Computational Issues for Optimal Shape Design in Hemodynamics

Olivier Pironneau

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Abstract

A Fluid-Structure Interaction model is studied for aortic flow, based on Koiter's shell model for the structure, Navier-Stokes equation for the fluid and transpiration for the coupling. It accounts for wall deformation while yet working on a fixed geometry. The model is established first. Then a numerical approximation is proposed and some tests are given. The model is also used for optimal design of a stent and possible recovery of the arterial wall elastic coefficients by inverse methods.

Introduction

Hemodynamics, a special branch of computational fluid dynamics, poses many problems of modeling, data acquisition, computation and visualization. However even as of now it is a valuable tools to understand aneurisms, to design stents and heart valves, etc (see for example [10, 4, 11]).

In this paper we shall focus on aortic flow, its modelisation, numerical simulation and inverse techniques.

Blood in large vessels like the aorta is newtonian and flows in a laminar regime with Reynolds number of a few thousands. The Navier-Stokes equation for incompressible fluid is a good model for it.

A blood vessel on the other hand is a complex structure for which linear elasticity is only a first crude approximation and for which the Lamé coefficients do not have a universal value and can vary with individuals.

Nevertheless, like many authors ([1, 8] for instance) we shall use Koiter's linear shell theory.

1 Koiter's Shell Model for Arteries

The following hierarchy of approximations for the displacement $\vec{d}$ of the aortic wall will be made:

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- Small displacement linear elasticity instead of large displacement (needed for the heart).
- No contact inequalities with the surrounding organs.
- Shell model for the mean surface,
- With reference to the mean surface, normal displacement of the walls only.

Let Σ be the shell surface representing the mean position of the blood vessel. Let $\vec{n}(x)$ be the normal at $x \in \Sigma$. Let $\vec{d}(x,t)$ be the displacement of the wall at x at time t. Normal displacement implies $\vec{d} = \eta\vec{n}$.

In [8] it is shown that under such conditions, Koiter's model reduces to the following equation of η on Σ

$$\rho_s h \partial_{tt}\eta - \nabla \cdot (\mathbf{T}\nabla\eta) - \nabla \cdot (\mathbf{C}\nabla\partial_t\eta) + a\partial_t\eta + b\eta = f^s, \tag{1}$$

where ρ_s is the density and h the thickness of the vessel, $\mathbf{T}$ is the pre-stress tensor, $\mathbf{C}$ is a damping term, a, b are viscoelastic terms and f^s the external normal force, i.e. the normal component of the normal stress tensor $-\sigma^{\mathbf{s}}{}_{nn}$. As with all second order wave type equations two conditions must be given at $t = 0$, for instance

$$\eta_{|t=0} = \eta_0, \qquad \partial_t\eta_{|t=0} = \eta_0'$$

Remark 1 *When $[h, T, C, a] << b$, (1) leads to the so-called **surface pressure model***

$$-\sigma^{\mathbf{s}}{}_{nn} = b\eta, \quad \textit{with } b = \frac{Eh\pi}{A(1-\xi^2)} \tag{2}$$

where A is the artery's cross section, E the Young modulus, ξ the Poisson coefficient.

Some typical values are (in the metric system MKSA) for a heart beat of one pulsation per second:

$$E = 3MPa, \quad \xi = 0.3, \quad A = \pi R^2, \ R = 0.013, \quad h = 0.001, \quad \rho^f = 9.81\ 10^6$$

leading to $b = 3.310^7 ms^{-2}$ and giving displacements in the range of 0.1 10^{-3}m and flow rates around 2 $10^{-5}\mathrm{m}^3\mathrm{s}^{-1}$

2 Fluid Equations

The *Navier-Stokes equations* in a moving domain $\Omega(t)$ define the velocity $\vec{u}$ and the pressure p:

$$\rho^f\left(\frac{\partial\vec{u}}{\partial t} + u \cdot \nabla u\right) + \nabla p - \mu\nabla \cdot (\nabla\vec{u} + \nabla\vec{u}^T) = 0, \quad \nabla \cdot \vec{u} = 0, \tag{3}$$

where ρ^f is the density of the fluid and μ its viscosity.

