Polycephalic Euclid?

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Collective aspects in the history of mathematics in the Bourbaki era.

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Abstract

In 1961, the French historian of mathematics Jean Itard ventured the idea that Euclid might have been no more than a *nom de plume* for a collective mathematical enterprise. This was anything but innocent, at the time when Bourbaki was so successful and well-known, and, more generally, collective aspects determined more and more the mathematical life of the period. In this paper, we look both at the place of collective practices in the historical writing of mathematicians around Bourbaki and at the role played by concepts representing the collective in their historiography. At first sight, although written collectively, Bourbaki's *Elements of the History of Mathematics*, which has been seen as an "internalist history of concepts", is an unlikely candidate for exhibiting collective aspects. But, as we shall show tension between individuals and collective notion, such as, most famously, "*Zeitgeist*" which is presented as orchestrating the development of infinitesimal calculus, are constant. It is interesting to unpack the way in which changes in mathematical practices impacted conceptions of the history of mathematics.

Introduction

> Difficulties which arise in the chronology when we admit the physical existence of a single Euclid lessen without vanishing when we accept to take his name as the collective title of a mathematical school. [Itard 1962, p.11][1]

The idea of Euclid as a collective mathematician, and more precisely its consideration as a mathematical "school" or "team"[2] was anything but innocent, especially at the time when Bourbaki was so successful and well-known ! The hypothesis concerning Euclid are well-explained and developed in the correspondant article in the MacTutor History of Mathematics archive[3] :

> The situation is best summed up by Itard who gives three possible hypotheses.
>
> (i) Euclid was an historical character who wrote the Elements and the other works attributed to him.

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[1] "Les difficultés qui surgissent à chaque instant dans la chronologie lorsque l'on admet l'existence physique d'un seul Euclide s'atténuent sans disparaître lorsque l'on accepte de prendre son nom comme le titre collectif d'une école mathématique."

[2] It is not easy to translate the French expression "école mathématique" into English. We choose to use the word "school" because it is the translation that was chosen by Bourbaki itself in the English publication of the *Elements of the history of mathematics*. We come back on the notion of school later in this article.

[3] http://www-history.mcs.st-andrews.ac.uk/Biographies/Euclid.html (March 16, 2013)

(ii) Euclid was the leader of a team of mathematicians working at Alexandria. They all contributed to writing the 'complete works of Euclid', even continuing to write books under Euclid's name after his death.

(iii) Euclid was not an historical character. The 'complete works of Euclid' were written by a team of mathematicians at Alexandria who took the name Euclid from the historical character Euclid of Megara who had lived about 100 years earlier.

Even if they choose the first solution, and consider that the third is "the most fanciful", they quote the Bourbaki project to kind of justify it :

> Although on the face of it (iii) might seem the most fanciful of the three suggestions, nevertheless the 20th century example of Bourbaki shows that it is far from impossible. Henri Cartan, André Weil, Jean Dieudonné, Claude Chevalley and Alexander Grothendieck wrote collectively under the name of Bourbaki and Bourbaki's Eléments de mathématiques contains more than 30 volumes.

In a recent online article F. Acerbi reminds that the idea of a collective Euclid comes from the 1950s, in the Bourbaki years, but cannot be plausible :

> Le manque de données biographiques produisit un autre mythe, celui-ci tout à fait moderne : Euclide ne serait que le nom de plume d'une équipe d'auteurs-compilateurs sur le modèle de Bourbaki. Cette hypothèse, de fait émise pour la première fois dans les années 1950, ne peut paraître plausible qu'à quelqu'un qui n'aurait pas la moindre idée des féroces revendications d'auteur qui caractérisent toute la littérature grecque, et le domaine des mathématiques en particulier.
> [Acerbi 2010]

The idea of a collective Euclid, that we will not discuss here, should thus be understood through the light of the collective project of Bourbaki[4]. More generally, as it is stressed in A.-S. Paumier in her Ph.D. thesis[Paumier 2013] , collective aspects determined more and more the mathematical life of the period.

We will present some aspects of the collective life of mathematics during the twentieth century, and then focus on the practice and content of history of mathematics done by the Bourbakis. Although the Bourbakis have individually written many texts concerning the history of mathematics, we will focus on the collective volume of the *Elements of the history of mathematics*[5]. We will look both at the place of the collective in the history of mathematics written by Bourbaki and collective practices in the writing of such histories. At first sight, although written collectively, Bourbaki's *Elements of History of Mathematics* is an unlikely candidate for exhibiting collective aspects. According to J. Pfeiffer [Dauben and Scriba 2002, p.40], it is an "internalist history of concepts", but, as she quotes, "Zeitgeist" is presented as orchestrating the development of infinitesimal calculus. This echoes Aubin's analysis where he tried to account for structuralist ideas that were "in the air" in terms of "cultural connectors" [Aubin 1997, p.298]. It would be interessing to unpack the way in which changes in mathematical practices impacted conceptions of the history of mathematics.

[4]The reverse idea, that is to say that Bourbaki is kind of a successor of Euclid, is mentioned by Beaulieu through the analysis of a poem in [Beaulieu 1998, p.112].

[5]Dieudonné and Weil are the most prolific authors in that field. We can just mention here that Dieudonné has written histories of functionnal analysis [Dieudonné 1981], algebraic and differential topology [Dieudonné 1989] and directed a collective volume [Dieudonné 1978] ; and that Weil has written a history of number theory [Weil 1984] and expressed his ideas about the way history of mathematics should be practiced and written [Weil 1978 a]. We will see some excerpts in the paper. About the way Weil used historical mathematical texts, see [Goldstein 2010]

1 Collective practices in twentieth century mathematics in France

> The huge scale of scientific research in the second half of the twentieth century is hard to ignore.(...) Seen from the inside – form the scientists' perspective – big science entails a change in the very nature of a life in science.
>
> [Galison 1992, p.1]

It is well-known that the mathematical community rapidly increases just after the Second World War in a numerical way. This numerical increase deeply transforms the organization of the mathematical life into a collectively-structured life.

The collective mathematical life can be efficientely studied by the biographical path, as it is stressed by A.-S. Paumier in her phD dissertation entitled "Laurent Schwartz et la vie collective des mathématiques" [Paumier 2013]. Let's try to present the collective life of mathematics experienced by the Bourbakis. We will focus widely on the years 1935-1960 ; 1935 is the year of the first Bourbaki congress in Besse-en-Chandesse and 1960 the year when the *Eléments d'histoire des mathématiques* was first published. This can be done because the Bourbakis were influent mathematicians at the time and acted in important ways in the collective structuration of the community.

1.1 A first naive conception of the collective work in mathematics

We could limit ourselves to present the mathematical work as a cumulative enterprise. But this can be ambiguous, as it is stressed in a polemical article written by Serge Lang [Lang 2001], in which he criticizes Weil when he, according to him, "transgressed certain standards of attribution several times throughout his life in significant ways." More than the polemic, what is interesting here is the conception of the mathematical collective work that is expressed :

> Concerning a comment at sfome Weil talk that proper credit was not given by Weil for some theorem, Knapp quoted Weil's answer "I am not interested in priorities", and added his own comment : "This was the quintessential Weil. Mathematics to him was a collective enterprise." I object. In the sense that mathematics progresses by using results of others, Knapp's assertion is tautologically true, and mathematics is a collective enterprise not only to Weil but to every mathematician.
>
> However, there is also another sense. Mathematics is often a lonely business. Public recognition of the better mathematicians is a fact. (...)
>
> The Knapp editorial and Rosen's comments prompt me to complement my *Notices* article by further historical remarks showing how Weil several times throughout his life did not properly refer to his predecessors, but was "interested in priorities". Theses constitute significant examples, when Weil does not regard mathematics as a "collective enterprise" in the sens that he hides the extent to which he uses previous work, and sets up or pokes fun at some of his predecessors, as we shall now document.

