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Equilibrium zinc isotope fractionation in Zn-bearing minerals from first-principles calculations

Manoj Ducher1,∗, Marc Blanchard1, Etienne Balan1.

Abstract

Isotopic composition analysis contributes significantly to the investigation of the biogeochemical cycle of zinc with its economical, environmental and health implications. Interpretation of isotopic measurements is however hindered by the lack of a set of equilibrium isotopic fractionation factors between Zn-bearing minerals. In this study, equilibrium mass-dependent Zn isotope fractionation factors in Zn-bearing minerals are determined from first-principles calculations within the density functional theory (DFT) scheme. A wide range of minerals belonging to sulfide, carbonate, oxide, silicate, sulfate and arsenate mineral groups are modelled to account for the natural diversity of Zn crystal-chemical environment. Calculated reduced partition function ratios (β-factors) span a range smaller than 2% at 22°C. All studied secondary minerals (adamite, gahnite, gunningite, hemimorphite, hydrozincite, zincite) but zinc carbonate (smithsonite) are isotopically heavier than zinc sulfide minerals (sphalerite and wurtzite) from which they could form through supergene processes. Zinc-aluminium spinel and zinc silicate are the isotopically heaviest minerals. The investigation of the crystal-chemical parameters at the origin of differences in isotopic properties shows an excellent linear correlation between ln β and Zn interatomic parameters. β-factors are also observed to increase when the Zn-first neighbour bond lengths decrease and charges on atoms involved in the bonding increase and vice versa. These findings are in line with the observation of heavy isotope enrichment in systems having the largest bond strength.

Keywords: zinc isotope fractionation, ab initio, equilibrium fractionation, Zn mineral

1. Introduction

Anthropic activities affect global Zn cycle and cause major environmental issues. Globally the anthropic Zn reservoir (4.9×10⁵ Gg Zn) and flux are of the same order of magnitude as the natural ones (Rauch and Pacyna, 2009). About half of extracted Zn are discarded from anthropic reservoir to soils, which are the third largest reservoir of this element after Earth’s mantle and crust (Rauch and Pacyna, 2009). Once in soils, Zn is either trapped or mobilised to other reservoirs like freshwater, oceans and atmosphere. Mechanisms such as Zn incorporation in minerals, precipitation, adsorption and dissolution control its mobility and would either attenuate or enhance Zn pollution. These processes are in turn controlled by environmental factors such as pH, the presence of bio-organisms and crystalline phases (clay minerals, transition element oxides and Zn-bearing minerals) that change the speciation of Zn. Moreover, Zn is a vital dietary element (Frassinetti et al., 2003; Mason et al., 2005; Wilkinson et al., 2005).

Speciation changes are known to induce isotopic fractionations. Thus, Zn isotope fractionation imposes itself as a relevant physical process that can be used to address these questions, and analytical techniques paved the way for precise Zn isotope measurements (Maréchal, 1998). Zn displays five stable isotopes with atomic masses 64, 66, 67, 68, 70 and relative abundances of 48.6%, 27.9%, 4.1%, 18.8% and 0.6% respectively. Zn isotope compositions are usually expressed as δ⁶⁶Zn defined as the measured isotope ratio in a sample with respect to JMC Lyon standard solution (Maréchal et al., 1999):

\[
\delta^{66}\text{Zn} = \left[ \frac{(\text{Zn}^{66}/\text{Zn}^{64})_{\text{sample}}}{(\text{Zn}^{66}/\text{Zn}^{64})_{\text{JMC Lyon}}} - 1 \right] \times 1000
\]