Continuity on Σ of fluid and solid velocities implies

$$\vec{u} = \frac{\partial d}{\partial t} := \vec{n}\frac{\partial \eta}{\partial t}, \text{ on } \Sigma$$

Continuity of normal stresses implies

$$\sigma^f_{nn} := \vec{n} \cdot (\mu(\nabla u + \nabla u^T) - p)\vec{n} = -\sigma^s_{nn} := b\eta$$

Notice that as a consequence of the hypothesis of normal displacements only of the structure, there is no provision to write the continuity of the tangential stresses.

For aortic flow there also an inflow and an outflow boundary Γ_i and Γ_o on which we will prescribe pressure and no tangential velocity. If $S = \Gamma_i \cup \Gamma_o$, then the boundary Γ is

$$\Gamma := \partial\Omega(t) = \Sigma \cup S = \Sigma \cup \Gamma_i \cup \Gamma_o$$

In [9] and many other authors, the matching conditions on Σ are written on the boundary of a fixed reference domain $\partial\Omega_0$ because Koiter's shell model works with a fixed mean surface Σ.

With the notations of [7], assume that the domain of the fluid is $\Omega_t = \mathcal{A}_t(\Omega_0)$ with $\mathcal{A}_t : x_0 \to x_t := \mathcal{A}_t(x_0)$. Let

$$u_\tau(x,t) = u(\mathcal{A}_t(\mathcal{A}_\tau^{-1}(x)), t), \ \forall x \in \Omega_\tau \tag{4}$$

Then in Ω_t at $t = \tau$, the Navier-Stokes equations are in ALE format

$$\begin{aligned} &\rho^f \frac{\partial \vec{u}_\tau}{\partial t} + (\vec{u}_\tau - \vec{c}_\tau) \cdot \nabla \vec{u}_\tau + \nabla p - -\mu \nabla \cdot (\nabla \vec{u}_\tau + \nabla \vec{u}_\tau^T) = 0, \\ &\nabla \cdot \vec{u}_\tau = 0, \quad \text{with} \quad c_\tau(x) = -\frac{\partial \mathcal{A}_t(\mathcal{A}_\tau^{-1}(x))}{\partial t}|_{t=\tau} \end{aligned} \tag{5}$$

3 Transpiration Conditions for the Fluid

3.1 Conservation of Energy

We begin with an important remark on the conservation of energy.

The variational formulation of (3)- divided by ρ^s - is, $\forall \hat{u}, \hat{p}$

$$\begin{aligned} \int_{\Omega(t)} [\hat{u} \cdot (\partial_t u \quad &+ u \cdot \nabla u) + \nabla p \cdot \hat{u} - \hat{p} \nabla \cdot u \\ &+ \frac{\nu}{2}(\nabla u + \nabla u^T) : (\nabla \hat{u} + \nabla \hat{u}^T)] = \int_{\Omega(t)} f^s \cdot \hat{u} \end{aligned} \tag{6}$$

An energy balance is obtained by taking $\hat{u} = u$ and $\hat{p} = -p$,

$$\partial_t \int_{\Omega(t)} \frac{u^2}{2} + \frac{\nu}{2} \int_\Omega |\nabla u + \nabla u^T|^2 = \int_\Omega f^s \cdot \hat{u} - \int_{\partial\Omega} pu \cdot n \tag{7}$$

because

$$\partial_t \int_{\Omega(t)} u \cdot w = \int_{\Omega(t)} \partial_t (u \cdot w) + \int_{\partial\Omega} v \; u \cdot w$$
$$\int_{\Omega} ((u\nabla u) \cdot u) = \int_{\partial\Omega} u \cdot n \frac{u^2}{2} = \int_{\partial\Omega} \frac{v}{2} u \cdot u \tag{8}$$

when $v = u \cdot n$, the normal speed of $\partial\Omega$.

