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Fluid friction and wall viscosity of the 1D blood flow model

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Abstract

We study the behavior of the pulse waves of water into a flexible tube for application to blood flow simulations. In pulse waves both fluid friction and wall viscosity are damping factors, and difficult to evaluate separately. In this paper, the coefficients of fluid friction and wall viscosity are estimated by fitting a nonlinear 1D flow model to experimental data. In the experimental setup, a distensible tube is connected to a piston pump at one end and closed at another end. The pressure and wall displacements are measured simultaneously. A good agreement between model predictions and experiments was achieved. For amplitude decrease, the effect of wall viscosity on the pulse wave has been shown as important as that of fluid viscosity. ¹

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Keywords: Pulse wave propagation; One-dimensional modeling; Fluid friction; Viscoelasticity

1 Introduction

Although the modeling of blood flow has a long history, it is still a challenging problem. Recently 1D modeling of blood flow circulation has attracted more attention. One reason is that it is a well balanced option between complexity and computational cost (see e.g. [2, 7, 17, 21, 26, 30]). It is not only very important to predict the time-dependent distributions of flow rate and pressure in a network, but it is also important to be able to predict mechanical properties of the wall (see [14]), it is clear that could help the understanding of cardiovascular pathologies.

The 1D fluid dynamical models are non nonlinear and are able to predict flow, area and pressure. Within the dynamical system there exist several damping factors, such as the fluid viscosity, the wall viscoelasticity, the geometrical changes of vessels, etc. Previous studies have shown that in vessels without drastic geometrical variations (i.e. no severe aneurysms or stenoses), the fluid viscosity and wall viscoelasticity are the most significant damping factors [16]. Comparisons between the 1D model and *in-vivo* data [11, 22] suggest that the predictions of a viscoelastic 1D model is significantly more physiological than those of an elastic one which contains high frequencies in the pulse which is not observed experimentally. But the comparisons were only qualitative or semi-quantitative due to the limited accuracy of associated non-invasive measurements and the lack of patient-specific parameter values of the 1D model for each subject.

Quantitative comparisons can be done with *in-vitro* experimental setups. Reuderink et al. [20] connected a distensible tube to a piston pump, which ejects fluid in pulse waves throughout the tube, and the experimental data were

26 compared against numerical predictions of several formulations of the 1D model.
27 In the first formulation, they proposed an elastic tube law and Poiseuille's theory
28 to account for the fluid viscosity, and their studies underestimated the damping
29 of the waves and predicted shocks, not observed in the experiments. In another
30 formulation, still linear, the fluid viscosity was predicted from the Womersley
31 theory with a viscoelastic tube law which gave a better match between the
32 predictions and the experiments. A similar experiments setup was proposed
33 by Bessems et al. [5] using a 3-component Kelvin viscoelastic model to model
34 the wall behavior, however in this work, both the convective and fluid viscosity
35 terms were neglected. Alastruey et al. [1] presented a comparative study using
36 an experimental setup with a network, they measured the coefficients of a Voigt
37 viscoelastic model by tensile tests instead of fitting them from the waves. For
38 the fluid viscosity term, they adopted a value from literature, which was fitted
39 from waves of coronary blood flow with an elastic wall model [24].

40 In this paper, we study the friction and wall viscoelasticity using the 1D
41 model and a similar experimental setup where pulse waves are propagating in
42 one distensible tube. However, there are three main differences between our
43 study and previous ones:

- 44 1. *Both of the two damping factors (fluid friction and wall viscosity) are*
45 *modelled.* Although there are several theories to estimate the friction term
46 (see, e.g. [6, 13, 18]), the value is rarely determined experimentally besides
47 the study of Smith et al. with an elastic model [24]. It is well known that
48 fluid viscosity and wall viscoelasticity have damping influences on the
49 pulse waves. These slight differences are discussed in [27], nevertheless it
50 is difficult to evaluate them separately from pulse waves. However, the
51 viscoelasticity has smoothing effect on the waveforms whereas the fluid
52 friction does not [3], we investigated this claim by only accounting for the
53 amplitude or the sharpness of the signal. The study shows the results of
54 including both effects, one, or the other.

- 55 2. *The viscoelasticity of the wall is measured in a new manner.* The vis-
56 coelasticity of a solid material is difficult to measure accurately, even in
57 an *in-vitro* setup. In our study, the viscoelasticity is determined through
58 the pressure-wall perturbation relation of the vessel under operating condi-
59 tions. The internal pressure is measured by a pressure sensor and the per-
60 turbation of the wall is measured by a Laser Doppler Velocimetry (LDV).
- 61 3. *A shock-capturing scheme is applied as the numerical solver.* In a non-
62 linear hyperbolic system, shocks may arise even if the initial condition is
63 smooth (even for small viscoelasticity values). The Monotonic Upstream
64 Scheme for Conservation Laws (MUSCL) scheme is able to capture shocks
65 without non-physical oscillations, and is applied to discretize the governing
66 equations and compared to the MacCormack scheme.

