Polycephalic Euclid?

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Polycephalic Euclid? Collective Practices in Bourbaki's History of Mathematics

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Abstract

In this paper, we argue that Bourbaki's historiography, which has been extremely influential among mathematicians and historians of mathematics alike, reflected the special conditions of its elaboration. More specifically, we investigate the way in which the collective writing practices of the members of the Bourbaki group in both mathematics and the history of mathematics help to explain the particular form taken by the *Elements of the History of Mathematics* (1960). At first sight, this book, which has been seen as an "internalist history of concepts," may seem an unlikely candidate for exhibiting collective aspects of mathematical practice. As we show, historical considerations indeed stood low on the group's agenda, but they nevertheless were crucial in the conception of some parts of the mathematical treatise. We moreover claim that tensions between individuals and notions related to a collective understanding of mathematics, such as "*Zeitgeist*" and "mathematical schools," in fact structured Bourbaki's historiography.

Introduction

> Incessant chronological difficulties, which arise when we suppose the physical existence of a single Euclid, lessen, without vanishing, if we accept to take his name as the collective title of a mathematical school [Itard 1962, p. 11].[1]

That the French historian of mathematics Jean Itard (1902–1979) retained the idea of Euclid as a collective mathematician, and more precisely as "the collective title of *mathematical school*," was anything but innocent. In the early 1960s, the "polycephalic" author Nicolas Bourbaki and the members of this group of mathematicians were dominating the mathematical scene in France and beyond. Many, at the time, saw in Bourbaki the latest incarnation of what constituted a true "mathematical school" (or in French *une école mathématique*), wholly original in nature, with clear methods, objectives, and ideology.[2]

Only in the context of the Bourbakist experience might the idea of a collective Euclid have had some sort of appeal.[3] As explained by the historian Fabio Acerbi, the lack of historical and biographical data about Euclid had produced yet another fantasy about

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[1] "Les difficultés qui surgissent à chaque instant dans la chronologie lorsque l'on admet l'existence physique d'un seul Euclide s'atténuent sans disparaître lorsque l'on accepte de prendre son nom comme le titre collectif d'une école mathématique." Unless when taken from a published translation, all translations are our own.

[2] For historical accounts of Bourbaki's domination in the 1960s, see [Corry 1996] and [Aubin 1997]. It was in a paragraph referring to the "école Bourbaki" that André Delachet first revealed to the world that Bourbaki was a "polycephalic" mathematician (his term) [Delachet 1949, p. 113–116].

[3] Note that following conventions established by Liliane Beaulieu, we will refer to members of the group as *Bourbakis* and their followers as *Bourbakists* [Beaulieu 1989].

the author of the *Elements*. A polycephalic Euclid indeed seemed highly implausible to scholars who were well aware of the staunch claims for authorship that characterized Greek literature, and mathematics especially [Acerbi 2010].[4] That Bourbaki saw itself as a new Euclid is rather obvious.[5] We would like to conjecture that such possibility was entertained only as a consequence of changing mathematical practices at the time of Bourbaki and the rise of collective undertakings. More generally, as was stressed recently by one of us [Paumier 2014], collective practices came to characterize more and more the mathematical life of the period.

Now, Bourbaki was not only a mathematician, but also a highly influential historian of mathematics. In 1960, the "historical notes" appended to each volume of Bourbaki's monumental treatise were gathered in a single volume titled the *Elements of the History of Mathematics*. Interestingly, therefore, this volume on the history of mathematics was the (collective) product of mathematicians who themselves felt that they were experimenting with original ways of (collectively) writing a mathematical treatise, while at the same time developing a new (collective) understanding of mathematics as a whole. Were these collective writing practices at all reflected in Bourbaki's historiography? Although written collectively, Bourbaki's *Elements of the History of Mathematics* in fact appear as an unlikely candidate for exhibiting collective aspects in the practice of mathematics. According to Jeanne Peiffer [Dauben and Scriba 2002, p. 40], it is an "internalist history of concepts" which has only little to say about the way in which mathematics emerged from the interaction of groups of people in specific circumstances.

In the following, we will try to unpack this intricate relation among various ways of seeing and not seeing the importance of collective practices in the history of mathematics. We first introduce some aspects of the collective life of mathematics during the twentieth century, and then focus on the practice and content of the history of mathematics written by the Bourbakis. Although the Bourbakis have individually written many texts concerning the history of mathematics, we will focus on the collective volume of the *Elements of the history of mathematics*[6]. We then examine the concrete practices involved in the writing of Bourbaki's historical notes, as far as it is possible to determine them. We investigate the place of collective practices in Bourbaki's historiography and examine in particular the role played by the notion of "mathematical school."

1 Collective practices in 20th–century mathematics

> The form of collaboration we have adopted is new; we did not limit ourselves to chop the topic into various pieces and to distribute among ourselves the writing of the diverse parts; on the contrary, after having been discussed and prepared at length, each chapter is assigned to one of us; the text thus obtained is seen by all, it is once gain discussed in details, and it is always reviewed at another time, and sometimes more. We are thus embarked on a truly collective enterprise which will have a deeply unified character.[7]

[4] For a recent examination of the polycephalic Euclid hypothesis, see the corresponding article in the MacTutor History of Mathematics archive (`http://www-history.mcs.st-andrews.ac.uk/Biographies/Euclid.html`, last viewed on March 16, 2013). Three possibilities are listed for Euclid: individual author, team leader, or mere pen name ("The 'complete works of Euclid' were written by a team of mathematicians at Alexandria who took the name Euclid from the historical character Euclid of Megara who had lived about 100 years earlier").

[5] This is made especially clear in Beaulieu's analysis of a poem written by some member of the Bourbaki collective [Beaulieu 1998, p. 112].

[6] Jean Dieudonné and André Weil are the most prolific authors in that field. Let us just mention here that Dieudonné has written histories of functionnal analysis [Dieudonné 1981], algebraic and differential topology [Dieudonné 1989] and directed a collective volume [Dieudonné 1978], and that Weil has written a history of number theory [Weil 1984] and expressed his ideas about the way history of mathematics should be practiced and written [Weil 1978]. About the way Weil used historical mathematical texts, see [Goldstein 2010].

[7] "La formule de collaboration que nous avons adoptée est nouvelle; nous ne nous sommes pas bornés à partager le sujet en tranches et à nous distribuer la rédaction de ces diverses parties; au contraire, chaque chapitre après avoir été longuement discuté et préparé, est confié à l'un d'entre nous; la rédaction ainsi obtenue est vue par tous, elle est à nouveau discutée en détails, elle est toujours reprise au moins une

In this letter, which has become famous, Szolem Mandelbrojt described Bourbakis' working rituals to several officials, in order to obtain funding for their congresses. There is little need now to insist on the originality of the enterprise.[8] The group of mathematicians who would adopt N. Bourbaki as a pen name met for the first time in Paris, in December 1934. At first, they sought to write a treatise of analysis that would serve as a university textbook for the next ten years. In the first months of its existence, the Bourbaki group is therefore referred to as the "Committee for a treatise of analysis" [*comité du traité d'analyse*]. As is also well known, the scope of the project widened considerably and the treatise, which started to appear under the title *Elements of mathematics* in 1939–1940, was announced to cover vast areas of mathematics. For mathematicians, Bourbaki soon came to represent a highly recognizable mathematical style, systematically relying on the axiomatic method to introduce mathematics in the most rigorous, abstract manner. The members of the Committee slightly varied from one meeting to the next, but more or less stabilized at Besse–en–Chandesse, in July 1935, at the first of what would become a series of annual, or semiannual, Congresses. Over the years, newsletters, meeting reports, and drafts of various parts of the chapters circulated among members and are now accessible online, thanks to the work of Liliane Beaulieu and her collaborators.

Although Bourbaki's experience may well have been unique, the period we will study here, roughly from 1934 to 1960, is characterized by great changes in mathematicians' working practices. This corresponds to a change of scale, which in other fields has been captured by the expression "Big Science." To quote from a book about the phenomenon, "Seen from the inside – from the scientists' perspective – big science entails a change in the very nature of a life in science" [Galison 1992, p. 1]. In mathematics, the change of scale was translated into an anxiety about keeping abreast with the rapid development of a field where the number of practitioners skyrocketed. Bourbaki expressed this in a famous article on "The Architecture of Mathematics," first published in 1948:

> The memoirs in pure mathematics published in the world during a normal year cover several thousands of pages. [...] No mathematician, even were he to devote all his time to the task, would be able to follow all the details of this development [Bourbaki 1950, p. 221].

This was not a mere marketing ploy to sell Bourbaki's treatise as an efficient solution to the fear of disunity in mathematics. As we shall see later, André Weil expressed the same idea in his private correspondance. Some early drafts of Bourbaki's treatise also put this in a blunter way:

> There is a single *mathématique*, [that is] one and indivisible: this is the *raison d'être* of the present treatise, which aims at introducing its elements in the light of a 25–century–old tradition.[9]

But, of course, Bourbaki was just one response to this anxiety. Another was the significant change in mathematicians' collective working practices. In her recent Ph.D. dissertation, Paumier has focused on the emergence, diffusion, and growing importance of various forms of collective organization such as the seminar, the specialized international conference, and the research center.

In their formative years at the École normale supérieure in Paris, the first generation of Bourbakis attended Jacques Hadamard's seminar at the Collège de France. Weil underscored how the seminar played a crucial role in his own training : "the *bibli* [library]

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fois, et quelques-fois plusieurs. Nous poursuivons ainsi une œivre véritablement collective, qui présentera un profond caractère d'unité." Mandelbrojt to Mme Mineur, Borel, and Perrin (1936); repr. in "Journal de Bourbaki N°6 ; 27/11/1936," Archives Bourbaki, `http://purl.oclc.org/net/archives-bourbaki/70`. In the following, we will refer to online documents from the Bourbaki Archives only by the file number (in this case `70`). All were last accessed on October 27, 2013.

[8] Historical literature on Bourbaki is rather extensive. we refer especially to [Beaulieu 1989]. In English, one is referred to, among others, [Beaulieu 1993, Beaulieu 1994, Corry 1996, Aubin 1997, Mashaal 2006].

[9] "Il y a une mathématique, une et indivisible : voilà la raison d'être du présent traité, qui prétend en exposer les éléments à la lumière de ving–cinq siècles." "Introduction au Livre I (état 3) [ou état 2]" (n.d.), p.1. Archives Bourbaki, `563`.

and Hadamard's seminar [...] are what made a mathematician out of me" [Weil 1992, p. 40]. At the time, the seminar represented a novelty for mathematicians in Paris.[10] In 1933, a small group among the future Bourbakis clearly understood the benefits to be reaped from this form of organization and set up a seminar under the moral authority of Gaston Julia who was a full professor at the Sorbonne. To quote Beaulieu, Julia's seminar was "the laboratory of a restricted team" of mathematicians working together on definite topics [Beaulieu 1989, p. 133–137]. This kind of collective organization for mathematical research would quickly explode: in the 1960s, more than 30 seminars met regularly in Paris and its surroundings. As a group or individually, the Bourbakis played a significant part in this development. Launched in 1945–1946, the Bourbaki Seminar quickly became a major social event for mathematicians in France and abroad. Soon, Paul Dubreil, Henri Cartan, and Laurent Schwartz, among others, also organized their own seminars.

