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ABSTRACT

Equations for cosmological evolution are formulated in a Weyl invariant formalism to take into account possible Weyl anomalies. Near two dimensions, the renormalized cosmological term leads to a nonlocal energy-momentum tensor and a slowly decaying vacuum energy. A natural generalization to four dimensions implies a quantum modification of Einstein field equations at long distances. It offers a new perspective on time-dependence of couplings and naturalness with potentially far-reaching consequences for the cosmological constant problem, inflation, and dark energy.

To define a path integral over metrics in a quantum theory of gravity, one must introduce a regulator. Since the metric itself is a dynamical field, it is not clear in which metric to regularize and renormalize the theory, and how to ensure that the resulting answer is coordinate invariant and background independent. For this purpose it is convenient to enlarge the gauge symmetry to include Weyl invariance in addition to general coordinate invariance. This can be achieved by introducing a Weyl compensator field and a fiducial metric which scale appropriately keeping the physical metric Weyl invariant. The number of degrees of freedom remains the same upon imposing Weyl invariance. The path integral can now be regularized and renormalized using the fiducial metric.

A Weyl-invariant formulation has an important conceptual advantage because it separates scale transformations from coordinate transformations. The path integral can be regularized maintaining coordinate invariance at the quantum level. Weyl invariance can have potential anomalies in the renormalized theory but since it is a gauge symmetry all such anomalies must cancel. Coordinate invariance of the original theory then becomes equivalent to coordinate invariance plus *quantum* Weyl invariance of the modified theory. This procedure is well-studied in two dimensions where the Liouville field plays the role of the Weyl compensator and quantum Weyl invariance implies nontrivial scaling exponents.

There are both theoretical and phenomenological motivations to develop a Weyl-invariant formulation of gravity in higher dimensions, especially in the context of cosmology. Our chief theoretical motivation is to formulate the cosmological constant problem [1] in a manifestly gauge invariant way. The problem is usually stated in the language of effective field theories as a 'naturalness problem' analogous to the Higgs mass problem in electroweak theory or the strong-CP problem in quantum chromodynamics. The cosmological constant is the coupling constant of the identity operator added to the effective action. Since the identity has dimension zero, the cosmological constant term is the most relevant operator and should scale as M_0^d in d space-time dimension where the ultraviolet cutoff scale M_0 is at least of the order of a TeV. To reproduce the observed scale of the cosmological constant of the order of an meV, it is necessary to fine tune the bare vacuum energy.

This formulation of the cosmological constant problem is not entirely satisfactory. While the *generation* of the cosmological constant in the effective action depends only on short-distance physics, its *measurement* relies essentially on long-distance physics spanning almost the entire history of the universe. The physics of the cosmological constant thus spans more than a hundred logarithmic length scales. Moreover, all scales are evolving in a cosmological setting, and there is no preferred time for setting the cutoff in a manner that respects coordinate invariance. Thus, even to pose the cosmological constant problem properly, it is desirable to develop a formalism that accesses all time-scales in a gauge-invariant fashion.

A chief phenomenological motivation is to explore the possibility of effective time variation of vacuum energy. There is a substantial body of cosmological evidence for a slowly varying vacuum energy which is believed to have been responsible for an inflationary phase of exponential expansion in the very early universe. Observations of cosmic microwave background radiation indicate that the power spectrum generated during inflation is not strictly scale-free but has a slight red tilt. This implies that vacuum

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energy was not strictly constant but was slowly decaying during the inflationary era. Cosmological data also indicates that 69% of present energy density is in the form resembling vacuum energy. Time variation of dark energy is not established observationally at present but could be observed in planned observations. Any theoretical insight into the magnitude, equation of state, and time dependence of dark energy is clearly desirable.