Several studies measured Zn isotope compositions in both synthetic and natural materials. In the latter, the lowest δ⁶⁶Zn value has been so far measured in lunar samples with δ⁶⁶Zn = −11.4‰ (Kato et al., 2015) while the largest value has been found in primitive meteorites by Moynier et al. (2011a) with δ⁶⁶Zn = 7.35‰. In comparison, δ⁶⁶Zn of terrestrial geological materials spread into a small range comprised between −0.80‰ and 2.5‰ (Pichat et al., 2003; Mason et al., 2005; Wilkinson et al., 2005;
Toutain et al., 2008; Chen et al., 2009; Moynier et al., 2009a; Pons et al., 2011; Gagnevin et al., 2012; Pašava et al., 2014; Little et al., 2014; Zhao et al., 2014) and the average $\delta^{66}$Zn of igneous rocks is $0.28 \pm 0.05\%$ (Maréchal, 1998; Cloquet et al., 2006; Herzog et al., 2009; Paniello et al., 2012; Chen et al., 2013). Zn isotope composition have been widely used to trace environmental processes, to study spatial extension of metal pollution and to identify some attenuation mechanisms. As mining and smelting activities contribute to a large extent in Zn pollution, a number of works have measured isotopic compositions in regions impacted by industrial activities (Borrok et al., 2008, 2009; Balistrieri et al., 2008; Mattielli et al., 2009; Juillot et al., 2011; Yin et al., 2015). Other experimental studies investigated some of the mechanisms that potentially fractionate Zn isotopes. Biotic processes involving uptake of Zn by living-organisms (plant, diatoms) have been shown to induce isotopic fractionation. Gélabert et al. (2006) and John et al. (2007) have observed that marine diatoms surface is preferentially enriched in heavy isotopes ($\Delta^{66}$Zn$_{\text{surface-solution}} = 0.3 \pm 0.1\%$) while cell-internalised Zn is lighter ($\Delta^{66}$Zn$_{\text{cell-solution}} = -0.2 \pm 0.8\%$). In plants, Weiss et al. (2005, 2007), Viers et al. (2007), Moynier et al. (2009b) and Aucour et al. (2011) have found roots to be enriched in heavy isotopes with respect to leaves and shoots. Studies by Pokrovsky et al. (2005), Balistrieri et al. (2008), Juillot et al. (2008), Bryan et al. (2015) and Veeramani et al. (2015) investigated isotopic fractionation induced by adsorption on oxides and hydroxides and have reported a preferential sorption of heavy isotopes on minerals surface. Guinoiseau et al. (2016) measured enrichment in heavy isotopes at phyllosilicates surface with $\Delta^{66}$Zn$_{\text{adsorbed-solution}}$ ranging from 0.11% at low pH and low ionic strength to 0.49% at high pH and high ionic strength. The knowledge of equilibrium fractionation factors are necessary to investigate the underlying processes leading to natural variations of Zn isotope composition. In a theoretical DFT-based study, Fujii and Albarède (2012) explained through the calculation of equilibrium fractionation factors, the preferential heavy Zn isotopes enrichment in roots and lighter ones in leaves of plants. The differences in complexation environment of Zn in phosphates that are found in roots, and malates and citrates present in leaves, are likely to be at the origin of the observed isotopic fractionation. Other studies of Fujii et al. (2010), Fujii et al. (2011), Black et al. (2011) and Fujii et al. (2014) based on Zn molecular clusters, have provided equilibrium fractionation factors for Zn in various aqueous solutions.

In minerals, equilibrium isotopic fractionation factors of Zn remain unknown. The knowledge of these factors are essential for instance, to understand isotopic composition modifications induced by post-depositional fractionation processes in contaminated soils (Mason et al., 2005; Weiss et al., 2007; Juillot et al., 2011). In order to provide a consistent set of equilibrium mass-dependent isotopic fractionation factors, a selection of Zn-bearing minerals from various mineral groups, covering a large range of Zn crystallographic environments, are studied from first-principles within density functional theory (DFT). Starting from Zn primary ore (sphalerite ZnS) the oxidation and alteration products (secondary minerals) are mainly formed through supergene processes. Several crystalline phases containing Zn have been identified in contaminated soils. Commonly observed minerals include zinc carbonate minerals (smithsonite ZnCO$_3$ and hydrozincite Zn$_5$(CO$_3$)$_2$(OH)$_6$) and zinc silicate minerals like hemimorphite (Zn$_4$Si$_2$O$_7$(OH)$_2$·H$_2$O) (Boni et al., 2005; Balassone et al., 2008; Jacquet et al., 2008, 2009). In acid mine drainage systems and smelter-impacted soils, zinc-iron and zinc-aluminium spinels namely franklinite (ZnFe$_2$O$_4$) and gahnite (ZnAl$_2$O$_4$) are found as well as the zinc oxide zincite (ZnO) (Kucha et al., 1996; Hochella et al., 1999; Sobanska, 1999; Juillot et al., 2003; Jacquet et al., 2009; Juillot et al., 2011). Steger and Desjardins (1980) show that oxidation of sphalerite could result in the formation of hydrated zinc sulfate, which could be gunningite (ZnSO$_4$·H$_2$O) (Jambor and Boyle, 1962). Rarely occurring zinc arsenate (adamite Zn$_4$AsO$_4$(OH)) can also be found. In all these minerals, Zn displays one oxidation state (+2) but is found in various crystallographic environments and with a coordination number ranging from 4 to 6. The calculation of equilibrium isotopic fractionation factors allows to identify some of the crystal-chemical parameters controlling these isotopic properties and provide a theoretical basis for the interpretation of isotopic measurements.