Another form of collective organization that spread after WWII was the international conference that focused on a research subfield. Looking for an efficient way to help the reconstruction of French mathematics, the Rockefeller Foundation gave money to the CNRS to sponsor such meetings. The series of the "*colloques internationaux du CNRS*" were supposed to be "small and informal" so as to foster effective collaboration. In Warren Weaver's own term, "the attendance of mature contributors [was] restricted to say 15; with provision, however for additional listening and observing audience of young men" [Zallen 1989, p. 6]. They were to include two to five non–French speakers. In the field of mathematics, the Bourbakis were again greatly involved in those conferences. The conference on Harmonic Analysis held in Nancy in 1947 for instance involved Schwartz, Mandelbrojt, and Roger Godement. Cartan, Charles Ehresmann, and Jean Leray took part in a conference on Algebraic Topology in Paris, also in 1947. Paul Dubreil was one of the organizers of a conference on Algebra and Number Theory in 1949, at which Weil and Jean Dieudonné spoke. About the Nancy conference, a report stated:

> Beyond scientific results [that were] improved, clarified, or established, beyond the long–lasting personal contact that will result from it, this scientific event has shown that it was possible for a small number of qualified people to work very fruitfully on a well circumscribed topic. The material format tried out on this occasion proved as useful as it was pleasant.[11]

In their mathematical work, members of the Bourbaki group very consciously explored new forms of collective organization. Before we try to assess whether this had an effect on the content of their historiography, let us examine the collective practices that went into the elaboration of Bourbaki's historical writings.

2 Bourbaki's Historical Notes as Collective Work

As far as editorial fortune goes in the history of mathematics, the *Eléments d'histoire des mathématiques* by N. Bourbaki was immensely successful: first published by Hermann in 1960 and reprinted in 1964, a second and a third editions were published, respectively, in 1969 and 1974 with corrections and additions; it was then reissued by Masson in 1984 and later reprinted by Springer as recently as 2007.[12] This book was also translated into Italian and Russian (1963), in German (1971), Spanish (1972), and Polish (1980). The English translation, the rather poorly rendered *Elements of the History of Mathematics*, only came out later [Bourbaki 1994]. This editorial success is all the more surprising when one realizes that most of these "elements" had been written much earlier, sometimes

[10] On Hadamard's seminar, see [Beaulieu 1989, p. 60–65] and [Chabert and Gilain forthcoming].

[11] "Outre, les résultats scientifiques améliorés, éclaircis ou établis, outre, les contacts personnels durables qui en résulteront, cette manifestation scientifique a montyé qu'il était possible de travailler très utilement sur un sujet bien délicité entre un petit nombre de personnes qualifyées. La forme matérielle in augurée en cette occasion s'est montrée aussi utile qu'agréable et il y a lieu d'insister sur l'honneur qui rejaillit sur l'Université de Nancy, du fait qu'elle ait été choisie comme théâtre de la premieèrerúnion de cette nature." Archives départementales de Meurthe–et–Moselle, Nancy, W 1018/96, Rapport sur le colloque.

[12] The editions and reviews are listed in [Beaulieu 1989, Annexe II B, Annexe III B] from 1960 to 1986. Some of the reviews will be quoted here.

	Chapter Title	Pages	Book	First publ.
1.	Fondations of Mathamatics; Logic; Set Theory	44 p.	I	1957
2.	Notations, Combinatorial Analysis	2 p.	?	?
3.	The Evolution of Algebra	10 p.	II	1942
4.	Linear Algebra and Multilinear Algebra	12 p.	II	1947-1948
5.	Polynomials and Commutative Fields	16 p.	II	1950
6.	Divisibility; Ordered Fields	8 p.	II	1952
7.	Commutative Algebra. Algebraic Number Theory	24 p.	VIII	1965
8.	Non Commutative Algebra	8 p.	II	1958
9.	Quadratic Forms; Elementary Geometry	14 p.	II	1959
10.	Topological Spaces	6 p.	III	1940
11.	Uniform Spaces	2 p.	III	1940
12.	Real Numbers	10 p.	III	1942
13.	Exponentials and Logarithms	2 p.	III	1947
14.	n Dimensional Spaces	2 p.	III	1947
15.	Complex Numbers; Measurement of Angles	4 p.	III	1947
16.	Metric Spaces	2 p.	III	1949
17.	Infinitesimal Calculus	32 p.	IV	1949
18.	Asymptotic Expansions	4 p.	IV	1951
19.	The Gamma Function	2 p.	IV	1951
20.	Function Spaces	2 p.	III	1949
21.	Topological Vector Spaces	12 p.	V	1955
22.	Integration in Locally Compact Spaces	12 p.	VI	1956
23.	Haar Measure. Convolution	6 p.	VI	1963
24.	Integration in Non Locally Compact Spaces	10 p.	VI	1969
25.	Lie Groups and Lie Algebra	22 p.	VII	1972
26.	Groups Generated by Reflections; Root Systems	6 p.	VII	1969

Table 1: Table of Contents of the *Éléments d'histoire des mathématiques* [Bourbaki 2007] with number of pages, book of the *Éléments de mathématique* and year of first publication.

in the early 1940s, if not before. Indeed, nothing was original in the volume when it first appeared in 1960. It merely gathered most, but not all, of the "historical notes," which had been published earlier in the corresponding volumes of Bourbaki's *Eléments de mathématique*.

2.1 A Choice Made Early

The first historical notes appeared in 1940, appended to a booklet consisting of chapters 1 and 2 of the *Elements of mathematics*, Book III ("General Topology"). This in fact was the very first booklet that truly belonged to the treatise, that is, apart from the digest of mathematical results on set theory [*fascicule des résultats*], published a year earlier.[13]. In the following volumes, historical notes were likewise appended to most chapter, although sometimes historical elements concerning several successive chapters were gathered in a single note. In their original form, historical notes had no titles. Chapter headings found in the *Eléments d'histoire des mathématiques* were given in 1960. These notes greatly varied in length (see table 1).

In the "*Mode d'emploi du traité*", a separate leaflet of instructions inserted in each volume of the mathematical treatise, Bourbaki explained what was the role of these historical notes. The first paragraph specified the objectives of the treatise and discussed norms adopted throughout concerning typography, terminology, symbols, etc.:

[13]The list of all published volumes can be found in [Beaulieu 1989, AnnexeII]. See also `http://archives-bourbaki.ahp-numerique.fr/publications`.

> To the reader, [...]
>
> 1. The Elements of Mathematics Series takes up mathematics at the beginning and gives complete proofs. In principle, it requires no particular knowledge of mathematics on the reader's part, but only a certain familiarity with mathematical reasoning and a certain capacity for abstract thought.[14]

About historical notes, it was further specified:

> 12. Since in principle the text consists of the dogmatic exposition of a theory, it contains in general no references to the literature. Bibliographical references are gathered together in *Historical Notes*. The bibliography which follows each historical note contains in general only those books and original memoirs that have been of the greatest importance in the evolution of the theory under discussion. It makes no sort of pretense to completeness.[15]

In a retrospective account, Henri Cartan discussed the meaning of those historical notes.

> Bourbaki often places an historical report at the end of a chapter. Some of them are quite brief, while others are detailed commentaries. Each pertains to the whole matter treated in the chapter. There are never any historical references in the text itself, for Bourbaki never allowed the slightest deviation from the logical organization of the work. It is only in the historical report that Bourbaki explains the connection between his text and traditional mathematics and such explanations often reach far back into the past. It is interesting to note that the style of the "Notes Historiques" is vastly different from that of the rigorous canon of the rest of Bourbaki's text. I can imagine that the historians of the future will be hard put to explain the reasons for these stylistic deviations [Cartan 1980, p. 178].[16]

Historical notes therefore had a distinct status from the rest of the treatise. Their main function was to provide a space for bibliographical references, which were banned from the treatise due to its self–contained nature. Added to the style difference underscored by Cartan, this suggest that writing processes for historical notes and the mathematical parts of the treatise also differed. Indeed, relatively little information can be found in the Bourbaki archives about the way members of the group decided to include and actually wrote historical notes.

2.2 The Conception of Bourbaki's Historical Notes

The accounts of the meetings of the "Committee for a treatise of analysis" show that in 1934–1935 the history of mathematics was a very minor concern of the participants. Indeed, one could go as far as saying that history was what they would be writing *against*. At the very first meeting, on December 10, 1934, a consensus was quickly reached on the principle of adopting a general abstract point of view. The scope of the preliminary

[14] We quote here from the published translation, to be found in every volume of the *Elements of Mathematics*. A draft can be found in "Bourbaki – Mode d'emploi de ce traité," Archives Bourbaki, 438. In this document, the original text is: "Le traité prend les mathématiques à leur début, et donne des démonstrations complètes. Sa lecture ne suppose donc en principe, chez le lecteur, aucune connaissance mathématique particulière, mais seulement une certaine habitude du raisonnement mathématique, et un certain pouvoir d'abstraction."

[15] The original text is a bit more specific and keeps the same content, viz. "Bourbaki – Mode d'emploi de ce traité," p .5. Archives Bourbaki, 438: "Le texte étant consacré, en principe, à l'exposé dogmatique d'une théorie, on n'y trouvera qu'exceptionnellement des références bibliographiques ; les références seront regroupées dans *un exposé historique*, placé le plus souvent *à la fin de chaque chapitre* et où l'on trouvera, le cas échéant, des indications sur les problèmes non résolus de la théorie. On se bornera à donner les références aux livres et mémoires originaux dont l'étude peut être le plus profitable au lecteur. Les références qui servent seulement à fixer des points de priorités sont presque toujours omises; à plus forte raison, le lecteur ne doit pas s'attendre à trouver ici de bibliographie complète des sujets traités."

[16] This talk was delivered in Düsseldorf on January 8, 1958 at the 76th Meeting of the *Arbeitsgemeinschaft für Forschung des Landes Nordrhein–Westfalen*.

"abstract package," on the other hand, was debated, and then "discussions became," according to the meeting report by Delsarte, "confused – historical and philosophical."[17] Clearly, historical considerations here were opposed to the "modern" (Delsarte's term), systematic approach that was desired.

Traces of discussions can nevertheless be found in the accounts of some of Bourbaki's early "congresses." Thus, at Besse–en–Chandesse in July 1935, we can find the following requirement among the long list of desiderata spelt out for the final form of the treatise: "Dictionary of used terminology (history and references)" [*Dictionnaire des termes usités (historique et références)*].[18] Already at this time, history was thus envisioned by Bourbaki as being inseparable from the need to refer to the existing literature, as well as from emphasis on terminology.

The various types of texts that would figure in Bourbaki's treatise are specified in more details at the next annual congress, called *Congrès de l'Escorial*, in September 1936. A document was drafted with editorial decisions reached by the Bourbakis. Among decisions concerning names to be given to theorems, mathematical notations, size of fonts, etc., the Bourbakis listed three points of interest to us. Before we explain them, let us first give the original French text:

x) *Laïus scurrile, toute latitude, en caractères normaux.*

y) *Laïus historique, en fin de chapitre, quand ce sera utile.*

z) *Laïus excitateur, en fin de chapitre, avec références. (Comme bon exemple voir Severi, Traité de géométrie algébrique).*[19]

As is often the case with Bourbaki's historical archives, a lot of unpacking may be necessary to understand the above. The term *laïus*, to start with, was frequently used by Bourbaki to refer to a written piece that was not purely mathematical, in the sense that it involved some rhetorical elements that could not be easily translated into formal language. According to the authoritative Littré dictionary, the term was slang at the Ecole Polytechnique for a long speech. The word *scurrile* is even more pedantic and of Latin origin. It can be translated in English as scurrilous, that is vulgar and clownesque, but it has no obscene connotation in French. A document from the 1935 Besse–en–Chandesse Congress explains that, at Cartan's insistance, this term (together with *futile*) "was adjoined to the mathematical vocabulary" together with their superlatives.[20] It soon became widely used in Bourbaki internal communications. The Bourbaki archives preserves an example of such "vulgar speech," which should have been printed in normal characters in the treatise: a document titled "*Ensembles – Décisions escoriales – Projet de laïus scurrile*," from 1936, opening with the following sentence as an introduction to the theory of sets:

> The object of a mathematical theory is a structure organizing a set of element: the words "structure," "set," 'éléments," being not subject to a definition but constituting primordial notions [that are] common to all mathematicians, will become self-evident as soon as structures will be defined, as this will be done as early as within this very chapter.[21]

[17]"La discussion devient confuse – historique et philosophique." Jean Delsarte, "Réunion du 10/12/1934," p. 3. Archives Bourbaki, 1.

