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## Angular Alignment and Control of SNDICE with the Canada-France-Hawaii Telescope

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## 1 Position of the problem

The SNDICE calibrated light source ("LED-head") is mounted on an altitude-azimuth (alt-az) motion, itself installed on the south-east platform fixed to the CFHT superior part of the dome in a overhanging position. The altitude and the azimuth motors are hold together by a right angle bracket. The azimuthal plate has been shimmed horizontally using a level. Both motors have an elementary step of $1^{\circ} / 400=9$ ". The optical axis of the LED-head is materialized by a parallel beam, the "artificial planet" (AP), which angular aperture is 40 ".


Figure 1: The artificial planet : left) centered inside full MegaCam field; right) defocussed and zoomed (x16). A 40" circle indicates planet size and arrows shows the resolution of horizontal and vertical motion

We have to coordinate the motions of the telescope and of SNDICE in order to center the light beam of the LEDs on the camera. This is defined by centering the AP on MegaCam focal plane as seen in Figure 1. A full calibration of the instrument includes a scan of the telescope aperture by the LED beams. It consists in a sequence of coordinates moves where successive positions are characterized by the position of the beam -a 30 cm spot- on the CFHT mirror and the corresponding orientation of the telescope with its optical axis parallel to the SNDICE axis. In addition this operation can be repeated at any position of the dome (defined by a rotation angle between $0^{\circ}$ and $360^{\circ}$ ). The dome rotation is controled with a $\approx 0.2^{\circ}$ accuracy and $\mathrm{a} \approx 0.02^{\circ}$ precision, while the resolution of SNDICE motion allows a $\approx 0.002^{\circ}$ alignment with CFHT. This precision is much higher than really needed for calibration, but we have already seen that it can be useful for other uses of SNDICE and it constitutes a good check of the system.

## 2 First order solution

At first order there is a very simple solution to our problem:

1. we transform by software the CFHT coordinates from their equatorial system into a virtual $\mathbf{a l t}_{\mathbf{1}} / \mathbf{a z}_{\mathbf{1}}$ system (in fact this feature exists already in the telescope control system.)
2. the motion of the dome moves sndice in the CFHT alt-az system on a parallel (at altitude $\approx 60^{\circ}$ ). Sndice azimuth is equal to the dome angle (defined as the azimuth of the vertical symmetry plane of the dome) plus a constant fixing the position of AP on sndice platform relative to the dome.
3. the orientation of sndice in CFHT system is obtained by combining the dome rotation and the own sndice $\mathbf{a l t}_{\mathbf{2}} / \mathbf{a z}_{\mathbf{2}}$ motion relative to the dome.

Our method is based on the idea that, the telescope control beeing precise and well checked, we have to situate sndice AP in the telescope coordinate system using the megacam large field of view. The centering of the AP on megacam depends only on the orientation of sndice relative to the telescope (not on their relative positions). The centering condition is expressed by the constancy of the altitude sum ( $\mathrm{alt}_{1}+\mathrm{alt}_{2}$ ) and azimuth sum $\left(\mathrm{az}_{1}+\mathrm{az}_{2}\right)$, expressing their "antipodal" nature (when the telescope is looking up to the right of sndice, sndice has to look down to the left of the telescope center as shown in Figure 13). This angular relation is simple and easy to check. Conversely computing the impact position of the sndice beam on the telescope mirror depends on geometrical details. We have developed an intrinsic autocollimation method which uses the spot produced on MegaCam by the reflexion of the AP on the focal plane followed by its reflexion on the camera optics. It yields a vector proportional to the angle of incidence of the AP beam on the CCDs, which is proportional to the vector defining the impact of the AP beam on the telescope mirror.

## 3 Analysis of febuary AP scan

We have taken well centered AP exposures (07-09 febuary) by illuminating the mirror at various angles within Megacam field (defined initially by the analysis of the survey data found in Section 8). Two sets of data have been taken at the privileged position of the dome corresponding to sndice placed at north $\left(\mathrm{az}_{1}=0^{\circ}\right)$ and south $\left(\mathrm{az}_{1}=180^{\circ}\right)$ of the telescope. The scan was performed along constant azimuth or constant altitude lines (see Figure 2), which, in north and south positions, correspond nearly to a constant right ascension (RA) or a constant declination (dec).