Slowly varying vacuum energy can be represented by a cosmological constant Λ to first approximation. However, any time variation cannot be reintroduced simply by making Λ time-dependent because that would not be coordinate-invariant. A simple way to obtain time-dependent vacuum energy is to represent it by a slowly-rolling condensate of a scalar field. This idea is central to most current models of varying vacuum energy. Such a slowly-rolling field is called the 'inflaton' during the inflationary era and 'quintessence' during the present era. Models with scalar fields have the virtue of simplicity, but among the plethora of models none is particularly more compelling than the others; and our understanding of many questions of principle such as the initial value problem or the measure problem is less than satisfactory.

The puzzles regarding the cosmological constant, inflation, and dark energy all concern the nature of slowly varying vacuum energy. Occam's razor suggests that perhaps the essential underlying physics is governed by the same fundamental equations. With these motivations, I develop a Weyl-invariant formulation of quantum cosmology to explore the possibility of slowly evolving vacuum energy that does not rely on fundamental scalars.

I start with a Weyl-invariant reformulation of classical general relativity in d spacetime dimensions by introducing a Weyl compensator field Ω and a fiducial metric $h_{\mu\nu}$. Given a UV cutoff M_0, the reduced Planck scale M_p and the cosmological constant Λ correspond to dimensionless 'coupling constants' κ^2 and λ defined by:

$$M_p^{d-2} := \frac{M_0^{d-2}}{\kappa^2} \qquad \Lambda := \lambda\kappa^2 M_0^2\,. \tag{1}$$

The gravitational action $I_K[h,\Omega]$ is given by

$$\frac{M_0^{d-2}}{2\kappa^2}\int dx\, e^{(d-2)\Omega}[R_h + (d-2)(d-1)|\nabla\Omega)|^2] \tag{2}$$

where all contractions are using the metric h and $dx := d^dx\sqrt{-h}$. The cosmological term is given by

$$I_\Lambda[h,\Omega] = -M_p^{d-2}\Lambda\int dx\, e^{d\Omega} = -\lambda M_0^d\int dx\, e^{d\Omega}\,. \tag{3}$$

All terms are coordinate invariant. Both I_K and I_Λ are separately invariant under Weyl transformations:

$$h_{\mu\nu}\to e^{2\xi}h_{\mu\nu}\,, \qquad \Omega\to\Omega-\xi\,. \tag{4}$$

Consequently both I_K and I_Λ satisfy the Ward identities for coordinate invariance:

$$\nabla^\nu\Big(\frac{-2\,\delta I_a}{\sqrt{-h}\,\delta h^{\mu\nu}}\Big) - \frac{1}{\sqrt{-h}}\frac{\delta I_a}{\delta\Omega}\nabla_\mu\Omega \equiv 0 \quad (a = K,\Lambda)\,, \tag{5}$$

and for Weyl invariance:

$$h^{\mu\nu}\Big(\frac{-2\,\delta I_a}{\sqrt{-h}\,\delta h^{\mu\nu}}\Big) - \frac{1}{\sqrt{-h}}\frac{\delta I_a}{\delta\Omega} \equiv 0 \quad (a = K,\Lambda)\,. \tag{6}$$

The physical metric $g_{\mu\nu} := e^{2\Omega}h_{\mu\nu}$ is Weyl invariant. In the 'physical' gauge we have $\Omega = 0$ and $h_{\mu\nu} = g_{\mu\nu}$ and (2) reduces to the Einstein–Hilbert action.

Consider a homogeneous and isotropic universe described by a spatially flat Robertson–Walker metric with scale factor $a(t)$, filled with a perfect fluid of energy density ρ and pressure p. The classical evolution of the universe is governed by the first Friedmann equation

$$H^2 = \frac{2\kappa^2\rho}{(d-2)(d-1)M_0^{d-2}} \tag{7}$$

and the conservation equation

$$\dot\rho = -(d-1)(p+\rho)H\,. \tag{8}$$

For a perfect fluid with a barotropic equation of state $p = w\rho$, the solutions to (7) and (8) are given by

$$\rho(t) = \rho_*\Big(\frac{a}{a_*}\Big)^{-\gamma}\,, \qquad a(t) = a_*\Big(1+\frac{\gamma}{2}H_*t\Big)^{\frac{2}{\gamma}}\,, \tag{9}$$

where ρ_*, H_*, a_* are the initial values of various quantities at $t = 0$, and $\gamma := (d-1)(1+w)$. For the classical tensor of the cosmological term, $\rho_* = \lambda_* M_0^d$, $w = -1$, and $\gamma = 0$. As $\gamma\to 0$, the solution approaches nearly de Sitter spacetime with nearly exponential expansion and nearly constant density. Note that the cosmological evolution equations depend analytically on d, so one can 'analytically continue' the FLRW cosmologies.