[18]"Desiderata," p. 5. Archives Bourbaki, 27. An example of such dictionary for topology is discussed in "Compte rendu du Congrès de Nancy (9–13 avril 1948)," p. 5. Archives Bourbaki, 93.

[19]"Décisions Escorial (typographie et rédaction)," Archives Bourbaki, 36.

[20]"Les termes *scurrile* et *futile* ainsi que leurs superlatifs, dont la nécessité est mise hors de doute par les remarques de Cartan, sont adjoints au vocabulaire mathématique." "Brève histoire des travaux de Bourbaki," p. 3. Archives Bourbaki, 19.

[21]"L'objet d'une théorie mathématique est une structure organisant un ensemble d'éléments : les mots 'structure', 'ensemble', 'éléments' n'étant pas susceptibles de définition mais constituant des notions premières communes à tous les mathématiciens, ils s'éclaireront d'eux-mêmes dès qu'on aura eu l'occasion de définir des structures, comme il va être fait dès ce chapitre même." "Ensembles – Décisions Escoriales [sic]," Archives Bourbaki, 29.

Another example can be found in the detailed outline titled "Topologica Bourbachica," which tries to set out general directions for Bourbaki's view on topology where two "scurrilous speeches" are planed in the main text to argue for the necessity of adopting a general point of view in topology.[22] Although this was generally not the case, a scurrilous speech could involve historical references. The introduction to Integration written by Chevalley in 1936 apparently mentioned Ancient Egyptians ["*son laïus scurrile sur l'intégration muni des égyptiens*"].[23] Discussions about this cropped up again in 1937 when historical examples had been excluded in favor of reflections by Dieudonné that were deemed to be "of a great philosophical weakness." In order to introduce the axioms of a σ–algebra [*tribu* in Bourbaki's terminology], Delsarte argued that "scurrilities" should be as short as possible and Chevalley agreed to write a "scurrilous counter–speech starting with the weight of thin plates."[24]

In other words, the early Bourbakis talked about "scurrilous speeches" when they referred to remarks that required common parlance and, as such, lay outside of Bourbaki's ambition to produce, not a mathematical text in formal language, but at least one that could be formalized almost automatically. Generally, such speeches figured in the introduction of each book of the treatise: in Weil's outline for Topology in 1936, the first paragraph was thus called "*introduction et scurrilités.*"[25] Nevertheless, scurrilous speeches were to be printed in standard characters, like most of the mathematical developments. As the above shows, scurrilous speeches were thoroughly involved in the original, collective thinking at the basis of each book of the treatise.

Very different was the understanding of the kinds of "speech" Bourbaki called "historical" and "*excitatoire*" ["excitatory," or perhaps even "arousing"]. From the outset, they were thought as appendices that were not logically needed to understand the argument of the treatise, nor even part of its original conception. As such, they could be gathered at the end of a chapter without any damage to the edifice. They could be written belatedly, after the final text of the chapter was unanimously adopted, sometimes even using the work of hired staff (something wholly unthinkable for other parts of the treatise). Indeed the 1936 list of desirata included the following statement about outside sources:

> *References external to Bourkaki.* Fundamental references will be carefully given in the historic or excitatory speeches; possibly in the text (this therefore concerns references [that have been] checked, [and are] complete and correct).
>
> Concerning technical references, a mere mention of the presumed author. Concerning other [references], as it can be done, without conferring too much importance to them (use of staff for screening).[26]

As a self-contained organic whole, Bourbaki's *Elements of Mathematics* contained a complex system of cross-references, but outside sources had to be kept at bay. Bourbaki therefore distinguished three levels of external references in his treatise. Only fundamental ones were properly considered. Others were to be only casually mentioned. In effect, all the bibliography would be placed in historical notes.

Examples and exercises were other parts of the text that had a special status. At the L'Escorial Congress, it was decided that exercices would be placed at the end of a paragraph and that examples and counterexamples would figure in the main text. However, to indicate their lower status with regards to the logical architecture of the treatise, both would be printed in smaller fonts. The "*Mode d'emploi*" had been very explicit about

[22] "Topologia Bourbachica," p. 4. Archives Bourbaki, 492.

[23] "Intégration escoriale," Archives Bourbaki, 33.

[24] "Journal de Bourbaki N°9 ; 16/03/1937," Archives Bourbaki, 73.

[25] André Weil, "Topologie 1 (Weil) (Exemplaire archétype) Plan général de topologie," p. 12, Archives Bourbaki, 399.

[26] "*Références extérieures à Bourbaki.* Pour les références fondamentales, elles seront données avec soin dans les laïus historiques ou excitateurs ; éventuellement dans le texte ; (il s'agit donc de références vérifiées, correctes et complète)

"Pour les références de technique, simple mention de l'auteur présumé. Pour les autres, comme on pourra, sans y attacher trop d'importance (emploi de nègres pour dépistage)" "Décisions Escorial (typographie et rédaction)," p. 3. Archives Bourbaki, 36.

this lower status: exercises were "results which have no place in the text but which are nonetheless of interest." Here again, however, there was a significant difference with the process that led to historical notes, since both exemples and exercises often figured in outlines and drafts examined by the Bourbaki group, at all stages of production.

2.3 A Lower Status for Historical Notes?

Unfortunately, no example of an early draft of an historical note seems to have survived.[27] We may take this hole in the records as an indication of the fact that historical notes were never extensively discussed by the group, nor taken into consideration when designing general arguments.

The quote from the L'Escorial desirata list above at least shows that the Bourbakis had an explicit model in mind as far as historical notes were concerned. Francesco Severi's Treatise of Algebraic Geometry was deemed a "good example." In his *Trattato di geometria algebrica*, he had indeed included a few "*notizie storico–bibliografiche*" [Severi 1926, p. 350], which clearly were a stylistic guide for Bourbaki. Although he also had bibliographical footnotes, Severi placed his historical notes at the end of corresponding numbered paragraphs. Varying greatly in length from a one–line sentence to two densely printed pages, they appeared in a smaller font and included many bibliographical references. It seems that the writers of Bourbaki's treatise took dearly the recommandation of following Serveri's model not only in the physical layout, but in style as well. Compared to Bourbaki's however, Severi's historical notes appear much more closely related to the mathematical material just introduced, more factual, narrower in scope, and mostly concerned with recent developments.[28]

In a review of the *Elements of the History of Mathematics*, the French historian of mathematics René Taton suggested another model for Bourbaki's historical notes. Praising the originality of the work, Taton recalled the French–German edition of the *Encyclopédie des sciences mathématiques*, originally edited by Felix Klein. More elementary in mathematical terms, Taton thought, the Encyclopedia was more erudite in the historical sense:

> Very schematically, one may characterize the different spirit by saying that the notes of Bourbaki's *Eléments de mathématique* have been written by mathematicians interested in the history of their discipline for the purpose of other mathematicians who are equally curious of the origins of their science, while those of the *Encyclopédie* were written with the active participation of professional historians of mathematics who, in part at least, wrote for historians' purpose [Taton 1961, p. 158–159].[29]

In 1984, Saunders Mac Lane's account of the book for the *Mathematical Reviews* was harsher. To him, historical notes were little more than Bourbaki's mathematics retroprojected in the past:

> In virtue of its origin as appendices to separate texts on topics of current interest, these elements of history are just that: Former mathematics as it seems now to Bourbaki, and not as it seemed to its practitioners then. In the terminology of historiography, it is "Whig history" [MR0782480].

Many elements (typography, place in the treatise, style, lack of discussion at Bourbaki meeting, etc.) therefore converge to give the strong impression that historical notes held a low status in the project as a whole. Archives, where only a few mentions of historical

[27] A document is mentioned in the Bourbaki Archives with the title "Note historique : topologie Ch. I," 425, but when we last checked, it was unavailable.

[28] Earlier books by the same author also included short historical discussions, e.g., [Severi 1921].

[29] "Pour caractériser d'une faqn très schématique cette différence d'esprit, on peut dire que les notices des *Eléments de mathématique* de N. Bourbaki ont été rédigées par des mathématicien s'intéressant à l'histoire de leur discipline, à l'intention d'autres mathématiciens également curieux des origines de leur science, tandis que celles de l'*Encyclopédie* l'ont été avec la participation active d'historiens des mathématiques professionnels qui écrivent, en partie du moins, à l'intention d'un public d'historiens."

notes can be found, partly confirms this impression. Admittedly, some consideration was given to the mathematical literature, for example what should be on avail on the location of early Congresses, which often took place in isolated spots.[30] When they are mentioned, historical notes seemed to have been discussed rather quickly, and late in the writing process. In July 1945, for instance, a new outline was adopted for chapters 5 and 6 of Topology, but the final text remained unchanged: "As usual, historical notes will be discussed later," the report said, "at the small Congress that will take place in Nancy in December 1945; Weil and Chevalley will send their remarks to this congress by correspondance."[31] Unfortunately, no record of this meeting, if indeed it took place, subsists.

Historical notes could sometimes be used to address an issue that was deemed unworthy of figuring in the main structure of the treatise:

> The resolution of equations by radicals is abandoned (even in an annex), but the historical note will have to make briefly the link between this question and modern field theory.[32]

To emphasize the lower status of historical notes, let us finally quote from one of the famous ironic comments adorning much of Bourbaki's internal documents where the notes are presented as if they were mere social entertainment:

> Read as the opening speech, the historical note to chap. II–III of Algebra put the Congress "*in the right mood*" [in English in the original] toward Algebra it glorified Fermat, dutifully followed the meanderings of the linear, and examined the influence of Mallarmé on Bourbaki.[33]

2.4 The Role of History in Collective Thinking

Contrary to what was just said, however, we want to claim that historical notes also came out, at least in part, from the necessity of working collectively through the mathematical material. In some rather rare occasions, collective thinking on mathematical topics involved historical considerations. Not always "dead" mathematics, historical notes sometimes were reflections of very lively debates among the writers of the treatise. Often, remnants of collective reflections about introductions to each book of the treatise found a place in historical notes. This is hardly surprising since historical considerations were often raised in the conception of these introductions. At Dieulefit, in 1938, it was for example specified that: "*Weil fait l'introduction et laïus historico-bibliographico-existants*" (or in more sober language, he promised to write, for October 15, the introduction and the history of topology).[34]

As seen above, integration was one of the topics where historical considerations were discussed in early outlines and drafts. On February 11, 1935, the Committee approvingly discussed Chevalley's historically–minded project for measure and integration theory. Chevalley's outline, which was given orally, "seemed rather intuitive; he reaches the notion of measure by following the historical order Egyptians → Archimedes → Lebesgue

[30] Two instances in the archives: Jean Delsarte, "Sous-commission bibliographique 13/04/1935" Archives Bourbaki, 10; and "Bourbaki's Dicktat [sic] – Congrès du 18-28/09/1936" Archives Bourbaki, 28.

