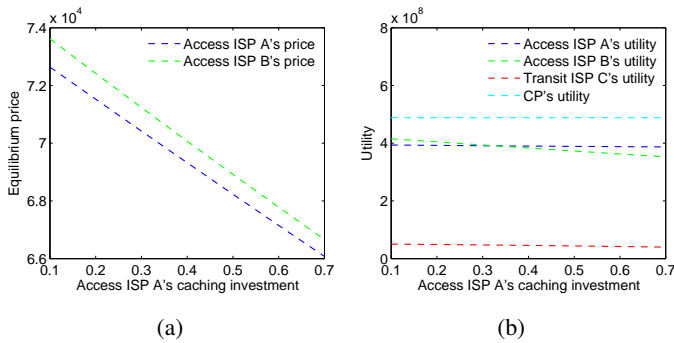


Fig. 8: Impact of caching investment of an ISP if transit ISP's price is negligible when compared to access ISP's price ( $p_C = 10000$ )

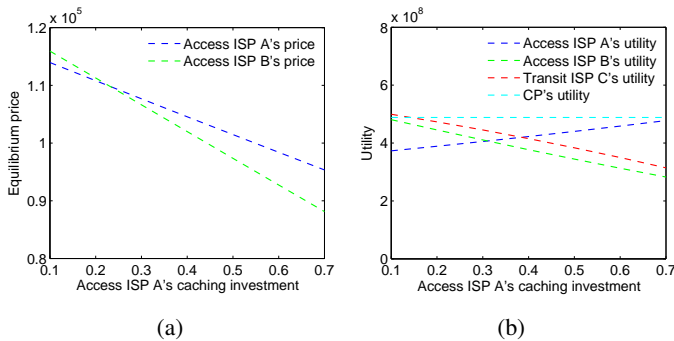


Fig. 9: Impact of caching investment of an ISP if transit ISP's price is significant when compared to access ISP's price ( $p_C = 60000$ )

proposed pricing model. The results show that access ISPs can benefit from caching investments when the transit price is high, and no entity of the market scenario can hold exclusive rights to establish its price.

Possible extensions of our results include the more detailed analysis taking into account specific caching schemes, or the utility distribution of various components in the light of different pricing models. Finally, we have analyzed caching and pricing in a noncooperative context in ICNs where ISPs compete for maximizing their benefits. It will be valuable to study also a cooperative context where ISPs form coalitions for sharing profits as in [24].

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#### REFERENCES

- [1] (2012) Information-centric networking research group. [Online]. Available: <http://irtf.org/icnrg/>
- [2] J. Rajahalme, M. Särelä, P. Nikander, and S. Tarkoma, "Incentive-compatible caching and peering in data-oriented networks," in *Proc. ACM CoNEXT 2008*, Madrid, Spain, Dec. 2008, pp. 62:1–62:6.

- [3] S. DiBenedetto, C. Papadopoulos, and D. Massey, "Routing policies in named data networking," in *Proc. ACM SIGCOMM Workshop on Information-Centric Networking (ICN 2011)*, Toronto, Canada, Aug. 2011, pp. 38–43.
- [4] S. Shakkottai and R. Srikant, "Economics of network pricing with multiple ISPs," *IEEE/ACM Trans. Netw.*, vol. 14, no. 6, pp. 1233–1245, Dec. 2006.
- [5] V. Valancius, C. Lumezanu, N. Feamster, R. Johari, and V. V. Vazirani, "How many tiers?: pricing in the internet transit market," in *Proc. ACM SIGCOMM 2011*, Toronto, Canada, Aug. 2011, pp. 194–205.
- [6] Y. Wu, H. Kim, P. Hande, M. Chiang, and D. Tsang, "Revenue sharing among ISPs in two-sided markets," in *Proc. IEEE INFOCOM 2011*, Shanghai, China, Apr. 2011, pp. 596–600.
- [7] L. He and J. Walrand, "Pricing and revenue sharing strategies for internet service providers," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 942–951, May 2006.
- [8] X.-R. Cao, H.-X. Shen, R. Milioto, and P. Wirth, "Internet pricing with a game theoretical approach: concepts and examples," *IEEE/ACM Trans. Netw.*, vol. 10, no. 2, pp. 208–216, Apr. 2002.
- [9] V. Jacobson, D. K. Smetters, J. D. Thornton, M. F. Plass, N. H. Briggs, and R. L. Braynard, "Networking named content," in *Proc. ACM CoNEXT 2009*, Rome, Italy, Dec. 2009, pp. 1–12.
- [10] T. Koponen, M. Chawla, B.-G. Chun, A. Ermolinskiy, K. H. Kim, S. Shenker, and I. Stoica, "A data-oriented (and beyond) network architecture," in *Proc. ACM SIGCOMM 2007*, Kyoto, Japan, Aug. 2007, pp. 181–192.
- [11] P. Jokela, A. Zahemszky, C. Esteve Rothenberg, S. Arianfar, and P. Nikander, "Lipsin: line speed publish/subscribe inter-networking," in *Proc. ACM SIGCOMM 2009*, Barcelona, Spain, Aug. 2009, pp. 195–206.
- [12] B. Ahlgren, M. D'Ambrosio, M. Marchisio, I. Marsh, C. Dannewitz, B. Ohlman, K. Pentikousis, O. Strandberg, R. Rembarz, and V. Vercellone, "Design considerations for a network of information," in *Proc. ACM CoNEXT 2008*, Madrid, Spain, Dec. 2008, pp. 66:1–66:6.
- [13] E. Rosensweig and J. Kurose, "Breadcrumbs: Efficient, best-effort content location in cache networks," in *Proc. IEEE INFOCOM 2009*, Rio de Janeiro, Brazil, Apr. 2009, pp. 2631–2635.
- [14] E. Altman, A. Legout, and Y. Xu, "Network non-neutrality debate: an economic analysis," in *Proc. IFIP Networking 2011*, vol. 2, Valencia, Spain, May 2011, pp. 68–81.
- [15] (2012) Visualizing global internet performance with Akamai. [Online]. Available: <http://www.akamai.com/html/technology/dataviz3.html>
- [16] B. Niven-Jenkins, F. L. Faucher, and N. Bitar, "Content distribution network interconnection (CDNI) problem statement," RFC 6707, Sep. 2012.
- [17] (2012) Edgecast licensed CDN. [Online]. Available: <http://www.edgecast.com/solutions/licensed-cdn/>
- [18] (2012) Akamai licensed CDN. [Online]. Available: [http://www.akamai.com/html/solutions/aura\\_licensed\\_cdn.html](http://www.akamai.com/html/solutions/aura_licensed_cdn.html)
- [19] L. Peterson. (2011) CDN interconnection. [Online]. Available: <http://www.verivue.com/blog/>
- [20] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*, 1st ed. The MIT Press, 1994.
- [21] S. Caron, G. Kesidis, and E. Altman, "Application neutrality and a paradox of side payments," in *Proc. ACM Re-Architecting the Internet Workshop (ReARCH 2010)*, Philadelphia, USA, Dec. 2010, pp. 9:1–9:6.
- [22] P. Milgrom and J. Roberts, "Rationalizability, learning, and equilibrium in games with strategic complementarities," *Econometrica*, vol. 58, no. 6, pp. 1255–1277, 1990.
- [23] D. Rayburn. (2012) Video CDN data: Pricing, contract, volume and market sizing trends. Content Delivery Summit 2012. [Online]. Available: <http://www.contentdeliverysummit.com/2012/Agenda.aspx>
- [24] R. Ma, D. Chiu, J. Lui, V. Misra, and D. Rubenstein, "On cooperative settlement between content, transit, and eyeball internet service providers," *IEEE/ACM Trans. Netw.*, vol. 19, no. 3, pp. 802–815, Jun. 2011.