Weyl invariance has potential anomalies at the quantum level. To gain intuition about these anomalies, we first consider spacetime near two dimensions, $d = 2+\epsilon$. To order ϵ, the total action I without matter is given by

$$\frac{q^2}{4\pi}\int dx\Big(\frac{R_h}{\epsilon} + |\nabla\Omega|^2 + R_h\Omega - \frac{4\pi\lambda M_0^2}{q^2}e^{2\Omega}\Big) \tag{10}$$

where the coupling constant q defined by

$$q^2 := \frac{2\pi\epsilon}{\kappa^2} \tag{11}$$

is held fixed as $\epsilon\to 0$. With $\chi := q\,\Omega$ and $\mu = \lambda M_0^2$, and ignoring the first term which depends only the fiducial metric, (10) is precisely the two-dimensional Liouville action with background charge q:

$$I[\chi] = \frac{1}{4\pi}\int dx\,\Big(|\nabla\chi|^2 + q\,R_h\,\chi - 4\pi\,\mu\,e^{2\beta\chi}\Big)\,. \tag{12}$$

The field χ is sometimes called the 'timelike' Liouville field because the kinetic term has a wrong sign, as expected for the conformal factor of the metric. Classical Weyl invariance implies $\beta = 1/q$, but this relation receives quantum corrections because the operator $e^{2\beta\chi}$ is a composite operator with short-distance singularities. It can be renormalized treating χ as a free field [2] with the Green function G_2 of the Laplacian Δ_2:

$$\Delta_2^x\, G_2(x,y) = \delta_2(x,y)\,. \tag{13}$$

There is a short-distance divergence arising from self-contractions which combine into an exponential of the coincident Green function $G_2(x,x)$. This divergence can be regularized by using a heat kernel with a short-time cutoff. Renormalization then consists in subtracting a logarithmically divergent term from the regularized $G_2(x,x)$. This procedure is manifestly local and coordinate invariant. In two dimensions, any metric is conformal to the flat metric $\eta_{\mu\nu}$: $h_{\mu\nu} = e^{2\Sigma}\eta_{\mu\nu}$. The renormalized operator $\mathcal{O}_h(x) := [e^{2\beta\chi}]_h$ depends on the fiducial metric used for regularization and satisfies

$$\mathcal{O}_h(x) = e^{-2\beta^2\Sigma(x)}\,\mathcal{O}_\eta(x)\,. \tag{14}$$

The scalar Σ is a *nonlocal* functional of the metric:

$$\Sigma[h](x) := \frac{1}{2}\int dy\, G_2(x,y)R_h(y)\,. \tag{15}$$

The defining property of Σ is its Weyl transformation

$$\Sigma[h] \to \Sigma[h] + \xi \quad \text{when} \quad h_{\mu\nu} \to e^{2\xi} h_{\mu\nu} \,. \tag{16}$$

Hence the renormalized operator has anomalous dimension $2\beta^2$. The total Weyl variation is given by

$$\mathcal{O}_h \to e^{-2(\beta q + \beta^2)\xi} \mathcal{O}_h \,. \tag{17}$$

The renormalized cosmological term is

$$I_\Lambda = -\mu \int dx \, \mathcal{O}_h(x) \,. \tag{18}$$

Weyl invariance of I_Λ now implies

$$2\beta q + 2\beta^2 = 2 \quad \text{or} \quad q = \frac{1}{\beta} - \beta \text{ and } \beta = \frac{1}{q} + \dots . \tag{19}$$