[31] "Comme d'habitude, les Notes historiques seront discutées ultérieurement, lors du petit Congrès qui se tiendra en décembre 1945 à Nancy ; Weil et Chevalley enverront leurs observations à ce Congrès par correspondance." "N° 11[sic] - 15 juillet 1945 CR du Congrès de Paris," p. 5. Archives Bourbaki, 88.

[32] "On renonce à faire (même en Appendice) la résolution des équations par radicaux, mais il faudra que la Note historique expose succinctement le lien entre cette question et la théorie moderne des corps." "Compte rendu du Congrès de Nancy (9–13 avril 1948)," p. 2. Archives Bourbaki, 93.

[33] "La Note historique des chap. II–III d'Algèbre, lue en guise de discours inaugural, mit le Congrès "in the right mood" quant à l'Algèbre : il glorifia Fermat, suivit docilement les méandres du linéaire et scruta l'influence de Mallarmé sur Bourbaki." "Observations du Congrès de Paris (18–20 Janvier 1947)," p. 1. Archives Bourbaki, 92. On the humorous tone of Bourbaki's documents, see [Beaulieu 1998].

[34] "[version ronéo de l'ensemble des textes]," Archives Bourbaki, 63. A preliminary document rather mentions "*laïus historico-bibliographico-excitants*," which is probably what was truly meant. See "Engagements de Dieulefit," Archives Bourbaki, 56.

→ de Possel."[35] On the next meeting, on February 25, Chevalley presented his report which began with a definition of measure as a "number reported to *some* sets" [*nombre rapporté à certains ensemble*], which was to be illustrated by "historical examples."[36] From what we find in some later draft, however, we may suspect that mentions of historical considerations remained cursory:

> the notions of *quality*, *quantity*, *magnitude*, and the notion of *measure*, which established the link between them all to the notion of number, have appeared very early in the history of the human thought. There are at the basis of civilized life and of present–day experimental science; [...] mathematics owes its origin and its most essentiel tools (such as the *real number* or the *integral*) to problems raised by the measure of magnitudes. The goal of this introductions to show how the abstract mathematical problems, which will be studied in the next chapters, can be, by means of the analysis of these *concrete* notions, disentangled from them.[37]

In the end, it was the lengthy note devoted to the history of logic ans set theory that accommodated some of these reflections rather than the introduction. Archival materials allows us partly to follow the collective decision process that led to this result. The introduction to Set Theory occupied various Bourbakis over the years and consensus about what it should be was especially hard to reach.

At the Congress of Nancy in April 1948, for example, this introduction, which was to serve as the general introduction to the treatise, was discussed at length. A pragmatic viewpoint on foundational issues was adopted, according to which the thorniest issues regarding the philosophy of language or the metaphysics of mathematical objects were simply to be set aside. In a separate article soon to be published, they deemed them irrelevant to the "working mathematician" [Bourbaki 1949]. Concerning the history of such debates, the report of the 1948 Congress bluntly declared: "State explicitly that we are not interested in the quarrels from the beginning of the century about sets and their paradoxes."[38] Although Weil was asked to come up with a first draft of this introduction, it was Chevalley's text that the Bourbaki Congress at Royaumont unhappily examined in April 1950. "For the explication of the axiomatic viewpoint," it was decided that "the best place is the historical note, which will have to be thorough [*chiadée*]."[39] At the next Congress in Pelvoux, this decision was confirmed: "Introduction. It is decided to make it very short, and to leave as many things as possible to the historical note."[40] Once again, Weil was assigned the task of writing the historical note on sets, with the help of the logician John Barkley Rosser, Sr. In October 1951, a reference to the 25 centuries of experience with elementary geometry and arithmetic that increased mathematicians' confidence in set theory (see the quote about the unicity of *mathématique* above, p. 3), was discarded.[41] The writing of this historical notice was assigned to a younger recruit, Pierre Samuel, who was to be "tutored" by Rosser.[42] The last mention of historical notes

[35] Jean Delsarte, "Comité du Traité d'analyse – Réunion du 11/02/1935," Archives Bourbaki, 5.

[36] Jean Delsarte, "Traité d' Analyse – Réunion du 25/02/1935," Archives Bourbaki, 6.

[37] "les notions de *qualité*, de *quantité*, de *grandeur*, et la notion de *mesure*, qui les relie à celle de nombre, sont apparues très tôt dans l'histoire de la pensée humaine. Elles sont à la base de la vie civilisé et de la science expérimentale actuelle; quant aux mathématiques, elles doivent leur origine et leurs outils les plus essentiels (comme le *nombre réel* ou *l'intégrale*) aux problèmes que pose la mesure des grandeurs. Le but de cette introduction et de montrer comment, par une analyse de ces notions *concrètes*, on peut en dégager les problèmes mathématiques abstraits qui seront étudiés dans les chapitres suivants." "Théorie de la mesure et de l'intégration: Introduction (Etat 2)," Archives Bourbaki, 414.

[38] "Dire explicitement que nous nous désintéressons des querelles du début du siècle sur les ensembles et leurs paradoxes." "Compte rendu du Congrès de Nancy (9–13 avril 1948)," p. 6. Archives Bourbaki, 93

[39] "N°22 Compte rendu du Congrès de la revanche" Archives Bourbaki, 100.

[40] "N°25 Compte rendu du Congrès oecuménique de Pelvoux (1951)," Archives Bourbaki, 102.

[41] "N°26 Compte rendu du Congrés croupion (oct 1951)," Archives Bourbaki, 103.

[42] In the report of the Congress in the summer 1952, one reads that: "*Note historique: Samuel se fera tapirer par Rosser à Ithaca.*" See "Pelvoux 25 juin–8 juillet 1952," Archives Bourbaki, 105. The verb *tapirer* is ENS slang for privately tutoring. Samuel was introduced to Bourbaki in 1945. See "N° 11 [sic] – 15 juillet 1945 CR du Congrès de Paris," p. 2. Archives Bourbaki, 88.

for logic and set theory occurred in October 1952 when Samuel was still in charge, but no due date was given for the manuscript.[43] We shall come back in the next section to these extended historical notes, first published in 1957.

In some cases, historical notes can therefore be read as the graveyard where earlier debates among Bourbakis were put to rest. Such debates were not limited to the various introductions, which always held a special status. They sometimes concerned core issues such as the role of Hilbert spaces and the emergence of Book V on Topological Vector Spaces. Considered a masterpiece [Dhombre 2006] or criticized as not the best of Bourbaki's books [Ferrier n.d.], Book V was acknowledged by all as a major restructuring of the field, more specifically with respect to its treatment of duality.[44] Although Book V was quite remote from integral equations, we think it is enlightening to follow original discussions in this way. Not only does this case help understand how the consideration of the historical literature was involved in the collective writing processes, it also provides clues as to what kind of historical reflections seemed useful to the writers of the treatise.

After an exchange of ideas on the theory of integral equations at the Committee meeting of March 25, 1935, Weil, Delsarte, Cartan, and Dieudonné reached the conclusion that the theory could be presented from three different "points of view". The first of these was that of Hilbert spaces "which gives a complete, perfectly esthetic theory." Then, there was Fredholm's "old" point of view. A final viewpoint was called "more modern, à la Rusz [sic, i.e. Frigyes Riesz] or à la Leray," and was placed in normed vector spaces. Acknowledging their poor command of the question, present members felt that, if Riezs's viewpoint covered Fredholm's and although it was more general, it nevertheless seemed not to "go as far in the results as Hilbert's point of view." But in Leray's absence, no decision was reached.[45] On May 6, 1935, Leray presented his own views on the matter before the Committee. He distinguished two parts: non symmetrical integral equations in Banach spaces and symmetric integral equations in Hilbert spaces. Although this raised some objections by Delsarte and Chevalley, Leray believed that there was no need to mention Fredholm's method.[46] In discussing Fredholm's theory, it is to be noticed that contrary to most manuscripts, participants to Bourbaki congresses often gave an exact reference, that of Riesz's article (published in *Acta mathematica* in 1918).[47] Lest we hastily conclude that this had nothing to do with proper historical work and that it was no more than mere bibliographical work, we should remember how history and bibliography were mixed in Bourbaki's practice. This clearly was the type of work Bourbaki considered history.

Although at the meeting at Besse–en–Chandesse in 1935, the report on integral equations was assigned to Mandelbrojt, a complete consideration of Hilbert spaces was only published as a part of Book V ("Topological Vector Spaces") in 1955.[48] At Besse–en–Chandesse, a section on "Fredholm determinant function" was planned as part of the theory of linear functional equations, for which the global existence theorem was assumed.[49] Later, Jean Coulomb was assigned the task of drafting this chapter, but declined in 1937.[50] In March, 1937, Delsarte presented a short draft concerning integral equations where considerations of Riesz's work was judged faulty and he was asked to rewrite the project.[51] The complete history of Book V's origins is fascinating and remains to be written, but it lies outside of the scope of this paper.[52]

[43] "N°29 Celles–sur–Plaine, 19–26 oct. 1952," Archives Bourbaki, 106.

[44] An otherwise critical reviewer thus wrote in 1956: "Chapter IV, entitled *Duality in topological vector spaces*, is [...] the most useful of all five chapters. Here is a complete and readable account of the various topologies for the space of continuous linear functionals on a topological vector space" [**?**, p. 508].

[45] Jean Delsarte, "Traité d'Analyse – Réunion du 25 mars 1935," Archives Bourbaki, 8.

[46] Jean Delsarte, "Traité d'Analyse – Comité de rédaction 06/05/1935," Archives Bourbaki, 11.

[47] For example in Jean Leray, "Théorie des systèmes de n équations à n inconnues. Théorie des équations fonctionnelles. A titre documentaire : Projet d'exposé des théorèmes d'existence topologiques par J. Leray (1935)," Archives Bourbaki, 619; or "Avant-projet – équations intégrales (Delsarte)," (dated September 18, 1936), Archives Bourbaki, 38.

[48] "Serment," Archives Bourbaki, 18.

[49] "Équations fonctionnelles linéaires," Archives Bourbaki, 26.

[50] "Journal de Bourbaki N°8 ; 16/02/1937," Archives Bourbaki, 72.

[51] "Journal de Bourbaki N°9 ; 16/03/1937," Archives Bourbaki, 73.

[52] This is the current research topic of a working group organized by Christian Houzel. See an abstract

Consideration about Fredholm theory however cropped out in the historical note long before Book V was finished that were appended to Chapter 1 of General Topology, published in 1940. In this note, Fredholm theory is presented as a way mathematicians got used to consider functions as elements of general topological spaces. Hilbert's work on Fredholm integrals and Erhard Schmidt generalization (1905) are praised as "memorable." In 1907, Riesz and Fréchet both developed general approaches to function spaces. But in Bourbaki's presentation of their work, the latter is said to have only succeeded in producing a cumbersome system of axioms, while the former's theory was judged to be "still incomplete, and remain[ing] besides as only a sketch" [Bourbaki 1994, p. 143]. Not discussed here, Riesz' 1918 paper was celebrated in the historical note that came with Book V and called "a chef-d'œuvre of axiomatic analysis [through which] the whole of Fredholm's theory (in its qualitative aspect) is reduced to a single fundamental theorem, namely that every normed locally compact space is finite dimensional" [Bourbaki 1994, p. 214]. It is interesting to note that a French reviewer of the 1940 booklet (A. Appert) regretted that Fréchet's contribution to general topology was minimized in the historical note, underscoring that Fréchet had done this while Riesz' work was unavailable to him due to war [JFM 66.1357.01].