This only shows that this first naive conception of mathematics as a collective enterprise is not all that simple. We will have to choose a more precise description of the collective practices in mathematics, focusing on those emerging in the twentieth century.

1.2 The collective life of mathematics experienced by the Bourbakis (1935-1960)

We will just deal here with two collectives that progressively structure the mathematical community throughout the twentieth century.

First of all, let's talk of the mathematical seminar. During their formation – most Bourbakis were at the Ecole Normale Supérieure –, they could only attend the well-known "séminaire Hadamard" at the Collège de France. It played an important role in André Weil's formation for instance, as he says many times[6]. This was something completely new at the time in France[7]. In 1933, around Gaston Julia who was a full professeur in La Sorbonne, this small group gives birth to a new type of séminaire de mathématiques, "le laboratoire d'une équipe restreinte", if we quote Liliane Beaulieu in her dissertation about Bourbaki[8]. In the 1960s there are some thirties mathematical seminars that take place around Paris every week ! Bourbaki as a group and as a collection of individuals played an important role in the fulgurant development of these mathematical seminars after the war : 1948 (1946) séminaire Bourbaki, 1947 : séminaire Dubreil, 1948 (1947) séminaire Cartan, 1953 : séminaire Schwartz...to give only some examples. The séminaire Bourbaki was for instance an opportunity for provincial mathematicians to meet three times a year in Paris.

In 1945, the Rockefeller Foundation looked for an efficient way to help the reconstruction of scientific research in France. As it is described in [Zallen 1989], France, under the behalf on the CNRS, received a grant (in fact 2 grants) for two purposes. First, to buy scientific equipment. Secondly to organize small international conferences : the "colloques internationaux du CNRS". These were supposed to be "small and informal"[9]. Most of these have been published. These were a great success, from the Rockefeller point of view but also the French mathematicians' [10]. Of course, the Bourbakis were very involved in the mathematical conferences. We will here mention the three first mathematical conferences, and the Bourbakis that have attended these :

- Analyse harmonique, Nancy 1947 (Schwartz, Mandelbrojt, (Julia), Godement)
- Topologie algébrique, Paris, 26 juin-2 juillet 1947. (Henri Cartan, Charles Ehresmann, Jean Leray)
- Algèbre et théorie des nombres (Paris, 25 septembre, 1 octobre 1949) (organized by Dubreil and Châtelet) (Chabeauty, Pisot, Weil, (Hadamard), Dieudonné, (Julia))

These short examples explain what we mean when speaking of collective practices in mathematics the twentieth century.

1.3 The particularity of the Bourbaki enterprise

There is no need to present Bourbaki ! We can just quote the first words of Liliane Beaulieu's pHD [Beaulieu 1989], entitled "Bourbaki. Une histoire du groupe de mathématiciens français et de ses travaux (1934-1944) "

> Le groupe de mathématiciens qui porte le nom de Nicolas Bourbaki s'est réuni pour la première fois en décembre 1934 et il poursuit encore ses activités. Cette équipe, composée surtout de mathématiciens français, se consacre

[6] For instance, see [Weil 1991, p.38]

> C'est entre la "bibli" et le séminaire Hadamard que je suis devenu mathématicien cette année-là et les suivantes.

[7] About the "séminaire Hadamard" see [Beaulieu 1989, p.60-65], [Chabert and Gilain à paraître].

[8] [Beaulieu 1989, p.133-137]

[9] [Zallen 1989, p.6] She quotes the Rockefeller archives : "the attendance of mature contributors restricted to say 15 ; with provision, however for additionnal listening and observing audience of young men"

[10] Archives départementales de Nancy, W 1018/96, Rapport sur le colloque.

> Outre, les résultats scientifiques améliorés, éclaircis ou établis, outre, les contacts personnels durables qui en résulteront, cette manifestation scientifique a montyé qu'il était possible de travailler très utilement sur un sujet bien délicité entre un petit nombre de personnes qualifyées. La forme matérielle in augurée en cette occasion s'est montrée aussi utile qu'agréable et il y a lieu d'insister sur l'honneur qui rejaillit sur l'Université de NANCY, du fait qu'elle ait été choisie comme théâtre de la premieèrerúnion de cette nature.

à l'élaboration d'un ouvrage intitulé *Eléments de mathématique* qui expose différents domaines des mathématiques. Pour les mathématiciens, Bourbaki a été le représentant d'un "style" mathématiques alliant l'axiomatique à un mode de présentation rigoureux et abstrait. Le groupe comme l'oeuvre ont connu la célébrité, surtout dans les années cinquante et soixante, alors qu'ils ont soulevé les passions les plus contradictoires chez les mathématiciens et qu'ils ont même attiré sur eux la curiosité de l'homme de la rue.

We will just keep in mind some aspects of the enterprise. First of all, it is a very specific initiative, at a specific time, by specific people. In 1934, there are not many occasions to meet, and the former student from the Ecole Normale may have felt the lack of collective meetings. This is why we should remember two aspects of the enterprise : the redaction of a treatise for the teaching but also the great amount of congresses, meetings and discussions of all the members. We have already mentionned the Bourbaki seminar, which is the public part of the enterprise.

2 Concrete aspects about the writing of the historical notes

Although the Bourbakis have written several books on the history of mathematics, we here focus on the *Eléments d'histoire des mathématiques* from N. Bourbaki. This volume was first published by Herman in 1960 (2nd edition 1969, third edition 1974). Then republished by Masson in 1984 (reedited by Springer in 2007). [11]

It was translated into English only in 1994 [Bourbaki 1994].

This volume is composed of a certain amount of historical notes, that first appeared in the volumes of the mathematical treatise of Bourbaki : *Eléments de mathématiques* (lots of volumes from 1936). Let's first examine the historical notes in the mathematical treatise.

2.1 An early choice in the treatise enterprise.

The *Eléments de mathématique* were published chapter after chapter (not in order) from 1939 on[12]. In the first real volume that was published in 1940[13], there is already a historical note. The notes do not have any name in the original books ; the names were given when the *Eléments d'histoire des mathématiques* was first published in 1960. The table 1 gives the years when each historical note (contained in the last edition of the *Eléments d'histoire des mathématiques*) was first published.

We can see some traces of discussions in the accounts of some Bourbaki's congresses. Thus, in 1935, at the first annual congress, Congrès de Besse en Chandesse, we can read as desidaratas [14] that a "Dictionnaire des termes usités (historique et références)‘" is wanted. We have more details in september 1936, the next annual congress, congrès de l'Escorial, in the document giving the decisions that have been taken there about the typography and the redaction [15].

The decisions and instructions for the redaction concern the name of the theorem, the notations or the size of the font and even a reference to Euclid for the terms to use for minor results. There are 13 general principles for typography and 26 for the redaction. Let us quote the three last points concerning the redaction :

[11] The editions and reviews are listed in [Beaulieu 1989, Annexe II B, Annexe III B] from 1960 to 1986. Some of the reviews will be quoted here. This book was also translated into italian and russian (1963), german (1971), spanish (1972) and polish (1980).

[12] The list can be found in [Beaulieu 1989, AnnexeII]

[13] III. Topologie générale ; chapitres 1. Structures topologiques et 2. Structures uniformes, Hermann Paris.