### 3.1 Angular analysis

A constant altitude line, e.g. alt $=56^{\circ}$, permits to fix a common altitude origin for the telescope and for the SNDICE angular systems. In our case this was done by fixing the origin of the SNDICE vertical scale (motor step count corresponding to the horizontal plane). The same statement applies to the azimuth line ( $\mathrm{az}=3^{\circ}$ ). These constants are determined independently for north scan and south scan. The difference in the origin of


Figure 2: Altitude/Azimuth scan for two dome orientations (north and south). The altitude and azimuth obtained for SNDICE after alignmenta on the telescope are matched with those set for CFHT (respectively o and *). SNDICE angles are computed with a 400 motor-unit/ degree scale and an offset fixed to match CFHT at alt $=56^{\circ}$ \& $\mathrm{az}=3^{\circ}$. For one third of the data the AP was exactly centered. For the other two third, the angles were corrected using the actual AP position within the Megacam field. The azimuthal bias of SNDICE as a function of the altitude is studied later.
a. alignment means centering artificial planet in MegaCam field
the altitude scale is $0.1^{\circ}$ (which could be explained by a $0.05^{\circ}$ angle between the axis of rotation of the dome and the «geographical» vertical of the telescope). The difference in the origin of the altitude scale is $0.4^{\circ}$ (which could be explained by the positioning error of the dome, although the repeatability of the dome position between the two consecutive nights of measurement in the north position was $0.01^{\circ}$ ).

It yields also some consistency checks:
a. the fit of alt=56 line in alt-az plane (cf.Figure 3) shows that the horizontal motor of SNDICE has been levelled better than was required $\left(0.01^{\circ}\right)$. North and south scans agree on that point.
b. the fit of $\mathrm{az}=3^{\circ}$ line in alt-az plane shows that the vertical motor of SNDICE is not orthogonal to the horizontal one, as it should be by construction. North and south scans agree on a second degree parametrization of the residual azimuthal shift as a function of altitude shown on Figure 4, that we shall evaluate in next section.
c. the azimuth and the altitude scale factor is measured precisely $\left(<10^{-3}\right)$ by comparing extreme values of the angular scan, which have been repeated several times. Then all turn out to be consistent with the same $2.5 \times 10^{-3}$ degree/step for both motions.
d. our «naive» estimation of the angular precision of our alignment, taken from residual of empirical fits such as Figure 4, is around $0.003^{\circ}$, which compatible with the resolution of the motors and the AP «eyeball» measurement and is a good surprise considering possible effects of mechanical stability (vibrations, mechanical play, thermal expansion, etc...).


Figure 3: The fit of the difference between SNDICE altitude and CFHT's as a function of azimuth yields the relative angle of horizontal planes from both systems.


Figure 4: The fit of the difference between SNDICE azimuth and CFHT's as a function of altitude yields a large relative angle of vertical planes from both systems. This seems in contradiction with the fact that horizontal and vertical motors are hold together by a right angle bracket.

### 3.2 Analysis of the AP beam parallax on the telescope

The two angles of incidence $\theta_{\mathrm{x}}$ and $\theta_{\mathrm{y}}$ of the AP beam on the focal plane are estimated by the coordinates $\Delta \mathrm{x}$ and $\Delta \mathrm{y}$ of the vector joining the centers of the AP spot and its satellite spot (resulting from the AP beam reflection on a CCD followed by its reflection on a window). The correlation between $\Delta x$ and $\Delta y$ and the azimuth and the altitude of the telescope yield the parallax - the distance between the axis of the telescope and the axis of SNDICE- (which has an x and a y component). Figure 5 shows a perfect linear correlation due to the particular geometry of north and south position. (The altitude is directly related to the declination and the declination axis is horizontal). Therefore the