67 2 Methodology

68 2.1 One-dimensional model

69 We use the 1D governing equations for flows passing through an elastic cylinder
70 of radius R expressed in the dynamical variables of flow rate Q , cross-sectional
71 area $A = 2\pi R$ and internal average pressure P . The 1D equations can be derived
72 by the integration over a cross-sectional area of the axy-symmetric Navier-Stokes
73 equations of an incompressible fluid at constant viscosity, giving the following
74 mass and momentum 1D conservation equations

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\alpha \frac{Q^2}{A} \right) + \frac{A}{\rho} \frac{\partial P}{\partial x} = -2\pi\nu \left[\frac{\partial v_x}{\partial r} \right]_{r=R}, \quad (2)$$

75 where v_x is the axial velocity, ρ is the fluid density and ν is the kinematic vis-
76 cosity of the fluid. The parameter α and the last term, the viscous or drag

77 friction, depend on the velocity profile. In general, the axial velocity is also
78 function of the radius coordinate r , v.i.z. $v_x = v_x(r, x, t)$. If we assume the
79 profile has the same shape $\Psi(r)$ in every vessel cross-section along the axial di-
80 rection, the velocity function can be separated as $v_x = U(x, t)\Psi(r)$, being U the
81 average velocity. If $\Psi(r)$ is known, the parameter α and the derivative $\frac{\partial v_x}{\partial r}$ that
82 appears in the friction term can be therefore calculated. The friction drag can
83 be approximated by $-C_f Q/A$. The radial profile $\Psi(r)$ is strongly dependent on
84 the Womersley number defined by $R\sqrt{\omega/\nu}$, where the quantity ω is the angular
85 frequency which characterizes the flow. If ω and ν are approximately constant,
86 only the radius R influences α and C_f , whose values should be determined by
87 experiments for vessels with various diameters. When the transient inertial force
88 is large, the profile is essentially flat, $\alpha = 1$ [24]. With a thin viscous bound-
89 ary layer, the inviscid core and a no-slip boundary condition, the friction term
90 can be estimated (see e.g. [6, 18]). When the transient inertial force is small,
91 the profile is parabolic, $\alpha = 4/3$; the viscosity force is then dominating and
92 $C_f = 8\pi\nu$. Using the power law profile proposed by Hughes and Lubliner [12],
93 Smith et al. [24] compute from coronary blood flow, $C_f = 22\pi\nu$ and $\alpha = 1.1$.
94 This value of C_f is used on other numerical works [1, 15] but setting $\alpha = 1$ for
95 simplification.

96 The viscoelasticity of the wall can be described using different viscoelastic
97 models, e.g. [11, 22, 25] with displaying disctint numerical problems [19, 25]. In
98 this study we use the two-component Voigt model, which relates the strain ϵ
99 and stress σ in the equation

$$\sigma = E\epsilon + \phi \frac{d\epsilon}{dt}, \quad (3)$$

100 where E is the Young's modulus and ϕ is a coefficient for the viscosity. In
101 reference [23, 28] we have shown that the model (i) fits experimental data and
102 (ii) it is able to filter high frequencies.

103 For a tube with a thin wall, the circumferential strain $\epsilon_{\theta\theta}$ can be expressed

104 as

$$\epsilon_{\theta\theta} = \frac{R - R_0}{(1 - \eta^2)R_0}, \quad (4)$$

105 where R_0 is the reference radius without loading and η is the Poisson ratio,
 106 which is 0.5 for an incompressible material. By Laplace's law, the transmural
 107 difference between the internal pressure P and the external pressure P_{ext} is
 108 balanced with the circumferential stress $\sigma_{\theta\theta}$ in the relation

$$P - P_{ext} = \frac{h\sigma_{\theta\theta}}{\pi R}. \quad (5)$$

109 Combining Eq. 3, 4 and 5, we get

$$P - P_{exp} = \nu_e(R - R_0) + \nu_s \frac{dR}{dt}, \quad (6)$$

110 with

$$\nu_e = \frac{Eh}{(1 - \eta^2)A_0}, \text{ and } \nu_s = \frac{\phi h}{(1 - \eta^2)A_0}.$$

111 Note that the radius R in the denominators of the two coefficients is approxi-
 112 mated by R_0 under the assumption that the perturbations are small.