[14] Archives Bourbaki, delbe-014, p.5. The archives of Bourbaki can be found online http://archives-bourbaki.ahp-numerique.fr.

[15] Archives Bourbaki, deles-09.

x) Laïus[16] scurrile, toute latitude, en caractères normaux.[17]

y) Laïus historique, en fin de chapitre, quand ce sera utile.

z) Laïus excitateur, en fin de chapitre, avec références. (Comme bon exemple voir Severi, Traité de géométrie algébrique)

The historical and excitatory notes are linked and are supposed to be at the end of a chapter. There is even a reference – of Severi, as we discuss just after – that is given as a good example. We will analyze more precisely these choices and the way they have been undertaken in the treatise. These quotations are here aimed to show the very early mentions of the existence of the historical notes.

There are even more precisions about the references from outside the treatise (the other volumes of the treatise are often referred to)[18] :

> Références extérieures à Bourbaki. Pour les références fondamentales, elles seront données avec soin dans les laï us historiques ou excitateurs ; éventuellement dans le texte ; (il s'agit donc de références vérifiées, correctes et complètes)
>
> Pour les références de technique, simple mention de l'auteur présumé. Pour les autres, comme on pourra, sans y attacher trop d'importance (emploi de nègres pour dépistage)

The references are supposed to be sent back to the historical notes.

2.2 An original choice.

Bourbaki's choice to write historical notes is widely discussed in the review that Taton makes for the publication of the *Eléments d'histoire des mathématiques* in 1960 :

> En fait, il s'agit là d'une œuvre entièrement originale, unique en son genre dans la littérature mondiale actuelle. L'élément de comparaison le plus valable est constitué par les notices historiques insérées en tête des différentes parties de l'édition franco-allemande de l'*Encyclopédie des sciences mathématiques pures* ; encore ne s'agissait-il là d'une présentation à la fois plus élémentaire au sens mathématique, du fait qu'elle ne recourait qu'à un vocabulaire assez général, et plus érudite au sens historique, grâce à ses notes très documentées et à ses références d'une rare précision. Pour caractériser d'une faqn très schématique cette différence d'esprit, on peut dire que les notices des *Eléments de mathématique* de N. Bourbaki ont été rédigées par des mathématicien s'intéressant à l'histoire de leur discipline, à l'intention d'autres mathématiciens également curieux des origines de leur science, tandis que celles de l'*Encyclopédie* l'ont été avec la participation active d'historiens des mathématiques professionnels qui écrivent, en partie du moins, à l'intention d'un public d'historiens.
>
> [Taton 1961, p.158-159]

[16] This is a very frequently used term by Bourbaki among others, to design something written that is not only mathematics or a speech for a seminar. It means a long speech. According to the Littré, it comes from the special vocabular of the Ecole Polytechnique :

> Un discours, dans l'argot des jeunes gens de l'école polytechnique. Piquer un laïus, prononcer un discours. "Dans le dialecte de l'école, tout discours est un laïus ; depuis la création du cours de composition française en 1804, l'époux de Jocaste, sujet du premier morceau oratoire traité par les élèves, a donné son nom au genre". [De la Bédollière, les Français peints par eux-mêmes, t. V, p. 116]

[17] We have a such example in deles-002 (Archives Bourbaki), entitled "Ensembles - Décisions escoriales - Projet de laïus scurrile", which was also written in the Congrès de l'Escorial in 1936. The first words, concerning the introduction of the theory of sets, are the following :

> L'objet d'une théorie mathématique est une structure organisant un ensemble d'éléments : les mots "structure"„ "ensemble", "éléments" n'étant pas susceptibles de définition mais constituant des notions premières communes à tous les mathématiciens, ils s'éclaireront d'eux-mêmes dès qu'on aura eu l'occasion de définir des structures, comme il va être fait dès ce chapitre même.

[18] Archives Bourbaki, deles-09

Although René Taton claims that this initiative is unique, Bourbaki gives a reference which inspired the historical notes. We can thus read "comme bon exemple voir Severi, Traité de géométrie algébrique". On can find, at the library of the Institut Henri Poincaré, the german version of this treatise [Severi 1921], which should be the book that the Bourbakis read. There are some historical that are inserted in the mathematical text, in smaller letters (size font between the one of the text and the one of the footnotes). For example, p. 113 :

> Die Literatur über den soeben bewiesen Satz, der in der algebraischen Behandlung der Theorie der linearen Scharen von BRILL und NOETHER eine fundamentale Rolel spielt, ist sehr umfangreich. Man findet viele Literaturangaben in des Encyklopädie der math. Wissenschaften sowie einer Note von BERTINI. Der NOETHERsche Satz is auch auf eine beliebige Anzahl von Formen mit beliebig vielen Veränderlichen ausgedehnt worden. (...)

These notes are very particular and specific. We can find about 15 of them, from a few lines to one or two pages. They do not have the same status as the footnotes, that are used to givie precise bibliographical references or mathematical explanations. An interesting point is also the reference to the *Encyklopädie der mathematischen Wissenschaften*, which is precisely the only point of comparison that is given by René Taton. Thus, we can compare the historical notes of the *Eléments de mathématique* of Bourbaki and the historical notes that are at the beginning of the chapters of this German-French[19]encyclopedy.

The historical notes are at the beginning in the encyclopedy, as something more simple than the rest of the chapter. As Taton says, it is more historically erudite and less mathematically complicated. In Bourbaki's books, the historical are at the end of the book, in smaller characters. There are references included, but not so many of them.

2.3 What do the typographical and layout choices express ?

In the Mode d'emploi du traité, which introduces each volume of the mathematical treatise, and starts with these nowadays well-known words :

> To the reader[20]
>
> 1. The Elements of Mathematics Series takes up mathematics at the beginning and gives complete proofs. In principle, it requires no particular knowledge of mathematics on the reader's part, but only a certain familiarity with mathematical reasoning and a certain capacity for abstract thought. Nevertheless it is directed especially to those who have a good knowledge of the first year or two of a university mathematics course.

The others introducing points are about the typography, the terminology, the place of examples, the symbol "dangerous bend", the logical framework, the composition of the treatise... There is also one point justifying the presence of the historical notes :

> Since in principle the text consists on the dogmatic exposition of a theory, it contains in general no references to the literature. Bibliographical references are gathered together in *Historical Notes*. The bibliography which follows each historical note contains in general only those books and original memoirs that have been of the greatest importance in the evolution of the theory under discussion. It makes no sort of pretence to completness.

We first learn that the first use of the historical notes is to give the necessary bibliographical references. We should read this point together with the point that present the exercises, which are also rejected to the end of a chapter :

> The Exercises are designed both to enable the reader to satisfy himself that he has digested the text and to bring to his notice results which has no place in the text but which are nonetheless of interest. The most difficult exercises bear the sign ¶

[19]Only the first volume have been translated and published in French, due to the first world war

[20]"To the reader" is the translation of "Mode d'emploi de ce traité".

The historical notes are supposed to fill the holes, as the exercises. The archives give us two very precise examples :

> On renonce à faire (même en Appendice) la résolution des équations par radicaux, mais il faudra que la Note historique expose succinctement le lien entre cette question et la théorie moderne des corps. [21]

and

> Introduction On décide de la faire très courte, et de reporter le plus de choses possibles à la note historique. [22]

2.4 A limited consideration for the historical notes

Henri Cartan [Cartan 1980, p.178] writes the following :

> The "Notes Historiques" and "Fascicules de Résultats" deserve special mention. Bourbaki often places an historical report at the end of a chapter. Some of them are quite brief, while others are detailed commentaries. Each pertains to the whole matter treated in the chapter. There are never any historical references in the text itself, for Bourbaki never allowed the slightest deviation from the logical organization of the work. It is only in the historical report that Bourbaki explains the connection between his text and traditional mathematics and such explanations often reach far back into the past. It is interesting to note that the style of the "Notes Historiques" is vastly different from that of the rigorous canon of the rest of Bourbaki's text. I can imagine that the historians of the future will be hard put to explain the reasons for these stylistic deviations.