With transpiration conditions we intend to work on a fixed domain with zero tangential velocity but non zero normal velocity $u \cdot n = w$. In that case, in order to preserve energy, (6) needs to be modify into

$$\int_{\Omega} [\hat{u} \cdot (\partial_t u + u \cdot \nabla u) + \nabla p \cdot \hat{u} - \hat{p} \nabla \cdot u$$
$$+ \frac{\nu}{2} (\nabla u + \nabla u^T) : (\nabla \hat{u} + \nabla \hat{u}^T)] - \int_{\partial\Omega} \frac{w}{2} u \cdot \hat{u} = \int_{\Omega} f^s \cdot \hat{u} \tag{9}$$

or equivalently into

$$\int_{\Omega} [\hat{u} \cdot (\partial_t u - u \times \nabla \times u) \quad + \nabla p \cdot \hat{u} - \hat{p} \nabla \cdot u$$
$$+ \frac{\nu}{2} (\nabla u + \nabla u^T) : (\nabla \hat{u} + \nabla \hat{u}^T)] = \int_{\Omega} f^s \cdot \hat{u} \tag{10}$$

Finally we recall an identity (see [3] for instance) which shows that we can use several forms for the viscous terms:

$$\int_{\Omega} [\nabla \times u \cdot \nabla \times v + \nabla \cdot u \nabla \cdot v] = \int_{\Omega} \nabla u : \nabla v$$
$$= \int_{\Omega} [\frac{1}{2} (\nabla u + \nabla u^T) : (\nabla v + \nabla v^T) - \nabla \cdot u \nabla \cdot v] \tag{11}$$

Hence a variational formulation adapted to the problem is to find u with $u \times n = 0$ and, for all $\hat{p}$ and all $\hat{u}$ with $\hat{u} \times n = 0$

$$\int_{\Omega} [\hat{u} \cdot (\partial_t u - u \times \nabla \times u) \quad - p \nabla \cdot \hat{u} - \hat{p} \nabla \cdot u + \nu \nabla \times u \cdot \nabla \times v]$$
$$+ \int_{\partial\Omega} p u \cdot n = \int_{\Omega} f^s \cdot \hat{u} \tag{12}$$

3.2 Transpiration

As the wall vessel is $\{x + \eta \vec{n} \; : \; x \in \Sigma\}$ and as, by Taylor,

$$\vec{u}(x + \eta \vec{n}) = \vec{u}(x) + \eta \nabla \vec{u} \cdot \vec{n}(x) + o(\eta)$$

matching the velocities of fluid and structure may be written as

$$u + \eta \frac{\partial u}{\partial n} = \vec{n} \frac{\partial \eta}{\partial t} + o(\eta) \text{ on } \Sigma, \quad u \times n = 0 \tag{13}$$

On a torus of small radius r and large radius R, at a point of coordinates $(R{+}r\cos\theta)\cos\varphi, (R{+}r\cos\theta)\cos\varphi, r\sin\theta)$, a straightforward calculation shows that

$$u\times n=0,\ \nabla\cdot u=0 \Rightarrow n\cdot\frac{\partial u}{\partial n}=(1+\frac{r}{R}\cos^2\theta)\frac{u\cdot n}{r}$$

So (13) becomes

$$u\cdot n=\partial_t\eta\big(1+\frac{\eta}{r}(1+\frac{r}{R}\cos^2\theta)\big)^{-1},\quad u\times n=0 \tag{14}$$

Similarly the normal component of the normal fluid stress tensor is

$$\sigma^f_{nn}=p+2(1+\frac{r}{R}\cos^2\theta)\frac{\mu}{r}u\cdot n$$

Therefore for a quasi toroidal geometry, for large R, (1) is

$$\begin{aligned}\rho_s h\partial_{tt}\eta- &\quad \nabla\cdot(\mathbf{T}\nabla\eta)-\nabla\cdot(\mathbf{C}\nabla\partial_t\eta)+a\partial_t\eta+b\eta\\ &=p+2(1+\frac{r}{R}\cos^2\theta)\frac{\mu}{r}\partial_t\eta\big(1+\frac{\eta}{r}(1+\frac{r}{R}\cos^2\theta)\big)^{-1}\end{aligned} \tag{15}$$

