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Abstract

We present a new algorithm for the problem of universal gathering mobile oblivious robots (that is, starting from any initial configuration that is not bivalent, using any number of robots, the robots reach in a finite number of steps the same position, not known beforehand) without relying on a common chirality.

We give very strong guaranties on the correctness of our algorithm by *proving formally* that it is correct, using the Coq proof assistant.

To our knowledge, this is the first certified positive (and constructive) result in the context of oblivious mobile robots. It demonstrates both the effectiveness of the approach to obtain new algorithms that are truly generic, and its managability since the amount of developped code remains human readable.

1 Introduction

Networks of mobile robots captured the attention of the distributed computing community, as they promise new applications (rescue, exploration, surveillance) in potentially dangerous (and harmful) environments. Since its initial presentation [18], this computing model has grown in popularity and many refinements have been proposed (see [11] for a recent state of the art). From a theoretical point of view, the interest lies in characterising the exact conditions for solving a particular task.

In the model we consider, robots are anonymous (*i.e.*, indistinguishable from each-other), oblivious (*i.e.*, no persistent memory of the past is available), and disoriented (*i.e.*, they do not agree on a common coordinate system). The robots operate in Look-Compute-Move cycles. In each cycle a robot "Looks" at its surroundings and obtains (in its own coordinate system) a snapshot containing the locations of all robots. Based on this visual information, the robot "Computes" a destination location (still in its own coordinate system) and then "Moves" towards the computed location. Since the robots are identical, they all follow the same deterministic algorithm. The algorithm is oblivious if the computed destination in each cycle depends only on the snapshot obtained in the current cycle (and not on the past history of execution). The snapshots obtained by the robots are not consistently oriented in any manner (that is, the robots local coordinate systems do not share a common direction nor a common chirality).

The execution model significantly impacts the solvability of collaborative tasks. Three different levels of synchronisation have been considered. The strongest model [18] is the fully synchronised (FSYNC) model where each phase of each cycle is performed simultaneously by all robots. On the other hand, the asynchronous model [11] (ASYNC) allows arbitrary delays between the Look, Compute and Move phases and the movement itself may take an arbitrary amount of time. In this paper, we consider the semi-synchronous (SSYNC) model [18], which lies somewhere between the two extreme models. In the SSYNC model, time is discretised into rounds and in each round an arbitrary subset of the robots are active. The robots that are active in a particular round perform exactly one atomic Look-Compute-Move cycle in that round. It is assumed that the scheduler (seen as an adversary) is fair in the sense that it guarantes that in any configuration, any robot is activated within a finite number of steps.

Related Work. The gathering problem is one of the benchmarking tasks in mobile robot networks, and has received a considerable amount of attention (see [11] and references herein). The gathering tasks consists in all robots (considered as dimensionless points in a Euclidian space) reaching a single point, not known beforehand, in finite time. A foundational result [18, 8] shows that in the FSYNC or SSYNC models, no oblivious deterministic algorithm can solve gathering for two robots without additional assumptions [13]. This result can be extended [18, 8] to the bivalent case, that is when an even number of robots is initially evenly split in exactly two locations. On the other hand, it is possible to solve gathering if $n > 2$ robots start from initially distinct positions, if robots are endowed with multiplicity detection: that is, a robot is able to determine the number of robots that occupy a given position. While probabilistic solutions [18, 12] can cope with arbitrary initial configuration (including bivalent ones), most of the deterministic ones in the literature [11] assume robots always start from distinct locations (that is, the initial configuration contains no multiplicity points). Some recent work was devoted to relaxing this hypothesis in the deterministic case. Dieudonné and Petit [10] investigated the problem of gathering from *any* configuration (that is, the initial configuration can contain arbitrary multiplicity points): assuming that the number of robots is odd (so, no initial bivalent configuration can exist), they provide a deterministic algorithm for gathering starting from any configuration. Bouzid *et al.* [6] improved the result by also allowing an even number of robots to start from configurations that contain

multiplicity points (albeit the initial bivalent configuration is still forbiden due to impossibility results in this case). In that sense, the algorithm of Bouzid *et al.* [6] is *universal* in the sense that it works for all gatherable configurations, including those with multiplicity points. Both aforementioned results assume that robots and are endowed with multiplicity detection and have a common chirality. A natural open question emerging from those works is whether any of those assumptions can be relaxed (not both of them can be relaxed at the same time, as impossibility results exist in this case [15]).