#### APPENDIX

##### *Proof of Proposition 2*

We sketch out the proof of the proposition. Because the role of ISPs  $A$  and  $B$  is similar, we will prove the proposition for ISP  $A$ . By using the utility functions of ISP  $A$  and  $B$  (3),

we find the first derivative of  $U_A$  with respect to  $p_A$  and the first derivative of  $U_B$  with respect to  $p_B$ . It is a system of two linear equations with respect to  $p_A$  and  $p_B$ . We solve the system using the Gauss elimination method. By differentiating the solution of  $p_A$  with respect to  $p_C$ , we obtain

$$\frac{\partial p_A}{\partial p_C} = \frac{1}{3(1 + \beta_A - \alpha_{(B,A)}\beta_A)(1 + \beta_B - \alpha_{(A,B)}\beta_B)} \times \begin{pmatrix} 3 - 2\alpha_{(A,A)} - \alpha_{(B,B)} \\ +3\beta_A - 2\alpha_{(A,A)}\beta_A - \alpha_{(B,A)}\beta_A - \alpha_{(B,B)}\beta_A \\ +3\beta_B - 2\alpha_{(A,A)}\beta_B - 2\alpha_{(A,B)}\beta_B - \alpha_{(B,B)}\beta_B \\ +2\beta_A\beta_B - 2\alpha_{(A,A)}\beta_A\beta_B - 2\alpha_{(A,B)}\beta_A\beta_B \\ +\beta_A\beta_B - \alpha_{(B,A)}\beta_A\beta_B - \alpha_{(B,B)}\beta_A\beta_B \\ +\alpha_{(B,A)}\alpha_{(B,B)}\beta_A + 2\alpha_{(A,A)}\alpha_{(A,B)}\beta_B \\ +2\alpha_{(A,A)}\alpha_{(A,B)}\beta_A\beta_B + \alpha_{(B,A)}\alpha_{(B,B)}\beta_A\beta_B \end{pmatrix} \quad (5)$$

By differentiating the solution of  $p_A$  with respect to  $p_O$ , we

obtain

$$\frac{\partial p_A}{\partial p_O} = \frac{\rho_O}{3\rho_A(1 + \beta_O)} \times \frac{1}{1 + \beta_B - \alpha_{(A,B)}\beta_B} \times \frac{1}{1 + \beta_A - \alpha_{(B,A)}\beta_A} \times \begin{pmatrix} -3 - 3\beta_A - 3\beta_B - 3\beta_A\beta_B \\ -\alpha_{(B,A)}\beta_A - \alpha_{(B,A)}\beta_A\beta_B \\ +\alpha_{(A,B)}\beta_B + \alpha_{(A,B)}\beta_A\beta_B \\ +3\alpha_{(A,B)}\alpha_{(B,A)}\beta_A\beta_B \end{pmatrix}. \quad (6)$$

Because  $\beta_A \in [0, 1]$ , and  $\alpha_{(A,A)}, \alpha_{(A,B)}, \alpha_{(B,A)}, \alpha_{(B,B)} \in [0, 1]$ , the right-hand side of equation (5) is positive and the right-hand side of equation (6) is negative, which prove Proposition 2.