It is effectively hard to understand the ambiguous status that has been given to these historical notes. Although the historical notes have been written by different people, they do not seem to have been discussed as the other parts of the treatise have. We find some rare mentions of the historical in the Bourbaki archives. The notes have sometimes been discussed, but very quickly, and are considered as non primordial in the treatise :

> Il est bien entendu qu'on ne revient pas sur le détail du texte actuel, qui reste inchangé aux menues retouches près qu'exigera le nouvel ordre des matières. Une fois ces retouches faites, le fascicule sera donc livré à l'impression, au plus tard en Octobre. Comme d'habitude, les Notes historiques seront discutées ultérieurement, lors du petit Congrès qui se tiendra en Décembre 1945 à Nancy ; Weil et Chevalley enverront leurs observations à ce Congrès par correspondance[23].

Moreover, the historical notes are sometimes considered as a distraction :

> La Note historique des chap.II-III d'Algèbre, lue en guise de discours inaugural, mit le Congrès 'in the right mood' quant à l'Algèbre : il glorifia Fermat, suivit docilement les méandres du linéaire et comuta l'influence de Mallarmé sur Bourbaki. Cela fait, on passa aux Chap. IV-V de l'Algèbre (...) [24]

The historical notes have a very ambiguous place in the treatise[25]. They do have a real place in the project but are not considered as interior to the mathematical text. We will now discuss in more details the content of the notes.

[21] Archives Bourbaki, nbt-017 (Tribu 16)
[22] Archives Bourbaki, nbt-026 (about théorie des ensembles)
[23] Archives Bourbaki, nbt-012 (about livre III Topologie Générale)
[24] Archives Bourbaki, nbt-016 : CR du Congrès de Noël 1947
[25] Beaulieu when examining the commemoration practices that are specific to Bourbaki points out that it should be compared to the implicit place that Bourbaki gives himself in the historical notes [Beaulieu 1998, p.114].

	Chapter Title	Pages	Book	First publ.
1.	Fondations of Mathamatics; Logic; Set Theory	44 p.	I	1957
2.	Notations, Combinatorial Analysis	2 p.	?	?
3.	The Evolution of Algebar	10 p.	II	1942
4.	Linear Algebra and Multilinear Algebra	12 p.	II	1947-1948
5.	Polynomials and Commutative Fields	16 p.	II	1950
6.	Divisibility; Ordered Fields	8 p.	II	1952
7.	Commutative Algebra. Algebraic Number Theory	24 p.	VIII	1965
8.	Non Commutative Algebra	8 p.	II	1958
9.	Quadratic Forms; Elementary Geometry	14 p.	II	1959
10.	Topological Spaces	6 p.	III	1940
11.	Uniform Spaces	2 p.	III	1940
12.	Real Numbers	10 p.	III	1942
13.	Exponentials and Logarithms	2 p.	III	1947
14.	n Dimensional Spaces	2 p..	III	1947
15.	Complex Numbers; Measurement of Angles	4 p..	III	1947
16.	Metric Spaces	2 p..	III	1949
17.	Infinitesimal Calculus	32 p.	IV	1949
18.	Asymptotic Expansions	4 p.	IV	1951
19.	The Gamma Function	2 p.	IV	1951
20.	Function Spaces	2 p.	III	1949
21.	Topological Vector Spaces	12 p.	V	1955
22.	Integration in Locally Compact Spaces	12 p.	VI	1956
23.	Haar Measure. Convolution	6 p.	VI	1963
24.	Integration in Non Locally Compact Spaces	10 p.	VI	1969
25.	Lie Groups and Lie Algebra	22 p.	VII	1972
26.	Groups Generated by Reflections; Root Systems	6 p.	VII	1969

Table 1: Table of Contents of the *Éléments d'histoire des mathématiques* [Bourbaki 2007] with number of pages, book of the *Éléments de mathématique* and year of first publication.

3 Collective Practices in Bourbaki's Historiography

When the *Elements of History of Mathematics* were published in 1960, Bourbaki made no pretense to present a complete history of mathematics. In an "*avertissement*" placed at the start of the volume, authors explained that seperate studies written for another purpose had merely been assembled without major revisions. As a consequence, many portions of the history of mathematics like differential geometry, algebraic geometry, and the calculus of variations were absent from the volume, because the corresponding parts in the *Element of Mathematics* had not been published yet. Bourbaki kept silent about branches of mathematics, such as probability and statistics or more generally all applied mathematics, that seemed of little relevance to the scheme the group was in the process of popularizing. Mostly they warned their readers that they would find in this book:

> no bibliographical or anecdotal information about the mathematicians in question; what has been attempted above all is for each thory to bring out as clearly as possible what were the guiding ideas, and how these ideas developed and interacted on the others [Bourbaki 1994, p. v].[26]

Although these notes reflected a historiography that was sometimes more than 20 years old already in 1960, historians of mathematics received the book with enthusiasm

[26]Note that the French version mentions *biographical* rather than *bibliographical* information [Bourbaki 2007, p. v].

and lauded its originality. While being critical on many rather minor points, Jean Itard for example emphasized that this was "a nice and good work"[27] . Itard also mentioned that one would do well to read Bourbaki's book in parallel with another collective project, the *Histoire générale des sciences*, directed by René Taton, whose chapter on the history of mathematics was partly written by a Bourbaki member, Jean Dieudonné. While underscoring more important inadequacies of Bourbaki's book (like the provisional character of the endeavor which was a direct consequence of the haphazard way in which it had been composed and the lack of reference to contemporary research by historians of mathematics), the historian René Taton repeated without comment the author's point of view expressed above. While pointing out that the history of "the working mathematician" was not to be found in Bourbaki's book which he found " factual rather than interpretative," the historian Ivor Grattan–Guinness nonetheless called it "the most important history yet produced of the mathematics of recent times."[28] Mathematicians likewise repeated approvingly Bourbaki's words about the lack of biographical information. As A.C. Aitken noted: "the work has no concern whatever with anecdote, legend or personality of the authors concerned: authors are related solely to theorems or contributions to theory which mathematics owes them"[29].

3.1 Collective Practices Emerging

The table of contents (table 1) greatly reinforced the view that Bourbaki's book was above all concerned with mathematical notions, and modern ones more especially. But as is well known, this focus on ideas erased much of the social dynamics at play in the historical development of mathematics. The collective result of a group of mathematicians who had embarked 25 years earlier on a highly original project of collectively and anonymously rewritting vast portions of mathematics, Bourbaki's *Elements of the History of Mathematics* thus appear at face value as a paradoxical product: a history of mathematics from which all collective dimensions seemed to have vanished. Let us see what a deeper examination can nevertheless dig up concerning collective aspects of mathematical practice and how discussions of the collective in Bourbaki's historiography might be a reflection of their practical experience as mathematicians.

3.1.1 Institutions

Unsurprisingly, there are few institutions in Bourbaki's account of the history of mathematics. Universities are barely mentioned twice in passing [Bourbaki 1994, p. 136 & 170], like the École polytechnique (p. 56 and 133). Most of the time, this is to recall that formal training play a crucial part in the flow of ideas from generation to the next.[30] Journals, academies and learned societies scarcely appear outside references to the literature. In a rare instance, Bourbaki recalled that Benjamin Pierce and Clifford made several references to their personal discussions at a meeting of the London Mathematical Society [Bourbaki 2007, p. 150n].