Another line of work that is related to our concern that of using formal methods in the context of mobile robots [5, 9, 3, 2, 14, 8]. Model-checking proved useful to find bugs in existing literature [3] and assess formally published algorithms [9, 3], in a simpler setting where robots evolve in a *discrete space* where the number of possible positions is finite. Automatic program synthesis (for the problem of perpetual exclusive exploration in a ring-shaped discrete space) is due to Bonnet *et al.* [5], and can be used to obtain automatically algorihtms that are "correct-by-design". The approach was recently refined by Millet *et al.* [14] for the problem of gathering in a discrete ring network. As all aforementioned approaches are designed for a discrete setting where both the number of positions and the number of robots are known, they cannot be used in the continuous space where robots positions take values in a set that is not enumerable, and they cannot permit to establish results that are valid for any number of robots. Developed for the Coq proof assistant,[1] the Pactole framework enabled the use of high-order logic to certify impossibility results [2] for the problem of convergence: for any positive ε, robots are required to reach locations that are at most ε apart. Another classical impossibility result that was certified using the Pactole framework is the impossibility of gathering starting from a bivalent configuration [8]. While the proof assistant approach seems a sensible path for establishing certified results for mobile robots that evolve in a continous space, to this paper there exists no *positive* certified result in this context. Expressing mobile robot algorithms in a formal framework that permits certification poses a double challenge: how to express the algorithm (that can make use of complex geometric abstractions that must be properly defined within the framework), and how to write the proof?

Our contribution

Motivated by open problems on the gathering side and on the proof assistant side, we investigate the possibility of *universal* gathering mobile oblivious robots (that is, starting from any initial configuration that is not bivalent, using any number of robots) without relying on chirality (unlike [10, 6]).

We present a new gathering algorithm for robots operating in a continuous space that *(i)* can start from any configuration that is not bivalent, *(ii)* does not put restriction on the number of robots, *(iii)* does not assume that robots share a common chirality. We give very strong guaranties on the correctness of our algorithm by *proving formally* that it is correct, using the Coq proof assistant. To this goal we use the formal model and libraries we develop, and that has been

[1] `http://coq.inria.fr`

previously sketched in [2] and [8].

To our knowledge, this is the first certified positive (and constructive) result in the context of oblivious mobile robots. It demonstrates both the effectiveness of the approach to obtain new algorithms that are truly generic, and its manageability since the amount of developped code remains human readable. Our bottom-up approach permits to lay sound theoretical foundations for future developments in this domain.

Roadmap.

The sequel of the paper is organised as follows. First, we recall the context of robot networks in Section 2. In Section 3, our algorithm is informally presented, along with the key points of its correctness proof. We present our formal COQ framework in Section 4, together with the formalization of the key concepts identified in the previous section. Section 5 investigates further some planned developments.

The actual development for COQ 8.5 is available at
`http://pactole.lri.fr/pub/certified_gathering1D.tgz`

2 Robot Networks

We borrow most of the notions in this section from [18, 1, 11]. The network consists in a set of n mobile entities, called robots, arbitrarily located in the space. Robots cannot communicate explicitely by sending messages to each others. Instead, their communication is based on vision: they observe the positions of other robots, and based on their observations, they compute destination points to which they move.

Robots are *homogeneous* and *anonymous*: they run the same algorithm (called *robogram*), they are completely indistinguishable by their appearance, and no identifier can be used in their computations. They are also *oblivious*, i.e. they cannot remember any previous observation, computation or movement performed in any previous step.

