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Josephson oscillation linewidth of ion-irradiated $\text{YBa}_2\text{Cu}_3\text{O}_7$ junctions

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Abstract. We report on the noise properties of ion-irradiated $\text{YBa}_2\text{Cu}_3\text{O}_7$ Josephson junctions. This work aims at investigating the linewidth of the Josephson oscillation with a detector response experiment at $\simeq 132$ GHz. Experimental results are compared with a simple analytical model based on the Likharev-Semenov equation and the de Gennes dirty limit approximation. We show that the main source of low-frequency fluctuations in these junctions is the broadband Johnson noise and that the excess ($\frac{1}{f}$) noise contribution does not prevail in the temperature range of interest, as reported in some other types of high- T_c superconducting Josephson junctions. Finally, we discuss the interest of ion-irradiated junctions to implement frequency-tunable oscillators consisting of synchronized arrays of Josephson junctions.

1 Introduction

Ion-irradiating technology for high temperature superconducting (HTS) integrated circuits is a very promising approach for a broad range of applications because of the possibility to design circuits that include a high density of arbitrarily located Josephson elements. Developments in the field include the fabrication of SQUIDs [1, 2], large arrays of Josephson junctions [3, 4], SQIFs [5], heterodyne mixers for THz frequency range [6, 7], and the recent realization of SIS tunnel junctions [8]. Today, one of the main challenges in the field is to build a THz frequency-tunable oscillator by synchronizing the oscillations of a large number of Josephson junctions. Indeed, when phase locking of N junctions is achieved, the maximum output power of the array increases as N^2 , enabling the generation of sizable power [9]. In addition, the linewidth Δf of the radiation can be considerably reduced since it decreases as $\frac{1}{N}$ [10]. This is particularly interesting in the case of practical HTS junctions, which have a rather broad Josephson oscillation linewidth at the typical temperature of operation (\sim GHz for $T \sim 50$ K). Recently, c-axis mesa of the layered high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ has emerged as a promising coherent THz oscillator. Its operation relies on the synchronization of the natural intrinsic Josephson junctions that are formed between CuO_2 superconducting planes separated by BiSrO insulating planes [11]. Engineering arrays of reproducible nanofabricated HTS junctions should enable the realization of similar oscillators with an additional degree of freedom regarding the architecture of the array and possibly the frequency tunability.

Unlike in common types of self-excited oscillators, the high-frequency linewidth Δf of the Josephson oscillation is mainly determined by low-frequency noise level [12]. This makes any prediction difficult since low-frequency noise depends not only on well understood thermal fluctuations but also on excess $\frac{1}{f}$ noise (critical current fluctuations), whose nature is still not fully understood and which varies significantly from one type of junctions to another [13–17]. This results in unpredictable linewidth behavior with temperature - some junctions demonstrate purely temperature defined noise level [18] while for others, generation linewidth might even drop down with temperature [19]. The aim of this work is to measure the linewidth of ion-irradiated Josephson junctions using the self-detected dc-response technique which was previously exploited for Josephson spectroscopy and oscillation linewidth measurements of several types of junctions [18–20] and to compare the results with analytical predictions for a pure thermal fluctuation source.

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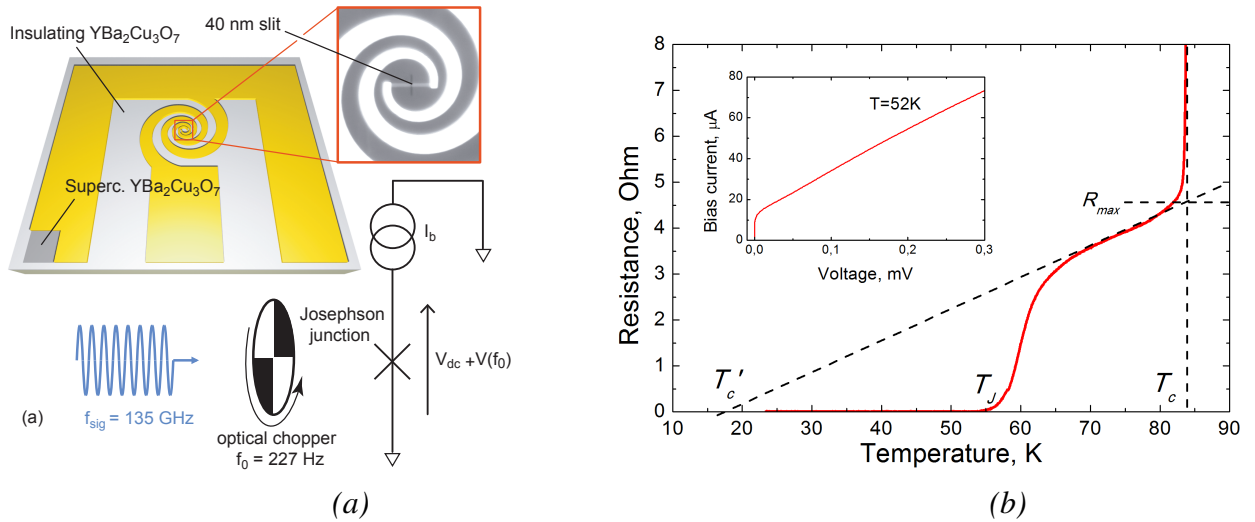


