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Kinetics of precipitation of non-ideal solid-solutions in a liquid environment.

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We present a theoretical formalism which, for the first time, accounts for the nucleation, growth and/or redissolution of binary non-ideal solid-solutions, whether mineral or bimetallic, in solution. It yields the time evolution of all ion activities, together with the particle population characteristics: number, size and composition profile of particles as a function of time and of their time of nucleation. It is shown that depending on the Guggenheim parameter values which drive the non-ideality of the solid-solution, on the ratio of the solubility products of the end-members and on initial conditions, different scenarios of precipitation may take place, in which particles display composition profiles which may be smooth or discontinuous. An illustration of the characteristics of precipitation in the various scenarios is given, by simulations performed under some simplifying assumptions and qualitative predictions are made for the precipitation of some mineral solid solutions of geochemical interest. To our knowledge, this is the first time, in the fields of both geochemistry and metallic alloys, that these out-of-equilibrium precipitation processes of non-ideal solid-solutions are fully described.

Highlights:

- Theoretical and numerical model for out-of-equilibrium precipitation of non-ideal solid solutions in a liquid medium.
- For the first time, nucleation, growth and/or redissolution processes are fully described.
- Particle size distribution functions and composition profiles are obtained.
- The formalism applies to mineral $A_{1-x}B_xC$ as well as bimetallic $A_{1-x}B_x$ nanoparticle formation.
- Four scenarios are highlighted; predictions for the precipitation of mineral solid solutions of geochemical interest are made.

Keywords: nucleation and growth, non-ideal solid solutions, miscibility gap, clay minerals, bimetallic nanoparticles, alloy nanoparticles, core-shell nanoparticles, wet-chemical synthesis, kinetic simulation, Nanokin code.

PACS numbers:

I. INTRODUCTION

In natural water-rock interaction systems on the Earth surface, primary minerals are often in a thermodynamic nonequilibrium state. This is the key-condition for the alteration which takes place in the water cycle, including both weathering processes near surface and hydrothermal alteration at depth. The resulting spontaneous dissolution of primary minerals leads to the formation of secondary minerals which are generally not defined compounds but often solid solution phases (SS), with compositions that adjust to the evolution of the chemical composition of the aqueous solution (AS). The most frequent example of this property is the formation of clay mineral phases in the alteration of rock-forming silicates (Millot, 1970; Meunier and Velde, 1989), but many oxides, carbonates, and sulfates also share this property (Drever, 1984; Geiger, 2001; Rhada and Navrotsky, 2013).

In another context, formation of bimetallic nanoparticles is often the aim of wet chemistry experiments in the laboratory, due to their interesting properties for plasmonics (Major et al., 2009), catalysis (Zhang et al.,

2011) or electrocatalysis (Peng and Yang, 2009) applications. Similarly to their mineral counterparts, these alloys may display an ideal SS behavior, or, alternatively, a tendency towards ordering or demixing (phase separation), depending upon the sign and strength of their mixing enthalpy of formation (Ferrando et al., 2008).

The equilibrium behavior of a SS in contact with an AS, whether ideal or non-ideal, is now well established (Lippmann, 1982; Glynn and Reardon, 1990), as reviewed by Ganguly (2001) or Prieto (2009). The relationship between the SS composition and the distribution of ions in the AS may be represented by the classical Lippmann's or Roozeboom's diagrams (Lippmann, 1980; Roozeboom, 1904). Recent theoretical studies of non-ideal mineral SSs at equilibrium mainly concern cements and concretes interacting with the AS, particularly in the field of nuclear waste storages and clay barriers (Börjesson et al., 1997; Walker et al., 2007) but carbonate SSs have also been considered (Kulik et al., 2010; Katsikopoulos et al., 2009).

As far as the kinetics of SS formation are concerned, experiments making use of counter-diffusion of reactants

through a porous medium (Prieto et al., 1997; Sánchez-Pastor et al., 2006) or in situ atomic force microscopy studies of the growth of SSs in a fluid cell (Pina et al., 2000; Putnis et al., 2002; Astilleros et al., 2003; Astilleros et al., 2006) have provided important information on growth mechanisms and particle composition for various mineral SSs.

Inclusion of kinetic effects in the modeling of a SS formation still remains a difficult task. In the water-rock interaction model KINDIS (Madé et al., 1994) and its extension for treating reaction and transport (Nourtier-Mazauric et al., 2005), kinetic dissolution and precipitation at equilibrium of ideal SSs were included but without considering nucleation and growth. In these works, a single SS was allowed to precipitate for a given set of end-members, corresponding to the least soluble phase or, equivalently, to the phase with the highest supersaturation. More recent approaches rely on empirical rate equations, not considering explicitly nucleation, size-dependent growth and nucleation (Shtukenberg et al., 2010, Brandt et al., 2015). The same was true in the coupled reaction and transport model by Lichtner and Carey (2006) who represented the SS by a discrete set of stoichiometric solids with fixed composition. On the other hand, atomistic Monte Carlo simulations of ideal SSs under constant supersaturation have specified how the distribution coefficients vary with the supersaturation at kink, step and terrace sites of the growing particles (Matsumoto and Kitamura, 2001; Matsumoto et al., 2005). Only in the work of Pina and Putnis (2002) did a generalized expression for the nucleation rate appear, and the composition of the critical nucleus was determined from the maximum of the nucleation frequency. However, growth and feed-back effects were not included in this work.

To our knowledge, only in our previous works (Noguera et al., 2010; Noguera et al., 2012), were the full dynamics of a SS formation fully accounted for, with the inclusion of nucleation processes, size dependent growth, particle population and out-of-equilibrium composition of the critical nuclei and deposited layers during growth. This has led to the creation of a second version of the NANOKIN code (Noguera et al., 2010), which previously could only account for the kinetics of formation of minerals with fixed composition (Fritz et al., 2009). However, this second version was restricted to ideal binary SSs.

It is our goal, in the present work, to propose a theoretical description of nucleation and growth of *non-ideal* binary SSs. As will appear clearly in the following, it does not consist in merely introducing activity coefficients in the nucleation and growth equations. Depending on the strength of the enthalpy of mixing, which will be represented by a Guggenheim expansion restricted to two terms (sub-regular SS), and depending on the composition of the AS, several scenarios may take place in which the composition profiles of the formed particles and the precipitation dynamics are distinctly different. Each of these scenarios will be exemplified by a numerical simu-

lation, under some simplifying assumptions and predictions will be made for various mineral SSs of geochemical interest to assess which scenario applies to each of them.

The formalism primarily aims at describing SSs of the $A_{1-x}B_xC$ type, relevant e.g. to mineral SSs with homovalent substitution, like $(Ba,Sr)CO_3$. However the generalization to SSs of the $A_{1-x}B_x$ type, such as bimetallic SSs, is straightforward because it only requires skipping the C activities. In that way, our work can also be useful in the field of metallic alloys in which, to our knowledge, only thermodynamic aspects of the formation of bimetallic nanoparticles in wet chemistry experiments have been considered. We will use the generic term "aqueous" solution to refer to the solution in which precipitation takes place, whether it contains water or not.

The paper is organized as follows. In section II, we introduce thermodynamic concepts which are required for describing binary non-ideal (sub-regular) SSs in contact with an AS. We propose a new representation of the stoichiometric saturation condition, distinct from the Roozeboom diagram. It is more compact than the latter and turns out to be extremely useful in understanding the scenarios of precipitation of strongly non-ideal SSs. In section III, IV and V we present the theoretical background and master equations for nucleation, growth and feed-back effects on the chemical composition of the AS, respectively. Then, in Section VI, we discuss the characteristics of the precipitation process as a function of the degree of non-ideality of the SS and the initial conditions. We highlight four possible scenarios of precipitation and we devise a diagram of their occurrence as a function of the solid and AS characteristics. Finally, we illustrate the characteristics of precipitation in the various scenarios, by simulations performed within some simplifying assumptions, and we make qualitative predictions of the precipitation characteristics of SSs of geochemical interest (Section VII), before concluding. The text is complemented by five appendices in which most of the formal equations are derived.

II. THERMODYNAMIC CONCEPTS

In this section we first recall some useful concepts relevant for a SS in contact with an AS of given composition. This will allow us to introduce quantities, such as the stoichiometric solubility product, the stoichiometric saturation state of the AS with respect to the SS, and the concept of stoichiometric saturation. Then we will discuss in detail how the latter depends on the non-ideality characteristics of the SS, which will be a useful step before addressing out-of-equilibrium processes.

We consider a SS of composition $A_{1-x}B_xC$ ($0 < x < 1$), with AC and BC its end-members. In the following, A, B and C will represent the relevant aqueous species in the AS and [A], [B] and [C] their activities, respectively. The solubility products of the end-members, denoted

164 K_{AC} and K_{BC} , are functions of the standard changes
165 in Gibbs free energy, ΔG_{AC} and ΔG_{BC} , for dissolution:206

$$\begin{aligned} K_{AC} &= \exp(-\Delta G_{AC}/RT) \\ K_{BC} &= \exp(-\Delta G_{BC}/RT) \end{aligned} \quad (1)$$

166 in which R is the gas constant and T the temperature.

167 Considering the SS as a single component stoichiomet-
168 ric solid, the change of Gibbs free energy $\Delta G(x)$ during
169 the dissolution of one mole of composition x may be writ-
170 ten as:

$$\begin{aligned} \Delta G(x) &= (1-x)\Delta G_{AC} + x\Delta G_{BC} + \Delta G_M^E(x) \\ &\quad -RT(x \ln x + (1-x) \ln(1-x)) \end{aligned} \quad (2)$$

171 The sum $(1-x)\Delta G_{AC} + x\Delta G_{BC}$ represents the change
172 of Gibbs free energy for a mechanical mixture of AC and
173 BC. It is complemented by the ideal entropy of mix-
174 ing (on the second line), assuming full disorder of the
175 A and B species in the SS. $\Delta G_M^E(x)$ is the excess free
176 energy of mixing, which includes the excess entropy of
177 mixing $\Delta S_M^E(x)$ and the enthalpy of mixing $\Delta H_M(x)$:
178 $\Delta G_M^E(x) = \Delta H_M(x) - T\Delta S_M^E(x)$. We will neglect
179 $\Delta S_M^E(x)$ which may originate from non-configurational
180 entropy (Benisek and Dachs, 2012) or deviations from
181 perfect randomness. As regards $\Delta H_M(x)$, whose varia-
182 tions with the SS composition are usually represented by
183 the Guggenheim expansion (Guggenheim, 1937), we will
184 only keep its first two terms, and thus restrict ourselves
185 to sub-regular SS:

$$\Delta H_M(x) = -RTx(1-x)[A_0 + A_1(2x-1)] \quad (3)$$

186 The two dimensionless parameters A_0 and A_1 charac-
187 terize the non-ideality of the SS. A_0 may be related to
188 first neighbor pairwise interactions. Its sign drives the
189 tendency to ordering (if negative) or to unmixing (if pos-
190 itive). When A_0 exceeds some critical value, the SS pos-
191 sesses a miscibility gap, which means a range of compo-
192 sitions in which phase separation takes place. The A_1
193 coefficient introduces an asymmetry of $\Delta G_M^E(x)$ about
194 $x = 1/2$.