2. Methods

2.1. Equilibrium isotopic fractionation factors

The equilibrium isotopic fractionation factor $\alpha(a, b, Y)$ between two phases $a$ and $b$ of an element $Y$ is defined as the ratio of isotope ratios. Equivalently, it can also be written as the ratio of the reduced partition function ratios:

$$\alpha(a, b, Y) = \frac{\beta(a, Y)}{\beta(b, Y)}$$

where $\beta(a, Y)$ is the reduced partition function ratio between phase $a$ and a perfect gas of atoms $Y$ (Bigeleisen and Mayer, 1947). In solids, equilibrium mass-dependent isotopic fractionation mainly arises from the vibrational motions of atoms. As described thoroughly by Blanchar et al. (2017 in press), the reduced partition function ratio, also called "$\beta$-factor", can be determined by computing vibrational frequencies using a harmonic ap-
Calculated frequencies (cm\(^{-1}\))

The Planck constant and \(k\) approximate force constants along three orthogonal directions of the harmonic restor-
tional spectrum is prevented by the large number of atoms.

In such cases, the calculation of the vibrational spectrum is not calcu-
lation (centre of Brillouin zone), the three translational motions at \(\Gamma\) is typically doubled and a difference of

\[
\beta(a,Y) = \prod_{i=1}^{3N_{at}} \prod_{q} \frac{\exp(-h\nu_{q,i}/k_B T)}{\nu_{q,i}} \frac{1 - \exp(-h\nu_{q,i}/k_B T)}{1 - \exp(-h\nu'_{q,i}/k_B T)} \frac{1}{(NqN)}
\]  

where \(\nu'_{q,i}\) and \(\nu_{q,i}\) are the vibrational frequencies with wavevector \(q\) and branch index \(i\) in 1, 3\(N_{at}\) in two isotopo-
logues. \(N_{at}\) is the number of atoms in the crystal unit-cell with \(N\) sites for the \(Y\) atoms. \(T\) is the temperature, \(\hbar\) the Planck constant and \(k_B\) the Boltzmann constant. The second product is performed on a sufficiently large grid of \(N_q\) \(q\)-vectors in the Brillouin zone. For phonon calculations at \(\Gamma\) (centre of Brillouin zone), the three translational modes with \(\nu_{q,i} = 0\) are not considered. Eq. 3 takes into account the "rule of the high-temperature product" also called "Redlich-Teller rule" (Bigeleisen and Mayer, 1947; Urey, 1947), which accounts for the fact that there should be no isotopic fractionation at the high-temperature limit.