Figure 5: The correlation between the altitude of the telescope and the angle of incidence $\theta_{\mathrm{y}}$ of the AP beam projected on the vertical plane yields the altitude of SNDICE (abscissa at origin). The slope has been fixed to 1.0 on the left plot by fixing the coefficient transforming $\Delta y$ (pixel number) into $\theta_{\mathrm{y}}$ (degree). The absence of spread is explained by the fact that for north and south positions the instant rotation center of the telescope is fixed in the vertical plane.
altitudes at normal incidence ( $58.83^{\circ}$ north and $59.63^{\circ}$ south) are precisely determined. Their difference being as big as $1.2^{\circ}$, it cannot be explained by an inclination of the artificial planet orbit on the horizontal plane of CFHT seen previously which is $0.11^{\circ}$. We consider the two AP lines of sight -north and south- issued from the point $\Omega_{2}$ (known from blueprints and reported in ). A quick check with survey data is drawn in Figure 6. It finds a radial location of SNDICE compatible with the survey at the centimeter level on an 8 meter distance and it yields a 24 cm parallax between the dome and the telescope.

When playing the same game with azimuthal coordinates, we do not find as good a correlation between telescope azimuth and the angle of incidence $\theta_{\mathrm{x}}$ as for altitude and $\theta_{\mathrm{y}}$ in Figure 5. Nevertheless we measure the AP azimuth at normal incidence for north and south dome positions to be respectively $-0.20^{\circ}$ and $180.19^{\circ}$. These two numbers are precise to $0.01^{\circ}$, but they cannot help fixing the geometrical model because the dome motion encoding is not sufficiently accurate. We can either take a mean value for the azimuth of SNDICE relative to dome motion such as Az (sndice) $=102.00^{\circ}$ ans say that either the actual dome azimuth were $258-0.20^{\circ}$ and $78+0.19^{\circ}$ instead of $258^{\circ}$ and $78^{\circ}$, or say that dome azimuth are strictly $258^{\circ}$ and $78^{\circ}$ but that the telescope axis is displaced by 2.7 cm to the east of dome axis, yielding a $0.20^{\circ}$ bias on azimuth of north and south sndice positions (as drawn on Figure 6 left)

Now we can compare the angles of incidence of the AP beam on the focal plane with the telescope and the SNDICE aiming angles. In Figure 7 we see that they are closely related. The difference is due to the parallax created when moving the telescope (because its polar and its declination axes do not intersect). It is a second order effect


Figure 6: Schematics of the surveying of SNDICE using CFHT as a theodolite compared with the laser survey of Figure 15. The radius of SNDICE «orbit» is $786.3=(798.9+773.7) / 2$ compared to laser's $796 . \mathrm{cm}$. A 12.6 cm southward offset (and 2.7 cm eastward) of the telescope axis relative to the dome axis reproduce angular measurement mismatches (see left drawing).


Figure 7: For the north and the south AP scans, the angular coordinates of the telescope in the local tangent SNDICE system $(\mathrm{O})$ are compared to the angle of incidence $\left(^{*}\right)$ on the focal plane determined using the double reflection AP spot. The origin of coordinates $(\oplus)$ corresponds to the normal incidence (virtual ray joining the center of the mirror to SNDICE AP led -not physical since it traverses the camera).

## 4 Second order effects

## 4.1 the skewed vertical axis of SNDICE

We parametrize the large bias seen in Figure 4 by introducing a skew angle $\pi / 2+\delta \theta$ between a vertical plane of SNDICE (az2=constant) and its horizontal plane (perpendicular to the azimuth axis, i.e. alt $2=0$ ). The small effect seen in Figure 3 is parametrized by a skew angle $\delta \phi$ between the SNDICE horizontal plane and the CFHT horizontal plane. A simple formula taken from spherical trigonometry represents both skew effects.