195 The stoichiometric solubility product of the SS:
196 $K(x) = \exp(-\Delta G(x)/RT)$ is equal to:

$$K(x) = K_{AC}^{1-x} K_{BC}^x (1-x)^{1-x} x^x e^{x(1-x)[A_0 + A_1(2x-1)]} \quad (4)$$

197 One can deduce the stoichiometric saturation state $I(x)$
198 of the AS with respect to a SS of composition x (some-
199 times called $\beta(x)$ (Prieto et al., 1993)), equal to the ratio
200 between the ionic activity product $Q(x) = [A]^{1-x}[B]^x[C]$
201 and $K(x)$:

$$I(x) = \left[\frac{I_{AC}}{(1-x)} \right]^{1-x} \left[\frac{I_{BC}}{x} \right]^x e^{-x(1-x)[A_0 + A_1(2x-1)]} \quad (5)$$

202 In this expression, I_{AC} and I_{BC} are the saturation states
203 of the AS with respect to the pure end-members AC and
204 BC, respectively:

$$I_{AC} = \frac{[A][C]}{K_{AC}} \quad ; \quad I_{BC} = \frac{[B][C]}{K_{BC}} \quad (6)$$

$I(x)$ can also be written in terms of the activity coeffi-
cients λ_{AC} and λ_{BC} of the end-members:

$$I(x) = \left[\frac{I_{AC}}{(1-x)\lambda_{AC}(x)} \right]^{1-x} \left[\frac{I_{BC}}{x\lambda_{BC}(x)} \right]^x \quad (7)$$

The coefficients λ_{AC} and λ_{BC} depend on x and, for a
sub-regular SS, are equal to (Glynn, 1991):

$$\begin{aligned} \lambda_{AC}(x) &= e^{x^2[A_0 + A_1(4x-3)]} \\ \lambda_{BC}(x) &= e^{(1-x)^2[A_0 + A_1(4x-1)]} \end{aligned} \quad (8)$$

Thermodynamic equilibrium between the SS and the
AS is reached when simultaneously $I(x) = 1$ (equivalent
to $\Delta G(x) = -RT \ln Q(x)$), and $I(x)$ is maximum with
respect to x . These two conditions determine the compo-
sition x_0 of the SS and that of the AS (through the
values of I_{AC} and I_{BC}) at thermodynamic equilibrium.
They can be recast under the standard form:

$$\begin{aligned} I_{AC} &= (1-x_0)\lambda_{AC}(x_0) \\ I_{BC} &= x_0\lambda_{BC}(x_0) \end{aligned} \quad (9)$$

In the following, we will focus on the characteristics
of the *stoichiometric saturation state*, obtained from the
single condition that $I(x)$ is maximum with respect to
 x . Indeed, the goal of our work is to describe the *ki-
netics* of precipitation and not the thermodynamic equi-
librium between an AS and a SS, which is only found
at infinite time of the precipitation process. Using the
stoichiometric saturation amounts to considering the SS
with respect to which the AS is the most supersaturated
(Prieto, 2009). Its composition x_{st} is the solution of the
implicit equation (Appendix A):

$$\frac{I_{AC}}{\lambda_{AC}(x_{st})(1-x_{st})} = \frac{I_{BC}}{\lambda_{BC}(x_{st})x_{st}} \quad (10)$$

Such a relationship is often graphically represented in a
Roozeboom plot $(x_{st}, [B]/([A]+[B]))$ (Mullin, 1993). The
discussion which follows, based on Equation 10, paves
the route to understanding the composition of the critical
nuclei which will be the subject of the following section.

As shown in Appendix A, Eq. 10 may have one or
three roots, depending on the values of A_0 , A_1 , and on
the composition of the AS. The latter enters in a compact
way through the ratio W , which is equal to:

$$W = \frac{I_{BC}}{I_{AC}} = \frac{[B]K_{AC}}{[A]K_{BC}} \quad (11)$$

When Eq. 10 has a single root, x_{st} varies smoothly as a
function of W . When there are three roots, two corre-
spond to maxima of $I(x)$ (i.e. minima of $-\ln I(x)$) and
one to a minimum (Figure 11 in Appendix A). The com-
position x_{st} is equal to the root associated with the low-
est value of $-\ln I(x)$. In Figure 1 are represented typical
variations of the three roots of Eq. 10 as a function of W
and of the corresponding three values of $-\ln(I(x)/I_{AC})$

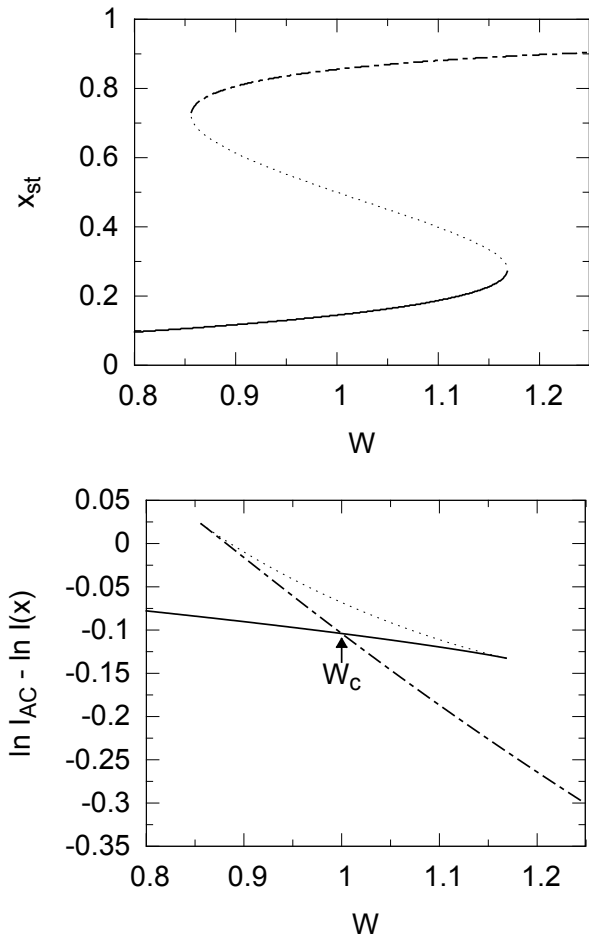


FIG. 1: Top panel: the three roots of Eq. 10, as a function of $W = I_{BC}/I_{AC}$, when $A_0 = 2.5$ and $A_1 = 0$. The full and dashed-dotted curves display the variations of the minima of $-\ln I(x)$ while the dotted one is associated with the maximum of this function. Bottom panel, corresponding values of $\ln I_{AC} - \ln I(x)$. The composition x_{st} of the SS at stoichiometric saturation corresponds to the lowest value of this function. The crossing of the curves at W_c is associated to a discontinuity of x_{st} .

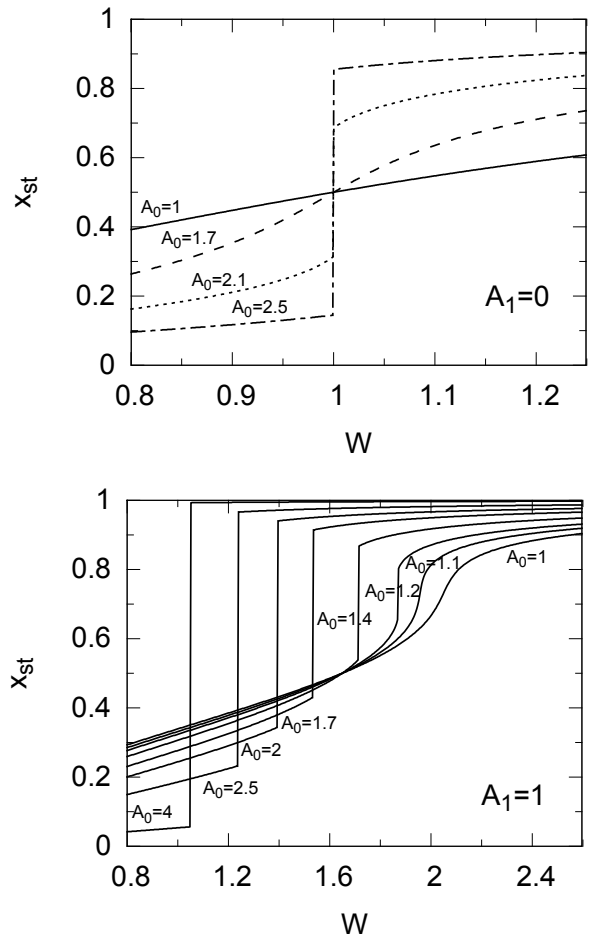


FIG. 2: Composition x_{st} of a SS at stoichiometric saturation with an AS whose composition is characterized by $W = I_{BC}/I_{AC}$, for various values of A_0 when $A_1 = 0$ (top panel) or $A_1 = 1$ (bottom panel).

in the case of a strongly non-ideal SS. Because the latter cross each other, a discontinuity in x_{st} between two values x_1 and x_2 takes place for some AS composition characterized by $W = W_c$, at which the two phases of composition x_1 and x_2 have the same Gibbs free energy per mole. When $W = W_c$, the solid phase may become spatially inhomogeneous and separate into two phases of compositions x_1 and x_2 .

Figure 2 shows the variations of x_{st} as a function of W for several values of A_0 and A_1 . In the case of regular SSs ($A_1 = 0$), x_{st} varies smoothly (single root in Eq. 10) as long as A_0 remains smaller than 2. x_{st} is less than 0.5 (which means that the SS is richer in A ions than in B ions) whenever $W < 1$ and larger than 0.5 in the opposite case. When A_0 exceeds 2, a discontinuity occurs at $W_c = 1$, whose height increases with A_0 (e.g. $x_2 - x_1 \approx 0.4$ for $A_0 = 2.1$ and 0.7 for $A_0 = 2.5$). The

symmetry of the miscibility gap about $x_{st} = 1/2$ is to be linked to the shape of the Gibbs free energy of mixing.

In sub-regular SSs ($A_1 \neq 0$), the transition between smooth and discontinuous variations of x_{st} occurs at smaller values of A_0 and the discontinuity (when it exists) takes place at varying values of W_c . x_1 and x_2 are no longer symmetric about 0.5.

The dependence of W_c on A_0 is represented in Figure 3 (top panel). W_c decreases (resp. increases) asymptotically towards $W_c = 1$, as A_0 becomes larger if $A_1 > 0$ (resp. $A_1 < 0$). On each curve, there exists a minimum value of A_0 below which the discontinuity disappears and x_{st} recovers a smooth variation as a function of the AS composition. The range of parameters $\{A_0, A_1\}$ for which x_{st} has no discontinuity lies inside the region delineated by the two curves drawn in Figure 3 (bottom panel).

Compared to the Roozeboom plot, the representation of x_{st} as a function of $W = ([B]K_{AC})/([A]K_{BC})$ that we propose presents several advantages. First it acknowledges the fact that, at constant values of the Guggenheim

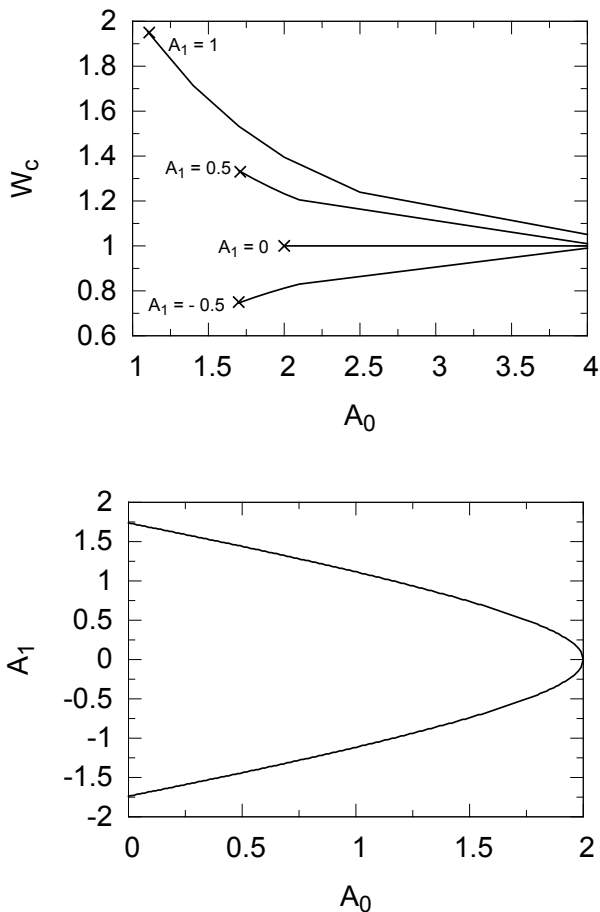


FIG. 3: Top: position W_c of the x_{st} discontinuity as a function of A_0 for $A_1=1, 0.5, 0,$ and -0.5 . At constant A_1 , the crosses mark the critical A_0 value above which a discontinuity starts taking place. This critical value is represented in the bottom panel as a function of A_1 . In the region between the two lines, Eq. 10 has a single root whatever W , which means that x_{st} varies smoothly as a function of the composition of the AS.

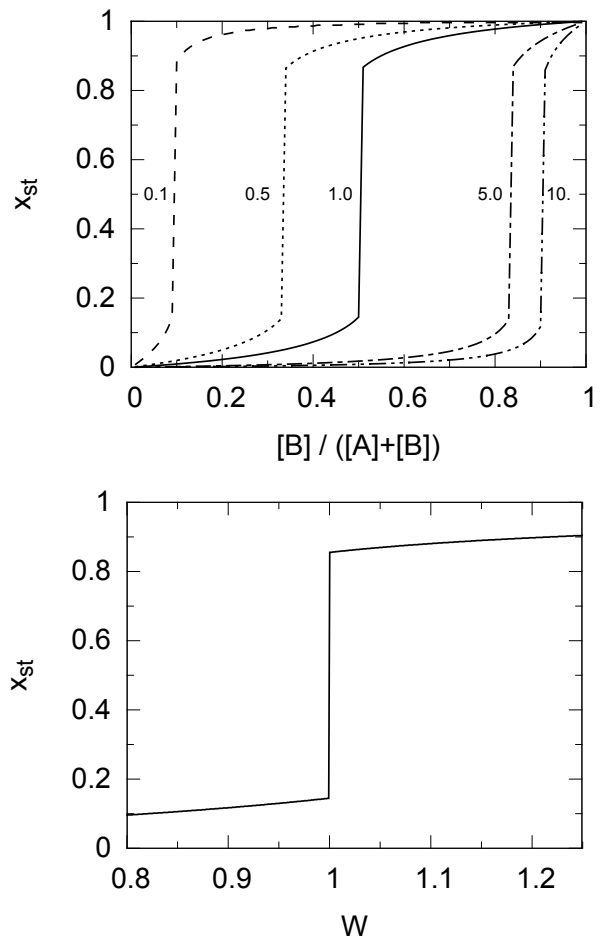


FIG. 4: Two representations of the relationship between the SS composition x_{st} and the AS composition. Top: Roozeboom plot as a function of $[B]/([A] + [B])$ for various values of K_{BC}/K_{AC} . Bottom: Representation as a function of $W = ([B]K_{AC})/([A]K_{BC})$. In both case the SS is characterized by Guggenheim parameters $A_0 = 2.5$ and $A_1 = 0$.