In some cases, the determination of the whole vibrational spectrum is prevented by the large number of atoms (electrons) in the unit-cell. In such cases, the calculation along three orthogonal directions of the harmonic restoring force constants \(F_i\) that tend to pull back the studied atom in its equilibrium position allows computing an approximate \(\beta\)-factor (Bigeleisen and Mayer, 1947):

\[
\beta \simeq 1 + \frac{3N_{at}}{2} \sum_{i=1}^{3} \frac{\nu_{i}^2 - \nu_i^2}{24} + \frac{m}{m} \frac{\hbar^2}{2k_B T^2} \sum_{i=1}^{3} F_i
\]

where \(\nu_i\) is the ratio \(h\nu_i/k_B T\). Eq 4 can be used in replacement of the full frequency calculation only if the ratio \(h\nu/k_B T\) is small (\(\ll 2\)). This approximation has been used in several studies investigating isotopic fractionation (Moynier et al., 2011b; Kowalski and Jahn, 2011; Kowalski et al., 2013).

2.2. Modelling approach

In order to compute the reduced partition function ratios in the harmonic approximation according to Eq. 3, the complete vibrational spectrum is required for each mineral. Starting from experimentally determined structural models, the atomic positions and lattice parameters are relaxed within the DFT scheme as implemented in the Quantum ESPRESSO suite of codes based on plane-waves and pseudopotentials (Giannozzi et al., 2009). For each mineral, the dynamical matrix is calculated within density functional perturbation theory (DFPT or linear response theory) and then diagonalised for two different \(Zn\) atomic masses (64 and 66) in order to obtain the vibrational frequencies of the two isotopologues.

For franklinite, the complete \textit{ab initio} vibrational spectrum is not calculated due to the large number of atoms in the unit-cell and Eq 4 is used instead. The force constants required in this approach, can be obtained within the DFPT scheme where all inequivalent \(Zn\) atoms are displaced one after the other.

All calculations are performed with the Perdew-Burke-Ernzerhof (PBE) parametrised generalised gradient approximation (GGA) functional (Perdew et al., 1996). Ultrasoft pseudopotentials from the GBRV library (Garrity et al., 2014) are used. The cut-off energies for wave-functions (80 Ry) and charge-density (720 Ry) are determined such as the total energies are converged within 1 mRy/atom. A regular grid in \(k\)-space according to Monkhorst and Pack (1976) scheme is used for Brillouin zone sampling. The size of the converged \(k\)-points grid for each mineral is indicated in Table 1 as well as the size of the \(q\)-points grid used for the calculation of the vibrational properties. For franklinite, calculations are spin-polarised to take into account the magnetic ordering. \(\beta\)-factor convergence with respect to \(q\)-points grid size has been tested for each mineral. Between the two most denser grids, the number of \(q\)-points is typically doubled and a difference of ln \(\beta\) of less than 0.1\% is targeted. Structural relaxation is done until the residual forces on atoms are less than 10\(^{-4}\) Ry/au and total energy difference between two consecutive SCF steps is less than 10\(^{-6}\) Ry.

3. Results

Calculated lattice parameters from DFT relaxed structures of the studied minerals compare well with their experimental counterparts. They are generally overestimated by around 1-2\%, which is the usual trend with
GGA PBE functional (Table 1). Only gunningite with its complex structure containing hydrogen bonds, displays a slightly larger distortion of the unit-cell with respect to the experimental values of Wildner and Giester (1991).

A good agreement is observed between calculated frequencies and experimental ones (Figure 1 and Table S1 of Supplementary Information) with a general underestimation of ~4%, commonly observed with GGA PBE functional (Méheut et al., 2009; Méheut and Schauble, 2014; Balan et al., 2014; He et al., 2014) related to the overestimation of the lattice parameters as shown by Favot and Dal Corso (1999). Equilibrium isotopic fractionation factors calculation is based on the knowledge of vibrational frequencies. Several studies rescaled the calculated harmonic frequencies to match the experimental ones (Schauble et al., 2006; Blanchard et al., 2009; Schauble, 2011; Widanagamage et al., 2014). This approach allows to obtain a better agreement between calculated and experimental frequencies, however, it may be hampered by either the lack of experimental vibrational spectra or an uncertain assignment of calculated and experimental vibrational modes. In those cases, the same scaling factor could be applied for all the studied systems but the calculated frequencies must have comparable deviations with respect to their experimental counterparts. Furthermore, Eq 3, which allows calculating the reduced partition function ratios, is only valid in the harmonic approximation. For these reasons, in our previous studies, we used the raw theoretical harmonic frequencies without any scaling (Méheut et al., 2007, 2009; Blanchard et al., 2010; Méheut and Schauble, 2014). In this study, we adopt the same approach by using unscaled vibrational frequencies.