It is illuminating to interpret these results in terms of a quantum effective action. In operator formalism, the renormalization of the cosmological operator corresponds to normal ordering in the conformal vacuum defined using the Klein–Gordon inner product in the metric $\eta_{\mu\nu}$. Evaluating the operator equation (14) around a classical background field Ω in the conformal vacuum and by using (19) and (18), we obtain the quantum effective action that replaces the classical action (3):

$$I_\Lambda = -\mu \int dx \, e^{2\Omega} \, e^{-2\beta^2(\Omega+\Sigma)} \,. \tag{20}$$

The term depending on Σ encapsulates the anomalous dimension of the operator and is nonlocal. The corresponding momentum tensor is

$$T^\Lambda_{\mu\nu}(x) = -\mu\,(1-\beta^2) h_{\mu\nu} \mathcal{O}_h(x) + 2\mu\beta^2 S_{\mu\nu}(x) \tag{21}$$

where $S_{\mu\nu}$ is nonlocal and traceless:

$$\begin{aligned} S_{\mu\nu}(x) = &\int dy \left[\nabla^x_\mu \nabla^x_\nu - \tfrac{1}{2} h_{\mu\nu} \nabla^x \cdot \nabla^x \right] G_2(x,y)\, \mathcal{O}_h(y) \\ &+ \int dy\, dz \Big[\nabla^x_{(\mu} G_2(x,y)\, \nabla^x_{\nu)} G_2(x,z) \\ &- \tfrac{1}{2} h_{\mu\nu}(x)\, h^{\alpha\beta}(x)\, \nabla^x_\alpha G_2(x,y)\, \nabla^x_\beta G_2(x,z) \Big] \mathcal{O}_h(y)\, R_h(z) \,. \end{aligned} \tag{22}$$

The nonlocality of the momentum tensor reflects the nonlocality of the action (20) and is in line with the interpretation of anomalies as the effect of regularization that cannot be removed by local counterterms. Since $(\Omega + \Sigma)$ is a Weyl-invariant scalar, the action (20) satisfies both Ward identities (5) and (6) and the quantum momentum tensor (21) is conserved.

The total quantum action in physical gauge is given by

$$I[g] = \frac{M_p^2}{2} \int d^{2+\epsilon}x \sqrt{-g} \left[R_g - 2\Lambda e^{-2\beta^2 \Sigma[g]} \right] . \tag{23}$$

The right-hand side of Einstein field equations now has a nonlocal momentum tensor. Correspondingly the Friedmann equations are replaced by new *nonlocal integro-differential equations* for cosmological evolution. One might worry that the nonlocality would lead to ghosts and causality violations. However, one must use the in-in effective action in the Schwinger–Keldysh formalism and not the in-out effective action. The corresponding boundary conditions naturally lead to retarded Green functions instead of Feynman propagators, thus ensuring causality of the quantum cosmological evolution.

For a spatially-flat Robertson–Walker metric, the second term in $S_{\mu\nu}$ vanishes and only time derivatives contribute to the first term. Consequently, the quantum momentum tensor evaluated on the Robertson–Walker ansatz turns out to be local, corresponding to a barotropic fluid with $w_\Lambda = -1 + 2\beta^2$ or $\gamma = 2\beta^2$. Using (9) we arrive at a dramatic conclusion that the vacuum energy decays and the effective coupling constant λ evolves as the universe expands:

$$\rho(t) = \rho_*(\frac{a}{a_*})^{-2\beta^2} \quad \text{or} \quad \lambda(t) = \lambda_*(\frac{a}{a_*})^{-2\beta^2} . \tag{24}$$

This provides a dynamical mechanism for the relaxation of the 'cosmological constant'.