Be that as it may, although our analysis of the path from Fredholm theory to Book V is sketchy and no more than provisional, it shows that, in the writing process of Bourbaki's treatise, reviews of recent literature—if not full historical considerations of the questions involved—were an essential part of the collective conception of the *Elements of Mathematics*. Moreover, our study underscores that traces of the process by which Bourbaki collectively restructured whole mathematical fields appeared in the historical notes. More precisely, elements taken from the historical notes, when reread in the light of discussions extracted from the archives, are witness to various stages in the organization of the treatise at various levels: general outline of the treatise, the order in which various topics were introduces, choices of viewpoints, etc. Let us now see whether this type of collective mathematical practices were reflected in the final product.

3 Collective Practices in Bourbaki's Historiography

When the *Elements of History of Mathematics* were published in 1960, Bourbaki made no pretense of giving a complete history of mathematics. In an "*avertissement*" placed at the start of the volume, authors explained that separate studies written for another purpose had merely been gathered here without major revisions. As a consequence, many portions of the history of mathematics like differential geometry, algebraic geometry, and the calculus of variations were absent from the volume, because the corresponding parts in the *Element of Mathematics* had not been published yet. Bourbaki kept silent about branches of mathematics, such as probability and statistics or more generally all applied mathematics, that seemed of little relevance to the scheme the group was in the process of diffusing. Mostly they warned their readers that they would find in this book:

> no bibliographic [sic] or anecdotal information about the mathematicians in question; what has been attempted above all is for each theory to bring out as clearly as possible what were the guiding ideas, and how these ideas developed and interacted on the others [Bourbaki 1994, p. v].[53]

Although these notes reflected a historiography that was sometimes more than 20 years old already in 1960, historians of mathematics received the book with enthusiasm and lauded its originality. While being critical on many relatively minor points, Itard for example emphasized that this was "a nice and good work" [Itard 1965, p. 123]. He also mentioned that one would do well to read Bourbaki's book in parallel with another collective project, the *Histoire générale des sciences*, directed by Taton, whose chapter

in Beaulieu et al. "Bourbaki et les espaces vectoriels topologiques: histoire et création mathématique," `http://hal.archives-ouvertes.fr/hal-00593455`.

[53]Note that the French version mentions *biographical* rather than *bibliographical* information [Bourbaki 2007, p. v].

on the history of mathematics was partly written by the Bourbaki Dieudonné. While underscoring more important inadequacies in Bourbaki's book (like the provisional character of the endeavor and the lack of reference to contemporary research by historians of mathematics), Taton repeated without comment Bourbaki's point of view quoted above. Pointing out that the history of "the working mathematician" was not to be found in Bourbaki's book which he deemed "factual rather than interpretative," the historian Ivor Grattan–Guinness nonetheless called it "the most important history yet produced of the mathematics of recent times" [Grattan–Guinness 1970]. Mathematicians likewise repeated approvingly Bourbaki's words about the lack of biographical information. As Alexander Craig Aitken noted: "the work has no concern whatever with anecdote, legend or personality of the authors concerned: authors are related solely to theorems or contributions to theory which mathematics owes them" [Aitken 1961].

Anecdotal evidence concerning Weil ironically exist to illustrate the point that this historiographical assumption hardly entailed a lack of attention to the collective aspects of mathematics:

> For a 1968–69 guest lecture in topology, the audience was packed into the lecture room of Old Fine Hall in Princeton and included Weil and many other notables. At one point someone in the audience rose to object that the lecturer was not giving proper credit for a particular theorem. The questioner went on in impassioned tones for what seemed an eternity. Finally Weil rose, turned to the questioner, and said in a loud voice, "I am not interested in priorities!" The discussion was over, and the lecturer resumed without further interruption. This was the quintessential Weil. Mathematics to him was a collective enterprise [Knapp 1999].

In this sense, the assertion was quite trivial. In a strong rebuttal piece, Serge Lang wrote: "I object. In the sense that mathematics progresses by using results of others, Knapp's assertion is tautologically true, and mathematics is a collective enterprise not only to Weil but to every mathematician" [Lang 2001, p. 46]. Polemically contending that Weil may have been disingenuous—or worse mischievous—on more than one occasion, and faulting him for not only neglecting other mathematicians' contributions but also deliberately misrepresenting their work, Lang at least showed that collective aspects in mathematics went much beyond the trivial conception put forward by Knapp.

3.1 Collective Practices Emerging

The table of contents (table 1) greatly reinforced the view that Bourbaki's book was above all concerned with mathematical notions, and modern ones more especially. The focus on ideas erased much of the social dynamics at play in the historical development of mathematics. The collective result of a group of mathematicians who had embarked 25 years earlier on a highly original project of collectively and anonymously rewriting vast portions of mathematics, Bourbaki's *Elements of the History of Mathematics* thus appeared, at face value, as a paradoxical product: a collectively written history of mathematics whose content eschewed any serious consideration of the collective social dimensions of mathematics. Of course, one may say that by keeping silent about collective social dynamics of the past, Bourbaki was merely mirroring its own practice as a group, which remained discrete, and even secretive, about its internal workings. We argue, however, that a deeper examination can nevertheless dig up crucial concerns for collective aspects of mathematical practice. Faint as it is, partial discussions on collective and social dynamics in Bourbakis' historiography is nonetheless a reflection of their practical experience as mathematicians.

3.1.1 Institutions

Unsurprisingly, there were few institutions in Bourbaki's account of the history of mathematics. Universities were barely mentioned twice and in passing [p. 136 and 170]; so is

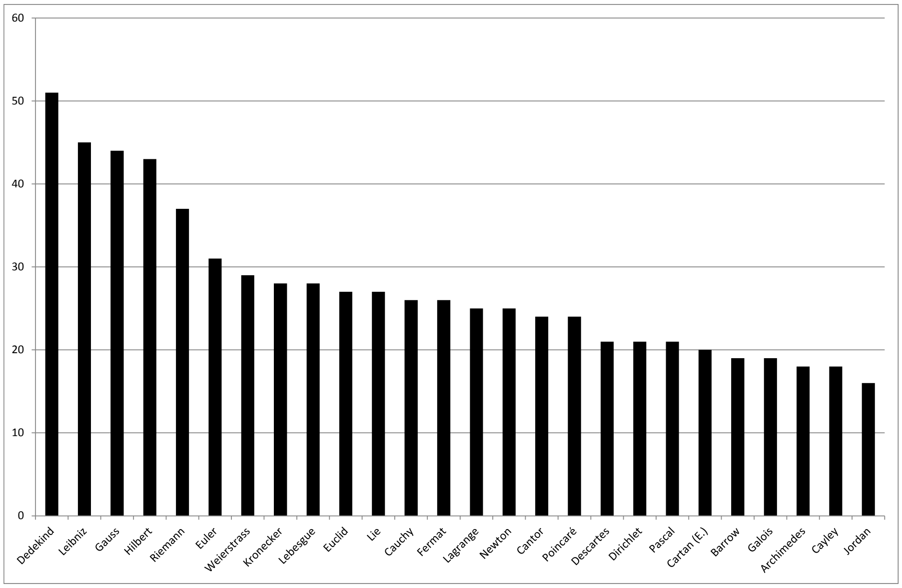

Figure 1: The 26 Mathematicians who are mentioned the most in Bourbaki's *Elements of the History of Mathematics* and the number of pages where their name occurs.

the École polytechnique [p. 59 and 133].[54] Most of the time, this was to recall that formal training played a crucial part in the flow of ideas from one generation to the next.[55] Journals, academies, and learned societies scarcely appear, except in titles of cited literature. In a rare instance, Bourbaki recalled the role played by personal discussions in the formal setting of the London Mathematical Society between Benjamin Pierce and William Clifford [p. 118n].

True to their belief in international exchanges, however, the Bourbakis paid a bit more attention to the International Congresses of Mathematicians. In their book, ICMs mostly served as vehicle for quickly diffusing ideas expressed by individual mathematicians. In Zurich (dated once erroneously 1896 and once correctly 1897), Hadamard and Hurwitz drew attention to the applications of set theory to analysis; in Paris in 1900, Hilbert included the problem on the noncontradiction of arithmetic among his famous list; in Rome in 1904, the same Hilbert attacked the same problem [p. 30, 142, 39 & 40]. Little more can be said about the now standard sociological foci of the historiography of mathematics. To have any chance of catching other ways in which social or collective aspects of mathematical practice may nonetheless surface in Bourbaki's historiography, our net needs a finer mesh.

3.1.2 Individual in Tension

While mathematical notions indeed structured Bourbaki's text, we note that very many individuals mathematicians were explicitly named. According to the index, which was added to the second edition of the work, 456 mathematicians were mentioned by name. As figure 1 exhibits, among the 26 most cited mathematicians geographical and chronological distributions were far from uniform. The first 8 in this list came from Germany and

[54] In the following, all pages numbers in square brackets without any other indication will be understood as taken from the English edition of the *Elements of the History of Mathematics* [Bourbaki 1994].

[55] The teachings of Cauchy at the École polytechnique, of Kronecker and Weierstrass at the university of Berlin where they introduced the "axiomatics" of determinants, and Gauss' courses followed by Riemann at Göttingen in 1846–1847 were thus mentioned [resp. p. 59, 64, and 129n].

Switzerland (out of a total of 10 who do). 10 of these 26 mathematicians were from France, 3 from Great Britain, 2 from the Ancient Greek world, and 1 one from Norway. Some of the names served as labels for mathematical ideas, notions or theorems. Sophus Lie's name for example arose frequently mostly due to the emphasis put on Lie groups and algebras in the treatise. One is also struck by the chronological imbalance of this list, where barely 9 mathematicians had died before the start of nineteenth century. Such statistics confirm common views about Bourbaki's image of mathematics. In total, German mathematicians received roughly 100 more citations than French ones. Recent developments in the German cultural sphere indeed were what Bourbaki valued most. At the same time, our numbers indicate that discussions of mathematical influences on Bourbaki might have downplayed the importance of French mathematicians, including twentieth–century ones, like Henri Poincaré, Henri Lebesgue, or Élie Cartan.

A mentioned, these names in Bourbaki's text represented mathematical ideas and seldom flesh–and–blood people. This deliberate viewpoint announced in the introduction was expressed in the text itself. In the chapter on "Polynomials and Commutative Fields," Bourbaki only reluctantly refrained from rehearsing the convoluted story of the Cardano formula for solving third–degree algebraic equations:

> We cannot describe here the picturesque side of this sensational discovery — the quarrels that it provoked between Tartaglia on the one hand, Cardan [sic] and his school on the other — nor the figures, often appealing, of the scholars who were its protagonists [p. 72].[56]

That this is a history of mathematics emphasizing ideas at the expanse of social practice and institutions is of course well known: what we would like to point out here is that, by doing so, Bourbaki inadvertently produced a very odd result. Indeed, in this text — as, for that matter, it is often the case in the history of ideas — much agency is placed in the hands (or minds) of (a selected set of) individuals. In the historical note on "Topological Sets," published in 1940, Bourbaki thus wrote: "It is Riemann who must be considered as the creator of topology; as of so many other branches of modern mathematics" [p. 139].[57] In another place, the authors underscored the sole agency of the inventors of the calculus in overcoming epistemological obstacles:

> it must be realised that this way was not open for modern analysis until Newton and Leibniz, turning their back on the past, accept that they must seek provisionally the justification for their new methods, not in rigorous proofs, but in the fruitfulness and the coherence of the results [p. 175].[58]

This view—according to which crucial innovations towards modernity "must await" the intervention of a chosen individual—was typical of Bourbaki's historiography. The authors for example wrote that we "must await" Cauchy [p. 129], Chasles [p. 131], or Möbius [p. 126] for the emergence of various mathematical concepts. Obviously, Bourbaki's historiography was filled with value judgments that emphasized the worth of great mathematicians and sometimes their "genius" [Poincaré on p. 35; Cantor's on p. 27]. For such luminaries, Bourbaki often preferred to talk of "mathematicians of the first rank" [p. 6, 11, and 61].