Temperature dependence of the reduced partition function ratios of Zn-bearing minerals are shown in figure 2 and parameters of empirical polynomial fits are given in Table 2. A quasi linear relation is observed between ln β and 1/T². At 22°C, the whole set of ln β ⁶⁶Zn/⁶⁴Zn values are within a range of 2‰. β-factor of Zn-bearing minerals are calculated according to Eq 3 for all minerals except franklinite for which the full frequency calculation is not done due to the large number of electrons in the unit-cell. From the force constant based approximate β-factor calculation for this mineral, the full frequency calculation β-factor is expected to be similar to hydrozincite Zn⁡₂ site β-factor for reasons explained in section 4.1. In figure 2, the corrected franklinite β-factor is shown. Zinc carbonate mineral (smithsonite), zinc ore mineral sphalerite and its high-pressure polymorph (wurtzite), are amongst those having the lowest β-factors. All studied minerals except wurtzite and smithsonite are isotopically enriched in ⁶⁶Zn with respect to sphalerite. The highest values of β-factors can be found in the zinc-aluminium spinel (gahnite) and the zinc silicate (hemimorphite) where Zn atoms are fourfold coordinated with O atoms. In the ferric counterpart of gahnite (franklinite), values of ln β are lower by 1.1‰ at 22°C. This value is even lower than that calculated for hydrozincite, which has three inequivalent Zn sites (1 tetra-coordinated and 2 hexa-coordinated sites). In both zincite and franklinite, Zn atoms are tetrahedrally coordinated to O atoms.
Table 1: Modelling parameters, calculated and experimental lattice parameters of the investigated minerals (in Å). (c), (h), (m) and (o) stand for cubic, hexagonal, monoclinic and orthorhombic crystal systems, respectively.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Structural formula</th>
<th>sym</th>
<th>k-points grid</th>
<th>q-points grid</th>
<th>This study</th>
<th>Exp.</th>
<th>Other theoretical studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphalerite ZnS</td>
<td>(c) 3x3x3</td>
<td>5x5x5</td>
<td>a = 5.444</td>
<td>5.411\textsuperscript{a}</td>
<td>5.449\textsuperscript{b}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wurtzite 2H ZnS</td>
<td>(h) 4x4x3</td>
<td>4x4x3</td>
<td>a = 3.842</td>
<td>3.823\textsuperscript{c}</td>
<td>3.755\textsuperscript{c}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>c = 6.303</td>
<td>6.261</td>
<td>6.168</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wurtzite 4H ZnS</td>
<td>(h) 4x4x2</td>
<td>3x3x3</td>
<td>a = 3.844</td>
<td>3.828\textsuperscript{d}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>c = 12.59</td>
<td>12.52</td>
<td></td>
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</tr>
<tr>
<td>Adamite Zn\textsubscript{2}AsO\textsubscript{4}(OH)</td>
<td>(o) 2x2x2</td>
<td>2x2x2</td>
<td>a = 8.491</td>
<td>8.306\textsuperscript{e}</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>b = 8.654</td>
<td>8.524</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>c = 6.160</td>
<td>6.043</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Franklinite ZnFe\textsubscript{2}O\textsubscript{4}</td>
<td>(c) 1x1x1</td>
<td>1x1x1</td>
<td>a = 8.505</td>
<td>8.431\textsuperscript{f}</td>
<td>8.453\textsuperscript{g}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gahnite ZnAl\textsubscript{2}O\textsubscript{4}</td>
<td>(c) 1x1x1</td>
<td>1x1x1</td>
<td>a = 8.183</td>
<td>8.081\textsuperscript{h}</td>
<td>8.231\textsuperscript{i}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gunningite ZnSO\textsubscript{4}H\textsubscript{2}O</td>
<td>(m) 2x2x2</td>
<td>2x2x2</td>
<td>a = 6.824</td>
<td>6.925\textsuperscript{j}</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>b = 7.923</td>
<td>7.591</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>c = 7.776</td>
<td>7.635</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(\beta = 117.3^\circ)</td>
<td>118.2\textsuperscript{c}</td>
<td></td>
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<td></td>
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<tr>
<td>Hemimorphite Zn\textsubscript{4}Si\textsubscript{2}O\textsubscript{7}(OH)\textsubscript{2}H\textsubscript{2}O</td>
<td>(o) 3x3x3</td>
<td>2x2x3</td>
<td>a = 8.486</td>
<td>8.367\textsuperscript{k}</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>b = 10.80</td>
<td>10.73</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>c = 5.255</td>
<td>5.115</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(\beta = 95.43^\circ)</td>
<td>95.83\textsuperscript{c}</td>
<td></td>
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</tr>
<tr>
<td>Hydrozincite Zn\textsubscript{5}(CO\textsubscript{3})\textsubscript{2}(OH)\textsubscript{6}</td>
<td>(m) 1x2x2</td>
<td>2x2x2</td>
<td>a = 13.73</td>
<td>13.62\textsuperscript{l}</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>b = 6.412</td>
<td>6.30</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>c = 5.456</td>
<td>5.42</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(\beta = 95.43^\circ)</td>
<td>95.83\textsuperscript{c}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smithsonite ZnCO\textsubscript{3}</td>
<td>(h) 2x2x2</td>
<td>2x2x2</td>
<td>a = 4.714</td>
<td>4.653\textsuperscript{m}</td>
<td>4.72\textsuperscript{n}</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>b = 15.28</td>
<td>15.03</td>
<td>15.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>c = 5.307</td>
<td>5.207</td>
<td>5.167</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a}Skinner (1961)  
\textsuperscript{b}Blanchard et al. (2009)  
\textsuperscript{c}Bachmann et al. (2012)  
\textsuperscript{d}Chao and Gault (1998)  
\textsuperscript{e}Hill (1976)  
\textsuperscript{f}Braestrup et al. (2008)  
\textsuperscript{g}Andersson and Stanek (2013)  
\textsuperscript{h}Mitchell et al. (1996)  
\textsuperscript{i}Wildner and Giester (1991)  
\textsuperscript{j}Libowitzky and Rossman (1997)  
\textsuperscript{k}Ghose (1964)  
\textsuperscript{l}Graf (1961)  
\textsuperscript{m}Wildner and Giester (1991)  
\textsuperscript{n}Kisi and Elcombe (1989)
dependence of isotopic fractionation with interatomic force