One expects that a composite operator like the determinant of the metric will have anomalous dimension even in four dimensions. The quantum action (23) suggests a natural generalization to four dimensions. Consider the Weyl-covariant quartic operator [3–5] defined by

$$\Delta_4 := \nabla^4 + 2\, R^{\mu\nu}\, \nabla_\mu \nabla_\nu + \frac{1}{3} (\nabla_\mu R)\, \nabla^\mu - \frac{2}{3} R\, \nabla^2 \tag{25}$$

and the corresponding Green function G_4 satisfying:

$$\Delta^x_4\, G_4(x,y) = \delta_4(x,y) \,. \tag{26}$$

Then Σ which transforms as in (16) is given by

$$\Sigma[h] := \frac{1}{4} \int dy\, G_4(x,y)\, F_4(y) \,; \tag{27}$$

$$F_4 := R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 - \frac{2}{3} \nabla^2 R \,. \tag{28}$$

Using these ingredients I propose a generalization of the Einstein–Hilbert action in the physical gauge:

$$I[g] = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[e^{-\gamma_K \Sigma[g]} R_g - 2\Lambda\, e^{-\gamma_\Lambda \Sigma[g]} \right] \tag{29}$$

where γ_K and γ_Λ are the anomalous dressings.

The resulting equations of motion are nonlocal and the momentum tensor has a complicated expression. Remarkably, for the Robertson–Walker metric, the momentum tensor simplifies again describing a barotropic perfect fluid with $w = -1 + \gamma/3$ and $\gamma = \gamma_\Lambda - \gamma_K$. Thus vacuum energy decays slowly for small positive γ. The slow-roll parameters are of order γ [6].

The ansatz (29) is motivated by the considerations of renormalization and quantum Weyl invariance. It would be important to compute the anomalous gravitational dressings in four dimensions from first principles. Even in two dimensions, such a computation is nontrivial. The use of free Green function (13) is justified ultimately by exact results obtained using conformal bootstrap. Analogous methods are not available in four dimensions, but we expect that semiclassical computations should be feasible and reliable on cosmological scales. From general renormalization group considerations, we expect non-vanishing anomalous dressings for any composite operator and the corresponding metric dependence through Σ. Since anomalous dressings in general can be scale dependent, they could be arbitrary functions of Σ [6].

We comment briefly upon the relation of our proposal to earlier work in the literature. The idea of vacuum energy decay caused by infrared quantum effects has been explored earlier in four-dimensional gravity by several physicists.[1] There is considerable divergence in the literature [19] about the final result [20–26] and

[1] An interesting related idea explored in the literature concerns possible nontrivial fixed points of gravity in the UV [7–16] and in the IR [17,18]. This is a different regime than what we consider. Our interest is in the long distance physics on cosmological scales in weakly coupled gravity near the *trivial* Gaussian fixed point. Some of the methods developed in these investigations could nevertheless be useful for the computation of Weyl anomalies especially in very early universe.

more generally about infrared effects in nearly de Sitter spacetime [27–40]. In the present work we consider nonlocal effective action for the background metric and study its time evolution to determine whether de Sitter spacetime is a solution to the quantum corrected equations of motion. This is considerably simpler than the earlier treatments of resuming quantum effects perturbatively starting with a de Sitter background. An important conceptual difference is that we consider the effect of Weyl anomalies arising from the renormalization of composite operators; this effect is distinct from particle creation in a time-dependent background.

The advantage of our two-dimensional model is that the important quantum effects can be computed explicitly with relative ease and without ambiguities to all orders in perturbation theory. They are clearly distinct from the effects of particle creation. The main lesson is that the physical coupling constants are the couplings of the *gravitationally dressed* operators. The anomalous dimensions of the dressed operators are in principle different from the anomalous dimensions of the undressed operators. For example, the cosmological constant, which is usually regarded as the coupling constant of the identity operator, is really the coupling constant of the square-root of the determinant of the metric with a nontrivial anomalous gravitational dressing. This anomalous dressing introduces additional dependence on the metric and affects the gravitational dynamics. This is the essential idea that we wish to generalize to four dimensions.