Let us draw the right spherical triangle ABC generated by the CFHT-horizon AB ( $2 \mathbb{H}_{1}=0$ ), a CFHT-meridian AC ( $\partial z_{1}=$ constant) and a SNDICE-meridian BC. Using the usual notation of spherical trigonometry, we have the relation (angles in radian):
$\tan B=\tan b / \sin c$, where $\mathrm{B}=\pi / 2-\delta \theta$ ( $\delta \theta$ is the vertical skew angle), yielding $a z=c=\operatorname{asin}(\tan \delta \theta \times \tan$ alt $)$
The same formula applies to the horizontal skew, with BC
 being the SNDICE equator and $\mathrm{B}=\delta \phi$. The altitude bias plotted in Figure 3 as a function of azimuth $a z_{1}$ is: $2 \mathbb{t}_{2}-a \mathrm{It}_{1}=\mathrm{AC}=\mathrm{b}=\operatorname{atan}(\sin \mathrm{c} * \tan \mathrm{~B}) \approx$ $B * \sin C=\delta \phi * \sin a z_{1}$.
Numerical application to Figure 3 yields an horizontal skew angle equal to $0.15^{\circ}$ for North scan and $0.27^{\circ}$ for south scan. Application to Figure 4, yields an excellent single parameter fit of $\delta \theta$ in the expression of $\partial z(a \mathrm{~J} t)-\partial z(56)$ shown in Figure 8. The dispersion


Figure 8: Fit of the vertical skew of Figure 4 with the analytical single parameter $(\delta \theta)$ formula taken from spherical trigonometry. Note that the effect of a $\delta \theta=1.38^{\circ}$ skew angle is a $5.7^{\circ}$ inclination of the vertical axis in the tangent reference system, because of the high altitude of SNDICE $\left(60^{\circ}\right)$.
of the residuals is 12 " (RMS) for a motor step of 9 " and a AP size 40 ".

## 4.2 the CFHT parallax

The parallax due to the fact that the polar axis and the declination axis are at a 60 cm distance is only seen by comparing the aiming angle and the incidence angle in Figure 7. If parallax is proportional to the aiming angle it just enters in a global proportionality constant. As it is seen we have chosen the north and south position to minimize parallax effects. We need east and west positions to test a parallax model. Let us just recall how to compute parallax effect.
The optical axis is subtended by 1) the polar axis, 2) the circle $\Gamma$ (radius $a=60 \mathrm{~cm}$ ) orthogonal to the polar axis and centered on it (the center is $\Omega_{1}$ in Figure 6 and Figure 15). The declination and the hour angle fix the position of the points of the optical axis on these two loci. The mirror is at a fix distance from the circle $\Gamma(1.915 \mathrm{~m})$ and from the prime focus ( 11.585 m ). The geometrical construction of the AP-ray of normal incidence on the focal plane defined earlier is as follows:

- given a dome azimuthal angle we find point $\mathrm{O}\left(\phi_{\mathrm{L}}\right)$ of the SNDICE-orbit (all in mm).

$$
\overrightarrow{\Omega_{1} O}=\overrightarrow{\Omega_{1} \Omega_{2}}+\overrightarrow{\Omega_{2} G}+13210 \cdot \vec{z}+7960 \cdot \vec{u}
$$

$$
\begin{gathered}
\overrightarrow{\Omega_{1} \Omega_{2}}=a \cdot \vec{g} \\
\overrightarrow{\Omega_{2} G}=240 \cdot \vec{k}+27 \cdot \vec{h}
\end{gathered}
$$

- take the plane P defined by the polar axis and O . The unit vector perpendicular to $\mathrm{P}, \vec{x}^{\prime}$ in Figure 14, is computed by the cross-product of polar vector $\vec{n}$ and vector $\mathrm{O} \Omega_{1}$.
- the normal incidence AP-ray is $\mathrm{O} \Omega_{2}{ }^{\prime}$, where $\Omega_{2}{ }^{\prime} \in \Gamma$ is the upper point common to P and $\Gamma$. It is constructed using the relation $\Omega_{1} \Omega_{2}^{\prime}=\mathrm{a} \vec{x}^{\prime} \wedge \vec{n}$ and the previous result (vector $\mathrm{O} \Omega_{1}$ ).
- the point $\Omega_{2}{ }^{\prime}$ defines the hour-angle HA (north dome position gives $\Omega_{2}$ situated in the vertical plane $\mathrm{HA}=\Omega_{2} \Omega_{2}{ }^{\prime}$ ) and the point D (intersection of polar axis and $\mathrm{O} \Omega_{2}{ }^{\prime}$ ) gives the declination $\operatorname{dec}=\operatorname{atan}\left(\mathrm{D} \Omega_{1} / \mathrm{a}\right)$.