Figure 1: (a) Schematic description of the sample used in the detector response experiment. (b) Experimental dependence of the junction resistance with temperature (solid red lines). An intersection of a dashed extrapolation line with X-axis can be considered as an estimation of T_c' - transition temperature for the damaged area of the junction. Inset: IV-curve of the junction for $T=52$ K.

2 Experiment

In this experiment we used ion-irradiated YBa₂Cu₃O₇ Josephson junctions embedded into a spiral log-periodic antenna and a 50Ω coplanar waveguide (CPW) transmission line (figure 1(a)). Details of the fabrication process can be found in references [7, 21]. In short, we start from a commercial 70 nm thick YBa₂Cu₃O₇ films grown on a Al₂O₃ substrate [22], and covered with a 200 nm gold layer. The spiral antenna and the CPW transmission line are first defined in the gold layer through a MAN e-beam patterned resist, followed by a 500-eV Ar⁺ Ion Beam Etching. Then, a 0.75-2 μ m wide channel located at the center of the antenna is patterned in a MAN e-beam resist, followed by a 70-keV oxygen ion irradiation at a dose of $2 \cdot 10^{15}$ at/cm². This process ensures that the regions of the film which are not protected either by the resist or by the gold layer become deeply insulating. Finally, the junction is defined at the center of the superconducting channel by irradiating through a 40-nm wide slit patterned in a PMMA resist with 110 keV oxygen ions at a dose of $3 \cdot 10^{13}$ at/cm². The results presented in the paper were obtained with a $L=0.75 \mu$ m-wide junction, though all measured samples demonstrated qualitatively a similar behavior.

Figure 1(b) shows the resistance of the junction as a function of temperature. The highest transition at $T_c=84$ K refers to the superconducting transition of the reservoirs, i. e. the non-irradiated part of the YBa₂Cu₃O₇ channel, whereas the second transition at the lower temperature $T_J=55$ K corresponds to the occurrence of a sizable Josephson coupling (i.e stronger than thermal fluctuations, [23]). The temperature $T_c' \simeq 20$ K indicates the superconducting transition of the irradiated part itself and separates the pure Josephson regime from a flux-flow regime. Junctions fabricated by this method have non-hysteretic current-voltage characteristics expected in the overdamped regime, defined by a McCumber dimensionless parameter much smaller than one (inset figure 1(b)).

In order to measure the Josephson oscillation linewidth, we used the indirect detector response technique proposed in reference [24]. The schematic diagram of the experiment is shown in figure 1(a). A Gunn diode generates a continuous $f_{sig}=132$ GHz signal which is mechanically modulated by an optical chopper at a frequency $f_0=227$ Hz. The sample was placed inside a closed cycle refrigerator, at the focal point of a Winston cone, and was exposed to the external signal through an optical window. The output voltage $V(f_0)$ was measured as a function of the dc bias with a lock-in amplifier synchronized at the chopper frequency f_0 . Figure 2(a) shows a typical result obtained in the Josephson regime at $T=40$ K. The linewidth is directly extracted by measuring the spectral width between the two response extrema or equivalently between the zeros of its first derivative. Its evolu-

tion with temperature is shown in figure 3(b). A clear broadening of the linewidth is observed when the temperature is increased which suggests a mechanism based on thermal noise consideration. In the next section, we discuss the dependencies of the critical current and thermal linewidth broadening of the junction on temperature in the framework of the de Gennes proximity effect model and the Resistively Shunted Junction (RSJ) model.