\[ \ln Z/F \text{calculated using Eq 4 are also shown in order to compare the full frequency calculation to the approximate one} \]

For Zn sulfide minerals, ln β and F are correlated in the same way as for the all other minerals. Sphalerite and wurtzite with average Zn-S bond lengths of 2.36 Å are out of the general trend of figure 4 because of differing chemical properties of Zn-S bond with respect to Zn-O bond. These two minerals display very close β-factor values with a ln β difference of 0.08% at 22°C, which is in agreement with the
Figure 3: $\ln \beta^{66}Zn^{64}Zn$ at 22°C as a function of ab initio interatomic force constants for each site in the studied minerals. Closed and open symbols refer to $\beta$-factors calculated according to Eq 3 and Eq 4 respectively. Solid and dashed lines are the corresponding linear fits: $10^3 \ln \beta^{66}Zn^{64}Zn = 1.5767 F + 0.1640$ for the full frequency calculation and $10^3 \ln \beta^{66}Zn^{64}Zn = 1.7314 F + 0.0067$ for the force constant based calculations. The full frequency calculation is not performed for franklinite. However, as hydrozincite $Zn2$ and franklinite display similar force constants, franklinite will have similar $\ln \beta$ as hydrozincite $Zn2$ if the full frequency calculation was done.

Figure 4: $\beta$-factors at 22°C as a function of average Zn-O bond lengths for each Zn site or mineral. Solid black line is a guideline emphasizing the decrease of $\ln \beta$ with the increase of bond lengths.