113 If we assume P_{ext} constant and inserting Eq. 6 into the 1D momentum
 114 equation to eliminate P , gives

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\alpha \frac{Q^2}{A} + \frac{\beta}{3\rho} A^{\frac{3}{2}} \right) = -C_f \frac{Q}{A} + C_v \frac{\partial^2 Q}{\partial x^2}, \quad (7)$$

115 where

$$\beta = \frac{\sqrt{\pi}Eh}{(1 - \eta^2)A_0}, \text{ and } C_v = \frac{\sqrt{\pi}\phi h}{2\rho(1 - \eta^2)\sqrt{A_0}}.$$

116 The 1D model was numerically solved by two approaches : MacCormack
 117 and MUSCL. More details on the integration schemes and on the treatment of
 118 the boundary condition are in [8, 27]. More precisely here the boundary condi-
 119 tion modeling the stainless rod in the experiment, a total reflection boundary
 120 condition, can be numerically achieved by imposing a mirror condition at the

121 end of the elastic tube.

122 **2.2 Experimental setup**

123 The experimental setup is shown in Fig. 1. The piston pump (TOMITA Engi-
124 neering) injects fluid (water) into a polyurethane tube. The output of the pump
125 is a sinusoidal function in time, whose period and duration can be programmed
126 through a computer. At the measurement points, a pressure sensor (Keyence,
127 AP-10S) is inserted into the tube. The perturbation of the tube wall is mea-
128 sured by a LDV (Polytec, NLV-2500). The pump, the pressure sensor and the
129 LDV are controlled by a computer, which synchronizes the operations of the
130 instruments and stores the measurement data at 10 *KHz*. The end of the tube
131 is closed by a stainless rod and thus a total reflection boundary condition is
132 imposed at the outlet. Pulse waves are bounced backward and forward in the
133 tube multiple times before the equilibrium state is restored. We measured at
134 two points, *A* and *B*, which are respectively close to the proximal and distal
135 ends of the tube. Table 1 summarizes the parameters of the elastic tube and
136 fluid: the thickness of the wall *h*, the reference diameter *D*, the total length of
137 the tube *L*, the distances from the inlet to the two measurement points *L_A* and
138 *L_B*, the fluid density ρ and the kinematic viscosity ν .

<i>h</i> (cm)	<i>D</i> (cm)	<i>L</i> (cm)	<i>L_A</i> (cm)	<i>L_B</i> (cm)	ρ (kg/cm ³)	ν (cm ² /s)
0.2	0.8	192	28.3	168.2	1.050×10^{-3}	1×10^{-2}

Table 1: Parameters of the tube and fluid.

139 To evaluate independently the Young's modulus of the elastic tubes we com-
140 plete the experimental setup with a tensile device. We prepared two specimens
141 of the polymer of the elastic wall to use in the tensile test (Shimadzu EZ test).
142 The specimens were elongated at a rate of 0.5 m/min and then released at the
143 same rate. We applied the least square method (linear regression) to fit the
144 curve against the function $F = C_0 + ES\Delta L/L$, where C_0 is a constant, E is
145 the Young's modulus, S is the cross-sectional area of the specimens and L is
146 the original length. Dividing the fitted slope of the curve by S , we can estimate

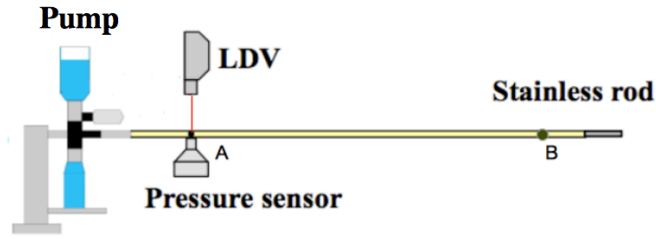


Figure 1: Experimental setup : the elastic tube (in yellow) is closed by a stainless rod at the right end (in grey). The points *A* and *B* indicate the measurement sites. Parameters of the tube and fluid are summarized in Table 1.

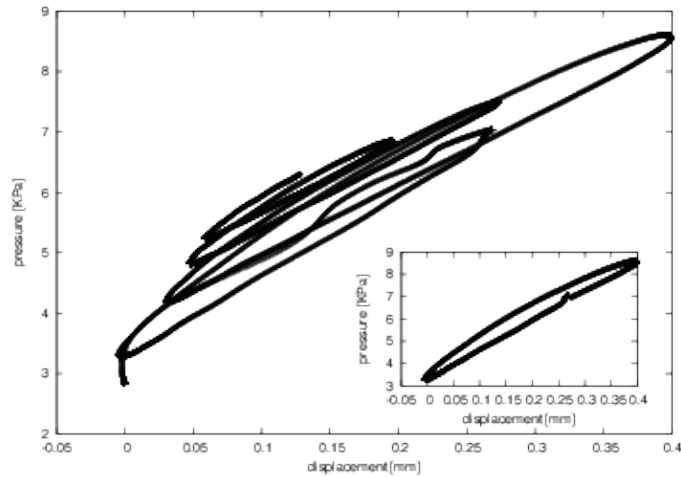


Figure 2: Experimental pressure-radius (P-R) loop. Insert : one period loop. Note that the system in the linear regime.