3 Models and discussions

In the following, we consider the junction as a Superconductor-Normal-Superconductor type to discuss its transport properties and generation linewidth. Notice that the normal part of the junction is itself superconducting with a reduced critical temperature T'_c , a situation often referred as a SS'S junction. A characteristic trait of ion-irradiated junctions is their non pure RSJ-like current-voltages curves ([25, 26]), which can be associated with temperature and bias current dependent length of the normal segment (and correspondingly normal resistance). Following this approach, we use a modified RSJ-model and the de Gennes proximity effect model [27] to analyze the experimental results. The approach is illustrated in figure 2(b) which shows the dependence of the $\text{YBa}_2\text{Cu}_3\text{O}_7$ critical temperature on both the coordinate x along the junction and the bias current I . For the sake of simplicity we consider the temperature and the bias current as independent variables neglecting therefore heating effect.

3.1 Critical current

Experimental dependence of the junction critical current on temperature is shown in figure 3(a). To describe the results we assume a linear dependence of the normal segment length with temperature T and bias current I :

$$l(I, T) = l_0(T) + \alpha \cdot I \quad (1)$$

$$l_0(T) = l_{max} \left(1 + \frac{T - T_c}{T_c - T'_c} \right) \quad T'_c < T < T_c \quad (2)$$

where α is a fitting parameter and l_{max} is the maximum length of the normal segment in the absence of bias current at $T = T_c$. T'_c is a temperature at which the entire damaged area becomes superconducting

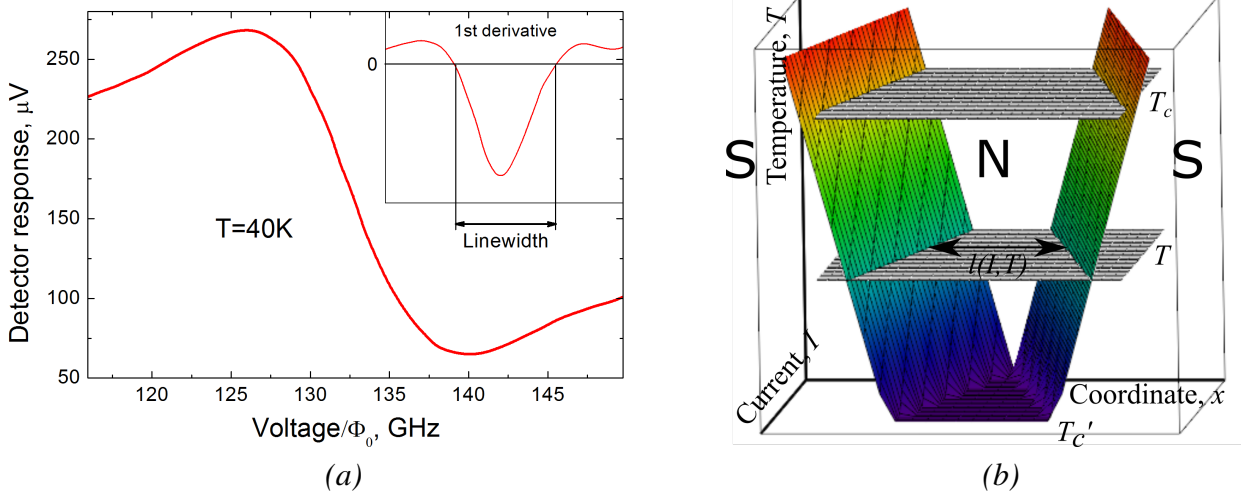


Figure 2: (a) Experimental detector response for temperature $T=40\text{K}$, Φ_0 is the magnetic flux quantum. The Josephson generation linewidth is given by the distance between the two extrema. Inset shows the first derivative of the response. (b) Linear dependence of the critical temperature of the ion-irradiated $\text{YBa}_2\text{Cu}_3\text{O}_7$ with bias current and coordinate assumed in the model.