III. NUCLEATION

In this section and the two following ones, we extend the formalism of nucleation and growth previously established for the precipitation of minerals with fixed composition (Noguera et al., 2006a,b) and ideal SSs (Noguera et al., 2010) to the formation of non-ideal SSs. Nucleation is treated within the framework of the classical theory of nucleation, in its continuum limit (Markov, 1995; Adamson, 1960). It is described in the following for homogeneous nucleation of spherical particles. The extension to heterogeneous nucleation (i.e. nucleation of particles on foreign solids) and non-spherical particle shapes is given in Appendix B. Moreover, we send the more complex treatment of precipitation of SSs whose surface energy depends on composition to Appendix C.

Under these conditions (homogeneous nucleation, spherical particles and constant surface energy), the

parameters, x_{st} is uniquely defined by the value of W , and not separately by the ratios $[B]/[A]$ and K_{BC}/K_{AC} as in the Roozeboom representation. The latter, for which one plot is needed for each value of K_{BC}/K_{AC} , is convenient when one considers a specific system. At variance, the representation of x_{st} as a function of W is unique whatever the value of K_{BC}/K_{AC} (Figure 4). It will help highlight the generic behavior of sub-regular SSs during precipitation, which is the goal of our work. As will be shown in the next sections, a representation of the same type will be extremely useful to characterize the composition of the critical nuclei and layers deposited during growth, and to discriminate the various scenarios of precipitation.

change in Gibbs free energy $\Delta G(n, x)$ in the formation of a nucleus containing n formula units of composition x is the sum of two terms (k_B the Boltzmann constant):

$$\Delta G(n, x) = -nk_B T \ln I(x) + n^{2/3} v(x)^{2/3} X \sigma \quad (12)$$

The first (bulk-like) term $-nk_B T \ln I(x)$, with $I(x)$ the stoichiometric saturation state given by Eq. 7, represents the gain (if $I(x) > 1$) of Gibbs free energy when ions from the AS condense into a solid phase. The second term $E_s = n^{2/3} v(x)^{2/3} X \sigma$ is the total surface energy of the nucleus. In this expression, $v(x)$ is the volume of a formula unit of composition x , that will be assumed to vary linearly between its end-member values (no excess molar volume): $v(x) = (1-x)v_{AC} + xv_{BC}$. The geometric factor X is equal to $X = (36\pi)^{1/3}$ for spherical particles and σ is the surface energy per unit area.

When $I(x) > 1$, $\Delta G(n, x)$ displays a maximum as a function of n , which defines the characteristics of the critical nucleus: its size $n_m(x)$ and the barrier to be overcome for its nucleation $\Delta G_m(x) = \Delta G(n_m(x), x)$:

$$n_m(x) = \frac{2u(x)}{\ln^3 I(x)} \quad \text{with } u(x) = \frac{4X^3 \sigma^3 v(x)^2}{27(k_B T)^3} \quad (13)$$

and :

$$\frac{\Delta G_m(x)}{k_B T} = \frac{u(x)}{\ln^2 I(x)} \quad (14)$$

Assuming that the flow of nuclei through size and composition space is confined to a path through this point only (Reiss and Shugard, 1976), the composition of the critical nuclei is determined by the condition that the nucleation frequency $F(x)$ is maximum with respect to x . $F(x)$ depends exponentially on the nucleation barrier $\Delta G_m(x)$:

$$F(x) = F_0 \exp\{-\Delta G_m(x)/k_B T\} \quad (15)$$

There have been attempts to theoretically estimate the prefactor F_0 for specific systems. However, in most cases, it has resulted in huge (several orders of magnitude) discrepancies with measured values, even in the case of minerals of fixed composition. For this reason, we will assume it to be a constant, with a value that must be empirically determined. The maximum nucleation rate is thus obtained when $\Delta G_m(x)$ is minimum with respect to x , in which case the critical nuclei correspond to a saddle point in the $\Delta G(n, x)$ energy surface.

Taking these expressions into consideration, after some algebra (Appendix D), the minimization of the nucleation barrier yields the critical nucleus composition x^* , solution of the implicit equation:

$$\left(\frac{I_{AC}}{(1-x^*)\lambda_{AC}(x^*)} \right)^{v_{BC}} = \left(\frac{I_{BC}}{x^*\lambda_{BC}(x^*)} \right)^{v_{AC}} \quad (16)$$

For ideal SSs, Eq. 16 has a single root. For regular or sub-regular SSs, there may be one or three roots and one

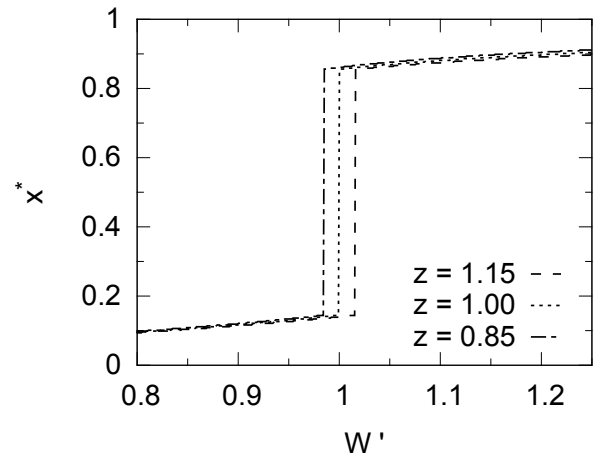


FIG. 5: Composition of the critical nuclei x^* , as a function of the ratio $W' = I_{BC}/I_{AC}^z$, for several values of $z = v_{BC}/v_{AC}$. All curves have been drawn for $A_0 = 2.5$ and $A_1 = 0$. When $z = 1$, $x^* = x_{st}$.

must determine the one which minimizes the nucleation barrier. To make a link with the previous section, one notes that, except for the exponents v_{AC} and v_{BC} , Eq. 16 strongly resembles Eq. 10, and, indeed, finding its solution(s) amounts to minimizing $-\ln I(x)/v(x)$. When $v(x)$ does not depend on x , the composition x^* of the critical nucleus is thus equal to x_{st} , determined previously. When v varies with composition, the minimization of $-\ln I(x)/v(x)$ is mathematically more involved (Appendix D). The solutions can be shown to be functions of A_0 , A_1 , $z = v_{BC}/v_{AC}$ (parameter related to the end-member structures), and of the ratio:

$$W' = \frac{I_{BC}}{I_{AC}^z} \quad (17)$$

function of the composition of the AS. Depending upon whether the Guggenheim parameter values are located outside or within the region of the miscibility gap, x^* varies smoothly as a function of the AS composition or displays a discontinuity at a value W'_c function of A_0 , A_1 and z . Usually z does not differ much from 1, especially for isomorphic end-members, because otherwise there would be no possibility to form an actual SS. As a result, in all cases, the composition x^* of the critical nucleus remains very close to x_{st} . Figure 5, for example, shows how x^* varies with W' for three values of z , in the case of a strongly non-ideal regular SS with $A_0 = 2.5$. The curve associated with $z = 1$ represents the variations of x_{st} . W'_c varies by $\pm 1.6\%$ when $z = 1 \pm 0.15$. The $\{A_0, A_1\}$ range, in which a discontinuity of x^* occurs, nearly exactly coincides with that for x_{st} (Figure 3 bottom panel) within a precision of 0.001 on the limiting values of A_0 and A_1 .

Nucleation may start as soon as $I(x^*) > 1$. However, it becomes efficient (more than one nucleus per second and liter of solution), only if $I(x^*)$ exceeds a critical value

388 $I_c(x^*)$ defined by:

$$\ln I_c(x^*) = \sqrt{\frac{u(x^*)}{\ln F_0}} \quad (18)$$

389 All quantities related to a given critical nucleus: x^* ,
390 $n^* = n_m(x^*)$, $\Delta G^* = \Delta G_m(x^*)$ and $F(x^*)$, depend on
391 the time t_1 at which nucleation occurs. This time depen-
392 dence comes from the instantaneous values of the satu-
393 ration states $I_{AC}(t_1)$ and $I_{BC}(t_1)$ of the AS with respect
394 to the pure SS end-members, entering Eq. 16 which de-
395 termines $x^*(t_1)$.

IV. GROWTH

397 Growth involves the condensation of ions from the AS
398 on the surface of the particles. A growth law which cor-
399 rectly describes such processes has to be size-dependent,
400 but its expression depends upon the rate limiting pro-
401 cess: diffusion in the liquid or the gaseous phase, contin-
402 uous interfacial effects, and two-dimensional nucleation
403 on flat faces or spiral growth (Burton et al., 1951; Baron-
404 net, 1982; Parbhakar et al., 1995). In the following, we
405 will restrict ourselves to a continuous growth mechanism,
406 limited by the incorporation of growth units at the sur-
407 face of a rough nucleus (Markov, 1995; Pina et al., 2000).
408 Furthermore, we will define an average rate of incorpora-
409 tion, so that the particle keeps the same shape as the
410 critical nucleus (Wulff or Wulff-Kaishev shape) during its
411 growth. Actually, the particle size may increase or de-
412 crease according to whether it is larger or smaller than
413 the instantaneous size of the critical nuclei. This is the
414 Ostwald ripening process (Ostwald, 1900; Lifschitz and
415 Slyozov, 1961). We will consider the two cases of positive
416 growth or redissolution (negative growth) separately.

A. Positive growth

418 The energetic cost to increase the dimensions of a par-
419 ticle may be related to its change of volume δV and its
420 change of total surface energy δE_s :

$$\delta \Delta G(x) = -\frac{\delta V}{v(x)} k_B T \ln I(x) + \delta E_s \quad (19)$$

421 In the following, we will make the assumption of local
422 equilibrium at the particle-solution interface. It amounts
423 to considering that short-range transport across the in-
424 terface is rapid enough to equilibrate the ions in the liquid
425 and solid layers in contact. It is valid provided that the
426 interface motion is slow enough (Aziz, 1988). When this
427 is the case, the chemical potentials of ions at the surface
428 of the particle are equal to those in the aqueous solu-
429 tion. The composition x of the layer which is deposited
430 is obtained from the condition that $\delta \Delta G(x)$ is minimum
431 with respect to x . The x dependence of $\delta \Delta G(x)$ is in-
432 cluded in $-\ln I(x)/v(x)$. Determining the composition

433 x of the incremental layer thus amounts to minimizing
434 this quantity. Because it is the same quantity which ap-
435 pears in the determination of the critical nucleus compo-
436 sition, the composition of the incremental layer at time t
437 is thus equal to $x^*(t)$.

Usually, and especially at low temperatures, solid state
diffusion is very slow compared to all other characteristic
times. We will neglect it, and assume that the composi-
tion of a given layer remains fixed once formed (Doerner-
Hoskins precipitation (Doerner and Hoskins, 1925)). The
particles thus display composition profiles due to the time
variation of x^* .

The dimensions and number of growth units of a par-
ticle at time t depend on two time indices: t_1 the time at
which the particle has nucleated, and t the time of ob-
servation. One has thus to write: $n(t_1, t)$ and $\rho(t_1, t)$ (ρ
the radius of the particle). At variance, the composition
of the outer layer of growing particles only depends on t ,
because it is the same for all particles.

In the regime of increasing particle size $d\rho(t_1, t)/dt >$
0, the growth equation used for particles of fixed compo-
sition can be straightforwardly generalized to SSs (Noguera
et al. 2010):

$$\frac{d\rho(t_1, t)}{dt} = \kappa \left(I(t, x^*(t)) - \exp \left[\frac{2u(x^*(t))}{n(t_1, t)} \right]^{1/3} \right) \quad (20)$$

In Eq. 20, it is the saturation index relative to the com-
position of the deposited layer, and thus relative to a SS
of composition $x^*(t)$, which must be used.