For simplicity, we assume that robots are *without volume*, *i.e.* they are modeled as points that cannot obstruct the movement or vision of other robots. Several robots can be located at the same point, a *tower* is a location inhabited by (one or) several robots. The multiplicity of a location l, that is the number of robots at this location, is denoted by $|l|$.

Visibility is *global*: the entire set of robots can always be seen by any robot at any time. Robots that are able to determine the exact number of robots occupying a same position (i.e., the multiplicity of a tower) enjoy *strong* multiplicity detection; if they can only know if a given position is inhabited or not, their multiplicity detection is said to be *weak*.

Each robot has its own local coordinate system and its own unit measure. Robots do not share any origin, orientation, and more generally any frame of reference, but it is assumed that every robot is at the origin of its own frame of reference.

At a given time, robots and their positions define a *configuration*. A configuration that consists of exactly two towers of same cardinalities is said to be *bivalent*.

The degree of asynchrony in the robot swarm is characterised by an abstract entity called the *demon* (or adversary). Each time a robot is activated by the demon, it executes a complete three-phases cycle: Look, Compute and Move. During the Look phase, using its visual sensors, the robot gets a snapshot of the current configuration. Then, based only on this observed configuration, it computes a destination in the Compute phase using its robogram, and moves towards it during the subsequent Move phase. Movements of robots are *rigid*, *i.e.* the demon cannot stop them before they reach the destination.

A *run* (or execution) is an infinite sequence of rounds. During each round, the demon chooses a subset of robots and activates them to execute a cycle. We assume the scheduling to be *fair*, *i.e.* each robot is activated infinitely often in any infinite execution, and *atomic* in the sense that robots that are activated at the same round execute their actions synchronously and atomically. An atomic demon is called fully-synchronous (FSYNC) if all robots are activated at each round, otherwise it is said to be semi-synchronous (SSYNC).

3 Setting and Robogram

We consider a set of nG anonymous robots that are oblivious and equipped with global strong multiplicity detection (that is, they are able to count the number of robots that occupy any given position). The demon is supposed to be fair, and the execution model is SSYNC.

The space in which they move is the real line $\mathbb{R}$. Robots do not share any common direction of the line, nor any chirality.

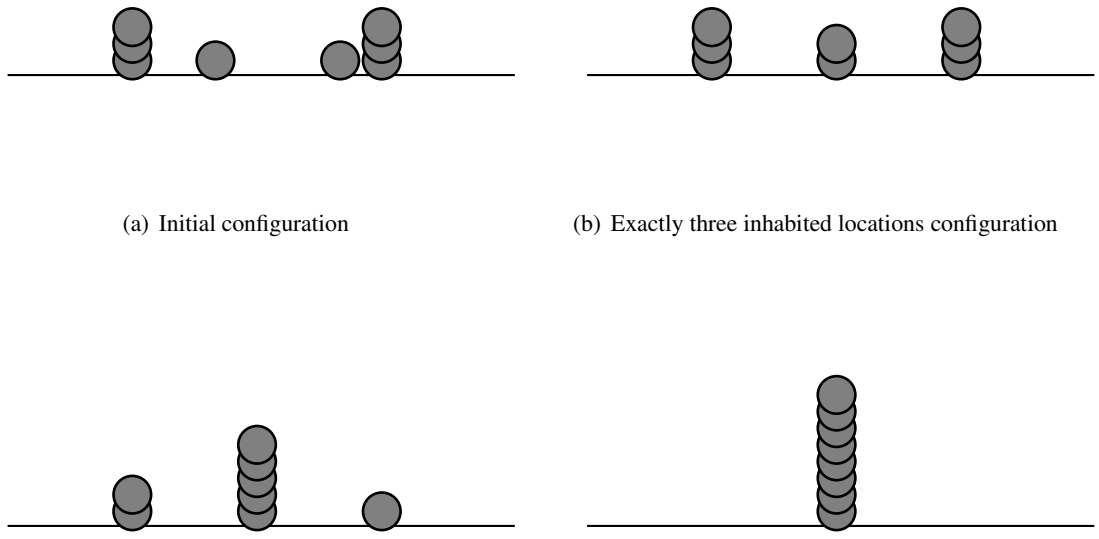
Any initial configuration is accepted as long as it is not bivalent (including those with multiplicity points). Indeed, [18] shows that gathering is not solvable for two robots, and a formal certified proof that the gathering problem cannot be solved if bivalent positions are tolerated is available [8].