As has often been emphasized, this conception of history certainly contained elements useful to Bourbaki's self–promotion and self–aggrandizing. Often starting with the Greeks

[56]"*Nous ne pouvons décrire ici le côté pittoresque de cette sensationnelle découverte — les querelles qu'elle provoqua entre Tartaglia,d'une part, Cardan et son école de l'autre — ni les figures, souvent attachantes, des savants qui en furent les protagonistes*" [Bourbaki 2007, p. 96].

[57]Early in the book, Bourbaki similarly underscored that Boole "must be considered to be the real creator of modern symbolic logic" [p. 8]. Galois was considered as the "real initiator" of the theory of substitution [p. 51]. The notion of tensor product of two algebras "must be attributed" to Benjamin Pierce [p. 118].

[58]One may note that Bourbaki is using the present tense to talk about the past and try to infer something from this unusual practice in English. In our view, this would be mistaken since this was then already a common practice in French historical writing to use the present tense.

or even the Babylonians, Bourbaki systematically saw the notions emphasized in its treatise and, at times, the work of Bourbakis themselves as the rightful culmination of a continuous, progressive history of mathematics. This has been discussed as the "royal road to me" historiography of mathematics, where mathematicians "confound the question , 'How did we get here?', with the different question, 'What happened in the past?' " [Grattan–Guinness 1990, p. 157].[59] Chevalley's initial plan for measure theory, whereby the line from the Egyptians to Bourbaki was almost direct (quoted above), was a caricature of this pattern. Deservedly or not, the work of individual members of the Bourbaki group was often given prominence. In Book V, as a reviewer noted at the time, Bourbaki "trace[d] the history of the subject from the contributions of D. Bernoulli to those of L. Schwartz" (a postwar recruit of Bourbaki's) [Hewitt 1956, p. 508]. Most striking perhaps, in the published record, was a development where work by Weil and Jean–Pierre Serre (another postwar recruit) was identified as having successfully dispelled mistrust regarding the theory of normed algebras developed by Israel Gelfand [p. 114–115].[60]

In Bourbaki's hand, the history of ideas therefore became a teleological account where agency was placed in the hands of individuals. This of course was paradoxical since nowhere was it explained how these special individuals could have been able to discern ahead of time the direction that history would take. From the text emerged a tension between individual contributions and collective aspects. This tension, we argue, was resolved in various ways that were not fully thought out.

3.1.3 Fluid Metaphors, the Practice of the Mathematicians, and *Zeitgeist*

Intent on capturing the flow of mathematical ideas, Bourbaki used a mixed bag of fluid metaphors to characterize the "stream of ideas" they wished to capture [p. 19, 228, 240, and 270]. The notion of existence at the beginning of the 20th century was to be at the center of a "philosophico mathematics maelstrom" [p. 24]. Ideas were said to be "bubbling" [*bouillonnement*] in algebra at the start of the 19th century [p. 52].

But, in some case, it was impossible to ignore that the flow of ideas had met some serious obstacles in the history of mathematics. The first chapter of the book, dealing with the foundations of mathematics and set theory, contained remnants of the discussions sketched above about the introduction to Book I on Set Theory. There, Bourbaki uncharacteristically acknowledged that one needed to pay attention to "problem[s] which visibly ha[ve] nothing anymore to do with Mathematics" [p. 15], such as deciding whether geometry corresponds to experimental reality. Experience, intuition, and the "practice of the mathematicians" [p. 10, 13, 14, and 35] entered the discussion, and, significantly, issues were debated in reference to conflicting collective understanding of the nature of mathematics. Discussing the *Grundlagen* crisis, Bourbaki identified several groups of mathematicians who held different views—rather than focusing, as usual, on various ideas for the foundation of mathematics. "Idealists" and "Formalists" looked for an axiomatic basis of mathematics [p. 31]; "empiricists," "realists," and "intuitionists" [p. 35] clung on to the need for inner certainty about the "existence" of mathematical objects.

> The intuitionist school, of which the memory is no doubt destined to remain only as a historical curiosity, would at least have been of service by having forced its adversaries, that is to say the immense majority of mathematicians, to make their position precise and to take more clearly notice of the reasons (the ones of a logical kind, the others of a sentimental kind) for their confidence in mathematics [p. 38].

Our goal is not to discuss the validity, or not, of Bourbaki's views on the foundational crisis here. We merely want to point out that, when pressed to provide an account for a diversity of opinions on a topic that was related to mathematics, Bourbaki decided to frame the question in collective terms.

[59] On Bourbaki's implicit place in the historical notes, see also [Beaulieu 1998, p. 114].

[60] In Bourbaki's history of mathematics, one also finds mentions of Bourbaki's *Elements of Mathematics*, as well as Henri Cartan's notion of filters [p. 160 & 180]. Chevalley is mentioned eight times in the text.

A similar issue sprang up in the chapter on the birth of differential calculus where Bourbaki struggled with famous priority disputes between Newton and Leibniz, which were difficult to ignore. Interestingly, the authors here emphasized, as being characteristic of the period, the fact that "mathematical creations, the arithmetic of Fermat, the dynamic of Newton, carried a strong *individual* cachet" [p. 173, our emphasis]. But Bourbaki wished to undermine individual idiosyncrasies in order to stress the unstoppable motion of history:

> it is very much the gradual and inevitable development of a symphony, where the '*Zeitgeist*,' at the same time composer and conductor hold the baton, that we are reminded by the development of the infinitesimal calculus of the XVIIth century: each [individual mathematician] undertakes his part with characteristic timbre, but no one is master of themes that he is creating for the listener, themes that a scholarly counterpoint has almost inextricably entwined. It is thus under the form of a thematic analysis that the history of this must be written [p. 173].

In this remarkable excerpt, history is likened to a symphony where individual performers are allowed to express their individuality within limits. An invisible director and composer (the spirit of the time, or *Zeitgeist*) is invoked to ensure that the individual priority claims at stake in the Newton–Leibniz debate remained as irrelevant to the history of mathematics as are quibbles between music performers to the rise of a symphonic theme. Deliberately confusing two different senses of the term "theme" (a short melodic subject and a subject of discourse), Bourbaki concluded that history of mathematics needed to be "thematic," that is, to follow the *Zeitgeist*'s lead rather than individual idiosyncrasy.[61]

Significantly, this is also where Bourbaki was the most explicit about his method as a historian. In a rare acknowledgment of the need for historians to pay attention to context, Bourbaki underscored that quarrels about the invention of the calculus have a lot to do with organizational "deficiencies" in 17th–century mathematics:

> The historian must take account also of the organisation of the scientific world of the time, very defective still at the beginning of the XVIIth century, whereas at the end of the end of the same century, by means of the creation of scholarly societies and scientific periodicals, by means of consolidation and development of the universities, it ends up by resembling strongly what we know today. Deprived of all periodicals until 1665, mathematicians did not have the choice in order to make their work known, of anything other than by way of letters, and the printing of a book, most often at their own cost, or at the cost of a patron if one could be found. The editors and printers capable of work of this sort were rare [...]. After the long delays and the innumerable troubles that a publication of this kind implied, the author had most often to face up to interminable controversies, provoked by adversaries who were not always in good faith, and carried on sometimes in a surprising bitterness of tone [p. 170–171].

Clearly, obstacles to the smooth flow of ideas came from social inadequacies. In the absence of proper scientific institutions, some "science amateurs, such as Mersenne in Paris, and later Collins in London" filled the void with a vast correspondence network, "not without mixing in with these extract stupidities of their own vintage" [ibid.]. Other types of social dynamics however might have had a more positive effect. In a thinly veiled allusion to their youthful travels sponsored by the Rockefeller Foundation, which had shaped their image of mathematics, the Bourbakis noted: "The studious youth journeyed, and more perhaps than today; and the ideas of such a scholar were spread sometimes

[61] In a letter to his sister, dated February 29, 1940, Weil developed a strikingly similar idea: "As for speaking to nonspecialists about my research or any other mathematical research, it seems it would be better to try and explain a symphony to a death person. This can be done: one uses images, speaks of themes that run after each other, that intermingle [...]: but what have we at the end? Sentences, or at most a problem, good or bad, but without relation to what it was meant to describe"[Weil 1979, p. 255].

better as a result of the journeys of his pupils than by his own publication" [p. 171]. From all these considerations, Bourbaki concluded that: "It is therefore in the letters and private papers of the scholars of the time, as much or even more than in their publications proper, that the historian must seek his documents" [p. 171].

While Bourbaki refrained from going in this direction, this implicitly recalls the division of tasks famously suggested by Weil at the Helsinki ICM in 1978: "The historian can help [since we mathematicians] all know by experience how much is to be gained through personal acquaintance when we wish to study contemporary work; our meetings and congresses have hardly any other purpose. [Weil 1978, p. 229]. As can be seen from the above, Weil's notorious article was a suitable development from Bourbaki's historiography. But a crucial reversal had occurred. At Helsinki, Weil indeed went on:

> It is also necessary not to yield to the temptation (a natural one to the mathematician) of concentrating upon the greatest among past mathematicians and neglecting work of only subsidiary value. Even from the point of view of esthetic enjoyment one stands to lose a great deal by such an attitude, as every art–lover knows; historically it can be fatal, since genius seldom thrives in the absence of a suitable environment, and some familiarity with the latter is an essential prerequisite for a proper understanding and appreciation of the former. Even the textbooks in use at every stage of mathematical development should be carefully examined in order to find out, whenever possible, what was and what was not common knowledge at a given time [Weil 1978, p. 335].

In Weil's view, social circumstances appeared no longer as blocks to the natural flow of ideas anymore; they were the ground on which the seed of genius was allowed to blossom. We would like to argue that the notion of school which is very prominent in Bourbaki's historiography prefigured this reversal.

3.2 The Notion of School

3.2.1 Schools in Bourbaki's Historiography

The notion of "school" was by far the most commonly used by Bourbaki in its *Elements of the History of Mathematics* to refer to social and collective aspects.[62] It occurred on 40 pages of the book (out of 274). On the very first page of the first chapter, a mention of the "Vienna School" appeared, alongside "the Sophists," reinforcing long–term resonances between Greek Antiquity and the modern period in the history of mathematics.[63] Significantly, also mentioned in the same sentence as these schools, were "controversies [...] which have never stopped dividing philosophers" [p. 1]. Like all extra-mathematical entities, schools of thought were thus associated with a lack of certainty undermining the mathematical enterprise as understood by Bourbaki.

In the *Element of the History of Mathematics*, the concept of mathematical schools was used to refer to rather specific entities, but its general signification was loosely defined. It is of course impossible to analyze, for each and every case, the criteria that were implicitly used to identify mathematical schools and whether this identification holds up to historical scrutiny. In the following, we merely want to exhibit the many occurrences of the term in Bourbaki's historiography. This will enable us more precisely to characterize the understanding of collective practices that was put forward.

First, it is to be noted that Bourbaki often linked schools with prominent names: Brouwer [p. 37], Riemann [p. 54], Banach [p. 66], Cardano [p. 72], Clebsch and M. Noether [p. 106], Gelfand [p. 114], Monge [p. 132], etc. If this list in itself was not enough to show that the school concept had a positive value in Bourbaki's eyes, the fact that Hilbert

[62] To examine the relationship between Bourbaki's notion of "school" and attempts by historians of science to give some substance to the notion of "research schools" [Geison & Holmes 1993] (and esp. Servos' paper therein) is a suggestive idea, but a path not taken here. For a discussion of the "modern mathematical research schools," see [Ferreirós 1999] and [Rowe 2003].