True to their belief in international exchanges, however, the Bourbakis paid more attention to the International Congresses of Mathematicians. But ICMs play the role of sounding boards for ideas expressed by individual mathematicians. In Zurich (dated once 1896 erroneously and once correctly 1897), Hadamard and Hurwitz drew attention to the applications of set theory to analysis; in Paris in 1900, Hilbert included the problem on the noncontradiction of arithmetic among his famous list; in Rome in 1904, the same Hilbert attacked the same problem (p. 30, 142, 39 & 40).

[27]Review in *Revue d'histoire des sciences et de leurs applications* **18** (1) (1965), p. 120–123; url:www.persee.fr/web/revues/home/prescript/article/rhs_0048-7996_1965_num_18_1_2402.

[28]Review of the *Élément d'histoire des mathématiques*, in *British Journal for the History of Science* **5** (1970), p. 190–191.

[29]Book Review, in *Proceedings of the Edinburgh Mathematical Society* **12** (1961), p. 217.

[30]The teachings of Cauchy at the École polytechnique, of Kronecker and Weierstrass at the university of Berlin where they introduced the "axiomatics" of determinants, and Gauss' courses followed by Riemann at Göttingen in 1846–1847 are thus mentioned [Bourbaki 2007, resp. p. 81, 87 and 163n].

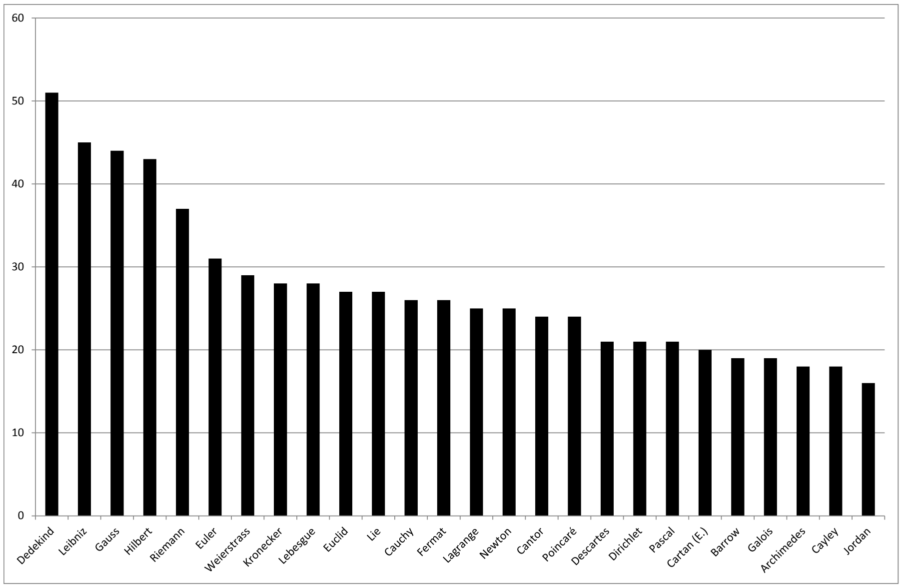

Figure 1: The 26 Mathematicians who are mentioned the most in Bourbaki's *Elements of the History of Mathematics* and the number of pages where their name occurs.

Little more can be said about the now standard sociological foci of the historiography of mathematics. What we need is to enlarge our net if we want to have any chance of catching other ways in which collective aspects of mathematical practices may nonetheless surface in Bourbaki's historiography.

3.1.2 Individuals in Tension

While mathematical notions indeed structure the text, we note that very many individuals mathematicians are named explicitly. According to the index, which was added in later editions of the work, 456 mathematicians are mentioned by names. As figure 1 exhibits clearly, among the 26 most cited mathematicians geographical and chronological distributions are far from uniform. The first 8 in this list come from Germany (out of a total of 10). In addition, there 10 mathematicians from France, 3 from Great Britain, 2 from the Ancient Greek world, and 1 one from Norway (Sophus Lie whose name appear frequently mostly due to the emphasis put on Lie groups and algebras). One is also struck by the chronological imbalance in this list, where barely 9 mathematicians having died before the start of nineteenth century appear.

Such statistics confirm common views about Bourbaki's image of mathematics. There is a total of around 350 citations for the 10 most cited German mathematicians compared to 225 for the French. Developments that had mostly occured recently in the German world indeed seem to be the most valued by Bourbaki. Such numbers however emphasize the importance of French mathematicians in this picture, whose role in Bourbaki's historiography is often downplayed, including twentieth–century mathemlaticians like Henri Poincaré, Henri Lebesgue or Élie Cartan.

As mentioned above, these individuals are mostly vehicles for mathematical ideas and seldom appear as flesh–and–blood people in Bourbaki's text. This deliberate viewpoint announced in the introduction is sometimes expressed in the text itself. In the chapter on "Polynomials and Commutative Fields," Bourbaki seems only reluctantly to refrain from telling the convoluted story of the way in which Italian algebrists in the Renaissance publicized what is now known as the Cardano formula for solving third–degree algebraic equations:

> We cannot describe here the picturesque side of this sensational discovery — the quarrels that it provoked between Tartaglia on the one hand, Cardan [sic] and his school on the other — nor the figures, often appealing, of the scholars who were its protagonists [Bourbaki 1994, p. 72].[31]

This is well known: what we would like to emphasize here is that by emphasizing ideas at the expanses of practices and institutions, Bourbaki perhaps inadvertantly produced a very odd result. Indeed, in this text – as, for that matter, it is often the case in the history of ideas – much agency is placed in the hands (or minds) of (a selected set of) individuals. In the first notice that was published in 1940, addressing the history of "Topological Sets," Bourbaki thus writes:

> It is Riemann who must be considered as the creator of topology; as of so many other branches of modern mathematics [Bourbaki 1994, p. 139].

Early in the book, Bourbaki similarly underscored that Boole "must be considered to be the real creator of modern symbolic logic" [Bourbaki 1994, p. 8]. Galois is considered as the "real initiator" of the theory of substitution [Bourbaki 1994, p. 51]. The notion of tensor product of two algebras "must be attributed" to Benjamin Pierce (p. 118). In still another passage, the authors write:

> it must be realised that this way was not open for modern analysis until Newton and Leibniz, turning their back on the past, accept that they must seek provisionally the justification for their new methods, not in rigorous proofs, but in the fruitfulness and the coherence of the results [Bourbaki 1994, p. 175].

This view according to which crucial innovations in the path to modernity must await the intervention of a chosen individual is quite typical of Bourbaki's historiography. The authors for example write that we "must await" Cauchy (p. 129), Chasles (p. 131), or Möbius (p. 126) for the emergence of various mathematical concepts. Obviously, Bourbaki's historiography is filled with value judgments that emphasize the worth of great mathematicians and sometimes their "genius" (Poincaré on p. 35; Cantor's on p. 27). Instead of genius, Bourbaki often preferred to talk of "mathematicians of the first rank" [Bourbaki 1994, p. 6, 11, & 61].

As has often been emphasized, this conception according to which selected individuals play a leading role in the history of mathematics certainly contains an element of selfpromotion as well as self-aggrandization. Often starting with the Greeks or even the Babylonians, Bourbaki systematically saw the notions emphasized in his treatise and, at times, the work of Bourbaki members as the rightful culmination of a continuous, progressive account of the history of mathematics.[32] Most striking perhaps in a passage where André Weil's and Jean–Pierre Serre's work are discussed as those which succeeded in dispelling all mistrust regarding the theory of normed algebras developed by Gelfand [Bourbaki 2007, p. 147–148][33].