B. Redissolution

417 Whenever the right hand-side of Eq. 20 is negative, i.e.
418 whenever $n(t_1, t) < n^*(t)$, the particles decrease in size
419 ($d\rho(t_1, t)/dt < 0$). During such a redissolution stage, lay-
420 ers formed at anterior times are progressively dissolved.
421 A layer corresponding to a radius $\rho(t_1, t)$ which reaches
422 the particle/solution interface at time t , had been de-
423 posited at time t_2 such that:

$$\rho(t_1, t_2) = \rho(t_1, t) \quad (21)$$

t_2 is specific to the particle and thus depends on t_1 and
 t . At time t_2 , the layer composition was equal to $x^*(t_2)$.
Consequently, in the redissolution regime, the growth
rate reads:

$$\frac{d\rho(t_1, t)}{dt} = \kappa \left(I(t, x^*(t_2)) - \exp \left[\frac{2u(x^*(t_2))}{n(t_1, t)} \right]^{1/3} \right) \quad (22)$$

As a whole, the growth laws written in Eqs. 20 and 22
allow positive or negative growth of particles, depending
on the relative value of their size with respect to the
critical nucleus size. The process of Ostwald ripening is
thus included in the present formalism. At variance, a

476 growth law of the type:

$$\frac{d\rho(t_1, t)}{dt} = \kappa(I^p(t, x(t_1, t)) - 1)^q \quad (23)$$

477 as often assumed in the literature (Lasaga, 1984; 524
478 Parkhurst and Appelo, 1999), is unable to lead the solid 525
479 phase toward equilibrium, whatever the values of the em- 526
480 pirical exponents p and q . In the long term, it correctly 527
481 drives the saturation state of the AS towards 1 if the 528
482 feed-back effect of growth on the AS composition is in- 529
483 cluded, but because all nucleated particles survive, the 530
484 total surface energy of the solid phase is not minimized. 531
485 The lowest energy configuration (a single particle with 532
486 all available matter in it) is never reached. 533

487 V. FEED-BACK EFFECT ON THE SOLUTION 534

488 At a given time t , the particle population consists of all 536
489 the particles which have nucleated at times $t_1 < t$, and 537
490 with nucleation frequencies and sizes equal to $F(t_1)$ and 538
491 $n(t_1, t)$ respectively. The amounts $q_M(t)$ of end-members 539
492 M (M=AC or BC) which have been withdrawn from the
493 AS at time t are thus equal to:

$$q_M(t) = \int_0^t F(t_1)(n^*(t_1) - 1)X_M(t_1)dt_1 + \int_0^t F(t_1)dt_1 \int_{t_1}^t dt_3 \frac{dn(t_1, t_3)}{dt_3} X_M(t_3) \quad (24)$$

494 The first term represents the contribution of nucle- 546
495 ation, with $X_M(t_1)$ the molar fractions of the end- 547
496 members equal to $(1 - x^*(t_1))$ and $x^*(t_1)$ for M=AC and 548
497 BC, respectively. The second term is due to the size evo- 549
498 lution of the particles. $n^*(t_1) - 1$ is written rather than 550
499 $n^*(t_1)$ to signal that more than one growth unit is neces- 551
500 sary to determine if a solid phase is formed. In the case of 552
501 redissolution $X_M(t_3)$ must be put equal to $X_M(t_2)$ with 553
502 t_2 determined by the condition that $\rho(t_1, t_2) = \rho(t_1, t_3)$ 554
503 (Equation 21). From these quantities and an ionic spe- 555
504 ciation model, one can calculate all activities in the AS 556
505 and the saturation indexes $I_{AC}(t)$, $I_{BC}(t)$ and $I(t, x)$. 557

506 Including feed-back effects on the AS allows an evolu- 558
507 tion of its composition towards thermodynamic equilib- 559
508 rium. When $t \rightarrow \infty$, $I(t, x^*)$ tends to 1 and, combining 560
509 Equations 5 and 16, it is easy to check that $x^* \rightarrow x_0$, as 561
510 it should. 562

511 The equation giving $q_M(t)$ (Equation 24), together 563
512 with those which fix $I(t, x)$, $x^*(t_1)$, $n^*(t_1)$, $\Delta G^*(t_1)$, 564
513 $F(t_1)$, and $\rho(t_1, t)$ form a complete set which, together 565
514 with the speciation equations, allow the full determina- 566
515 tion of the precipitate and aqueous solution characteris- 567
516 tics at all times. 568

517 The present formalism represents an important ad- 569
518 vance with respect to our previous work (Noguera et 570
519 al., 2010) which was restricted to ideal solid solutions,

spherical particles and homogeneous nucleation. The de-
velopment of the NANOKIN code to include these new
functionalities is presently under progress, and its ap-
plication to a realistic precipitation process will be the
subject of a forthcoming paper. In the following, we will
highlight some generic characteristics of the precipitation
of non-ideal SS, and, under some approximations, we will
present some numerical simulations exemplifying various
scenarios which may be encountered in the precipitation
of SSs of geochemical interest.

VI. PRECIPITATION SCENARIOS

In this section, we first discuss the characteristics of
the precipitation process as a function of the degree of
non-ideality of the SS and we evidence four possible pre-
cipitation scenarios (Section VIA). We then discuss their
conditions of occurrence, under some simplifying assump-
tions, and we represent them graphically as a function of
the ratio of the solubility products of the end-members
and the Guggenheim coefficient A_0 (Section VIB).

A. The four scenarios

First we recall that when the Guggenheim coefficients
 A_0 and A_1 belong to the zone included in between the two
lines drawn in Figure 3 (lower panel), the equations which
fix x_{st} and x^* have a single root and the precipitation
scenario bears strong resemblances to that of an ideal
SS. When $A_0 > 0$, the only difference with truly ideal
SSs lies in the corrections due to the activity coefficients
 λ_{AC} and λ_{BC} . In that case the precipitation scenario
will be called *Precipitation Scenario #1* (Sc. #1).

When the contribution of the enthalpy of mixing of the
SS to the Gibbs free energy of dissolution is negative, the
SS displays a tendency towards ordering. It is generally
associated with negative values of the first Guggenheim
coefficient ¹ and is usually interpreted as resulting from
short range attraction between dissimilar first neighbors,
which favors A-B pairs over A-A or B-B pairs. Which
order is actually achieved depends on a contribution of
the entropy of mixing which is specific to each case. Be-
cause at the present stage our study remains generic, we
do not introduce it, so that this limit is not well-treated
by our approach and will not be further discussed.

In the limit of strong non-ideality of the SS, a miscibil-
ity gap is present which is revealed by a jump of x^* from
 x_1 to x_2 at a critical value W'_c of $W' = I_{BC}/I_{AC}^z$ (see e.g.
Figure 5). This occurs when the Guggenheim coefficients
 A_0 and A_1 belong to the regions of the diagram in Figure
3 above or below the two lines. x_1 , x_2 and W'_c are solely
determined by the values of A_0 , A_1 and $z = v_{BC}/v_{AC}$
(See Appendix D). For example, in the special case where
 $A_1 = 0$ (regular SSs) and $z = 1$, x_2 is equal to $1 - x_1$
and the critical value for W is equal to $W'_c = 1$ (see Sec-
tions 2 and 3). The question of phase separation in the

¹ For sub-regular SSs, the A_1 coefficient should also be taken into
account. However, in the logics of the Guggenheim expansion
and its truncation, A_1 is expected to be smaller than A_0 , in
absolute value, so that extension of the zone of existence of a
miscibility gap towards negative A_0 values in Figure 3 appears
meaningless.

critical nuclei or in the deposited layers becomes relevant only when $W' = W'_c$ because then the nucleation barrier (Eq. 14) takes equal values for $x^* = x_1$ and $x^* = x_2$ and the same is true for the interfacial Gibbs free energy for growth (Eq. 19).

Consequently, the scenario of precipitation depends on whether and how the condition $W' = W'_c$ is met during the time evolution of the system. The initial conditions (embedded in the value of W' at time $t = 0$) and the sign of dW'/dt in the vicinity of the discontinuity are the relevant factors in that respect. We first note that, in the presence of a miscibility gap, $W'(t)$ has a slope discontinuity at $W' = W'_c$, due to the different values x_1 and x_2 of the SS composition for $W' < W'_c$ and $W' > W'_c$, respectively (Appendix E). We will note dW'_1/dt and dW'_2/dt the associated two time derivative values of $W'(t)$, respectively. This allows discrimination of the following scenarios:

- conditions are such that during the precipitation process the discontinuity is not met. This takes place if, at $t = 0$, $W' < W'_c$ and close to the discontinuity $dW'_1/dt < 0$, or if, at $t = 0$, $W' > W'_c$ and close to the discontinuity $dW'_2/dt > 0$. This scenario will be referred to, in the following, as *Precipitation Scenario #2* (Sc. #2).
- conditions are such that, during the precipitation process, the discontinuity is met but dW'/dt has the same sign on both sides of the discontinuity. The discontinuity is thus crossed but the time spent by the system at $W' = W'_c$ is irrelevant. No phase separation takes place. This scenario will be referred to as *Precipitation Scenario #3* (Sc. #3).
- finally, it may be that the discontinuity is met but dW'/dt has opposite signs on both sides of the discontinuity, which tends to bring it back towards $W' = W'_c$ on both sides. There is then a conflict between the variations of W' forcing it to stay constant and equal to W'_c (Lyapunov stable equilibrium point (Lyapunov, 1992)). It is in this case that phase separation takes place, in order to allow the condition $dW'/dt = 0$ to be fulfilled. This scenario will be referred to as *Precipitation Scenario #4* (Sc. #4).

To go further, one has to determine the sign of dW'/dt on the left and right of the discontinuity. It is possible to derive formal expressions for dW'_1/dt and dW'_2/dt from the feed-back equations, as described in Appendix E, and also to deduce the relative percentages α and $(1 - \alpha)$ of the two SSs with composition x_1 and x_2 when phase separation occurs. These expressions may then be quantitatively estimated for specific cases.

Aside from this numerical approach, in the following in order to gain some physical insight into practical conditions of occurrence of the four scenarios, we restrict the discussion to regular SSs and some simplified conditions of precipitation. This will allow us to devise a dia-

gram of occurrence of the scenarios as a function of some parameters (ratios of the solubility products of the two end-members, degree A_0 of non-ideality of the SS and initial conditions of precipitation), and to make qualitative predictions of the precipitation characteristics of realistic SSs of geochemical interest.

B. Conditions of existence of the four scenarios for regular SSs

The simplifying assumptions are the following:

- the SS is regular ($A_1 = 0$)
- the formula unit volume as well as the surface energy of the SS are assumed to be independent of x . For example they may be set equal to the average of the corresponding values of the end-members. Consequently, there are no surface excess quantities (Appendix C) and $z = 1$.
- the A, B and C species are the dominant forms of the elements in the AS, and no mineral other than the SS may dissolve or precipitate. The time evolution of the [A], [B] and [C] activities thus only comes from the precipitation of the SS under consideration. Moreover, for SSs with a miscibility gap, we assume that, at the time when the x^* discontinuity is met, redissolution is negligible. Both hypotheses imply that the contribution W'' to dW'/dt which is continuous at W'_c vanishes (Appendix E).

Under these hypotheses, in strongly non-ideal SSs ($A_0 > 2$), the miscibility gap is symmetric ($x_2 = 1 - x_1$) and $W'_c = 1$. At the discontinuity, the last equality means that $[B]/[A] = K_{BC}/K_{AC}$. Moreover, close to the discontinuity (Appendix E):

$$\frac{1}{W'} \frac{dW'}{dt} \propto - \left(x^*(t) - (1 - x^*(t)) \frac{K_{BC}}{K_{AC}} \right) \quad (25)$$

with x^* equal to x_1 or x_2 when W'_c is smaller or larger than 1, respectively.

As a consequence:

- $dW'_1/dt > 0$ if $x_1/(1 - x_1) < K_{BC}/K_{AC}$
- $dW'_1/dt < 0$ otherwise
- $dW'_2/dt > 0$ if $x_2/(1 - x_2) < K_{BC}/K_{AC}$
- $dW'_2/dt < 0$ otherwise

Using these inequalities, the conditions of occurrence of Scenario #2 read:

$$\frac{[B(t=0)]}{[A(t=0)]} < \frac{K_{BC}}{K_{AC}} < \frac{x_1}{1 - x_1} \quad (26)$$

669 OR:

$$\frac{x_2}{1-x_2} < \frac{K_{BC}}{K_{AC}} < \frac{[B(t=0)]}{[A(t=0)]} \quad (27)$$

670 Scenario #3 requires that the three ratios $[B(t=0)]/[A(t=0)]$, $x_1/(1-x_1)$ and $x_2/(1-x_2)$ are simultaneously either smaller than K_{BC}/K_{AC} or larger than it. Finally, Scenario #4 takes place when the following inequalities are fulfilled:

$$\frac{x_1}{1-x_1} < \frac{K_{BC}}{K_{AC}} < \frac{x_2}{1-x_2} \quad (28)$$

675 whatever the value of $[B(t=0)]/[A(t=0)]$

676 These conditions are graphically represented in Figure 6, as a function of $\ln K_{BC}/K_{AC}$ and the degree A_0 of non-ideality of the SS. The boundaries between the zones of existence of the scenarios #1, #2, #3, and #4 include the vertical line $A_0 = 2$ on the left of which only Scenario #1 takes place, and the two lines $\ln K_{BC}/K_{AC} = \ln x_1/(1-x_1)$ and $\ln K_{BC}/K_{AC} = \ln x_2/(1-x_2)$. In between the two latter, phase separation takes place within Scenario #4.