So, in fine, the domain Ω no longer varies with time but on part of its boundary

$$\begin{aligned}&u\cdot n=\partial_t\eta\big(1+\frac{\eta}{r}(1+\frac{r}{R}\cos^2\theta)\big)^{-1},\quad u\times n=0,\\ &\rho_s h\partial_{tt}\eta-\nabla\cdot(\mathbf{T}\nabla\eta)-\nabla\cdot(\mathbf{C}\nabla\partial_t\eta)+a\partial_t\eta+b\eta=p\end{aligned} \tag{16}$$

where a is a non linear function of η.

Remark 2 *Notice that $\eta << r$, i.e. large vessels, allows us to eliminate η and write everything in terms of $\partial_t p$ and $u_n := u\cdot n$. It suffices to differentiates the last equation with respect to t and use the first one and integrate in time:*

$$\begin{aligned}p= &\quad p_0+\mathcal{L}(u\cdot n):=\\ &\int_0^t\big(\rho_s h\partial_{tt}u_n-\nabla\cdot(T\nabla u_n)-\nabla\cdot(C\nabla\partial_t u_n)+a\partial_t u_n+bu_n\big)\end{aligned} \tag{17}$$

4 Variational Formulation and Approximation

Coming back to (4) and using (17):

Continuous Problem Find u with $u\times n=0$ and, for all $\hat p$ and all $\hat u$ with $\hat u\times n=0$

$$\boxed{\begin{aligned}&\int_\Omega[\hat u\cdot(\partial_t u-u\times\nabla\times u)-p\nabla\cdot\hat u-\hat p\nabla\cdot u+\nu\nabla\times u\cdot\nabla\times v]\\ &\qquad+\int_\Sigma\Big(p_0+\mathcal{L}(u\cdot n)\Big)u\cdot n=-\int_S p_\Gamma\hat u\cdot n\end{aligned}}$$

4.1 Approximation in Time

From now on, for clarity, we consider only the case of the surface pressure model, i.e. $h = T = C = a = 0$, $\mathcal{L}(u \cdot n) = bu \cdot n$. However everything below extends to the full model.

So define

$$U(t) = \int_0^t u(s)\mathrm{d}s \text{ and use the integration rule } U^{m+1} = U^m + u^{m+1}\mathrm{d}t.$$

Then

Time discrete Problem $p(t) = p_0 + bU(t)$ and we seek $u^{m+1} \in V, \hat{p}^{m+1} \in Q$, satisfying for all $\hat{u} \in V, \hat{p} \in Q$,

$$\begin{aligned} &\int_\Omega \Big[\hat{u} \cdot (\frac{u^{m+1} - u^m}{\delta t} - u^{m+\frac{1}{2}} \times \nabla \times u^m) \\ &-p^{m+1}\nabla \cdot \hat{u} - \hat{p}\nabla \cdot u^{m+\frac{1}{2}} + \nu\nabla \times u^{m+\frac{1}{2}} \cdot \nabla \times \hat{u}\Big] \\ &+ \int_\Sigma [b\hat{u} \cdot \vec{n}(u^{m+\frac{1}{2}}\delta t + U^m) \cdot \vec{n}] = -\int_S p_\Gamma \hat{u} \cdot n \end{aligned} \tag{18}$$

where $u^{m+\frac{1}{2}} = \frac{1}{2}(u^{m+1} + u^m)$.

4.2 Convergence

A convergence analysis was done in [2]; we recall the results. We denote u_δ the linear in time interpolate of $\{u^m\}_1^M$ on $(0, T) = \cup_1^M [\,(m-1)\delta t, m\delta t]$. For clarity let's assume that $S = \emptyset$.