147 experimentally the Young's modulus as $1.92 \pm 0.06 \cdot 10^5$ Pa.

148 2.3 Parameter estimation

149 We present the method used for the evaluation of the Young's modulus, the wall
150 viscosity and the fluid friction.

151 2.3.1 Young's modulus

152 In order to estimate the Young's modulus E we propose two different methods:
153 using numerical simulations and by integration of the experimental pressure-
154 radius curve shown in Figure 2. We note that the system is in the linear zone.

155 The values of E computed in each approach will be compared to those given by
 156 the tensile test.

157 **Numerical simulations** In the first approach, using the fact the velocity
 158 of pulse wave is directly related to the stiffness through the Moens-Korteweg
 159 formula [9], we vary Young's modulus in numerical simulations to match the
 160 wave peaks coming from experimental signal taken in points A and B . The best
 161 fit will give the optimal Young's modulus E_0 .

162 **Integration of the experimental pressure-radius signal** In the second
 163 approach we use the experimental data and impose a sinusoidal wave of only
 164 one full period strictly. The net volume of fluid injected into the tube was
 165 zero, and the tube returned to the original state with the amplitude dampened
 166 roughly in a oscillatory way. In this situation the energy loss is due to the wall
 167 viscosity. Integrating the viscoelastic tube law (6) times the wall velocity $\frac{dR}{dt}$
 168 from the starting time t_0 to the final time t_e we found that the work done by
 169 the mechanical system is

$$\int_{t_0}^{t_e} (P - P_{ext}) \frac{dR}{dt} dt = \int_{t_0}^{t_e} \nu_e (R - R_0) \frac{dR}{dt} dt + \int_{t_0}^{t_e} \nu_s \left(\frac{dR}{dt} \right)^2 dt. \quad (8)$$

170 From the time series of the pressure $P(t)$ and the wall displacement $R(t)$ the
 171 evaluation of the viscoelastic term ν_s is straightforward as long as both the
 172 external pressure P_{ext} and the work done by the elastic component (the 1st term
 173 of the rhs of equation (8)) are zero. Once the viscosity coefficient ν_s is calculated,
 174 the tube law (6) can be rearranged to give $P - P_{ext} - \nu_s(dR/dt) = \nu_e(R - R_0)$
 175 and the elastic coefficient ν_e can be estimated by linear regression. We note
 176 that we have additionally estimated the viscoelastic term.

177 2.3.2 Viscoelastic parameters

178 For the estimation of the viscoelasticity parameters, we introduce a cost func-
 179 tion defined by the normalized root mean square (NRMS) error between the

180 experimental signal of pressure P_{exp} and the numerical predictions P_{sim}

$$NRMS = \frac{1}{\max(P_{exp}) - \min(P_{exp})} \sqrt{\frac{\sum_N (P_{sim} - P_{exp})^2}{N}},$$

181 where N is the number of temporal data points and P_{sim} depends on the fluid
182 friction and wall viscosity for fixed Young's modulus E_0 . For each run we obtain
183 numerically the temporal series of the cross-sectional area A from equation (1)
184 and compute the numerical prediction of the pressure using equation (6). In
185 practice, we fixed C_f for different values from $8\pi\nu$ to $33\pi\nu$, and for each value,
186 we fitted the parameters ϕ by minimizing the NRMS. As C_f was fixed for each
187 step we only did an one dimensional minimization by doing small variations of ϕ
188 to find the minimum. This is particular case of the Steepest Descent approach
189 for a functional minimum, where the new search direction is orthogonal to the
190 previous. The parameter optimisation was done on the two measurement points
191 A and B , and the consistency of the results estimated from the two sets of data
192 was checked.

193 **3 Results**

194 In this Section we present the results of the parameter estimations using the
195 methods described before. Please note that the final state on the experimental
196 data as well as the numerical results has a higher pressure than the initial state.
197 That is because we imposed a half sinusoidal wave at the inlet and thus a net
198 volume of about 4.5 cm^3 fluid was injected into the tube. Only in the case when
199 we do the integration of the experimental pressure-radius signal to computed
200 the wall viscosity and the fluid friction we impose a complete period at the inlet
201 in order of to have no net extra volume inside the elastic tube.