677 Outside these regions, Scenarios #2 or #3 may take place, depending on the initial conditions. When $\ln K_{BC}/K_{AC} > \ln x_2/(1-x_2)$, the discontinuity is not crossed (Scenario #2) if $[B(t=0)]/[A(t=0)] > K_{BC}/K_{AC}$ and otherwise it is crossed (Scenario #3). Symmetrically, when $\ln K_{BC}/K_{AC} < \ln x_2/(1-x_2)$, the discontinuity is not crossed (Scenario #2) if $[B(t=0)]/[A(t=0)] < K_{BC}/K_{AC}$ and otherwise it is crossed (Scenario #3). The difference between the two scenarios, i.e. the existence of a composition discontinuity inside the particles, is thus only fixed by the initial conditions in these regions.

697 VII. NUMERICAL SIMULATION AND RELEVANT EXAMPLES

698 In this section, we present results of numerical simulations which highlight the generic characteristics of the precipitation kinetics under conditions such that scenarios #1, #2, #3 or #4 take place. We will make use of the same assumptions as in subsection VIB and also assume that the particles have a spherical shape. Although these assumptions are rather simplistic, they help provide a first insight into the precipitation characteristics of SSs of geochemical interest.

708 A. Weakly non-ideal SS: precipitation Scenario #1

709 When the Guggenheim coefficient $A_0 < 2$, the SS which forms is weakly non-ideal. The only difference from truly ideal SSs lies in the corrections due to the activity coefficients λ_{AC} and λ_{BC} . The dynamics of precipitation presents many common characteristics with that of ideal

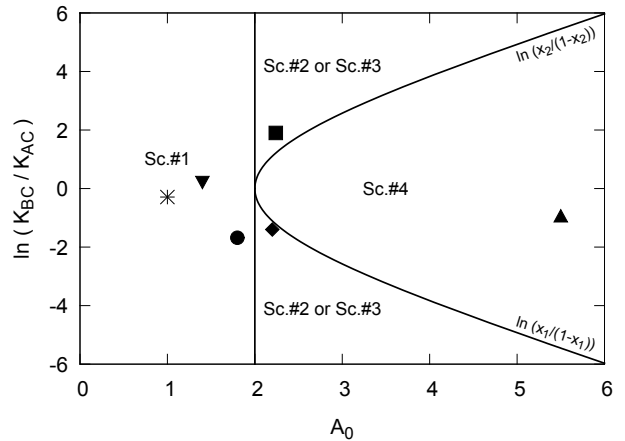


FIG. 6: Diagram $\ln K_{BC}/K_{AC}$ as a function of the degree A_0 of non-ideality of the SS, representing the zones of existence of the scenarios #1, #2, #3, and #4. $K(\text{Cl,Br})$, $(\text{Mg,Fe})\text{CO}_3$, $(\text{Ba,Ra})\text{SO}_4$, $(\text{Ca,Zn})\text{CO}_3$, $\text{Ca}(\text{SO}_4, \text{SeO}_4)$ and $(\text{Ca,Sr})\text{CO}_3$ SSs are located in this diagram, represented by triangle-down, circle, star, diamond, square and triangle-up, respectively (see text).

SSs, which we have analyzed in a previous work (Noguera et al., 2010). Two typical examples are shown in Figure 7, for $A_0 = 1$ and two values of the solubility product ratio K_{BC}/K_{AC} .

In both cases, due to nucleation and growth, the AS is impoverished in A, B, and C species as time passes and $I(t, x^*(t))$ decreases towards 1, until thermodynamic equilibrium is reached in the long term. The time variation of the critical nucleus composition is very dependent on the ratio K_{BC}/K_{AC} . When the solubility products of the two end-members are close to each other, the range of x^* values is small and the composition profile of the particles is smooth, as exemplified in Figure 7 (left panels) where K_{BC}/K_{AC} was chosen equal to 1. At variance, when the two end-members have largely different solubility products, x^* varies in a larger range as shown in Figure 7 (right panels) for which $K_{BC}/K_{AC} = 10$. The surviving particles may display either a core-shell structure, with a smooth interface between core and shell, or nearly constant composition, depending on the initial conditions. In any case, when thermodynamic equilibrium is reached, the last surviving particle has a non-homogeneous composition, at variance with thermodynamic models of SSs which assume an homogeneous composition.

$\text{KCl}_{1-x}\text{Br}_x$ was shown to be a quasi-regular SS, with $A_0 = 1.4$ and a very small A_1 coefficient (Glynn et al., 1990) and the two end-members have a small solubility product ratio ($K_{BC}/K_{AC} = 1.68$ (Blanc et al., 2012)). The particles formed during precipitation are thus expected to have a smooth composition profile. This is also the case for the $(\text{Ba,Ra})\text{SO}_4$ SS with a ratio of solubility products K_{BC}/K_{AC} equal to 0.512 (Hummel

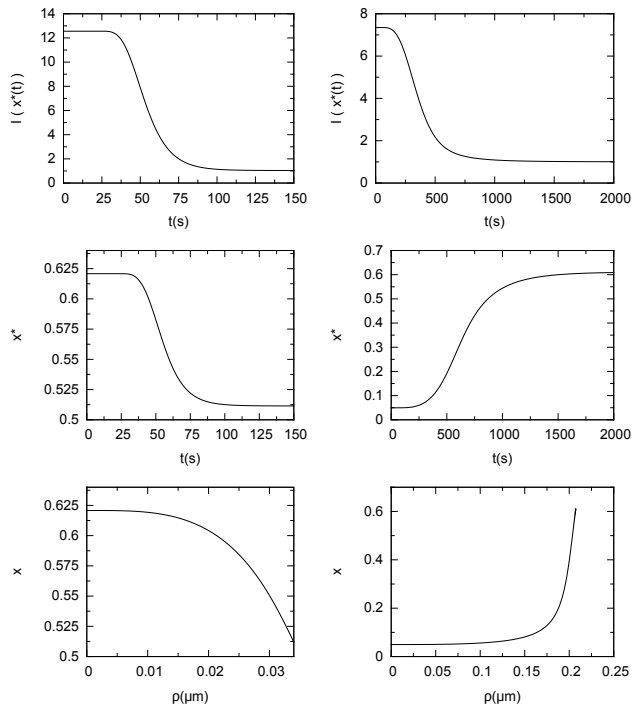


FIG. 7: From top to bottom: time dependence of the saturation state $I(t, x^*(t))$, time dependence of the critical nucleus composition x^* and concentration profile of the surviving particles at the end of the simulation. Left and right panels refer to $K_{BC}/K_{AC} = 1$ and 10, respectively. All curves have been drawn for $A_0 = 1$, $A_1 = 0$, $F_0 = 10^{19}$ particles per second and liter of solution, $\sigma = 50$ mJ/m² independent on composition, $\kappa = 10^{-10}$ m/s, $v_{AC} = v_{BC} = 50$ Å³, $K_{AC} = 10^{-6}$ and initial activities: $[A(t=0)] = 7 \cdot 10^{-4}$, $[B(t=0)] = 9 \cdot 10^{-4}$, $[C(t=0)] = 10^{-2}$.

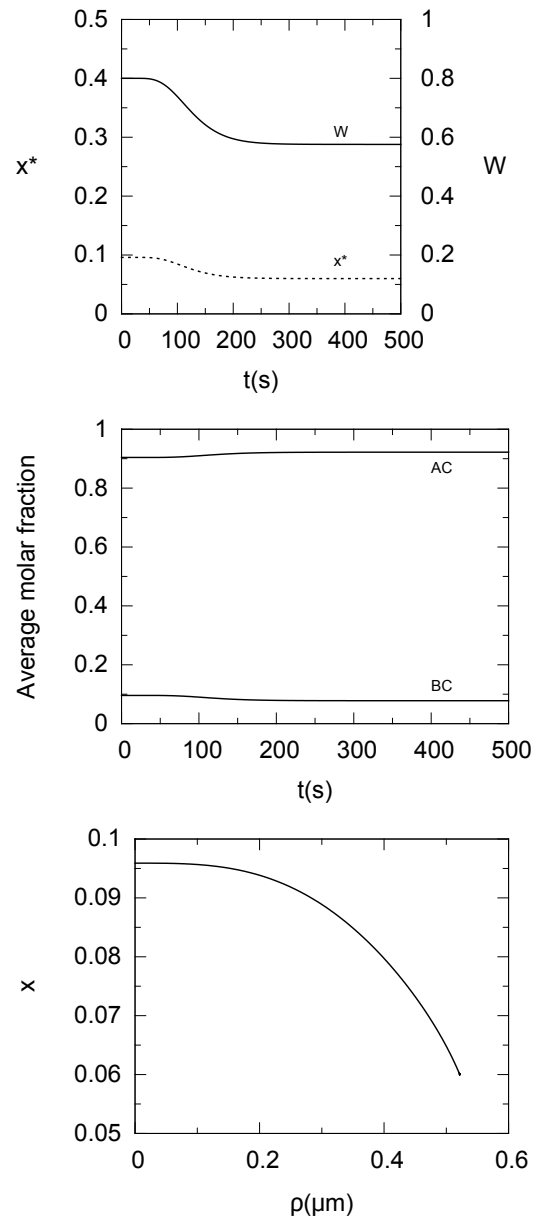


FIG. 8: From top to bottom: time dependence of W and the critical nucleus composition x^* , time dependence of the end-member average molar fractions $q_{AC}/(q_{AC} + q_{BC})$ and $q_{BC}/(q_{AC} + q_{BC})$, and composition profile of long lasting particles. All curves have been drawn for $A_0 = 2.5$, $A_1 = 0$, $F_0 = 10^{19}$ particles per second and liter of solution, $\sigma = 50$ mJ/m² independent on composition, $\kappa = 10^{-9}$ m/s, $v_{AC} = v_{BC} = 50$ Å³, $K_{AC} = 10^{-6}$, $K_{BC} = 10^{-7}$ and initial activities: $[A(t=0)] = 6 \cdot 10^{-4}$, $[B(t=0)] = 48 \cdot 10^{-6}$, $[C(t=0)] = 10^{-2}$.

747 et al., 2002). This SS has recently been studied by
 748 Brandt et al. (2015) who confirmed a Guggenheim co-
 749 efficient $A_0 = 1$ as theoretically predicted (Vinograd et
 750 al., 2013). At variance, particles of $\text{Mg}_{1-x}\text{Fe}_x\text{CO}_3$, for
 751 which $A_0 = 1.8$ (Chai and Navrotsky, 1996) should dis-
 752 play a core-shell structure because the solubility products
 753 of the end-members differ by about two orders of magni-
 754 tude ($K_{BC}/K_{AC} = 0.02$ (Blanc et al., 2012)). The infor-
 755 mation for the three SSs described above are reported in
 756 Figure 6.