[63] There also were several mentions of the Pythagorean "school" [p. 2, 69, 147], whose heirs, the Peripathetics, opposed the "Megaric and Stoic schools" [p. 5].

and his school appeared prominently confirmed this impression: "a whole school of young mathematicians take part (Ackermann, Bernays, Herbrand, von Neumann)" in his work on proof theory [p. 40].[64]

Second, schools in the *Elements* could also be associated with cities or countries as we have seen apropos the "Vienna School," but also "the school of Moscow" in general topology [p. 143]. Mostly, schools were identified by one or several countries, the most prominent being of course the German school(s). Let us give a few examples:

- "the work of the modern German school: begun by Dedekind and Hilbert in the last years of the XIXth century, the work of axiomatisation of Algebra was vigorously pursued by E. Steinitz, then, from 1920, under the impulsion of E. Artin, E. Noether and the algebraists of their schools (Hasse, Krull, O. Schreier, van der Wearden)" [p. 55].
- "the German school of the XIXth century (Dirichlet, Kummer, Kronecker,Dedekind, Hilbert) of the theory of algebraic numbers, coming out of the work of Gauss" [p. 53];
- "the German school around E. Noether and E. Artin, in the period 1921–1931 which sees the creation of modern algebra" [p. 122].[65]

Other national schools were mentioned, often in relation with German schools. The development of abstract algebra was attributed to both the American (Wedderburn and Dickson) the German (E. Noether and Artin) schools [p. 67]. In analysis, Bourbaki lumped together "the French and German schools of the theory of functions (Jordan, Poincaré, Klein, Mittag–Leffler, then Hadamard, Borel, Baire, ...)" [p. 142]. There are many other examples.[66] Finally, as seen in passing in the above, some schools were on occasions identified by mathematical criteria. One can for instance find: the formalist and intuitionist school [p. 32 and 38]; a school of "fanatical 'quaternionists' " [p. 62]; a school studying quadratic forms [p. 61]; or the "school of 'synthetic geometry' " [p. 131].

From this survey, we conclude that the notion of school in Bourbaki's historiography was indeed very vague. Above all, it was a catchall, never defined nor discussed generally. Bourbaki had no wish to develop the notion of mathematical school nor to explain its meaning. But its use was systematic: this was the principal tool with which Bourbaki dealt with collective aspects of mathematical research. The only place in the whole book where the term is used, not to refer to one or several specific "schools," but more generally, shows that schools were the repository of some mathematical values. In this instance, Bourbaki saw in Euclid's Book VIII "the rigidity of the rather pedantic reasoning that does not fail to appear in all mathematical schools where 'rigour' is discovered or believed to have been discovered" [p. 13].

Where did this use of the term come from is not too clear. The historian José Ferreirós claims that it was already common in the 19th century to speak of mathematical "schools" [Ferreirós 1999, p. xviii]. In the French language, the term was commonly used to refer to "a sect or doctrine of a few individuals," especially in philosophy and art and in reference to actual schools like Plato's Academy or Raphael's workshop.[67] In the 19th century, one found occurrences of the phrase "*l'école mathématique française*" [Fourier 1825, p. xvi],

[64] There were many more instances where schools are associated with individual mathematicians: Booles's system as the basis for an active school of logicians [p. 9]; "the Peano school" suffering a "heavy blow" from Poincaré's "unjustified" criticism that "became an obstacle to the diffusion of his [Peano's] doctrine in the world" [p. 10]; Zariski and his school of algebraic geometry [p. 52]; a school working on Lie algebra in Leipzig [p. 119 and 254]; a school whose main representative was von Staudt [p. 134].

[65] And also: the "German school of number Theory" [p. 98]; and "the German school of Geometry in the years 1870–1880" [p. 104].

[66] One can find mentions of: the "American school, around E. H. Moore and L. E. Dickson" for the study of finite fields [p. 120]; an "the anglo-American school" in algebra [p. 118] (partly overlapping with the English school of algebraists, "most notably Morgan and Cayley" [p. 52, 117]); an Italian school (Dini and Arzela) as well as a German school (Hankel, du Bois-Reymond) on uniform convergence [p. 205]; the Russian and Polish schools in topology [p. 156]; and, from a different time period, the "Italian school" at the beginning of the 16th century, solving algebraic equations by radicals [p. 50 and 73].

[67] *Dictionnaire de l'Académie française*, 4th ed. (1762); online edition (accessed October 28, 2013): `http://artfl-project.uchicago.edu/`.

and, at the start of the 20th century, frequent mentions of other national schools in mathematics. Although the expression "school of thought" obviously is classic in English as well, it seemed that its more systematic usage among historians of science may have been related to the mathematicians' usage [Rowe 2003, p. 121].

A tantalizing possibility would be that the Bourbakis themselves felt that they formed a "school." In historiographical terms, one may wonder how their own understanding of the collective work they had undertaken informed their discussion of the importance of research schools for the development of mathematics. We may at least say that, in a positive or in a negative light, they were indeed regarded as forming a school at the time of Bourbaki's greatest fame. As we have mentioned, the "*École Bourbaki*" was the term used in early characterizations of the collective enterprise [Delachet 1949, p.113–116]; one may even find earlier mentions of it [Bouligand 1947, p. 318]. Even abroad, this seemed clear, as witnessed by the Hungarian mathematician Béla Szőkefalvi-Nagy: "Without doubt, Bourbaki will form a school [*fera son école*] and will have a considerable influence on the development of mathematics' [Szőkefalvi-Nagy et al. 1950, p. 258]. In a more critical way, some older mathematics professor at the Sorbonne harshly criticized the Bourbakis' clanic behavior: "I fear your absolutism, your certainty of holding the true faith in mathematics, your mechanical move to take out the sword to exterminate the infidel to the Bourbakist Coran [...]. We are many to think that you are despotic, capricious, and sectarian."[68]

3.2.2 Schools in Bourbakist Historiography and Beyond

Albeit loosely defined, mathematical schools played a crucial role in some of the Bourbakis' understanding of the dynamical development of mathematics. In 1940, while imprisoned in Finland for having refused to be drafted in the army, Weil wrote to his sister Simone:

> The current organization of science does not take into account [...] the fact that very few persons are capable of grasping the entire forefront of science, of seizing not only the weak points of resistance, but also the part that is most important to take on, the art of massing the troops, of making each sector work toward the success of the others, etc. Of course, when I speak of troops the term (for the mathematician, at least) is essentially metaphoric, each mathematician being himself his own troops. If, under the leadership given by certain teachers, certain "schools" have notable success, the role of the individual in mathematics remains preponderant [Weil 2005, p. 341].

In other words, Weil though of mathematical schools as extensions of the powers of the individual mathematician, like armies extended the power of army generals, with the crucial difference being that while a general without his army was powerless, a single mathematician was able to accomplish much. At a time when "it is not possible to have someone who can master enough of both mathematics and physics at the same time to control their development alternatively or simultaneously" [ibid.], schools were individuals writ large and often relied on charismatic leaders. Schools were natural extensions of individual agency in mathematics and, indeed, another way to address the anxiety caused by the unfettered growth of science.

Among Bourbaki's founding generation, Weil and Dieudonné were, as we know, the most prolific producers of historical texts under their own names. They paid distinct attention to historical contextualization, which generally aroused much less interest from Dieudonné's part, while Weil always remained critical of "extra–mathematical" asides in the history of mathematics [Weil 1984]. Both Dieudonné and Weil used the notion of school in their individual work in ways we shall not study here, besides suggesting that Weil might have been more careful in doing so.[69] Other mathematicians also reflected on

[68]"*Je redoute votre absolutisme, votre certitude de détenir la vraie foi en mathématiques, votre geste mécanique de tirer le glaive pour exterminer l'infidèle au Coran bourbakiste. [...] Nous sommes nombreux à vous juger despotique, capricieux, sectaire.*" Arnaud Denjoy to Henri Cartan (May 22, 1954). Archives de l'Académie des sciences, fonds Montel, carton 1.

[69]See, e.g., [Dieudonné 1981, p. 39, 81, 83, and 212],[Dieudonné 1989, p. 19n, 39n, 52, 68, 198 and 288], and [Weil 1992, p. 50].

the term , e.g. [Mordell 1959, p. 41].

In the wake of Bourbaki, historians of mathematics widely adopted this term and, perhaps independently, "research schools" entered the vocabulary of the critical historian of science as well [Morrell 1972, Geison & Holmes 1993]. But many felt uneasy about the loose use fo the term:

> That the word "school," which has often been invoked in the history of mathematics, has been understood in a loose sense is indicated by the pervasive usage of the word in quotation marks [Parshall 2004, p. 271].

And some historians of mathematics tried better to circumscribe historically and conceptually the meaning of a "mathematical research school."

> [C]ollaborative research presupposes suitable working conditions and, in particular, a critical mass of researchers with similar backgrounds and shared interests. A work group may be composed of peers, but often one of the individuals assumes a leadership role, most typically as the academic mentor to the junior members of the group. This type of arrangement—the modern mathematical research school—has persisted in various forms throughout the nineteenth and twentieth centuries [Rowe 2003, p. 120].[70]

As a group, Bourbaki certainly fitted Rowe's description, albeit without a clear leader, hence perhaps, the necessity of inventing a fictitious one. In any case, a term casually used by mathematicians has become an inescapable descriptor for some social dynamics in mathematics. Even if Bourbaki cannot be held accountable for originating the wide use of the term, we suggest that the group's writings may have helped to diffuse it widely. Mostly, we venture that "schools," as a loose concept, had become a useful way to resolve a historiographical tensions between individual and collective agencies in the history of mathematics precisely because "schools," in the more restricted sense quoted above, now corresponded to a social situation that was experienced more commonly than ever.

Conclusion

Historical notes held an ambiguous status in Bourbaki's *Elements of Mathematics*. The original emphasis on history in a mathematical treatise clearly played a part in shaping the "image of mathematics" the group wanted to project [Corry 1996]. In a self–contained whole from which all reference to the literature, to historical development, or even to the mathematicians themselves had all but vanished, historical notes allowed the Bourbakis to re–humanize mathematics somewhat. We have moreover established here that this historical work sometimes impacted the architecture of the mathematical enterprise. In these cases, historical notes can be read as vestiges of mathematical discussions from which the treatise is a result. Entering through the backdoor, historical notes were however never allowed to take precedence over real Bourbakist mathematics. This relatively lower status was reflected in the different treatment historical notes received in the writing process as opposed to the other parts of the treatise. Although historical notes, like the rest of the treatise, certainly emerged through collective writing practices, this was achieved through much less back–and–forth motion among the various authors.

Despite being collectively conceived, the historical notices that were assembled in the *Elements of the History of Mathematics* exhibited great unity and belonged to a well–defined historiographical genre that stressed the stream of ideas from the remotest Antiquity to the Bourbakist present while emphasizing the contributions of a selected set of individuals. As we have shown, while collective aspects of mathematical work hardly surfaced in this book, the notion of "school" was used extensively for the purpose of capturing some of these aspects. For Bourbaki, the consideration of loosely–defined "mathematical schools," while often insisting on charismatic leaders, was a way to resolve

[70]For other attempts at restricting the notion of school in the historiography of mathematics, see [Ferreirós 1999, p. xviii–xx] and [Parshall 2004, p. 271–274].

the historiographical tension between streams of idea and individual agency. Its success in Bourbakist historiography also stemmed from the term's appropriateness as a reflection of the group's self–understanding in terms of social dynamics.