Those familiar with geometry, will notice the properties of the bundle constituted of positions of the telescope axis recalled above, which favor RaDec variables over AltAz when studying the parallax effect. They explain why parallax is not seen in Figure 5. In the case of north and south dome position shown in this figure Alt and Dec are confused. These properties are also seen in Figure 9. One sees that the apparent altitude of CFHT center seen from SNDICE is constant for positions around north and south and its apparent azimuth varies less rapidly around south than north position.

## 5 Conclusions

We have checked the stability of the SNDICE attitude during an exposure (seconds) or from one exposure to the next (for several hours) and then we tested its reproducibility from one night to the next. We have reached a 3" precision, using the parallel beam of the artificial planet and the megacam frames packed at a $16 \times 16$ pixel scale. The accuracy of the SNDICE angular motion was measured using the CFH telescope. It is just at the 9 " level corresponding to a motor step. Starting from scratch it was possible to align the optical axis of SNDICE with the telescope axis by catching the AP in the megacam field first, then centering it by trial and error. This pointing process can be automatized using


Figure 9: apparent azimuth (left) and altitude (right) of CFHT center seen from SNDICE
a geometrical model which is able to perform ray tracing at a 10 " precision between SNDICE and CFHT. The point where an AP ray impinges upon the mirror is located within millimeters and it can be correlated with the angle of incidence on the focal plane which is measured using a satellite spot on megacam resulting from a reflexion of the AP beam on the CCDs followed by a reflexion on a window. This last measurement permits to locate by triangulation the position of SNDICE in the CFHT coordinate system 10 times more precisely than allowed by the a-priori knowledge of the dome position (millimeters instead of centimeters). These results lead to an improved pointing model, however this model cannot be completed with the presently limited set of data (only two orientations of SNDICE with respect to the telescope). The two main difficulties encountered in the analysis are respectively:

1) the «vertical» plane of the SNDICE motion is not orthogonal to its horizontal one ( $1.38^{\circ}$ skew angle)
2) the CFHT equatorial motion has no rotation center ( 60 cm distance between both axes).
The first problem favoring the use of alt-az angular coordinates and the second one favoring RA-dec coordinates, we choose a system using both alt-az and RA-dec interfaces.

## 6 Propositions for the motion control of the SNDICE system

When we try to decompose SNDICE motion control into elementary steps, we come to the following list:

- set dome angle
- move AP (LED) (alt-az or RaDec)
- check AP
- center AP
- check alt (Dec) motion (strt,step)
- check az (RA) motion (strt,step)
- center system
-prepare transforms; prepare slow control panel
-computes SNDICE and CFHT motions
-read AP.fits, perform slow-control (reset?Y/N.)
-iterate move+check
-iterate move+centered AP along angular ramp
-same
-iterate move+centered AP along cross

We expect that the corresponding routines which are developped and tested offline with the existing data, could be implemented easily under the Director framework developped at CFHT
//I/I/


Figure 10: Mirror scan by varying declination at constant HA : rotation around $\Omega_{2}{ }^{\prime}$


Figure 11: Mirror scan by varying HA at constant dec: rotation around $\Omega_{3}\left(\Omega_{2}{ }^{\prime} \Omega_{3}=0.6 \mathrm{~m} /\right.$ cos(dec))

## 7 Appendix A: SNDICE geometry

We have to relate three different motions:

- CFHT equatorial motion with two axes which do not intersect: Right Ascension (RA) and Declination (Dec)
- Dome rotation with one vertical axis: Dome Azimuth (DA)
- SNDICE alt-az motion with two axes: one vertical and one horizontal

They act on three solid bodies (see Figure 12):

- Telescope relative to ground
- Dome relative to ground
- SNDICE relative to dome


Figure 12: Schematic top-view and side-view of the SNDICE setup at the dome «garage» position (opening oriented north)

We should distinguish between the effect of a given rotation on the orientation of a solid body ("attitude") and the effect on its position. The analytic formulation of the problem is done in $\S 7.2$ after introducing in $\S 7.1$ a reference trihedron for each solid. With the complexification of the model, the dome rotation angle is no more the azimuth of SNDICE.