757 B. Strongly non-ideal SS: precipitation Scenario 758 #2 without composition discontinuity

759 We present here simulation results for the precipita-
 760 tion of a strongly non-ideal SS ($A_0 = 2.5$, $x_1 = 0.1448$
 761 and $x_2 = 0.8552$), under conditions relevant for scenario
 762 #2 (Conditions 26 or 27). In Figure 8, initial conditions
 763 and ratios of solubility products have been chosen such
 764 that $W' < 1$ at $t = 0$ and $dW'_1/dt < 0$. The system
 765 thus does not encounter the composition discontinuity
 766 and the time dependence of x^* is smooth. The critical

nuclei and the deposited layers remain AC rich during the whole precipitation process, consistent with average molar fractions of the AC and BC end-members in the precipitate, $q_{AC}/(q_{AC} + q_{BC})$ and $q_{BC}/(q_{AC} + q_{BC})$, close to 90% and 10%, respectively. A typical particle composition profile at the end of the process is shown in Figure

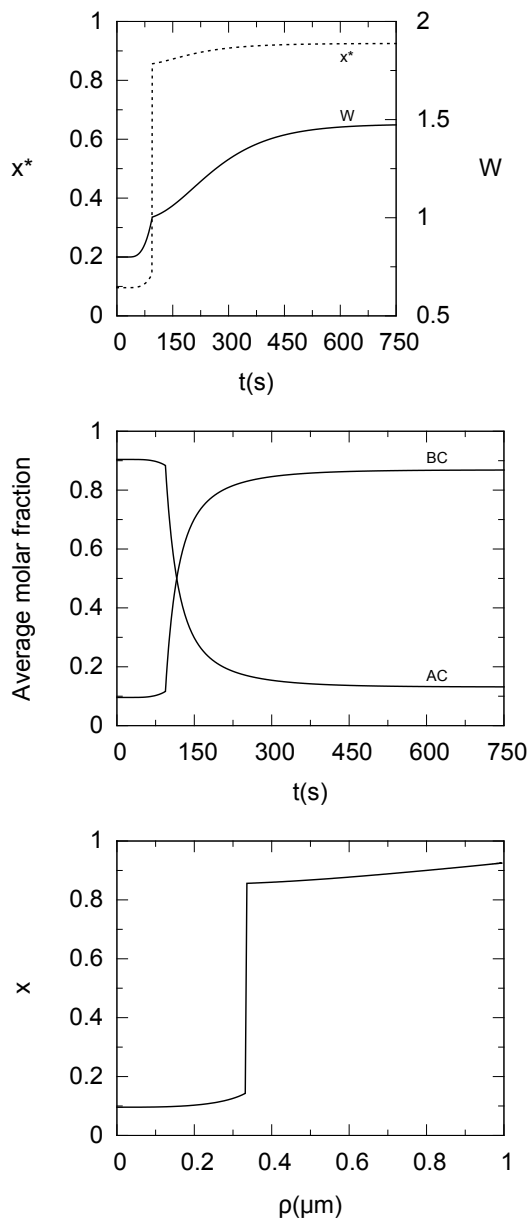


FIG. 9: From top to bottom: time dependence of W and the critical nucleus composition x^* , time dependence of the end-member average molar fractions $q_{AC}/(q_{AC} + q_{BC})$ and $q_{BC}/(q_{AC} + q_{BC})$, and composition profile of long lasting particles. Same parameter values as in Figure 8, except $K_{BC} = 10^{-5}$ and $[B(t=0)] = 48.10^{-4}$.

C. Strongly non-ideal SS: precipitation Scenario #3 with a crossing of the composition discontinuity

This is another example of precipitation of a strongly non-ideal SS, with the same characteristics as the previous one ($A_0 = 2.5$, $x_1 = 0.1448$ and $x_2 = 0.8552$), but under conditions relevant for scenario #3, i.e. such that the discontinuity in the x^* versus W' curve is met and crossed (same sign of dW'/dt on both sides of the discontinuity).

The precipitation characteristics shown in Figure 9 have been obtained under conditions close to those of the previous subsection, except for the initial activity $[B(t=0)]$ and for the ratio of solubility products. At $t = 0$, $W' < 1$ and $dW'_1/dt > 0$. The system thus meets the composition discontinuity at the time t_c when $W' = 1$. This clearly shows up in the time dependence of x^* which displays a 70% jump, and in the slope discontinuity in the time dependence of W' . Before t_c , the critical nucleus and deposited layer compositions are AC rich ($x \approx 15\%$). They become BC rich ($x \approx 85\%$) after t_c . This sudden change is reflected in the time dependence of the molar fractions of the end-members in the precipitate which displays a crossing point at some time posterior to t_c . The composition profile of the particles at the end of the process also reflects the discontinuity which has occurred at t_c . The particles have a core-shell structure, with an AC-rich core and a BC-rich shell and an abrupt interface between core and shell (Figure 9, bottom panel).

$\text{Ca}_{1-x}\text{Zn}_x\text{CO}_3$ and $\text{Ca}(\text{SO}_4)_{1-x}(\text{SeO}_4)_x$ SSs may display precipitation scenarios #2 or #3 depending on the initial value of the $[\text{Zn}]/[\text{Ca}]$ or $[\text{SeO}_4]/[\text{SO}_4]$ activity ratios. Indeed, for both SSs, the Guggenheim coefficients A_0 are equal to 2.2 (Glynn and Reardon, 1990) and 2.24 (Fernández-González et al., 2006), respectively, and their solubility products locate them, respectively, in the lower and upper regions of Figure 6 where scenarios #2 or #3 take place. ($K_{BC}/K_{AC} = 0.03$ for $\text{Ca}_{1-x}\text{Zn}_x\text{CO}_3$ (Crocket and Winchester, 1966) and $K_{BC}/K_{AC} \approx 80$ for $\text{Ca}(\text{SO}_4)_{1-x}(\text{SeO}_4)_x$ (Parkhurst and Appelo, 1999)). During their formation by precipitation, the particles are thus expected to display smooth profiles or core-shell structure depending on initial conditions which will determine whether scenarios #2 or #3 apply.

D. Strongly non-ideal SS: precipitation Scenario #4 with phase separation

Keeping the characteristics of the SS unchanged with respect to the two previous subsections ($A_0 = 2.5$, $x_1 = 0.1448$ and $x_2 = 0.8552$), we now consider conditions relevant for scenario #4, i.e. such that the discontinuity is met but dW'/dt has opposite signs on both sides of the discontinuity. This happens if, at $t = 0$, $W' < 1$ and close to the discontinuity $dW'_1/dt > 0$ and $dW'_2/dt < 0$, or if, at $t = 0$, $W' > 1$ and close to the discontinuity

8 bottom panel. The particles are non-uniform in com-
position and become more AC-rich close to their surface.
However, the variation of composition x between the core
and the surface is small, of the order of 4%.

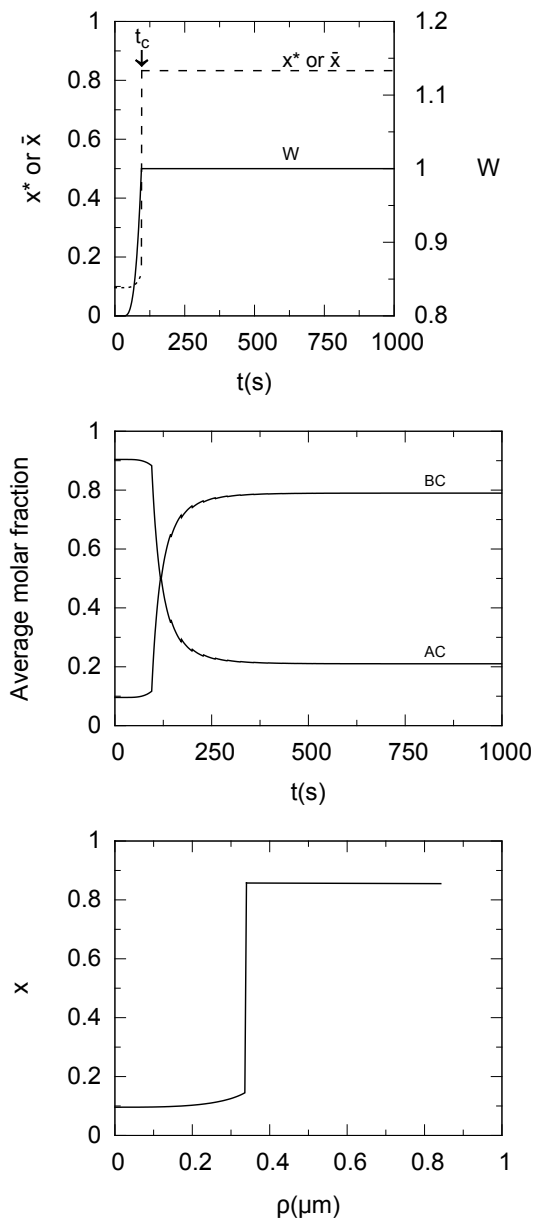


FIG. 10: From top to bottom: time dependence of W , time dependence of the critical nucleus composition x^* (when $t < t_c$) or $\bar{x} = \alpha x_1 + (1 - \alpha)x_2$ (when $t > t_c$); time dependence of the end-member average molar fractions $q_{AC}/(q_{AC} + q_{BC})$ and $q_{BC}/(q_{AC} + q_{BC})$; composition profile of a long lasting particle. Same parameter values as in Figure 8, except $K_{BC} = 5.10^{-6}$ and $[B(t=0)] = 24.10^{-4}$.

$dW'_1/dt < 0$ and $dW'_2/dt > 0$ (Condition 28).

For the example shown in Figure 10, initial conditions and ratios of solubility products have been chosen so that $W' < 1$ at $t = 0$, and simultaneously $dW'_1/dt > 0$ and $dW'_2/dt < 0$ at the discontinuity. All other parameters are equal to those of the preceding examples. The system encounters the composition discontinuity at a time t_c . The conflicting variations of W' on the left and right of the discontinuity force W' to remain constant and equal

to 1 at all posterior times (Figure 10, top panel). For all times $t > t_c$, the ratio of B and A activities remains constant ($[B(t)]/[A(t)] = K_{BC}/K_{AC}$), although the saturation states I_{AC} and I_{BC} of the AS with respect to the end-members decrease. Phase separation between phases of compositions x_1 and x_2 takes place at $t > t_c$, with relative amounts α and $1 - \alpha$ (Eq. 61 in Appendix E). It is not easy to tell how the two phases will be spatially organized, but an average composition in the critical nucleus or the instantaneous deposited layers may be defined at each time $t > t_c$ as $\bar{x} = \alpha x_1 + (1 - \alpha)x_2$. Because α remains constant, the same is true for \bar{x} , which in the present example is equal to 0.833. Because it is closer to x_2 than to x_1 , the end-member molar fractions $q_{AC}/(q_{AC} + q_{BC})$ and $q_{BC}/(q_{AC} + q_{BC})$ strongly vary for $t > t_c$ and tend to approximately $1 - \alpha$ and α , respectively, in the long term. A typical particle profile is shown in the lowest panel of Figure 10, highlighting a core-shell structure and an abrupt interface between them.

$\text{Ca}_{1-x}\text{Sr}_x\text{CO}_3$ is a strongly non-ideal SS characterized by a Guggenheim coefficient equal to 5.5 (Casey et al., 1996), indicating poor solubility of Sr in aragonite and a miscibility gap occupying most of the phase diagram ($x_1 \approx 0.004$ and $x_2 \approx 0.996$). Whatever the initial conditions of precipitation, a phase separation is expected, as in the example shown in Figure 10.

VIII. CONCLUSION

We have developed a formalism which describes the precipitation kinetics of non-ideal SSs from an initially supersaturated AS. It treats the time evolution of the AS composition and the formation, growth or redissolution of particles. It extends our previous work, which was restricted to ideal SSs, spherical particles and homogeneous nucleation. The formalism is relevant to both mineral SSs and bimetallic nanoparticle formation. To our knowledge, it is the first time, in the fields of both geochemistry and metallic alloys, that these out-of-equilibrium processes are fully taken into account for non-ideal SSs.

This work highlights how particle composition and size vary with time, resulting in composition profiles which may be smooth or discontinuous, depending on the Guggenheim parameter values which drive the non-ideality of the SSs, and the ratio of the solubility products of the end-members. We have shown that even for strongly non-ideal SSs, phase separation is not the general case and that other scenarios may take place. We have specified their characteristics and under which conditions they may be encountered. Numerical simulations have been performed to exemplify them for a regular SS, under a few simplifying assumptions, and qualitative predictions of the precipitation characteristics of some mineral SSs have been made.

The development of the NANOKIN code to include these new functionalities is presently under progress, and its application to a realistic precipitation process will be

895 the subject of a forthcoming paper. In the context of
 896 water-rock interactions, our work provides enhanced possibilities
 897 for analyzing precipitation processes for various
 898 SS types, such as carbonates, sulfates or clay minerals,
 899 among others.

900 Appendix A: Condition of stoichiometric saturation

901 In this appendix, we analyze the mathematical properties
 902 of the function $-\ln I(x) = \ln K(x) - \ln Q(x)$ whose
 903 minimum determines the stoichiometric saturation condition
 904 (Eq. 10 in the text). $-\ln I(x)$ reads:
 905

$$\begin{aligned} -\ln I(x) = & -(1-x)\ln I_{AC} - x\ln I_{BC} \\ & +x(1-x)[A_0 + A_1(2x-1)] \\ & +x\ln x + (1-x)\ln(1-x) \end{aligned} \quad (29)$$

906 or:

$$\begin{aligned} -\ln \frac{I(x)}{I_{AC}} = & -x\ln W + x(1-x)[A_0 + A_1(2x-1)] \\ & +x\ln x + (1-x)\ln(1-x) \end{aligned} \quad (30)$$

907 after introducing the ratio W of the saturation states of
 908 the AS with respect to the pure end-members:

$$W = \frac{I_{BC}}{I_{AC}} \quad (31)$$

909 In the following we analyze the variations of the func-
 910 tion $f(x)$ equal to the right hand side of Eq. 30. Its first
 911 derivative is:

$$\frac{df(x)}{dx} = \ln \frac{x}{W(1-x)} + A_0(1-2x) - A_1(1-6x+6x^2) \quad (32)$$

912 and its second derivative is:

$$\frac{d^2f(x)}{dx^2} = -2A_0 + 6A_1(1-2x) + \frac{1}{x} + \frac{1}{1-x} \quad (33)$$

913 There are regions of the parameter space $\{A_0, A_1\}$
 914 where $d^2f(x)/dx^2 > 0$ whatever x . In that case, $f(x)$
 915 is a convex function, with a single minimum. This hap-
 916 pens, for example when $A_1 = 0$ and $A_0 < 2$. Otherwise,
 917 $f(x)$ may display one minimum, or two minima and a
 918 maximum, depending upon the value of W .