Recalling that collective research practices were also increasingly experienced by historians of mathematics, too, in the same period, it comes as no surprise that this usage of the term "schools" was widely adopted rather uncritically at first, with more subtlety later. In 1948, the "*Séminaire d'histoire des mathématiques*" was indeed launched by Taton, among others, at the Institut Henri Poincaré, where the Bourbaki Seminar also took place. Taton would soon be called to direct the ambitious collective project of the *Histoire générale des sciences* in four thick volumes [Taton 1957–1964], to which he "devoted so many hours" [Huard 1959, p. 74].[71] Itard with whose remarks about Euclid we have opened this paper was part of both of these collective undertakings. By then, it seemed not only that Bourbaki had replaced the old Euclidean approach to mathematics based on intuition and experience, but also that the mere appearance of Bourbaki as a "polycephalic mathematician" was enough to cast doubt on the old master's very existence. Euclid's metamorphosis into a "school" with no identified leader was the culmination of both Bourbakist mathematics and Bourbaki's historiography. Perhaps this was the crime of *lèse–majesté* Dieudonné had in the back of his mind when he famously exclaimed at a European conference on the teaching of geometry in secondary schools: "Down with Euclid!" [Dugac 1995, p. 15]?

References

[Acerbi 2010] Acerbi, Fabio. 2010. Euclide. *Images des Mathématiques*, CNRS, url: `http://images.math.cnrs.fr/Euclide.html`.

[Aitken 1961] Aitken, Alexander Craig. 1961. Review of Bourbaki, *Éléments d'histoire des mathématiques*. In: *Proceedings of the Edinburgh Mathematical Society* 12:217.

[Aubin 1997] Aubin, David. 1997. The Withering Immortality of Nicolas Bourbaki: A Cultural Connector at the Confluence of Mathematics. *Science in Context* 10:297–342.

[Beaulieu 1989] Beaulieu, Liliane. 1989. *Bourbaki: une histoire du groupe de mathématiciens français et de ses travaux (1934–1944)*. Ph.D. thesis, Université de Montréal.

[Beaulieu 1993] Beaulieu, Liliane. 1993. A Parisian Café and Ten Proto–Bourbaki Meetings (1934–1935), *The Mathematical Intelligencer* 15(1):27–35.

[Beaulieu 1994] Beaulieu, Liliane. 1994. Questions and Answers about Bourbaki's Early Work (1934-1944). In: *The Intersection of History and Mathematics*, ed. Sasaki Chikara et al.: 241–252. Basel: Birkhäuser.

[Beaulieu 1998] Beaulieu, Liliane. 1998. Jeux d'esprit et jeux de mémoire chez N. Bourbaki. In: *La mise en mémoire de la science: pour une ethnographie historique des rites commémoratifs*, ed. P. Abir-Am: 75–123. (ed.). Amsterdam: Éd. des archives contemporaines .

[Bouligand 1947] Bouligand, George. 1947. Le mathématicien au seuil de la métaphysique. *Revue de Métaphysique et de Morale* 52:308–321.

[Bouligand 1951] Bouligand, George. 1951. L'École Bourbaki en face des secteurs décisifs de l'analyse. *Revue générale des sciences pures et appliquées et Bulletin de la Société Philomathique*, 58:65–66.

[Bourbaki 1949] Bourbaki, Nicolas. 1949. Foundations of Mathematics for the Working Mathematician, *Journal of Symbolic Logic* 14:1–8.

[71] For a list of talks delivered at the Seminar since 1948, see `http://www.ihp.fr/seminaire/SHM-histoire`.

[Bourbaki 1950] Bourbaki, Nicolas. 1950. The Architecture of Mathematics, *American Mathematical Monthly* 57:221–232.

[Bourbaki 1994] Bourbaki, Nicolas. 1994. *Elements of the History of Mathematics*, trans. John Meldrum. Berlin: Springer, 1994.

[Bourbaki 2007] Bourbaki, Nicolas. 2007. *Éléments d'histoire des mathématiques*. Berlin: Springer, 2007; repr. of Paris: Masson, 1984.

[Cartan 1980] Cartan, Henri. 1980. Nicolas Bourbaki and contemporary mathematics. *The mathematical Intelligencer* 2:175–181.

[Chabert and Gilain forthcoming] Chabert, Jean-Luc and Gilain, Christian. Forthcoming. Sans amour et sans haine: le difficile principe de Jacques Hadamard.

[Corry 1996] Corry, Leo. 1996. *Modern Algebra and the Rise of Mathematical Structures*. Basel: Birkhäuser.

[Dauben and Scriba 2002] Dauben, Joseph W. and Scriba, Christoph J., eds. 2002. *Writing the History fo Mathematics : Its Historical Development*. Basel: Birkhäuser.

[Delachet 1949] Delachet, André. 1949. *L'Analyse mathématique*. Paris: Presses Universitaires de France, "Que sais-je ?"

[Dhombre 2006] Dhombres, Jean. 2006. Analyse fonctionnelle. In: *Dictionnaire d'histoire et philosophie des sciences*, ed. Dominique Lecourt, 4th ed., Paris: Presses universitaires de France.

[Dieudonné 1978] Dieudonné, Jean, ed. 1978. *Abrégé d'histoire des mathématiques*, 2 vol. Paris: Hermann.

[Dieudonné 1981] Dieudonné, Jean. 1981. *History of functional analysis*. Amsterdam: North–Holland Publishing.

[Dieudonné 1989] Dieudonné, Jean. 1989. A History of Algebraic and Differential Topology, 1900–1960. Basel: Birkhäuser.

[Dieudonné 2000] Dieudonné, Jean. 2000. L'école mathématique française du XXe siècle, *Development of mathematics, 1950–2000*, ed. Jean–Paul Pier: 329–358. Basel: Birkhäuser.

[Dugac 1995] Dugac, Pierre. 1995. *Jean Dieudonné, mathématicien complet*. Paris: Jacques Gabay.

[Ferreirós 1999] Ferreir'os, José. 1999. *Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics*. Basel: Birkhäuser.

[Ferrier n.d.] Ferrier, Jean–Pierre. n.d. La dualité dans les EVT de Bourbaki : entre exposition et création, online unpublished manuscript `http://www.iecn.u-nancy.fr/ ferrier/EVT/DualEVT.pdf`.

[Fourier 1825] Fourier, Joseph. 1825. Analyse des travaux de l'Académie royale des sciences pendant l'année 1824: partie mathématiques. *Mémoires de l'Académie des Sciences de l'Institut Impérial de France* 5: i–xci.

[Galison 1992] Galison, Peter. 1992. *Big Science: The Growth of Large-Scale Research*. Stanford: Stanford University Press.

[Geison & Holmes 1993] Geison, Gerald L., and Holmes, Frederick L., eds. 1993. *Research Schools: A Reappraisal*, special issue of *Osiris* 8.

[Goldstein 2010] Goldstein, Catherine. 2010. Des passés utiles: mathématiques, mathématiciens et histoires des mathématiques. *Noesis* 17:137–152.

[Goldstein, Shappacher and Schwermer (eds) 2006] Goldstein, C. and Schappacher, N. and Schwermer, J., eds. 2006. *The Shaping of Arithmetic after C. F. Gauss's Disquisitiones Arithmeticae*. Heidelberg: Springer.

[Grattan–Guinness 1970] Grattan–Guinness, Ivor. 1970. Review of Bourbaki, *Élément d'histoire des mathématiques*. In: *British Journal for the History of Science* 5:190–191.

[Grattan–Guinness 1990] Grattan–Guinness, Ivor. 1990. Does the History of Science Treat of the History of Science? The Case of Mathematics, *History of Science* 28:149–173.

[Hewitt 1956] Hewitt, Edwin. 1956. Review of Bourbaki's *Espaces vectoriels topologiques* (Actualités scientifiques et industrielles, nos. 1189, 1229, 1230). In: *Bulletin of the American Mathematical Society* 62:507–508.

[Huard 1959] Huard, Pierre. 1959. Review of René Taton, *Histoire générale des sciences*. In: *Revue d'histoire des sciences* 12:71–74.

[Itard 1962] Itard, Jean. 1962. *Les Livres arithmétiques d'Euclide*. Paris: Hermann.

[Itard 1965] Itard, Jean. 1965. Review of Bourbaki, *Éléments d'histoire des mathématiques* (1964 ed.). In: *Revue d'histoire des sciences et de leurs applications* 18:120–123.

[Knapp 1999] Knapp, Anthony W. Editorial, *Notices of the American Mathematical Society* 46:420.

[Lang 2001] Lang, Serge. 2001. Comments on Non–References in Weil's works. *Gazette des Mathématiciens* 90:46–52.

[Mashaal 2006] Mashaal, Maurice. 2006. *Bourbaki: A Secret Society of Mathematicians*, trans. Anna Pierrehumbert. Providence, R.I.: American Mathematical Society.

[Mordell 1959] Mordell, Louis J. 1959. *Reflections of a Mathematician*. Montreal: École polytechnique.

[Morrell 1972] Morrell, J. B. 1927. The Chemist Breeders: The Research Schools of Liebig and Thomas Thomson, *Ambix* 19:1–46.

[Paumier 2014] Paumier, Anne–Sandrine. 2014. *Laurent Schwartz et la vie collective des mathématiques*. Thèse de doctorat, Université Pierre et Marie Curie, Paris.

[Parshall 2004] Parshall, Karen Hunger. 2004. Defining a Mathematical Research School: The Case of Algebra at the University of Chicago, 1892?1945, *Historia Mathematica* 31:263–278.

[Rowe 2003] Rowe, David E. 2003. Mathematical Schools Communities, and Network. In: *Cambridge History of Science*, vol. 5, ed. Mary Jo Nye: 113–132. Cambridge: Cambridge University Press.

[Severi 1921] Severi, Francesco. 1921. *Vorlesungen über algebraische Geometrie*. Leipzig: Teubner.

[Severi 1926] Severi, Francesco. 1926. *Trattato di geometria algebrica*, vol. 1, part. 1, Bologne: Nicola Zanichelli.

[Szőkefalvi-Nagy et al. 1950] Szőkefalvi-Nagy, Béla, et al. 1950. Review of Bourbaki, *Éléments de mathématique*. In: *Acta scientiarum mathematicarum* 13:258–263.

[Taton 1957–1964] Taton, René, ed. 1957–1964. *Histoire générale des sciences*, 4 vols. Paris: Presses Universitaires de France.

[Taton 1961] Taton, René. 1961. Review of Bourbaki, *Eléments d'histoire des mathématiques*. In: *Bulletin des Sciences Mathématiques* 85:150–155.

[Weil 1978] Weil, André. 1978. History of Mathematics: Why and How, *ICM Proceedings, Helsinki 1978*, 227–236.

[Weil 1979] Weil, André. 1979. *Œuvres scientifiques*, vol. 1. New York: Springer.

[Weil 1984] Weil, André. 1984. *Number theory. An approach through history: From Hammurapi to Legendre*. Basel: Birkhäuser.

[Weil 1992] Weil, André. 1992. *The Apprenticeship of a Mathematician*, trans. Jennifer Gage, Birkhäuser, Basel.

[Weil 2005] Weil, André. 2005. A Letter of André Weil on Analogy in Mathematics, trans. Martin H. Krieger, *Notices of the American Mathematical Society* 52:334–341.

[Zallen 1989] Zallen, D. T. 1989. The Rockfeller Foundation and French Research. *Cahiers pour l'histoire du CNRS*, vol. 5.