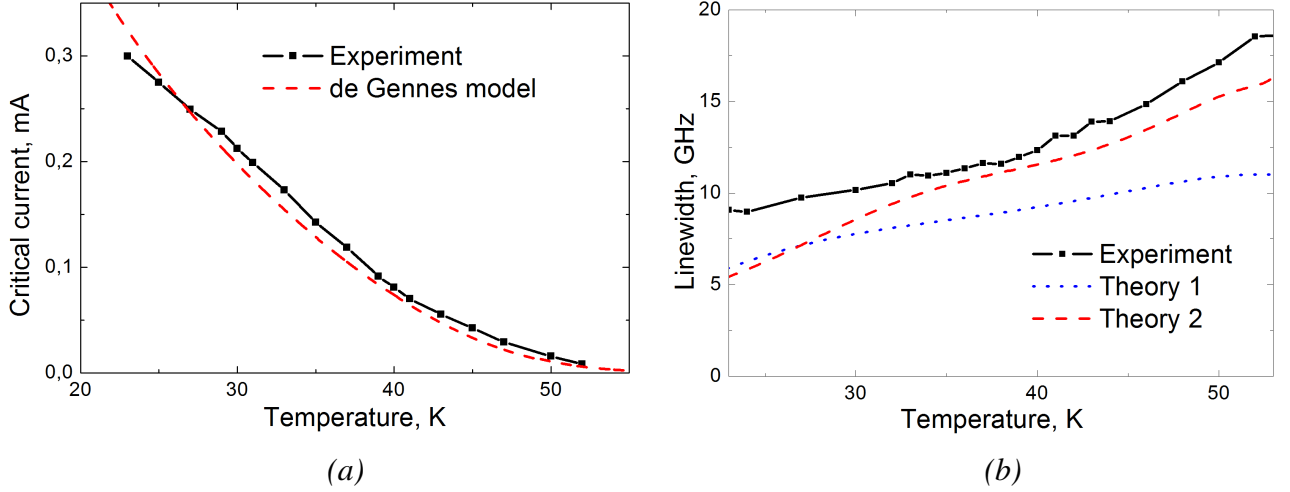


Figure 3: (a) Experimental critical current I_c as a function of temperature T (solid black line) fitted by de Gennes proximity effect model (dashed red line); (b) Generation linewidth a function of temperature (black solid line). Dashed red and blue dotted lines correspond to the two theoretical models based on formula (6).

($l(T'_c)=0$). We can estimate $T'_c \sim 20\text{K}$ and $l_{max} \sim 100\text{nm}$ (i.e. much larger than the slit width, see e.g. [28]) from the $R(T)$ curve presented on the figure 1(b) and Monte-Carlo (TRIM) simulation.

For conventional s-wave superconductors, $I_c(T)$ characteristics were extensively investigated for different types of junctions (see e.g. review [29]) including $SS'S$ -structures with $T_c \neq T'_c$ ([30, 31]). However, the applicability of these models for d-wave superconductors is not straightforward. Here, we use a simple proximity effect approximation as described by the de Gennes model:

$$I_c(I, T) = I_0(I, T) \cdot \left(1 - \frac{T}{T_c}\right)^2 \cdot \frac{\kappa(I, T)l(I, T)}{\sinh(\kappa(I, T)l(I, T))} \quad (3)$$

where $\kappa(T)$ is the reversed decay length given by

$$\kappa(T) = \left(\frac{D\hbar}{2\pi k_B T}\right)^{-1/2} \cdot \left(1 + \frac{2}{\ln(T/T'_c{}^{\text{eff}})}\right)^{-1/2}. \quad (4)$$

Here, D is the electron diffusion constant, k_B is the Boltzmann constant and $T'_c{}^{\text{eff}}$ is an effective critical temperature corresponding to an average of T'_c along the junction. $I_0(I, T)$ is the $T=0$ value of the critical current which is given by the expression :

$$I_0(I, T) = \frac{\pi\Delta_0^2}{4eR_N(I, T)k_B T_c} \quad (5)$$

where Δ_0 is the $\text{YBa}_2\text{Cu}_3\text{O}_7$ energy gap and R_N the normal resistance of the junction. The experimental critical current has to be compared with the one extracted for each particular temperature as a solution of the equation $I_c(I, T) = I$. Although the approach is expected to be strictly accurate only in the high temperature limit, a good agreement is obtained in the whole Josephson regime (see red dashed line on the figure 3(a)).

3.2 Generation linewidth

To analyze the experimental generation linewidth we describe the junction dynamics within the simple RSJ model and the Likharev-Semenov model [12] for nonlinear noise power transformation from high to low frequency. For simplicity we only take into consideration the thermal noise source

with a current spectral density¹ $S_I(0) = \frac{k_B T}{\pi R_N}$. The voltage spectral density can then be written as $S_V(0) = S'_I(0) \cdot R_d^2$, where R_d is the differential resistance of the junction and $S'_I(0) = S_I(0)(1 + \frac{I_c^2}{2I^2})$. In the frame of the Likharev-Semenov model the generation linewidth can be expressed as

$$\Delta f = \frac{4\pi}{\Phi_0^2} \cdot \frac{k_B T R_d^2}{R_N} \cdot \left(1 + \frac{I_c^2}{2I^2}\right) \quad [Hz] \quad (6)$$

where Φ_0 is the magnetic flux quantum.