Lemma 1 If Ω is $\mathcal{C}^{1,1}$ or polyhedral and $u_0 \in L^2(\Omega)^3$, $p_0 \in H^{1/2}(\Sigma)$, then the weak solution of the continuous problem verifies $u \in L^2(\mathbf{H}^2)$, $\partial_t u \in L^2(\mathbf{L}^2)$, $p \in L^2(H^1)$, and $u \times n = 0$ in $L^2(L^4(\Sigma))$, $\partial_t p = bu \cdot n$ in $L^2(H^{1/2}(\Sigma))$, $p(0) = p_0$

Theorem 1 *The solution of the time discretized variational problem satisfies*

$$\begin{aligned} \|u_\delta\|_{L^\infty(\mathbf{L}^2)} + \sqrt{\nu}\,\|u_\delta\|_{L^2(\mathbf{H}^1)} + \quad & b\,\|\delta t \sum_{k=1}^{n+1} u^k \cdot n\|_{L^\infty(\mathbf{L}^2(\Sigma))} \\ & \le C\left(\|u_0\|_{0,2,\Omega} + \frac{1}{\sqrt{\nu}}\|p_0\|_{L^2(\Sigma)}\right) \end{aligned}$$

Theorem 2 *If Ω is simply connected, there is a subsequence $(u_{\delta'}, p_{\delta'})$ which converges to the continuous problem in $L^2(\mathbf{W}) \times H^{-1}(L^2)$ where*

$$\mathbf{W} \quad = \quad \{w \in L^2(\Omega) \,|\, \nabla \times w \in L^2(\Omega),\ \nabla \cdot w \in L^2(\Omega),\ n \times \mathbf{w}_{|_\Sigma} = \mathbf{0}\,\}.$$

```
problem bb([u,v,w,p],[uh,vh,wh,ph], solver=UMFPACK)
            = int3d(th)((u*uh+v*vh+w*wh)/dt2
                        + nu*(dx(u)*dx(uh)+dy(u)*dy(uh)+dz(u)*dz(uh)        // rot u.rot uh
                            + dx(v)*dx(vh)+dy(v)*dy(vh) +dz(v)*dz(vh)
                            + dx(w)*dx(wh)+dy(w)*dy(wh) +dz(w)*dz(wh)
                            -(dx(u)+dy(v)+dz(w))*(dx(uh)+dy(vh)+dz(wh))
                         ) - (dx(uh)+dy(vh)+dz(wh))*p - (dx(u)+dy(v)+dz(w))*ph //-ph div u - p div uh
- v*(dx(vold)-dy(uold))*uh -w*(dy(wold)-dz(vold))*vh -u*(dz(uold)-dx(wold))*wh // u x rot u
+ w*(dz(uold)-dx(wold))*uh +u*(dx(vold)-dy(uold))*vh + v*(dy(wold)-dx(vold))*wh
                        )
              - int3d(th)( (uold*uh + vold*vh + wold*wh)/dt2 )                // u^m/dt
              + int2d(th,1)((     (u*N.y-v*N.x)*(uh*N.y-vh*N.x)                // uxn.uhxn/eps
                                + (v*N.z-w*N.y)*(vh*N.z-wh*N.y)
                                + (w*N.x-u*N.z)*(wh*N.x-uh*N.z)    )/eps)
              + int2d(th,1)(b*dt2*(u*N.x+v*N.y+w*N.z)*(uh*N.x+vh*N.y+wh*N.z)) // b dt u.n uh.n
              + int2d(th,1)(b*(Uold*N.x+Vold*N.y+Wold*N.z)*(uh*N.x+vh*N.y+wh*N.z)) // b n. int_0^t u
              - int2d(th,2)(p0*wh ) - int2d(th,3)(p1*vh )
              + on(2,3,u=0) + on(2,v=0) +on(3,w=0);
```

Figure 1: An implementation using freefem++ for problem (18)

4.3 Spatial Discretization with Finite Elements

The easiest is to use penalization to enforce $u \times n = 0$ by adding to the boundary integral $\frac{1}{\epsilon}\int_\Sigma u^{m+1} \times n \cdot \hat{u} \times n$. Then we may use conforming triangular or tetrahedral elements P^2 or P^1+bubble for the velocities and P^1 for the pressure.