In Bourbaki's hand, the history of ideas therefore becomes a teleological account where agency is placed in the hands of individuals. This of course is paradoxical since nowhere is explained the reason why these special individuals are able to discern ahead of time the direction history will take. From the text, therefore emerges a tension between individuals contributions and aspects of the collective that nevertheless are allowed to permeate Bourbaki's historiography of mathematics as a history of ideas, a tension that seldom has been discussed before.

[31]"*Nous ne pouvons décrire ici le côté pittoresque de cette sensationnelle découverte — les querelles qu'elle provoqua entre Tartaglia,d'une part, Cardan et son école de l'autre — ni les figures, souvent attachantes, des savants qui en furent les protagonistes*" [Bourbaki 2007, p. 96].

[32]This has been labelled as the "royal road to me" historiography of mathematics, which mathematicians "confound the question , 'How did we get here?', with the different question, 'What happened in the past?' " [Grattan-Guinness 1990, p. 157].

[33]One also finds mentions of Bourbaki's *Elements of Mathematics* as well as Henri Cartan's notion of filters [Bourbaki 2007, p. 160 & 180].

3.1.3 Fluid Metaphors, the Practice of the Mathematicians, and *Zeitgeist*

Intent on capturing the flow of mathematical ideas, Bourbaki used a mixed bag of fluid metaphors to characterize the "stream of ideas" they wished to capture [Bourbaki 1994, p. 19, 228, 240, & 270]. The notion of existence at the beginning of the 20th century is at the center of a "philosophico mathematics maelstrom" [Bourbaki 1994, p. 24]. Ideas are "bubbling" [*bouillonnement*] in algebra at the start of the 19th century [Bourbaki 1994, p. 52].

In the first chapter of the book, on the foundations of mathematics and set theory, however, another image is dominant. In this case, the flow of ideas seems to be broken by "problem[s] which visibly ha[ve] nothing anymore to do with Mathematics" [Bourbaki 1994, p. 15] such that of deciding whether a geometry corresponds to experimental reality. In this chapter, experience, intuition, and the "practice of the mathematicians" [Bourbaki 1994, 10, 13, 14 & 35] are allowed to enter the discussion, and, significatively, issues are debated in reference to conflicting collective understanding of the nature of mathematics. Discussing the *Grundlagen* crisis, rather than focusing on various ideas for the foundation of mathematics, Bourbaki identified several groups of mathematicians who held different those views. "Idealists" and "Formalists" looked for an axiomatic basis of mathematics [Bourbaki 1994, p. 31]; "empiricists," "realists," and "instuitionists" [p. 35] clung on the need for inner certainty concerning the "existence" of mathematical objects.

> The instuitionist school, of which the memory is no doubt destined to remain only as a historical curiosity, would at least have been of service by having forced its adversaries, that is to say the immense majority of mathematicians, to make their position precise and to take more clearly notice of the reasons (the ones of a logical kind, the others of a sentimental kind) for their confidence in mathematics [Bourbaki 1994, p. 38].

We do not want to discuss the validity, or not, of Bourbaki's views on the foundational crisis here. We merely want to point out that to account correctly for diversity of opinions on a topic that is related to mathematics Bourbaki decided here to frame the question in collective terms.

The same type of issues shows up in their discussion of the birth of differential calculus. In this chapter, Bourbaki struggles with the fact that it seems quite impossible to dispense from considering priority disputes about the discovery of the calculus. Interestingly, the authors here point that at the time some "mathematical creations, the arithmetic of Fermat, the dynamic of Newton, carried a strong *individual* cachet" [Bourbaki 1994, p. 173, our emphasis]. The authors indeed want to undermine individual idiosyncracies compared to the unstoppable discovery of diffential calculus which was like "the gradual and inevitable development of a symphony, where the '*Zeitgeist*,' at the same time composer and conductor hold the baton [...] of the infinitesimal calculus of the XVIIth century" (ibid.).

> each [individual mathematician] undertakes his part with characteristic timbre, but no one is master of themes that he is creating for the listener, themes that a scholarly counterpoint has almost inextricably entwined. It is thus under the form of a thematic analysis that the history of this must be written (ibid.).

In this remarkable quote, history is likened to a symphony where individual performers are allowed express their individuality. But this does not seem to matter since a mysterious director and composer, the spirit of the time, or *Zeitgeist*, is invoked to claim that individual claims for priority are as irrelevant in the history of mathematics as quarrels between music instruments about the exact timing of apparition of a theme in a symphony. Deliberately confusing two different senses of the term "theme" (a short melodic subject and a subject of discourse), Bourbaki concluded that history of mathematics needed to be "thematic," that is, to follow the leads of *Zeitgeist* rather than individual idiosyncracies.[34]

Interestingly, this note on differential calculus had started with one of the passages where Bourbaki was the most explicit about his method as a historian. In a rare acknowledgment of the fact that historians have to pay attention to the context in which mathematicians exerted their trade, Bourbaki underscored that quarrels about the invention of the calculus have a lot to do with the "deficiencies" of the 17th–century organization of mathematics:

> The historian must take account also of the organisation of the scientific world of the time, very defective still at the beginning of the XVIIth century, whereas at the end of the end of the same century, by means of the creation of scholarly societies and scientific periodicals, by means of consolidation and development of the universities, it ends up by resembling strongly what we know today. Deprived of all periodicals until 1665, mathematicians did not have the choice in order to make their work known, of anything other than by way of letters, and the printing of a book, most often at their own cost, or at the cost of a patron if one could be found. The editors and printers capable of work of this sort were rare [...]. After the long delays and the innumerable troubles that a publication of this kind implied, the author had most often to face up to interminable controversies, provoked by adversaries who were not always in good faith, and carried on sometimes in a surprising bitterness of tone [Bourbaki 1994, p. 170–171].

In the absence of proper scientific institutions, therefore, some "science amateurs, such as Mersenne in Paris, and later Collins in London" filled in the void by keeping a vest correspondence network, "not without mixing in with these extract stupidities of their own vintage" (ibid.). Clearely, obstacles to the smooth flow of ideas therefore come from social inadequacies.

In a thinly veiled allusion to their youthful travels sponsored by the Rockefeller Foundation which had such an effect on their latter conception of mathematics, the Bourbakis noted here:

> The studious youth journeyed, and more perhaps than today; and the ideas of such a scholar were spread sometimes better as a result if the journeys of his pupils that by his own publication [Bourbaki 1994, p. 171].

From all these considerations, Bourbaki concluded that: "It is therefore in the letters and private papers of the scholars of the time, as much or even more than in their publications proper, that the historian must seek his documents." While Bourbaki refrains from going in this direction, especially in the cases where private papers have not been published—they for instance constrast the case of Huygens where papers have been published and that of Leibniz, this implicitly recalls the division of tasks famously suggested by Weil at the Helsinki ICM, whereby historians can help mathematicians get a better sense of the environment in which mathematics was done:

> The historian can help [since we mathematicians] all know by experience how much is to be gained through personal acquaintance when we wish to study contemporary work; our meetings and congresses have hardly any other purpose. The life of the great mathematicians of the past may often have been dull and unexciting, or may seem so to the layman; to us their biographies are of no small value in bringing alive the men and their environment as well as their writings

[34] In a letter to his sister, dated 29 February, 1940, Weil developed a strikingly similar idea: "*Quant à parler à des non-spécialistes de mes recherches ou de toute autre recherche mathématique, autant vaudrait, il me semble, expliquer une symphonie à un sourd. Cela peut se faire ; on emploie des images, on parle des thèmes qui se poursuivent, qui s'entrelacent, qui se marient ou divorcent ; d'harmonies tristes ou de dissonances triomphantes : mais qu'a-t-on fait quand on a fini ? Des phrases, ou tout au plus un poème, bon ou mauvais, sans rapport avec ce qu'il prétendait décrire.* [*Œuvres scientifques* Weil 1979, p.255].