202 3.1 Young’s modulus

203 We vary Young’s modulus E in different simulations imposing a half sinusoidal
 204 wave at the inlet. Numerical simulations were done for E starting from $2.00 \times$
 205 10^5 Pa to 2.15×10^5 Pa, with a step of 0.01×10^5 Pa. We have found that for
 206 the value of $E \sim E_0 = 2.08 \times 10^5$ Pa, the difference of the arrival times between
 207 the experimental signal and predictions at the measurements points A and B
 208 was minimal (smaller than 0.02 s for each of the first ten peaks). The Figure 5
 209 shows the variations of the arrival times when we change the Young’s modulus.

method	E (10^5 Pa)	ϕ (kPa · s)
Numerical	2.08	1.0
Integration P-R data	1.45—2.90	0.97—1.94
Tensile test	1.92 ± 0.06	-

Table 2: Young’s modulus and Viscoelasticity of the polymer computed using three different approaches : 1D model optimisation, Pressure-radius experimental data and tensile test.

210 This value is in the range estimated with the integrated method [1.45 –
 211 2.9 10^5] and is about 8% bigger than those give by the tensile device (1.92 ± 0.06
 212 10^5). Besides the measurement error, the variance in the home-made polymer
 213 tubes may also contribute to the difference.

214 3.2 Fluid friction and wall viscosity

215 The friction and wall viscosity terms are both damping factors in the model
 216 equation. The key point is to be able of discriminate them when we are looking
 217 for the optimal values.

218 First we used an pure elastic model (the wall viscosity ϕ is set to 0) and we
 219 varied the friction coefficient C_f . Fig. 3 presents the runs (called waves) with
 220 three values of the friction coefficient C_f : $8\pi\nu$, $22\pi\nu$ and $33\pi\nu$. Using the first
 221 value, derived from a parabolic velocity profile, the predicted pressure wave has
 222 two main unrealistic features: (i) we have an overestimated pressure amplitude
 223 and (ii) we develop discontinuities or shocks, in contradiction to the experimen-
 224 tal measurement (blue line, Fig. 3). The second value comes from Smith et

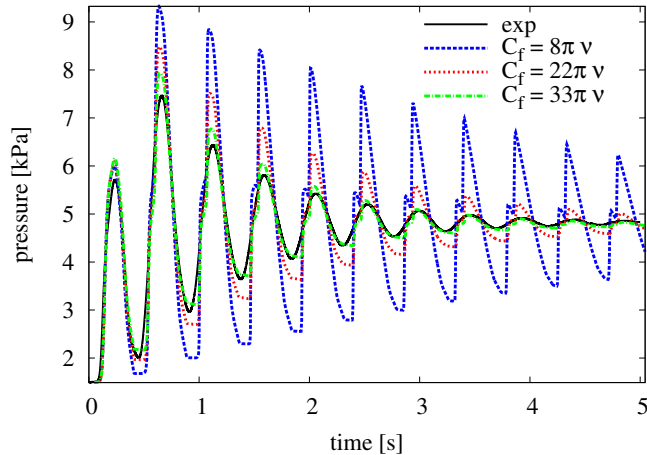


Figure 3: Pressure time series at measurement point A. The elastic model predicts shocks. Increasing the friction term can damp the amplitude effectively, but the shocks still exist. $E = 2.08 \times 10^5$ Pa and $\phi = 0$.

225 al. [24], and we can see the amplitude becomes closer to the experimental one
 226 (red line, Fig. 3). The third value gives the best prediction in terms of pressure
 227 amplitude but there are still discontinuities or shocks (green line, Fig. 3). We
 228 recall that, for a pure elastic model, we have always a finite time discontinuities,
 229 which is proper to the hyperbolic structure of the governing equations.

	wave1	wave2	wave3	wave4	wave5	wave6	wave7
$C_f(\pi\nu)$	8	14	18	22	26	30	33
$\phi(\text{kPa} \cdot \text{s})$	2.0	1.6	1.3	1.0	0.8	0.5	0.4
NRMS (%)	1.96	1.75	1.66	1.64	1.74	1.92	2.15

Table 3: Parameters of fluid friction and wall viscosity and the corresponding NRMS. Each wave correspond to a different run.

230 Table 3 summarizes the runs (wave 1 to 7) for different values of C_f , to-
 231 gether with the optimal value of ϕ found by optimization and the corresponding
 232 residuals of NRMS. We observe for increasing values of C_f increases that the
 233 parameter ϕ decreases. The minimal residual of NRMS achieves for wave4 and
 234 the limit cases (wave 1 and wave 7) are the worsts.