A freefem++ implementation (see [6]) is shown on Figure 1

5 Optimization and Inverse Problems

5.1 Optimal Stents with the Surface Pressure Model

A stent is a device to reinforce part of a cardiac vessel and/or to change the topology of the flow by its rigidity. This results in a change of the coefficient b. So with a first order scheme in time we can consider

$$\begin{aligned}
&\min_{b(x)} J = \int_{\Sigma\times(0,T)} F(p)\mathrm{d}x\mathrm{d}t \ : \ \text{Subject to} \\
&\int_\Omega [\hat{u}\cdot(\frac{u^{m+1}-u^m}{\delta t} - u^{m+1}\times\nabla\times u^m) - p^{m+1}\nabla\cdot\hat{u} - \hat{p}\nabla\cdot u^{m+1}] \\
&+\int_\Omega \nu\nabla\times u^{m+1}\cdot\nabla\times\hat{u} + \int_\Sigma (u^{m+1}b\delta t + p^m n)\cdot\hat{u} = -\int_S p_\Gamma\hat{u}\cdot n \\
&\forall\hat{u}\in V_h, \hat{p}\in Q_h \text{ with } \hat{u}\times n|_\Gamma = 0
\end{aligned} \tag{19}$$

For instance $F = |p|^4$ will minimize the time averages pressure peak on Σ.

5.1.1 First order discretization and adjoint

Consider the adjoint state

$$\begin{aligned}
&\int_\Omega [\hat{v}\cdot\frac{v^m - v^{m+1}}{\delta t} - \hat{v}\times\nabla\times u^{m-1}\cdot v^m - u^{m+1}\times\nabla\times\hat{v}\cdot v^{m+1} \\
&+\nu\nabla\times v^m\cdot\nabla\times\hat{v} + \nabla\hat{q}\cdot v^m - q^m\nabla\cdot\hat{v}] + \int_\Sigma \delta t b v^m\cdot\hat{v}
\end{aligned}$$

$$= \int_\Sigma F'(p^m)\hat{q} \tag{20}$$

for all $\hat{v}, \hat{q}$ such that $\hat{v} \times n = 0$ on $\partial\Omega$.

Letting $\hat{v} = \delta u^m, \hat{q} = \delta p^m$ and summing in m, from 1 to M gives

$$\sum_1^M \int_\Sigma F'(p^m)\delta p^m \delta t = \sum_1^M \delta t \int_\Omega \delta u^m \cdot \frac{v^{m-1} - v^m}{\delta t}$$
$$+ \sum_1^M \delta t \int_\Omega \Big(- \delta u^m \times \nabla \times u^{m-1} \cdot v^{m-1} - u^{m+1} \times \nabla \times \delta u^m \cdot v^m$$
$$+ \nu \nabla \times v^m \cdot \nabla \times \delta u^m \Big)$$
$$+ \sum_1^M \delta t \int_\Omega (\nabla \delta p^m \cdot v^m - q^m \nabla \cdot \delta u^m) + \int_\Sigma \delta t b v^m \cdot \delta u^m \tag{21}$$

5.1.2 Optimality Conditions

As $\delta u^0 = 0$ and by choosing $v^M = 0$ it is also

$$\delta J = \sum_0^{M-1} \delta t \int_\Omega \left(v^m \frac{\delta u^{m+1} - \delta u^m}{\delta t} + \nu \nabla \times v^{m+1} \cdot \nabla \times \delta u^{m+1} \right)$$
$$- \sum_0^{M-1} \delta t \int_\Omega \left(\delta u^{m+1} \times \nabla \times u^m \cdot v^m + u^{m+1} \times \nabla \times \delta u^m \cdot v^m \right)$$
$$- \sum_0^{M-1} \delta t \Big(\int_\Omega [\delta p^{m+1} \nabla \cdot v^{m+1} + q^{m+1} \nabla \cdot \delta u^{m+1}]$$
$$+ \int_\Sigma (\delta t b \delta u^{m+1} + \delta p^{m+1} n) \cdot v^{m+1} \Big) \tag{22}$$