> [...]
>
> In other words, a quality of intellectual sympathy is required, embracing past epochs as well as our own. Even quite distinguished mathematicians may lack it altogether [...]. It is also necessary not to yield to the temptation (a natural one to the mathematician) of concentrating upon the greatest among past mathematicians and neglecting work of only subsidiary value. Even from the point of view of esthetic enjoyment one stands to lose a great deal by such an attitude, as every art-lover knows; historically it can be fatal, since genius seldom thrives in the absence of a suitable environment, and some familiarity with the latter is an essential prerequisite for a proper understanding and appreciation of the former. Even the textbooks in use at every stage of mathematical development should be carefully examined in order to find out, whenever possible, what was and what was not common knowledge at a given time [Weil 1978 a, p. 229].

Weil's notorious article is a suitable development from Bourbaki's historiography. But a crucial reversal seems to have occurred from considering social circumstances essentially as blocks to the natural flow of ideas to acknowledging that this is the ground on which genius is allowed to blossom. There is in fact a notion in Bourbaki that prefigured this acknowldegement.

3.2 The Notion of "School"

By far the most commonly used notion to refer to collective aspects of the history of mathematics is that of "school"[35]. The "Vienna School" thus appears along side "the Sophists" on the first page of the first chapter, reinforcing a view according to which there was great continuity in the history of mathematics over a long period. Also mentioned in the same sentence as these "schools" however were "controversies [...] which have never stopped dividing philosophers" [Bourbaki 1994, p. 1]. This remark obviously associates schools of thought, like all extra-mathematical entities, with a lack of certainty that undermines the mathematical project as Bourbaki conceived it.

In the *Element of the History of Mathematics*, the concept of mathematical schools is used to refer to specific but often losely defined entities. School often are associated with prominent names: Brouwer [p. 37], Riemann [p. 54], Banach [p. 66], Cardan [p. 72], Clebsch and M. Noether [p. 106], Gelfand [p. 114], and Monge [p. 132]. If this list in itself was not enough to show that the school concept has a positive value in Bourbaki's eyes, the fact that Hilbert and his school appears prominently confirms this impression: "a whole school of young mathematicians take part (Ackermann, Bernays, Herbrand, von Neumann)" to his work on proof theory [p. 40]. There are more instances where schools are associated with individual mathematicians:

- a school whose principal representative is von Staudt [p. 134].
- Zariski and his school of algebraic geometry [p. 52];
- Booles's system as the basis for an active school of logicians [p. 9];
- "the Peano school" suffering a "heavy blow" from Poincaré's "unjustified" criticism that "became an obstacle to the diffusion of his [Peano's] doctrine in the world" [p. 10];
- a school working on Lie algebra [p. 119]: "Lie occupied at Leipzig the chair left vacant by Klein and had Engel as assitant; this circumstance favoured the flowering of an active mathematical school as well as the diffusion of the ideas of Lie [p. 254];

[35] To examine the relationship between Bourbaki's notion of "school" and attempts by historians of science to give some substance to the notion of "research schools" [Geison & Holmes 1993] (and esp. Servos' paper therein) is a suggestive idea but a path not taken here. For a discussion of the "modern mathematical research schools," see [Rowe 2003].

- or finally, in a very different era, the Pythagorean school [p. 2, 69, 147]; whose heirs are the Peripathetics who are opposed to the "Megaric and Stoic schools" [p. 5].

In the *Elements*, schools can also be associated with cities as we have seen apropos the "Vienna School," but also "the school of Moscow" in general topology [p. 143]. Mostly, schools are associated with one or several countries, the most prominent being of course the German school(s):

- "the work of the modern German school: begun by Dedekind and Hilbert in the last years of the XIXth century, the work of axiomatisation of Algebra was vigorously pursued by E. Steinitz, then, from 1920, under the impulsion of E. Artin, E. Noether and the algebraists of their schools (Hasse, Krull, O. Schreier, van der Wearden)" [p. 55].
- "the German school of the XIXth century (Dirichlet, Kummer, Kronecker,Dedekind, Hilbert) of the theory of algebraic numbers, coming out of the work of Gauss [p. 53];[36]
- "the German school around E. Noether and E. Artin, in the period 1921–1931 which sees the creation of modern algebra" [p. 122];
- the "German school of number Theory" [p. 98];
- "the German school of Geometry in the years 1870–1880" [p. 104];

Some schools are associated to other countries as well, which can work together or, more rarely, in opposition to the German school(s):

- "the anglo-American school" [p. 118], partly overlapping with the English school of algebraists, "most notably Morgana nd Cayley" [p. 52, 117];
- the development of the theory of algebra by the American (Wedderburn and Dickson) and espacially the German (E. Noether and Artin) schools [p. 67]; the study of finite fields is given a vigorous impulsion by the "American school, around E. H. Moore and L. E. Dickson" [p. 120].
- the Russian and Polish schools [p. 156];
- an Italian school (Dini and Arzela) opposed to the German school (Hankel, du Bois-Reymond) [p. 205].
- "the French, German or English schools of projective geometry" [p. 133].
- "the French and German schools of the theory of functions (Jordan, Poincaré, Klein, Mittag–Leffler, then Hadalmard, Borel, Baire, ...)" [p. 142];
- and again, from a different time period, the "Italian school" at the beginning of the 16th century, solving algebraic equations by radicals [p. 50]; again [p. 73].

Finally, as seen in passing above, some schools are more occasionally identified by mathematical criterias. One can find:

- the intuitionist school [p. 38] and the formalist school [p. 32],
- a school of "fanatical 'quaternionists' " [p. 62],
- a school studying qudratic forms [p. 61],
- and the "school of 'synthetic geometry' " [p. 131].

[36]On this, see [Goldstein, Shappacher and Schwermer (eds) 2006].

Schools in Bourbaki is a pragmatic concept that is never defined nor discussed generally. Only a single statement does not address one or several specific "schools," when Bourbaki mentions "the rigidity of the rather pedantic reasoning that does not fail to appear in all mathematical schools where 'rigour' is discovered or believed to have been discovered" [Bourbaki 1994, p. 13].

That the role of mathematical school was crucial in the mind of some of Bourbaki's founders can be derived from the following letter. In 1940, while he was imprisoned in Finland for having refused to be drafted in war, André Weil wrote to his sister Simone:

> The current organization of science does not take into account [...] the fact that very few persons are capable of grasping the entire forefront of science, of seizing not only the weak points of resistance, but also the part that is most important to take on, the art of massing the troops, of making each sector work toward the success of the others, etc. Of course, when I speak of troops the term (for the mathematician, at least) is essentially metaphoric, each mathematician being himself his own troops. If, under the leadership given by certain teachers, certain "schools" have notable success, the role of the individual in mathematics remains preponderant [Weil 2005, p. 341].