"very small fractionation" found by Schauble (2003). The same study reported a calculated value of $\ln \beta^{68}Zn^{66}Zn$ in zincite larger by $\sim 1\%$ with respect to sphalerite at room temperature, which is consistent with our calculations where zincite is found 0.95% heavier than sphalerite at 22°C for these isotopes.

Smithsonite, where Zn atoms are octahedrally coordinated, has a $\beta$-factor value lower than sphalerite by 0.1% meaning that at equilibrium, it would be slightly enriched in light Zn isotopes with respect to coexisting sphalerite. In the meanwhile, zinc hydroxy-carbonate hydrozincite, which belongs to the same mineral group as smithsonite, displays a $\ln \beta$ of 3.20% that is 0.90% larger than smithsonite and consistently, its average Zn-O bond strength is larger than in smithsonite by 0.6 Ry/Å$^2$ and average Zn-O bond length is smaller by 0.06 Å. An in-depth examination of site specific $\beta$-factors and bond lengths (Figure 4) shows that Zn in smithsonite has a smaller $\beta$-factor than Zn sitting in $Zn1$ and $Zn2$ octahedral symmetry sites in hydrozincite by 0.50% and 0.65% respectively, even if Zn-O bond lengths are comparable. The investigation of the first-neighbor shell of Zn atoms points out electronic differences in the bonding environment, which in turn explain contrasting isotopic properties between Zn site in smithsonite and $Zn1$ and $Zn2$ sites in hydrozincite. As shown in figure 5, in smithsonite, all the six first-neighbours of a Zn atom are O atoms, while around Zn in $Zn1$ and $Zn2$ sites in hydrozincite there are four hydroxyl groups amongst the six first-neighbours. In the polar covalent O-H bond, O atom gains a partial negative charge due to charge transfer from less electronegative H atom. Such an effect is absent in smithsonite and present in hydrozincite. In order to support this observation, we performed Löwdin charge calculation. The average charge on O atoms bonded to Zn is 2.4% smaller in smithsonite than in hydrozincite (for $Zn1$ and $Zn2$ sites). Thus, Zn-first neighbour bonds are weaker in smithsonite than in hydrozincite resulting in lower interatomic force constants and lower $\beta$-factors in smithsonite with respect to hydrozincite.

In adamite, $Zn2$ site has a trigonal bipyramidal symmetry formed by four O atoms and one OH group. As a result of this bonding environment, $\ln \beta$ is equal to 3.11% that is an intermediate value between minerals where Zn is tetrahedrally and octahedrally coordinated. The other site ($Zn1$) has an octahedral symmetry, with a corresponding $\ln \beta$ of 2.84% that is comparable to values calculated for $Zn1$ and $Zn2$ sites in hydrozincite.

In the modelling of franklinite, the normal spinel structure is considered, which is both the experimental and DFT ground-state structure (Braestrup et al., 2008; Andersson and Stanek, 2013). Zn atoms occupy the tetrahedral and Fe atoms the octahedral sites. In figure 3, franklinite appears to have lower values of $F$ and $\ln \beta$ than expected for a mineral in which Zn is tetrahedrally coordinated. In figure 4, franklinite does not appear to be in the general trend. Franklinite $\beta$-factor is below the
calculated value for zincite (Figures 2, 3 and 4), although bond lengths and local environment around Zn atoms in both minerals are very similar, with Zn being coordinated to four O atoms. Results shown in figures 2, 3 and 4 for franklinite assumes a spin ordering as in model 1 shown in figure 6 but the antiferromagnetic ground-state can be obtained through various spin orderings (Cheng, 2008). The effect of magnetic ordering on $\beta$-factor is investigated by considering a second model (model 2) as shown in figure 6. Both models are found to give a similar lattice parameter and similar Zn-O bond lengths. Model 1 is however lower in energy by 230 meV with respect to model 2 and has a bandgap of 0.3 eV while in the latter, it is 0.1 eV. Between these two models, a negligible change in $\ln \beta$ of less than 0.001% is observed. Consequently, franklinite $\beta$-factor remains doubtlessly lower than the zincite one, meaning that at equilibrium franklinite will be isotopically lighter than zincite. The first neighbour oxygen shells around Zn are identical in both minerals while the nature of the second neighbours is different, i.e. second neighbours are Fe atoms in franklinite and Zn atoms in zincite. In order to investigate the effect of second neighbours on the $\beta$-factor, a new model is made from relaxed franklinite, where all Fe atoms are replaced by Al atoms. The resulting fictitious mineral is chemically identical to gahnite ($\text{ZnAl}_2\text{O}_4$) but structurally similar to franklinite. Then, without any relaxation, the restoring force constants acting on Zn are calculated through DFPT and $\beta$-factor is computed using Eq 4. At 22°C, $\ln \beta$ has now increased from 2.95% to 3.64% that is closer to the expected value for tetrahedrally coordinated Zn. The total charge on O atoms is calculated to be 3.8% lower in franklinite than in the fictitious mineral. The calculated total charge difference is the consequence of a charge transfer from O to Fe atoms that disappears when Al for Fe substitutions are made. In the optimised gahnite, Zn-O bonds are the shortest and the average charge on O atoms is the highest amongst all investigated minerals. As a consequence, gahnite has the largest $\beta$-factor. Therefore Zn isotope fractionation in spinel minerals cannot be considered as a purely local property but is also a function of the medium-range environment of Zn.