### 7.1 Conventions

$\Omega_{T}$ is the telescope rotation center, $\mathrm{O}_{L}$ is the LED source rotation center, $\Omega_{P}$ is the platform rotation center, $\overrightarrow{A_{T}}$ is the telescope axis, $\overrightarrow{A_{L}}$ is the LED source axis, $\overrightarrow{A_{D}}$ is the dome axis, $\phi_{L}$ is the LED azimuthal angle (platform+dome angles), $\vec{r}_{T}$ is the LED impact vector in telescope system $\left(\overrightarrow{A_{T}} \perp \overrightarrow{r_{T}}\right)$
Problem: express $\overrightarrow{A_{L}}$ as a function of $\overrightarrow{r_{T}}$ and then $\overrightarrow{A_{T}}$


Figure 13: 3-D view of trihedra attached to telescope, dome and sndice. The sndice centering condition, i.e. sndice optical axis $A_{L}$ parallel to telescope axis $A_{T}$, is shown. The present motion model considers the trajectory of sndice center $\mathrm{O}_{\mathrm{L}}$ as a horizontal circle slightly off-centered respectively to the telescope (anyway the position of the telescope center $\Omega_{\top}$ depends on its direction). The parametrization is given by the position of the projection G of the circle on the horizontal plane passing through $\Omega_{2}$ as shown in Figure 6.

### 7.2 Coordinates systems

Figure 14 shows the transformations from one coordinate system to the next.


Figure 14: Sequence of rotations able to center sndice on cfht focal plane. An extra rotation of the telescope around its own axis (a «roll», depending only on dome orientation) would be needed to fix the sndice field orientation on telescope focal plane.

## 8 Appendix B: Survey of CFHT , dome and SNDICE positions

### 8.0.1 CFHT and SNDICE plans

The mechanical blueprints of CFHT and the sndice logbook have been examined in order to fix the positions of the rotation axes with respect to the reference trihedra introduced above. These data are represented in Figure 15.

### 8.0.2 Horizontal plane measurements

A few measurements have been done on jan 24th, with a meter tape and a laser distancemeter by T.Benedict and P.Repain.
Starting by the horizontal plane survey shown in Figure 16 they are :
-dome diameter --> radial coordinate for inner platform corner $=621 \pm 0.5 \mathrm{~cm}$ (dome center at midpoint).
-corner-to-corner distance between east and west platforms --> azimuthal position of platforms in the dome (a platform subtend $20^{\circ}$ at azimuth $-20^{\circ}$ ). This is conform to the azimuthal sector structure of the dome (total platform width not measured see Figure 15).

### 8.0.3 Vertical plane measurements.

Shown in Figure 17, we find :
-elevation above main floor $=1970 \pm 5 \mathrm{~cm}$
-platform lower guard bar is $1970+58$ and upper guard bar $1970+109 \mathrm{~cm}$
-distance measured from Led position to the temporary Clap position is 1416 cm compatible with other survey data and conveniently close to $\mathrm{f}_{\mathrm{CFHT}}=13.5 \mathrm{~m}$.
In Figure 17, we show in magenta CFHT pointing at a Sndice beam (red) which falls at $\mathrm{r}_{\text {mirror }}=1.3 \mathrm{~m}$

Conclusion: we have the data needed :
a) for pointing the LEDs on the mirror, we need

- LED altitude $=60^{\circ}$
- distance rotation-center-to-LED $=15.4 \mathrm{~m}$
b) for pointing the telescope on the LEDs, we need
- Led's azimuth $=-18^{\circ}$ ).


Figure 15: SNDICE : Origins \& parallaxes taken from original CFHT blueprints and P.Repain notes.


Figure 16: Horizontal plane measurements:blue for laser, black for meter, yellow for geometrical computation North is the symmetry plane of cupola (with this definition our platform is south-east, with LEDs at azimuth $108^{\circ}$ and altitude $59.1^{\circ}$ )


Figure 17: Vertical plane measurements