3.1 Robogram

In this particular case of the considered space being $\mathbb{R}$, even if there is no common frame of reference, we have that, for any configuration, the set of inhabited locations that are the *most external* is the same for all robots. Hence, those most external inhabited location define the same *center of extrema* to all robots, as well as the same set of (strictly) interior inhabited locations. Based on this remark, we can define the robogram embedded in each robot as follows:

1. If there is a unique location with highest multiplicity, the destination is that location,
2. Otherwise, if there are exactly three inhabited locations, the destination is the one in between,
3. Otherwise, if not already at one of the most external locations, the destination is the center of the most external ones.
4. Otherwise, the destination is the origin (do not move).

An example execution of our robogram is presented in Figure 1. In the initial configuration (see Figure 1.*(a)*), only the third condition is enabled. The inner robots move toward the middle of the extremal robots. When there are three inhabited locations (see Figure 1.*(b)*), only the second condition is enabled, and extremal robots move toward the inner inhabited location. When a single highest multiplicity point is reached (see Figure 1.*(c)*), only the first condition is enabled,and all robots move toward it. After all robots gather (see Figure 1.*(d)*), only the fourth condition apply, and the configuration is final.



(a) Initial configuration

(b) Exactly three inhabited locations configuration

(c) Unique highest multiplicity point configuration

(d) Gathered configuration

Figure 1: Example execution of our robogram.

This description of the protocol is obviously informal, however we present in Section 4.1 its formal version, that is, the COQ definition of our algorithm.

3.2 Key points to prove correctness

Some properties are fundamental in our proof that the algorithm is a solution to the problem of gathering. Namely, that robots move towards the same location, that a legal configuration cannot

evolve into a forbidden (that is: bivalent) one, and finally that the configuation is eventually reduced to a single inhabited location.

Robots that move go to the same location.

Note that by robots "that move" we explicitely mean robots the destination of which is not their original location, and *not* robots that are activated (some of which may not move). Robots enjoy global strong multiplicity detection, hence they all detect if there is a unique tower with the highest multiplicity, thus sharing the destination (Phase 1). If they do not find such a tower, they can all count how many locations are inhabited. Should they detect that there are only three of them (Phase 2) then, as previously remarked, sharing the notion of tower in between, they also share the destination. Finally if there is more than three inhabited locations none of which holding more robots than the others (Phase 3), as most external towers are the same for all robots, robots to move go the location defined as the center of those external towers, that is the same destination again.

Further note that we actually just showed that all moving robots are in the same phase of the robogram, and that the resulting destination does not depend on the frame of reference of the robot.

Bivalent positions are unreachable.