235 We plotted waves 1, 4 and 7 in Fig. 4(a). First we noticed that the disconti-
 236 nuities or shocks disappear and that the amplitude of the three waves are close

237 to the experimental data. However, in the first two seconds of the temporal se-
 238 ries, the wave-front of wave 7 is steeper than the others. This difference is more
 239 clear when we plot the power spectrum of the time series (Fig. 4(b)), which
 240 shows that the high frequency components of wave 7 are underdamped. This is
 241 because the damping effect of wall viscosity is stronger on high frequency waves
 242 while that the fluid friction does not depend on the frequency in our model. In
 243 the last part of the time records, only the main harmonic is still present, thus
 244 the difference between the three simulated waves is very small. The viscoelastic
 245 parameters estimated by the presented methods are summarized in Table 2.
 246 The values estimated by the data fitting with the 1D model fall into the range
 247 measured by the integrated approach of the pressure-radius (P-R) series data.

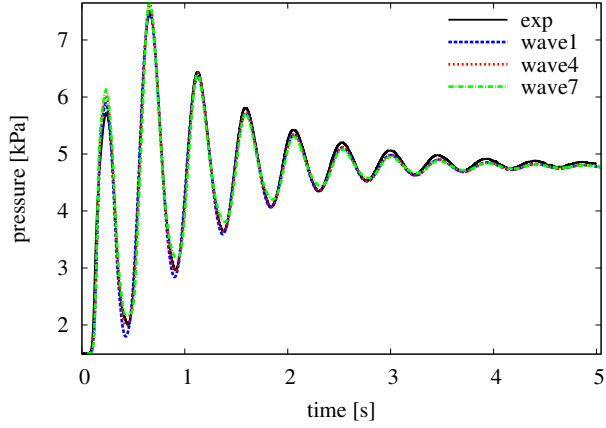
248 **3.3 Sensitivity study**

249 Fig. 5 presents the parameter sensitivity for Young's modulus E having a vari-
 250 ance of 10% around E_0 . The arrival time of each peak is significantly later when
 251 E decreases and vice versa.

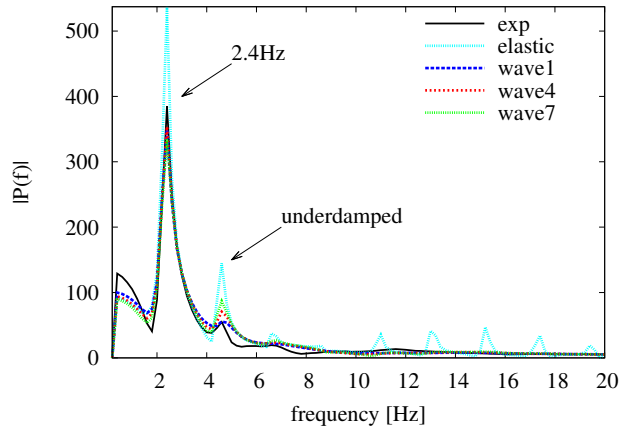
252 We also tested the sensitivity of the model to C_f , ϕ and α . For C_f and ϕ ,
 253 an uncertainty of about 20% produces a moderate variance on the predicted
 254 wave (see Fig. 6(a) and 6(b)). The sensitivity of the output to C_f and ϕ is
 255 in the same order. In contrast, when α is tested in the range from 1.0 to 1.3,
 256 there is no noticeable difference between the numerical predictions. Thus, the
 257 value of α can be set to 1.0. There exists indeed more sophisticated sensitivity
 258 techniques [29] but it is beyond the presented study.

259 **3.4 Integration schemes**

260 We tested two different integration schemes : MacCormack and MUSCL. We
 261 compared the performances for a pure elastic as well as for a viscoelastic model.
 262 In Fig. 7, we plotted the pressure waves for the numerical predictions against
 263 the experiments data at the two measurement points: left column for point A
 264 and right column for point B .



(a)



(b)

Figure 4: Experiments (line labelled exp) and simulations at measurement point A. Left (a): pressure time series. Right (b): spectrum of the pressure series (only frequencies less than 20 Hz are shown). $E = 2.08 \times 10^5$ Pa. For the elastic case, $C_f = 22\pi\nu$ and $\phi = 0$. The values of C_f and ϕ for the three viscoelastic waves are shown in Table 3.

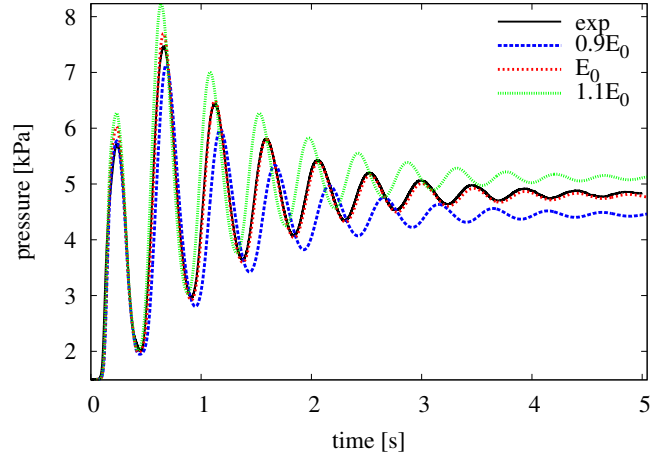


Figure 5: Sensitivity study. Pressure time series at measurement point A. E_0 is the best fit for the Young's modulus. If E_0 is perturbed 10%, the arrival time of each peak changes significantly. $C_f = 22\pi\nu$ and $\phi = 0.9 \text{ kPa} \cdot \text{s}$.