919 Figure 11 exemplifies this latter case when $A_0 = 2.5$
 920 and $A_1 = 0$. At low or high values of W (typically less
 921 than 0.86 or more than 1.14), $f(x)$ has a single minimum
 922 at x smaller or larger than 0.5, respectively. For inter-
 923 mediate values of W , $f(x)$ has two minima. The compo-
 924 sition x_{st} which corresponds to stoichiometric saturation
 925 is the one for which $f(x)$ is the lowest. x_{st} displays a dis-
 926 continuity between x_1 and x_2 at a critical value $W_c = 1$
 927 for which $f(x_1) = f(x_2)$. This behavior is represented in
 928 Figure 1 in the main text.

929 Similar reasoning for different values of A_0 and A_1
 930 leads to Figure 2, which shows that W_c remains equal

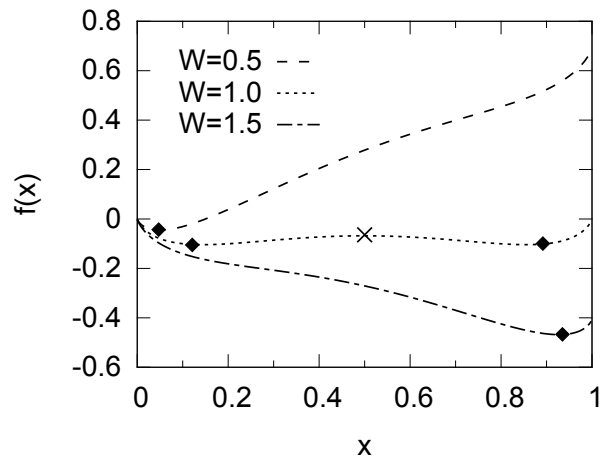


FIG. 11: Curve representative of $f(x)$ (right hand side of Eq. 30) for $A_0 = 2.5$, $A_1 = 0$ and $W = 0.5, 1.$ and 1.5 . The three curves exemplify the cases when $f(x)$ presents a single minimum at small value of x , two degenerate minima or a single minimum at large x value. The diamonds on the curves mark the positions of the minima and the cross the maximum.

931 to 1 whatever the value of $A_0 > 2$ if $A_1 = 0$, but varies
 932 with A_0 when $A_1 \neq 0$.

933 Finally, the limiting values of A_0 and A_1 between re-
 934 gions of discontinuities in x_{st} and regions where it varies
 935 smoothly, are obtained from the condition that simul-
 936 taneously $d^2f(x)/dx^2 = 0$ and, when increasing W ,
 937 $df(x)/dx = 0$ has, for the first time, three roots. They
 938 are represented in Figure 3, bottom panel in the main
 939 text.

940 Appendix B: Precipitation of particles with various shapes

941 The formalism associated with the homogeneous nucle-
 942 ation and growth of spherical particles has been presented
 943 in the main text. However, most solids, except amor-
 944 phous ones, are non-isotropic and their external shape,
 945 which departs from the sphere, reflects the relative ener-
 946 gies of their low index faces, as recognized by Wulff
 947 (Müller and Kern, 2000 and references therein). Indeed,
 948 Wulff theorem states that, at equilibrium, the distance
 949 from the center of a particle to its external facets is pro-
 950 portional to the surface energy of these facets. For ex-
 951 ample, according to Wulff theorem, the aspect ratio of
 952 tetragonal particles (basal dimensions $l \times l$ and thickness
 953 e , Figure 12), is given by the ratio between the surface
 954 energies of the basal and lateral faces (σ_{bas} and σ_{lat} , re-
 955 spectively):

$$\frac{e}{l} = \frac{\sigma_{bas}}{\sigma_{lat}} \quad (34)$$

This result can be extended to the case of particles in
 equilibrium with a substrate on which they lie on their

basal face. In this case, σ_{bas} is relevant for the face in contact with the AS, and $\sigma_{bas} - W_{adh}$ for the one in contact with the substrate (W_{adh} the adhesion energy). Their aspect ratio is then given by the Wulff-Kaishev theorem:

$$\frac{e}{l} = \frac{\sigma_{bas} - W_{adh}/2}{\sigma_{lat}} \quad (35)$$

If, instead of the basal face, one of the lateral faces is in contact with the substrate, their equilibrium shape involves three inequivalent dimensions l , l' et e . The ratio between these lengths is then equal to:

$$\frac{e}{\sigma_{bas}} = \frac{l}{\sigma_{lat}} = \frac{l'}{\sigma_{lat} - W_{adh}/2} \quad (36)$$

The corresponding expression for rhombohedral particles lying on a substrate on their basal face is:

$$\frac{e}{l} = \sqrt{3} \frac{\sigma_{bas} - W_{adh}/2}{\sigma_{lat}} \quad (37)$$

Similar reasoning can be done for other particle shapes.

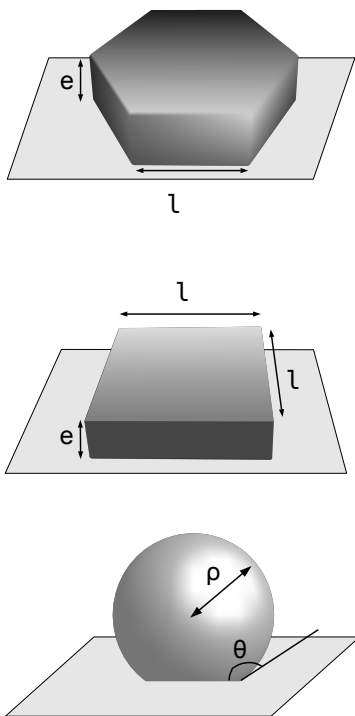


FIG. 12: Representation of rhombohedral, tetragonal and spherical cap particle shapes (from top to bottom).

With these elements in mind, one can consider homogeneous as well as heterogeneous nucleation of particles assumed to have the Wulff (homogeneous nucleation) or

Wulff-Kaishev (heterogeneous nucleation) shapes (Müller and Kern, 2000). The generalized expression of the change in Gibbs free energy for the formation of a critical nucleus then reads:

$$\Delta G(n, x) = -nk_B T \ln I(x) + n^{2/3} v(x)^{2/3} X \bar{\sigma} \quad (38)$$

The geometric factor X and the average surface energy $\bar{\sigma}$ have the following expressions for simple nucleus shapes and for homogeneous or heterogeneous nucleation (Fritz et al., 2009):

- spherical particles:

$$X = (36\pi\Phi(\theta))^{1/3} ; \bar{\sigma} = \sigma \quad (39)$$

θ is the wetting angle with the substrate, given by the Young-Dupré equation $-\sigma \cos \theta = \sigma - W_{adh}$, and $\Phi(\theta) = (1 - \cos \theta)^2 (2 + \cos \theta) / 4$. For a strong adhesion to the substrate, the wetting angle is equal to 0° , and it is equal to 180° when no wetting occurs (which is also the case for homogeneous nucleation).

- tetragonal particles lying on their basal face:

$$X = 6 ; \bar{\sigma} = (\sigma_{lat}^2 (\sigma_{bas} - W_{adh}/2))^{1/3} \quad (40)$$

- tetragonal particles lying on their lateral face:

$$X = 6 ; \bar{\sigma} = (\sigma_{lat} \sigma_{bas} (\sigma_{lat} - W_{adh}/2))^{1/3} \quad (41)$$

- hexagonal particles lying on their basal face:

$$X = 36^{1/3} \sqrt{3} ; \bar{\sigma} = (\sigma_{lat}^2 (\sigma_{bas} - W_{adh}/2))^{1/3} \quad (42)$$

To obtain the time evolution of all dimensions during growth, one assumes that the particles keep their equilibrium shape, which allows the use of equations similar to 20 and 22 of the main text for one dimension, deduction of the others from the relationships written above for the ratios between e , l and l' , and estimation of the volume V of the particle and finally the number of formula units $n = V/v$.

Appendix C: Precipitation of a SS with composition dependent surface energy

This appendix specifies the modifications to introduce in the formalism which describes SS precipitation when their surface energy depends on composition. The main difference from the simplified treatment, presented in the main text, comes from the existence of surface enrichment effects n_{ACs} and n_{BCs} of AC and BC composition. Starting from a reference state in which the nuclei have a sharp boundary with the aqueous solution (Gibbs dividing surface), such surface excess quantities have to be introduced, so that the Gibbs adsorption equation can be fulfilled (Adamson, 1960; Laaksonen et al., 1999; Noppel et al., 2002; Gaman et al., 2005).

1015 The change in Gibbs free energy $\Delta G(n, x)$ for nucle-1050
1016 ation now reads:

$$\Delta G(n, x) = -nk_B T \ln I(x) + n^{2/3} v(x)^{2/3} X \overline{\sigma(x)} - n_{BCs} k_B T \ln \left(\frac{I_{BC}}{x \lambda_{BC}(x)} \right) - n_{ACs} k_B T \ln \left(\frac{I_{AC}}{(1-x) \lambda_{AC}(x)} \right) \quad (43)$$

1017 The X parameter and the average surface energy $\overline{\sigma(x)}$ 1053
1018 have been defined in Appendix B for particles of various 1054
1019 shapes. Here $\overline{\sigma}$ is explicitly a function of x . The two last 1055
1020 terms in Eq. 43 involve surface excess quantities n_{ACs} 1056
1021 and n_{BCs} , multiplied by the corresponding changes in 1057
1022 chemical potential $\Delta\mu_{AC}$ and $\Delta\mu_{BC}$. The surface ex- 1058
1023 cesses are algebraic quantities, which can take positive 1059
1024 as well as negative values.

1025 The expression of the critical nuclei size $n_m(x)$, 1060
1026 obtained from the maximum of $\Delta G(n, x)$ with respect to
1027 x is the same as in Equation 13. In order to obtain the
1028 critical nucleus composition, via the maximum of the nu-
1029 cleation rate, the derivative of $\Delta G_m(x)$ with respect to
1030 x has to be performed, with $\Delta G_m(x) = \Delta G(n_m(x), x)$
1031 equal to:

$$\frac{\Delta G_m(x)}{k_B T} = \frac{u(x)}{\ln^2 I(x)} - n_{BCs} k_B T \ln \left(\frac{I_{BC}}{x \lambda_{BC}(x)} \right) - n_{ACs} k_B T \ln \left(\frac{I_{AC}}{(1-x) \lambda_{AC}(x)} \right) \quad (44)$$

1032 and $u(x)$ given in Equation 13. The part which de-1064
1033 pends on $d\overline{\sigma(x)}/dx$ and the excess quantities n_{ACs} and 1065
1034 n_{BCs} vanishes, because it represents the Gibbs adsorp-1066
1035 tion isotherm equation (Adamson, 1960):

$$n(x)^{2/3} v(x)^{2/3} X \frac{d\overline{\sigma(x)}}{dx} = -n_{ACs} \frac{d\Delta\mu_{AC}}{dx} - n_{BCs} \frac{d\Delta\mu_{BC}}{dx} \quad (45)$$

1036 A particularly simple choice for the Gibbs dividing sur-1068
1037 face is that for which the surface energy does not de-1069
1038 pend upon the curvature of the surface, in which case 1070
1039 the surface excesses fulfill the relationship $n_{ACs} v_{AC} + 1071$
1040 $n_{BCs} v_{BC} = 0$ (Laaksonen et al., 1999). Associated with 1072
1041 Eq. 45, it leads to the following values of n_{ACs} and n_{BCs} : 1073

$$n_{ACs}(x) = \frac{n(x)^{2/3} X x (1-x) v_{BC}}{k_B T v(x)^{1/3} [1-2x(1-x)\{A_0 + A_1(6x-3)\}]} * \frac{d\overline{\sigma(x)}}{dx} \quad 1074$$

$$n_{BCs}(x) = -\frac{n(x)^{2/3} X x (1-x) v_{AC}}{k_B T v(x)^{1/3} [1-2x(1-x)\{A_0 + A_1(6x-3)\}]} * \frac{d\overline{\sigma(x)}}{dx} \quad 1075$$

$$(46) \quad 1076$$

1042 They depend on x and are proportional to the critical 1077
1043 nucleus area, while $n(x)$ is proportional to the nucleus
1044 volume. They vanish if the surface energy is composition
1045 independent. In the absence of detailed information on
1046 the x dependence of $\sigma(x)$, a linear law may be assumed
1047 between the end-member values of σ .