The same is found by linearizing (19) and taking $\hat{u} = v^m$, $\hat{q} = q^m$, except that there is an additional term due to δb. In fine

$$\delta J = \delta t^2 \int_\Sigma \delta b \left(\sum_0^{M-1} u^{m+1} \cdot v^m \right) \tag{23}$$

5.1.3 Preliminary Computer Experiments

Experiment 1 This is only a feasibility test with $F = p^4$; The geometry is a quarter of a torus with R=4 and r=1. It is discretized with 1395 vertices and 6120 elements. The number of unknown of the coupled system $[\vec{u}, p]$ is 23940 with the P^1-bubble/P^1 element and Crank-Nicolson implicit scheme. The viscosity is $\nu = 0.01$; we chose $\epsilon = \nu$. The final time is $T = 1$, the time step is d$t = 0.1$ and the pressure difference imposed at Γ_i (top) and Γ_o bottom is $6\cos^2(\pi t)$.

The flow is stored on disk at every iteration ready to be reused backward in time for the adjoint equations.

Starting with b=200, after 3 iterations of steepest descent with fixed step size, the cost function is decreased from 1200 to 900. But as there is no constraint b is much reduced at the top near Γ_i. Consquently the vessel wall becomes fragile as shown by a simulated wall motion by $x \to x + \sum u^m \cdot n dt$ at every time step, as shown on Figure 2.

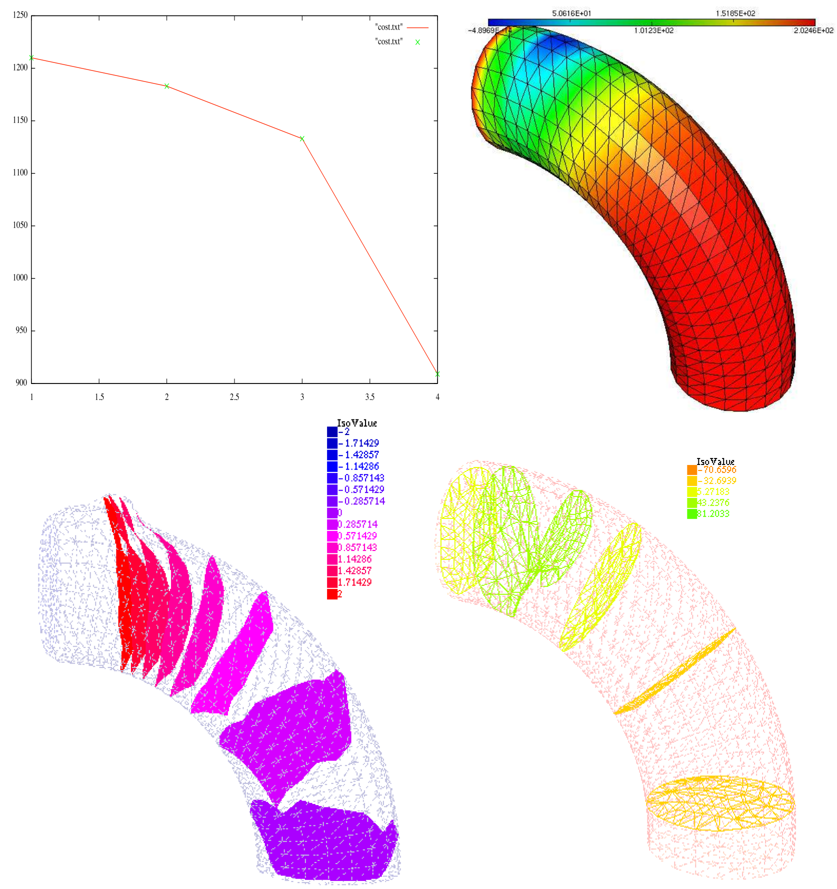

Figure 2: *Top left: Optimization criteria versus iteration number. Top right: the coefficient $b(x)$ after 3 iterations. Bottom Left: effect of the change of b on the dilatation of the vessel and some iso surfaces of constant pressure. Bottom right: a snap shot of the adjoint pressure and some iso surfaces.*

Experiment 2 The same computations has been made but now b is constrained to be greater than $b_0/2$. A mesh double the size of the previous one has been used, with 191808 degrees of freedom. The initial value of b is $b_0 = 200$. After 10 iterations, similar to Experiment 1 but with a projected gradient method for the optimization, the results of Figure 3 are found.