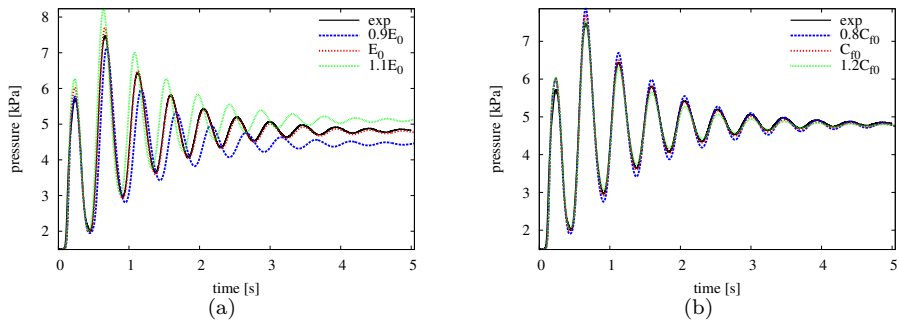


Figure 6: Time series of pressure with a 20% uncertainty of C_f (left) and ϕ (right).

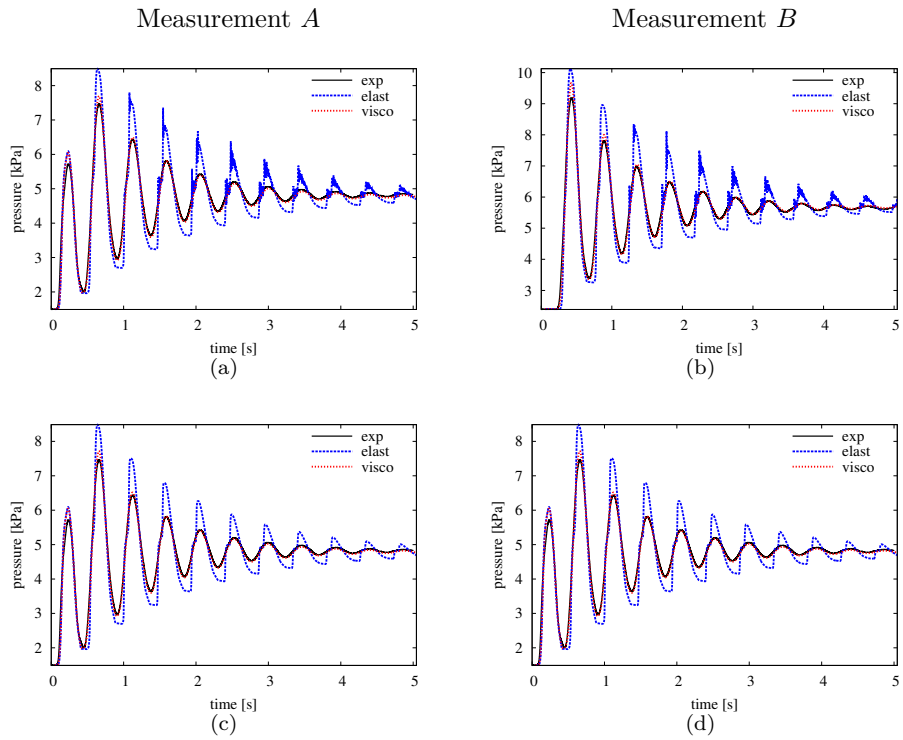


Figure 7: Pressure time series at the two measurement points with two numerical schemes. Left column: point *A*, right column: point *B*. Top row: MacCormack method, bottom row: MUSCL method. The viscoelastic model predicts much better than the elastic model at both the measurement points. The MUSCL method depresses the numerical oscillations when there are shocks. The parameters are: $E = 2.08 \times 10^5$ Pa, $C_f = 22\pi\nu$, and $\phi = 1.0$ kPa \cdot s (visco).

265 The discontinuities or shocks predicted by the elastic model are very obvious.
266 The MacCormack scheme produces numerical oscillations (top row) whereas the
267 MUSCL scheme depresses them because it includes a slope limiter (bottom row).
268 For the viscoelastic model, the shocks disappear and a much better agreement
269 is found at both locations A and B . If the solution is quite smooth, there is
270 essentially no difference between the two numerical schemes in accuracy. The
271 consistency between the two locations make us confident in the agreement be-
272 tween experiments and numerical simulations.