We require that the initial configuration does not consist of exactly two towers with the same multiplicity. One of the key points ensuring this algorithm's correctness is that there is no way to reach a position that is bivalent from a position that is *not* bivalent. Consider two configurations C_0 and C_1, C_1 being bivalent and resulting from C_0 by some round. Let us denote by $|x|_0$ (resp. $|x|_1$) the multiplicity of location x in C_0 (resp. in C_1). By definition, C_1 consists of two locations l_1 and l_2 such that $|l_1|_1 = |l_2|_1 = \frac{nG}{2}$. As all moving robots go to *the same location*, we can assume without loss of generality that robots moved to, say, l_1, adding to its original multiplicity $|l_1|_0$ (which might have been 0). Since the configuration is now bivalent, this means that l_2 was inhabited in C_0 and such that $|l_2|_0 \geq \frac{nG}{2}$ (some robot in l_2 might have moved to l_1). There cannot have been only one inhabited location l distinct from l_2 before the round because either $|l|_0 = |l_2|_0 = \frac{nG}{2}$ but we supposed the configuration was not bivalent, or $|l|_0 < \frac{nG}{2} < |l_2|_0$ but then by Phase 1 robots would have moved to l_2 and not l_1. Hence C_0 consisted of l_2 and several inhabited l_i ($i \neq 2$) amongst which the robots not located in l_2 were distributed, but then none of the l_i could have held more than $\frac{nG}{2} - 1$ robots, hence Phase 1 should have applied and robots should have moved to l_2, a contradiction.

Eventually no-one moves.

The termination of the algorithm is ensured by the existence of a measure decreasing at each round involving a moving robot for a well-founded ordering. We then conclude using the assumption that the demon is fair.

The measure is defined as follows: we map any configuration C_i to a $(p_i, m_i^{p_i}) \in \mathbb{N} \times \mathbb{N}$ such that p_i is the phase number of the moving robots, and:

- m_i^1 is the number of robots that are *not at* the (unique) location of highest multiplicity,
- m_i^2 is the number of robots that are *not at* the inhabited location in between,
- m_i^3 is the number of robots that are *neither at* a most external location *nor at* their center.

Let $>_{\mathbb{N}}$ be the usual ordering on natural numbers, the relevant ordering $\succ$ is defined as the lexicographic extension of $>_{\mathbb{N}}$ on pairs:

$$(p, m) \succ (p', m') \quad \text{iff} \quad \begin{cases} \text{either} & p >_{\mathbb{N}} p', \\ \text{or} & p =_{\mathbb{N}} p' \text{ and } m >_{\mathbb{N}} m'. \end{cases}$$

It is well-founded since $>_{\mathbb{N}}$ is well-founded. We show that for any round on a configuration C_k resulting in a *different* configuration C_{k+1}, $(p_k, m_k^{p_k}) \succ (p_{k+1}, m_{k+1}^{p_{k+1}})$, hence proving that eventually there is no more change in successive configurations.

4 A Formal Model to Prove Robograms

To certify results and to guarantee the soundness of theorems, we use Coq, a Curry-Howard-based interactive proof assistant enjoying a trustworthy kernel. The (functional) language of Coq is a very expressive λ-calculus: the *Calculus of Inductive Constructions* (CIC) [7]. In this context, datatypes, objects, algorithms, theorems and proofs can be expressed in a unified way, as terms.

The reader will find in [4] a very comprehensive overview and good practices with reference to Coq. Developing a proof in a proof assistant may nonetheless be tedious, or require expertise from the user. To make this task easier, we are actively developing (under the name Pactole) a formal model, as well as lemmas and theorems, to specify and certify results about networks of autonomous mobile robots. It is designed to be robust and flexible enough to express most of the variety of assumptions in robots network, for example with reference to the considered space: discrete or continuous, bounded or unbounded...

We do not expect the reader to be an expert in Coq but of course the specification of a model for mobile robots in Coq requires some knowledge of the proof assistant. We want to stress that the framework eases the developer's task. The notations and definitions we give hereafter should be simply read as typed functional expressions.

The formal model we rely on, as introduced in [2], exceeds our needs as in particular it includes Byzantine robots, which are irrelevant in the present work. The reader is invited to check that the actual code is almost identical.

4.1 The Formal Model

The Pactole model[2] has been sketched in [2, 8] to which we refer for further details; we recall here its main characteristics.

We use two important features of COQ: a formalism of *higher-order* to quantify over programs, demons, etc., and the possibility to define *inductive* and *coinductive* types [17] to express inductive and coinductive datatypes and properties. Coinductive types are in particular of invaluable help to express infinite behaviours, infinite datatypes and properties on them, as we shall see with demons.