1048 Excess quantities also contribute to the variation of 1078
1049 Gibbs free energy during growth. The energetic cost to 1079

change the dimensions of a particle $\delta\Delta G(x)$ reads:

$$\delta\Delta G(x) = -\frac{\delta V}{v(x)} k_B T \ln I(x) + \delta E_s - \delta n_{BCs} k_B T \ln \left(\frac{I_{BC}}{x \lambda_{BC}(x)} \right) - \delta n_{ACs} k_B T \ln \left(\frac{I_{AC}}{(1-x) \lambda_{AC}(x)} \right) \quad (47)$$

It is related to its change of volume δV , its change of
total surface energy δE_s (now a function of x through
 $\overline{\sigma(x)}$), and its change in excess surface quantities δn_{ACs}
and δn_{BCs} . In the minimization of $\delta\Delta G(x)$, the part
which depends on $d\overline{\sigma(x)}/dx$ and the excess quantities
 δn_{ACs} and δn_{BCs} is formally similar to that written for
nucleation, and yields similar expressions for δn_{ACs} and
 δn_{BCs} (Equation 46).

Finally, excess quantities have to be taken into account
in the feed-back equations (M=AC or BC):

$$q_M(t) = \int_0^t F(t_1) (n^*(t_1) - 1) X_M(t_1) dt_1 + \int_0^t F(t_1) dt_1 \int_{t_1}^t dt_3 \frac{dn(t_1, t_3)}{dt_3} X_M(t_3) + \int_0^t F(t_1) n_{Ms}(t_1) dt_1 \quad (48)$$

Appendix D: Composition of the critical nuclei

The methodology to determine the composition x^* of
the critical nucleus is very similar to that used to find the
composition x_{st} of the SS at stoichiometric saturation
(Appendix A). Indeed, x^* is obtained from the minimum
of the nucleation barrier $\Delta G_m(x)$:

$$\frac{\Delta G_m(x)}{k_B T} = \frac{u(x)}{\ln^2 I(x)} \propto \left(\frac{v(x)}{\ln I(x)} \right)^2 \quad (49)$$

In the right hand side of this equality, we have evidenced
the terms which depend on x . Minimizing $\Delta G_m(x)$ with
respect to x thus amounts to minimizing $-\ln I(x)/v(x)$.

By comparison with Appendix A, it first appears ob-
vious that the composition x^* of the critical nucleus is
equal to x_{st} when the formula unit volume $v(x)$ of the
SS is independent on x , because then only the minimiza-
tion of $-\ln I(x)$ must be found. Moreover, we note that
the minimum of $-\ln I(x)/v(x)$ coincides with that of
the function $g(x)$ equal to:

$$g(x) = -\frac{\ln I(x)}{v(x)} + \frac{\ln I_{AC}}{v_{AC}} \quad (50)$$

$g(x)$ can also be written:

$$v(x)g(x) = x \ln x + (1-x) \ln(1-x) + x(1-x)[A_0 + A_1(2x-1)] - x \ln W' \quad (51)$$

Aside from the parameters A_0 and A_1 , it depends on
the ratio $z = v_{BC}/v_{AC}$ of the end-member formula unit

volumes ($v(x) = v_{AC}[1 - x + zx]$) and on the composition of the aqueous solution, which enters in a compact way via the ratio $W' = I_{BC}/I_{AC}^z$.

The derivative of $g(x)$ is such that:

$$\frac{v(x)^2}{v_{AC}} \frac{dg(x)}{dx} = \ln x - z \ln(1-x) - \ln W' + A_0 [(1-x)^2 - zx^2] + A_1 [4x^3(1-z) + 3x^2(z-3) + 6x - 1] \quad (52)$$

The terms which depend on A_0 and A_1 turn out to be equal to $\ln \lambda_{BC} - z \ln \lambda_{AC}$, so that equating $dg(x)/dx$ to zero leads to the implicit equation which determines x^* (Equation 16 in the main text):

$$\left(\frac{I_{AC}}{(1-x^*)\lambda_{AC}(x^*)} \right)^{v_{BC}} = \left(\frac{I_{BC}}{x^*\lambda_{BC}(x^*)} \right)^{v_{AC}} \quad (53)$$

The composition x^* of the critical nucleus is thus obtained when simultaneously $dg(x)/dx = 0$ and $g(x)$ is minimal. The discussion proceeds along steps similar to those relevant for x_{st} . Depending upon the regions of parameter space $\{A_0, A_1\}$ (which now depends on z), $g(x)$ may be a convex function with a single minimum. Alternatively, it may display one minimum, or two minima and a maximum, depending upon the value of W' . When the latter case occurs, the composition of the critical nuclei is equal to the root x which corresponds to the lowest value of $g(x)$.

Appendix E: Conditions of occurrence of the various scenarios of precipitation

In this appendix, we derive formal relationships allowing the determination of dW'/dt , a crucial quantity to assess which scenario will take place. Moreover, we specify the relative percentage of each phase when phase separation takes place in Scenario #4.

We recast the feed-back equation under the following form:

$$\begin{aligned} \frac{d[A]}{dt} &= -D_0(t)(1-x^*(t)) + D_A(t) \\ \frac{d[B]}{dt} &= -D_0(t)x^*(t) + D_B(t) \\ \frac{d[C]}{dt} &= -D_0(t) + D_C(t) \end{aligned} \quad (54)$$

In these expressions, $D_0(t)$ represents the time derivative of the number of formula units withdrawn from the AS with composition x^* , while $D_A(t)$, $D_B(t)$ and $D_C(t)$ are the variations of A, B and C activities due to either particle redissolution (thus with a surface composition different from $x^*(t)$) or dissolution/precipitation of other minerals present in the AS.

The time derivative $\ln W'$ reads:

$$\frac{1}{W'} \frac{dW'}{dt} = \frac{1}{[B]} \frac{d[B]}{dt} - \frac{z}{[A]} \frac{d[A]}{dt} + \frac{(1-z)}{[C]} \frac{d[C]}{dt} \quad (55)$$

which, after some algebra and using Eq. 54, may be recast under the following form:

$$\frac{1}{W'} \frac{dW'}{dt} = W'' - D_0(t) \left(\frac{x^*(t)}{[B]} - \frac{z(1-x^*(t))}{[A]} \right) \quad (56)$$

with:

$$W'' = \frac{D_B}{[B]} - \frac{zD_A}{[A]} + \frac{(1-z)D_C}{[C]} - D_0(t) \frac{(1-z)}{[C]} \quad (57)$$

We have separated the contribution W'' to dW'/dt which does not present a discontinuity, from the one (second term on the right hand side of Eq. 56) which does present a discontinuity, due to the jump of $x^*(t)$ between x_1 and x_2 . In Eqs. 54, 56 and 57, all terms relative to $[C]$ have to be skipped when precipitation of bimetallic $A_{1-x}B_x$ particles is considered.

Whenever Scenario #4 takes place, the cancellation of dW'/dt when $W' = W_c$ yields the relative amounts $D_0(t)\alpha$ and $D_0(t)(1-\alpha)$ of the two SSs with composition x_1 and x_2 when phase separation occurs. From Eq. 56, one obtains:

$$\begin{aligned} \frac{1}{W'} \frac{dW'}{dt} = 0 = & W'' - D_0(t)\alpha \left(\frac{x_1}{[B]} - \frac{z(1-x_1)}{[A]} \right) \\ & - D_0(t)(1-\alpha) \left(\frac{x_2}{[B]} - \frac{z(1-x_2)}{[A]} \right) \end{aligned} \quad (58)$$

and thus:

$$\alpha = \frac{W'' - D_0(t) \left(\frac{x_2}{[B]} - \frac{z(1-x_2)}{[A]} \right)}{D_0(t)(x_1 - x_2) \left[\frac{1}{[B]} + \frac{z}{[A]} \right]} \quad (59)$$

All these expressions may be easily evaluated numerically to assess which is the scenario relevant for the case under study and, in the case where Scenario #4 applies, Eq. 59 gives the extent of phase separation and its time dependence.

In particular, under the assumptions made in Sections VIB and VII, dW'/dt takes the simplified form:

$$\frac{1}{W'} \frac{dW'}{dt} \propto \left(\frac{x^*(t)}{[B]} - \frac{z(1-x^*(t))}{[A]} \right) \quad (60)$$

which allows the slope discontinuity between $W' < W'_c$ and $W' > W'_c$ to be evaluated by replacing x^* by x_1 or x_2 , respectively. Moreover, under the same assumptions, when phase separation takes place, the relative percentages α and $(1-\alpha)$ of the two SSs with composition x_1 and x_2 are given by:

$$\alpha = - \frac{(x_2 - (1-x_2) \frac{K_{BC}}{K_{AC}})}{(1-2x_2) \left(1 + \frac{K_{BC}}{K_{AC}} \right)} \quad (61)$$

and are independent of time.

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LIST OF SYMBOLS

Latin characters

A_0, A_1	First and second coefficients of the Guggenheim expansion (dimensionless)
$[A]$	Activity of the aqueous species A
D_0	Time derivative of the number of formula units withdrawn from the AS with composition x^*
D_A, D_B, D_C	Contributions to the time derivative of [A], [B] or [C] which are continuous at W'_c
E_s	Total surface energy (J)
$F(x)$	Nucleation frequency (number of nuclei/s/liter of solution)
F_0	Prefactor of the nucleation frequency (number of nuclei/s/liter of solution)
ΔG_M	Standard changes in Gibbs free energy for the dissolution of the M end-member (M=AC or BC)
$\Delta G(x)$	Change of Gibbs free energy for a SS of composition x during precipitation or growth
$\Delta G_M^E(x)$	Excess change of Gibbs free energy of mixing for a SS of composition x
$\Delta G(n, x)$	Change in Gibbs free energy for the formation of a nucleus containing n formula units of composition x
$\Delta G_m(x)$	maximum of $\Delta G(n, x)$ with respect to n
$\Delta H_M(x)$	Enthalpy of mixing for a SS of composition x
I_M	Saturation state of the AS with respect to the pure end-member M (M=AC or BC)
$I(x)$	Stoichiometric saturation state of the AS with respect to a SS of composition x
$I_c(x^*)$	Critical saturation state of the AS with respect to a SS of composition x
k_B	Boltzmann constant ($1.3806504 \times 10^{-23}$ J/K)
K_M	Solubility product of the end-member M (M=AC or BC)
$K(x)$	Stoichiometric solubility product of a SS of composition x
$l(t_1, t)$	Lateral length at time t of a rhomboedral or a tetragonal particle created at time t_1 (m)
$n_m(x)$	Number of growth units in a particle of composition x at the maximum of $\Delta G(n, x)$ with respect to n
$n(t_1, t)$	Number of growth units at time t in a particle created at time t_1
$n_M(x)$	Surface excess quantities of end-member M in a particle
$Q(x)$	Ionic activity product of a SS of composition x
$q_M(t)$	Amount of end-member M withdrawn at time t from the AS (formula unit/liter of solution)
R	Gaz constant (8.314472 J/K/mol)
T	Temperature (K)
x_0	Composition of a SS at thermodynamic equilibrium with an AS
x_{st}	Composition of a SS at stoichiometric saturation with an AS
x^*	Critical nucleus composition
x_1, x_2	Values of the SS composition at the limit of the discontinuity in strongly non-ideal SSs
\bar{x}	Mean composition value of the SS when phase separation occurs
v_M	Volume of one formula unit of end-member M (m^3)
$v(x)$	Volume of one formula unit of a SS of composition x (m^3)
W	Ratio of the saturation states of the pure end-members $W = I_{BC}/I_{AC}$
W_c	Critical value of W at the composition discontinuity
W'	$W' = I_{BC}/(I_{AC})^z$
W'_c	Critical value of W' at the composition discontinuity
$dW'_1/dt, dW'_2/dt$	Slopes of dW'/dt on the left and right of the discontinuity, respectively
W''	Contribution to dW'/dt which is continuous at W'_c
W_{adh}	Adhesion energy, in the case of heterogeneous nucleation (J/m ²)
X	Geometric factor entering the total surface energy of the particles
z	Ratio of end-members formula unit volumes $z = v_{BC}/v_{AC}$

Greek characters

α	Percentage of the two SSs of composition x_1 and x_2 when phase separation occurs
$\Delta\mu_M$	Change in chemical potential of one formula unit of the M end-member during precipitation (M=AC or BC)
κ	Linear growth constant (m/s)
λ_M	Activity coefficient of the end-member M in the SS (M=AC or BC)
$\rho(t_1, t)$	Radius at time t of a spherical particle created at time t_1 (m)
$\sigma(x)$	Mean surface energy per unit area of SS particles of composition x (J/m ²)
$\sigma_{lat}, \sigma_{bas}$	Lateral and basal surface energies per unit area of non-spherical particles (J/m ²)
θ	Wetting angle