273 4 Discussion

274 We evaluated the stiffness and friction within a nonlinear 1D fluid dynamical
275 model with a viscoelastic law for the wall mechanics against experimental data.

276 The value of vessel stiffness estimated by the 1D model was compared to
277 values measured using a tensile test. We note that a small variance in stiffness
278 can significantly change the mean pressure, pulse pressure and wave velocity.
279 Under the operating pressure within our experiment, the nonlinearity seems not
280 large as shown in Figure 2. However, we note that the nonlinearity may be
281 more significant under physiological conditions. More studies have to be done
282 to evaluate the nonlinear elasticity of the arteries under real conditions.

283 The fluid friction and wall viscosity were fitted from experimental data using
284 the 1D model. We obtained good agreement between the 1D model results and
285 experiments. If experimental uncertainties are considered, it can be estimated
286 that $C_f = 22 \pm 4\pi\nu$ and $\phi = 1.0 \pm 0.3$ kPa·s (determined by the runs wave3 and
287 wave5 in Table 2). Our results confirm that in cases of blood flow with a similar
288 characteristic Womersley number, the Poiseuille model underestimates the fluid
289 friction (see e.g. [23]). The widely used value $C_f = 22\pi\nu$ in large arteries is
290 then acceptable. However, in smaller arteries, the Womersley number can be
291 less than one, so a parabolic velocity profile is more likely to appear, which
292 implies that C_f decreases to $8\pi\nu$. Thus the friction term should vary through

293 the whole cardiovascular system and a smaller value of C_f should be considered
294 if the Womersley number is smaller.

295 In our experimental study, the frequency of the main harmonic is 2.4 Hz
296 (see Fig. 4(b)) and thus the Womersley number is about 15.5. This value is
297 only slightly bigger than the Womersley number at the ascending aorta which is
298 13.2 [10]. Under *in vivo* conditions, the wall viscosity is much larger as measured
299 by Armentano et al. [4]. However the surrounding tissues of the vessel such as
300 fat may also damp the waves attributed wall viscosity. The viscoelasticity of
301 the arteries is mostly attributed to the collagen and elastin fibers in the wall,
302 which is different from the polymer tube.

303 The viscoelasticity of the wall dampens the high frequency components of the
304 wave, thus the waveform is not very front-steepened, which has been pointed
305 out by many previous studies (see e.g. [1, 11]). A perturbation of 20% on
306 wall viscosity introduces moderate variances on the pressure waveform, which
307 is similar to the fluid friction (see Fig. 6(a) and 6(b)). The output of the 1D
308 model is not very sensitive to uncertainties of the two damping factors. Thus it
309 is possible to use general values of those two parameters even in patient-specific
310 simulations with the 1D model.

311 We solved the nonlinear 1D viscoelastic model with MacCormack and MUSCL
312 schemes. The elastic model predicts shocks, which are captured by the MUSCL
313 method without non-physical oscillations.

314 Some limitations of our approach are : while the flow rate may be simi-
315 lar, material properties are likely different and the *in vivo* (invasive) pressure
316 measurements could hardly to including in a clinical protocol. One could ad-
317 vance that in real arteries under normal physiological conditions, discontinu-
318 ities or shocks are not present but in pathological situations (anastomoses,
319 artheromes) or after surgeries (i.e. stent) the discontinuities on the Young's
320 modulus of the arterial wall can lead to flow discontinuities. Concerning the
321 boundary conditions, arteries never display this type of vessel ending but it is
322 not unreasonable to image a clinical protocol with a short stopping blood flow

323 to observe localized backward waves.

324 **5 Conclusion**

325 We studied and evaluated the parameters of the nonlinear 1D viscoelastic model
326 using data from an experimental setup. The 1D model was solved by two
327 schemes, one of which is shock-capturing.

328 The value of vessel stiffness, estimated by the 1D model was consistent with
329 values obtained by an integrated method using experimental data (pressure-
330 radius time series) and tensile tests. The fluid friction and wall viscosity were
331 fitted from data measured at two different locations. The estimated viscoelas-
332 ticity parameters were consistent with values obtained with other methods. The
333 good agreement between the predictions and the experiments indicate that the
334 nonlinear 1D viscoelastic model can simulate the pulsatile blood flow very well.
335 We showed that the effect of wall viscosity on the pulse wave is as important as
336 that of fluid viscosity.

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342 **Conflict of interest**

343 All the authors have been involved in the design of the study and the inter-
344 pretation of the data and they concur with its content. There are no conflicts
345 of interest between the authors of this paper and other external researchers or
346 organizations that could have inappropriately influenced this work.

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