Robots are anonymous, however we need to identify some of them in the proofs. Thus, we consider given a finite set of *identifiers*, isomorphic to a segment of $\mathbb{N}$. We hereafter omit this set `G` unless it is necessary to characterise the number of robots. Robots are distributed in space, at places called *locations*. We call a *configuration* a *function* from a set of identifiers to the space of locations. The set of locations we consider here is the real line $\mathbb{R}$.

Note that from that definition, there is information about identifiers contained in configurations, in particular, equality between configurations does *not* simply boil down to the equality of the multisets of inhabited locations.

Now if we are under the assumption that robots are anonymous and indistinguishable, we have to make sure that those identifiers are not used by the embedded algorithm.

Spectrum. The computation of any robot's target location is based on the perception they get from their environment, that is, in an SSYNC execution scheme, from a configuration. The result of this observation may be more or less accurate, depending on sensors' capabilities. A robot's perception of a configuration is called a *spectrum*. To allow for different assumptions to be studied, we leave abstract the type *spectrum* (`Spect.t`) and the notion of spectrum of a position. Robograms will then output a location when given a spectrum (instead of a configuration), thus guarantying that assumptions over sensors are fulfilled. For instance, the spectrum for anonymous robots with *weak* global multiplicity detection could be a set of inhabited locations, i.e., without any multiplicity information. In a *strong* global multiplicity setting, a multiset of inhabited locations is a suitable spectrum; that is what we use in this work.

In the following we will distinguish a *demon* configuration (resp. spectrum), that is expressed in the global frame of reference, from a *robot* configuration (resp. spectrum), that is expressed in the robot's own frame of reference. At each step of the distributed protocol (see definition of `round` below) the demon configuration and spectrum are transformed (i.e., recentered, rotated and scaled) into the considered robots ones before being given as parameters to robograms. Depending on assumptions, the zoom and rotation factors may be fixed for each robot or chosen by the demon at each step. They may also be shared by all robots or not, etc.

Robogram. Robograms may be naturally defined in a *completely abstract manner*, without any concrete code, in our COQ model as follows. They consist of an actual algorithm `pgm` that takes

[2] Available at `http://pactole.lri.fr`

a spectrum as input and returns a location, and a compatibility property `pgm_compat` stating that target locations are the same if equivalent spectra are given (for some equivalence on spectra).

```
Record robogram := {
  pgm :> Spect.t → Location.t;
  pgm_compat : Proper (Spect.eq ⇛ Location.eq) pgm}.
```

Of course it is possible to instanciate the robogram by giving a concrete definition of the program, and proving that the compatibility property holds. In our case, the type of locations is `R.t` (from the COQ library on axiomatic reals) and the program as described in Section 3.1 is:

```
Definition robogram_pgm (s: Spect.t) : R.t :=
  match Spect.support (Smax s) with (* Locations of max multiplicity *)
    | nil ⇒ 0                       (* Only happens if no robot      *)
        if beq_nat (length (Spect.support s)) 3 then
          List.nth 1 (sort (Spect.support s)) 0  (* Phase 2: between*)
        else if is_extremal 0 s then 0           (* ... stay...     *)
             else extreme_center s               (* Phase 3: center *)
  end.
```

Note that this is almost exactly an ML code.

The resulting instanciated robogram is defined under the name `gathering_robogram`.

4.2 Formalising Key Points and the Main Theorem

The key steps of our proof can be expressed as relatively straightforward statements. Theorem `same_destination` states that two robots id_1 and id_2 that are in the set of moving robots (i.e., the destination of which is not their current location) compute the same destination location (in the demons's frame of reference).

```
Theorem same_destination : ∀ da config id1 id2,
  In id1 (moving gathering_robogram da config)
  → In id2 (moving gathering_robogram da config)
  → round gathering_robogram da config id1 =
      round gathering_robogram da config id2.
```

By case on the phases of the robogram, and on the structure of the provided code. The formal proof is about 30 lines of COQ long.