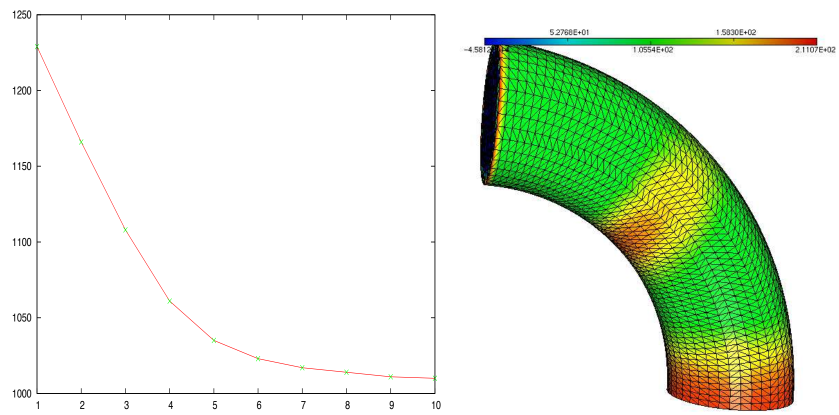

Figure 3: *Left: Optimization criteria* $\int_{\Sigma\times(0,T)} p^4$ *versus iteration number. Right: the coefficient* $b(x)$ *after 4 iterations. Right: effect of the change of* b *on the dilatation of the vessel.*

5.2 Identification of b

Finally we run an identification test of b from the observation of the wall displacement, ideally, $u \cdot n$. However the formulation does not allow it becausze the extra integral in the adjoint variational formulation is in competition with a similar term from the surface pressure model, so we used p/b. For this first test the criteria is

$$J = \int_{\Sigma\times(0,T)} |p - p_d|^2 \mathrm{d}x\mathrm{d}t$$

where p_d is obtained from a reference computation (introduction of b in the criteria makes the problem harder) with

$$b = 200 + 100 \cos x \cos y \cos z.$$

The results are shown on figure 4.

Because of the computing cost, we made only an initial study; the target is not reached, but 5 iterations go into the right direction. To do better one would have to used a varying step size gradient method and a better computer (this being done on a macbook pro, takes about 15 min).

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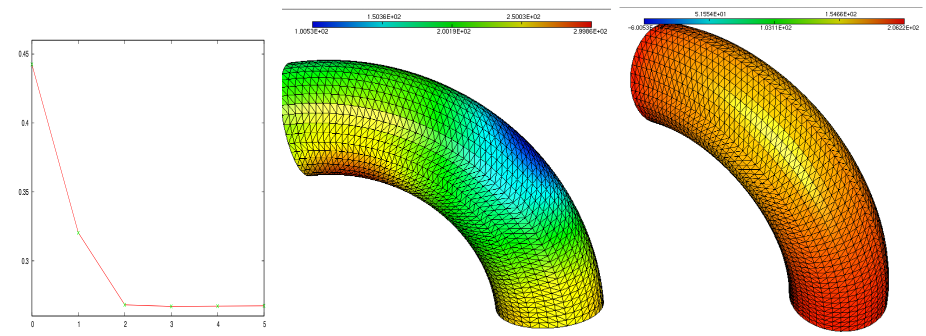

Figure 4: *Left: Optimization criteria* $\int_{\Sigma\times(0,T)}(p-p_d)^2$ *versus iteration number. Right: the coefficient* $b(x)$ *after 5 iterations. middle: The target* b.

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