As a first order approximation we can directly apply formula (6) using the experimental normal resistance $R_N = V/I$ and the differential resistance $R_d = dV/dI$ of the junction at the working point. The result of this simple estimation is shown in figure 3(b) with blue dots-line and referenced as "theory 1". A qualitative agreement is obtained with the experimental linewidth (black solid line) including in the low temperature range where the junction approaches a flux-flow regime. However, "theory 1" demonstrates a slower increase with temperature than the experimental one in the high temperature range area where the RSJ-model is expected to be more accurate.

To refine our analysis we propose another approach that explicitly takes into account the temperature and the current dependence of R_N . Following equations (1) and (2), we assume a linear dependence

$$R_N(I, T) = R_0(T) + \beta \cdot I \quad (7)$$

$$R_0(T) = R_{max} \left(1 + \frac{T - T_c}{T_c - T'_c}\right) \quad T'_c < T < T_c \quad (8)$$

where $R_0(T)$ is the normal segment resistance in the absence of bias current and R_{max} is the value of the resistance at $T = T_c$ (see figure 1(b)).

According to the simple RSJ-model, when the junction is biased with a dc current I the voltage across the junction is $V = R_N \sqrt{I^2 - I_c^2}$ and correspondingly the differential resistance is $R_d = \frac{dV}{dI} = \frac{R_N I}{\sqrt{I^2 - I_c^2}}$. Introducing equations (7) and (8) in the previous expressions of the voltage and the differential resistance allows calculating the generation linewidth (6) with only one fitting parameter β . The fit presented on figure 3(b) (red dashed line) and referenced as "theory 2" shows a quantitative agreement of the model with the experimental data. In particular, the temperature dependence of the generation linewidth is correctly reproduced. This remarkable behavior indicates that the noise of the junction is mainly limited by the broadband thermal noise and that additional sources of noise can be neglected at first order. Small discrepancies between experiment and theoretical model could originate from external noise absorbed by the unshielded junction in the refrigerator.

In this analysis, we used the Likharev-Semenov model which has been proposed for short Josephson junction corresponding to the regime $L < \lambda_J$ where $\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0 j_c d}}$ is the Josephson penetration length (λ_L is the London penetration length, $d = 2\lambda_L + l$, j_c is the critical current density and μ_0 is the vacuum permeability). Assuming a London penetration depth $\lambda_L(0) \sim 200$ nm for our YBa₂Cu₃O₇ junctions [32], the Josephson length is $\lambda_J \approx 600$ nm for a critical current of 100 μ A which is comparable to the width of the junction $L=750$ nm. Therefore, for the lower part of the temperature range investigated here, the junction is not strictly short and the Likharev-Semenov model may not give a very accurate result. To our knowledge there is no commonly accepted general theory on generation linewidth for long Josephson junctions, although known results ([33], [34]) suggest that it is somewhat broader compared to the short junction case.

¹We define spectral density for both negative and positive frequencies through expression $\langle I_{fluct}^2 \rangle \Big|_{d\omega} = 2S_I(\omega)d\omega$ for time averaged value of squared fluctuation current.

4 Conclusion

In conclusion, we have measured the transport characteristics as well as the Josephson linewidth dependence with temperature for ion-irradiated $\text{YBa}_2\text{Cu}_3\text{O}_7$ junctions. A good agreement is obtained with the Likharev-Semenov model which indicates that the linewidth of the junction is mainly limited by the broadband thermal noise. As already mentioned, a long term goal in the field is to build a frequency-tunable oscillator by synchronizing a large number of Josephson junctions. In previous experiments ([6, 7]), we demonstrated that ion-irradiated junctions could be operated as broadband Josephson mixers up to ~ 400 GHz and possibly higher frequencies provided a better impedance matching with the antenna and the CPW line. According to the experimental data shown in figure 3(b), the ion-irradiated junctions display a typical Josephson linewidth of approximately 10 GHz at 40K. Synchronizing an array of 10×10 junctions should therefore allow generating a coherent $\simeq 100$ MHz linewidth oscillation in the lower part of the THz spectrum. Combined with a Josephson mixer such arrays could be used as a frequency-tunable local oscillator to build a fully integrated detector.

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