Theorem `never_forbidden` says that for all demonic action *da* and configuration *conf*, if *conf* is not bivalent, then the configuration resulting from *conf* after the round defined by *da* and our robogram is not bivalent.

```
Theorem never_forbidden :
  ∀ da conf, ¬ forbidden conf
  → ¬ forbidden (round gathering_robogram da conf).
```

Proof is done by a case analysis on the set of towers of maximum height at the beginning. If there is none, this is absurd; if there is exactly one, the resulting configuration will have the same highest tower, a legal configuration. Now if there are at least two highest towers, then

if the resulting configuration is bivalent, at least one robot has moved (otherwise the original configuration would be bivalent, to the contrary of what is assumed), and all robots that move go to the same of the resulting two towers. The rest is arithmetics, as described on page 7. The proof of this key point is less than 100 lines of COQ script.

It remains to state that for all demonic action *da* and configuration *conf*, if *conf* is not bivalent, and if there is at least one robot moving, then the configuration resulting from the round defined by *da* and our robogram on *conf* is smaller than *conf*. The ordering relation on configurations, called `lt_conf`, being the one described in section 3.2. This is directly translated into the following theorem.

```
Theorem round_lt_conf : ∀ da conf,
  ¬ forbidden conf → moving gathering_robogram da conf ≠ nil
  → lt_conf (round robogram da conf) conf.
```

A general description on how to characterise a solution to the problem of gathering has been given in [8]. We specialise this definition here to take into account that an initial configuration is not bivalent. This is straightforward: any robogram r is a solution w.r.t. a demon d if for all configuration *conf* that is not bivalent, there is a point pt to which all robots will eventually gather (and stay) in the execution defined by r and d, and starting from *conf*.

```
Definition solGathering (r : robogram) (d : demon) :=
  ∀ conf, ¬ forbidden conf → ∃ pt : R, WillGather pt (execute r d conf).
```

The theorem stating the correctness of our robogram is then simply: for all demon d that is fair, `gathering_robogram` is a solution with reference to d.

```
Theorem Gathering_in_R :
  ∀ d, Fair d → solGathering gathering_robogram d.
```

The proof is led by well-founded induction on the `lt_conf` relation. If all robots are gathered, then it is done. If not, by fairness some robots will have to move, thus a robot will be amongst the first to move. (Formally, this is an induction using fairness.) We conclude by using the induction hypothesis (of our well-founded induction) as this round decreases the measure on configurations (theorem `round_lt_conf`). This proof of the main theorem is interestingly small as it is only about 20 lines of COQ.

The whole file dedicated to specification and certification of our algorithm (`RDVinR.v`) is about 2300 lines long. It includes 460 lines of definitions, specification and intermediate lemmas, and approximately 1460 lines of actual proof.

5 Perspectives

We proposed a new algorithm to gather anonymous and oblivious robots on a continuous unbounded space: the real line $\mathbb{R}$, without relying on a shared orientation or chirality, and allowing for any initial configuration that is not bivalent. This protocol is certified correct for any positive number of robots (more than 2) using our actively developed COQ framework for networks of mobile robots, which is publicly available to the research community.

A next step would be to add more dimensions to the considered Euclidian space, first by considering gathering in $\mathbb{R}^2$. As the framework is highly parametric, specifying another space in which robots move is not a dramatic change: the type of locations is a parameter, it is left abstract throughout the majority of the formalism, in which a concrete instance is not needed.

Another interesting evolution would be to take into account the more general ASYNC model, that is when Look-Compute-Move cycles and phases are not atomic anymore. Describing behaviours that are ASYNC in COQ may nonetheless add to the intricacy of formal proofs, and relevant libraries to ease the task of the developer will have to be provided accordingly.

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