In other words, mathematical schools are for Weil an extension of the powers of the individual mathematician. At a time when "it is not possible to have someone who can master enough of both mathematics and physics at the same time to control their development alternatively or simultaneously" (ibid.), schools are individuals writ large and often rely on charismatic leaders. Indeed, as natural extensions of individual agency in mathematics, the school concept, useful as it is, has a wholly positive value in Bourbaki's eyes and needs not to be defined or examined carefully.

Where did this use of the term come from is not too clear. As David Rowe noted, it seemed that it was traditional among mathematicians [Rowe 2003, p. 121]. We note that on the occasion of the International Congress of Mathematicians, held in Bologna in 1928, Ettore Bortolotti wrote a booklet titled *L'École Mathématique de Bologne. Apercu Historique*.

Bourbaki does not developp the notion of mathematical school nor does he explains what it means by this notion. But he uses it in a systematic way : it is its principal tool for dealing with collective aspects of mathematical research. A tantalizing possibility would be that Bourbaki mathematicians themselves felt that they formed a "school." In historiographical terms, we may wonder how their own undersanding of the collective work they had undertook informed their discussion of the importance of research schools in mathematics. We may at least say that, in a positive or in a negative light, they were indeed regarded as forming a school at the time or Bourbaki's greatest fame. "*L'École Bourbaki*" was indeed the term used in the early account of the story [Delachet 1949, 113–166], [Bouligand 1947],[Bouligand 1951]. Even abroad, this seemed clear, as witnessed by the Hungarian mathematician B. Sz.-Nagy: "*Sans doute, Bourbaki fera son école et aura une influence considérable sur le développement des mathématiques.*"[37]

In a more critical way, some older mathematics professor at the Sorbonne harshly reproached the Bourbakis their clanic behavior:

> I fear your absolutism, your certainty of holding the truth faith in mathematics, your mecanical move to take our the sword to exterminate the infidel to the Bourbakist Coran [...]. We are many to think that your are despotic, capricious, and sectarian.[38]

The fact that Bourbaki is almost immediately considered as a school may not be surprising. But it enhances the fact that, when using frequently the term and notion of

[37]Review of Bourbaki's *Éléments de mathématiques*, in *Acta scientiarum mathematicarum* **13** (1950), p. 258.

[38]"*Je redoute votre absolutisme, votre certitude de détenir la vraie foi en mathématiques, votre geste mécanique de tirer le glaive pour exterminer l'infidèle au Coran bourbakiste. [...] Nous sommes nombreux à vous juger despotique, capricieux, sectaire.*" Arnaud Denjoy to Henri Cartan (22 May, 1954). Archives de l'Académie des sciences, fonds Montel, carton 1.

school to describe historical collective dynamical practices, Bourbaki then expresses its own collective practices.

Conclusion

How did the peculiar experience of collective work shared by the members of the group shape Bourbaki's influential historiography of mathematics? We have shown here that historical notes in Bourbaki's *Elements of Mathematics* have an ambiguous status. On the one hand, this original emphasis on history clearly played a crucial part in shaping the "image of mathematics" they wanted to project [Corry 1996]. In a treatise from which all reference to the literature, historical development or even mathematicians as such, had all but vanished, historical notes allowed the Bourbakis to rehumanize mathematics somewhat.

Entering through the backdoor, historical notes, on the other hand, were never allowed to take precedence over real Bourbakist mathematics. This relatively lower status was reflected in the different treatment notes received in the writing process as opposed to the other parts of the treatise, which, as was established by Liliane Beaulieu, were submitted to gruelling examination processes. Although collective writing practices were certainly deployed for historical notices, like for rest of the treatise, there was much less back–and–forth motion here. Nonetheless, it seems safe to say that the notices were written by several members of the group, including Weil and Samuel.

As we know, the most prolific producers of historical texts under their own names among Bourbaki's founding generation were Weil and Dieudonné. But they paid distinct attention to historical contextualization, the latter being much less interested in it than the former even if critical of "extra–mathematical" asides.[39] The historical notices that were assembled in the *Elements of the History of Mathematics* however betray a rather well-defined historiography that stress the stream of ideas from the remotest Antiquity to the Bourbakist present while emphasizing the role of selected individuals. As we have shown, collective aspects of mathematical work hardly surface in this book, the only concept that is used extensively in that respect being that of "school." Although we have not been able to locate precisely the origin of this emphasis on schools, it seems that this may have been common parlance already in the interwar period. For Bourbaki, the use of the term "school" with its insistance on charismatic leaders merely was a way to resolve the historiographical tension between streams of idea and individual agency.

In the wake of Bourbaki, or perhaps independently, "mathematical schools" entered the historian of mathematics' vocabulary. In his *Histoire du calcul* (p. 42, 4th ed. from 1961), the French historian of science René Taton thus naturally described Bourbaki as a part of a "young French mathematical school":

> *une partie de la jeune école mathématique française, groupée sous le pseudonyme collectif de N. Bourbaki, aborde un nouvel et gigantesque travail de refonte des théories générales de l'analyse.*

More recently, David Rowe tried better to circumscribe historically and conceptually the meaning of a "mathematical research school."

> [C]ollaborative research presupposes suitable working conditions and,in particular, a critical mass of researchers with similar backgrounds and shared interests. A work group may be composed of peers, but often one of the individuals assumes a leadership role, most typically as the academic mentor to the junior members of the group. This type of arrangement—the modern

[39]Weil wrote in 1978: "when discipline, intermediary in some sense between two already existing ones (say A and B) becomes newly established, this often makes room for the proliferation of parasites, equally ignorant of both A and B, who seek to thrive by intimating to practitioners of A that they do not understand B, and vice versa. We see this happening now, alas, in the history of mathematics." [Weil 1978 b]. In [Weil 1984], Weil did focus on a few "extra–mathematical facts" to explain the development of the mathematical ideas he was mostly interested in.

mathematical research school—has persisted in various forms throughout the nineteenth and twentieth centuries [Rowe 2003, p. 120].

As a group Bourbaki certainly fitted Rowe's description, albeit without a clear leader. And thus has a term casually used by mathematicians become an inescapable descriptor for some social dynamics in mathematics. We have suggested that Bourbaki may be seen as an important intermediary in this process, not only because it helped them resolve a historiographical tension that they were facing but also perhaps because it reflected a crucial aspect of their experience as mathematicians, namely that collective activities had greatly gained in importance in the course of their lifetime.

Recalling that collective practices among historians of mathematics also were put to the fore in the same period, it may not come as such a surprise that this usage of the term "schools" was adopted rather uncritically at first, with more subtlety later. The "*Séminaire d'histoire des mathématiques*" was indeed launched at the Institut Henri Poincaré in 1948, the same year as the Bourbaki Seminar, by Taton, among others. Taton would soon be called to direct the amibitious collective project of the *Histoire générale des sciences* [Taton 1957–1964] in four thick volumes, to which he "devoted so many hours".[40]

Jean Itard with whose remarks about Euclid we have opened this paper was part of both of these undertakings at the time. By then, it seemed not only that Bourbaki had replaced the old Euclidean approach to mathematics based on intuition and experience, but also that the mere appearence of Bourbaki as a "polycephalic mathematician" was enough to cast doubt on the old master's very existence. Euclid had been transformed into a "school" with no identified leader. Perhaps this was the crime of lese–majesty Dieudonné trully had in mind when he famously exclaimed at a European conference on the teaching of geometry in secondary schools: "Down with Euclid!" [Dugac 1995, p. 15]?

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[40]For a list of talks delivered at the Seminar since 1948, see `http://www.ihp.fr/seminaire/SHM-histoire`. Quoted from a review by P. Huard, *Revue d'histoire des sciences* **12** (1) (1959), 71–74, on p. 74.

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