In Liouville theory, an operator $\mathcal{O}_i$ of mass dimension Δ_i can be coupled in a Weyl invariant way with an action

$$I_i = -\lambda_i M_0^{2-\Delta_i} \int dx\, \mathcal{O}_i\, e^{(2-\Delta_i)\Omega} e^{-\gamma_i(\Omega+\Sigma)} \,. \tag{30}$$

Here γ_i is the 'anomalous gravitational dressing' which is the anomalous dimension of the composite operator $[\mathcal{O}_i\, e^{(2-\Delta_i-\gamma_i)\Omega}]$. For the identity operator, $\Delta_\Lambda = 0$ and $\gamma_\Lambda = 2\beta^2$. Another example is the mass term for a fermion corresponding to the operator $\bar{\psi}\psi$ with $\Delta_F = 1$:

$$I_F = -\lambda_m M_0 \int dx \left[\bar{\psi}\psi\, e^{2\alpha q \Omega}\right] . \tag{31}$$

Weyl invariance implies

$$2\alpha q + 2\alpha^2 = 1 \,, \tag{32}$$

and the anomalous dressing is $\gamma_F = 2\alpha^2$.

In four dimensions, one expects similar anomalous dressings for matter. This has a striking consequence in an expanding universe: dimensionless ratios determined by naive mass dimensions can change with time. For example, for the 2d fermion mass we have

$$\lambda_m(t) = \lambda_{m*}\left(\frac{a}{a_*}\right)^{-2\alpha^2} . \tag{33}$$

This anomalous time evolution is similar to the slow time decay of vacuum energy (24). Even though the fermion mass and vacuum energy are of order one at the cutoff scale M_0 in the beginning, their effective values today can be smaller because of the anomalous time evolution.

I propose a 'Cosmological Naturalness Principle': "*If there is a very small dimensionless parameter occurring in nature, then the anomalous gravitational dressing of the associated operator in the effective action is such that the smallness of the parameter is a consequence of its anomalous time evolution in an expanding universe*." Lack of evidence for supersymmetry thus far could be an indication that the Higgs mass problem has a primarily cosmological explanation. This would allow for a higher scale of supersymmetry breaking. It would be interesting to compute the relevant gravitational dressings in the microscopic theory. Since gravitational dressings depend on the microphysics, their cosmological manifestations would provide a useful IR window into the UV physics.

Even a tiny positive γ would solve the cosmological constant problem with vanishingly small vacuum energy at late times. More generally, the mechanism of vacuum energy decay could have far-reaching consequences for our understanding of inflation and dark energy. Slowly decaying vacuum energy in the early universe with small γ can drive slow-roll inflation without an inflaton field. By itself, it would lead to an empty universe, but with matter fields, one can imagine scenarios for a 'graceful exit' into a hot big bang. For example, there can be a phase transition with large latent heat triggered by the mass-squared of a scalar field turning negative, to start a radiation-dominated hot big bang.

Such 'inflation without the inflaton' driven by the dynamics of the Omega field could be called 'Omflation'. It seems though that unless the vacuum energy is much smaller than the radiation energy *after* the latest phase transition, it would dominate radiation too soon to be compatible with big bang nucleosynthesis. At late times γ may be too small, of order $G\Lambda$. Since the omflation is a gauge degree of freedom, primordial scalar curvature perturbations would have to be generated by curvaton-like scalars [41]. It thus remains to be seen whether one can construct a consistent evolution of the universe that incorporates omflation followed by a big bang to terminate with a small remnant dark energy, and whether γ can be large enough during this evolution in the microscopic theory. At any rate, with our novel mechanism for vacuum energy decay, it becomes a dynamical question.

The new action (29) implies a quantum modification of Einstein field equations at long distances and possibly observable deviations from the predictions of Einstein Gravity. It also predicts mild time-dependence for dark energy today which could be detectable if the anomalous dressings are sufficiently large. Gravitational dressings of other fields can lead to slow variations of coupling constants over time which could be constrained by observation and may possibly be detectable. A detailed analysis of the theoretical and observational implications will be presented in forthcoming publications [6].

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