4.2. Comparison between theoretical and experimental fractionation

Only very few works exist in which Zn isotope fractionation has been measured in controlled experiments involving Zn-bearing minerals. Veeramani et al. (2015) studied Zn isotope fractionation between aqueous solutions and minerals such as sphalerite, hydrozincite and hopeite, which is a Zn phosphate. Sphalerite and hydrozincite were precipitated from ZnSO$_4$ and ZnCl$_2$ solutions respectively, to which Na$_2$S$_2$O$_7$ and NaHCO$_3$ with varying concentration, were added in batch experiments, respectively. According to a Rayleigh fractionation model, sphalerite and hydrozincite were found to be isotopically lighter and heavier than initial solutions at room temperature.
with $\varepsilon = -0.30\%$ and $\varepsilon = +0.18\%$ respectively. $\varepsilon$ is defined as:

$$
\varepsilon = \left( \frac{R_{\text{product}}}{R_{\text{reactant}}} - 1 \right) \times 1000
$$

(5)

where R is the isotopic ratio. A quantitative comparison of mineral-solution Zn isotope fractionation between experimental and theoretical studies requires several ful-
filments: 1) a precise knowledge of Zn speciation in precipi-
tated and dissolved phases, 2) the isotopic analysis of all reaction products, 3) and a reliable and consistent mod-
eling of isotopic properties of both the solid and dissolved phases. In those experiments, the evolution of Zn speci-
ation in the two different solutions is unknown and is re-
quired to deduce the relative enrichment between the two minerals. Indeed, during sphalerite precipitation, the pH varies from 3.87 to 11.45, which would likely modify the speciation of aqueous Zn. For the initial ZnSO$_4$ solution, a geochemical calculation at 22°C, pH 4 and in equilibrium with atmospheric CO$_2$ using PHREEQC (Parkhurst and Appel, 1999) and WATEQ4F database (Ball and Nord-
strom, 1991) indicates 70 mol% and 29 mol% of Zn in
Zn$^{2+}$ and ZnSO$_4$(aq) forms respectively. The calculation
for the initial ZnCl$_2$ solution results in almost 98 mol%
of Zn in Zn$^{2+}$. Zn$^{2+}$ is commonly believed to be present
as a hexaquo complex (Zn(H$_2$O)$_6^{2+}$). ZnSO$_4$(aq) has been
modelled by Black et al. (2011) as both inner and outer-
sphere complexes and associated $\beta$-factors have been cal-
culated. Rudolph et al. (1999) through Raman spectra
measurements have shown that at low temperature and
low ZnSO$_4$ concentrations (0.034 mol kg$^{-1}$) the forma-
tion of a 1:1 contact ion pair (Zn$^{2+}$OSO$_3^-\)$ is unlikely.
Furthermore, in experiments, the outer-sphere complex
is undistinguishable in vibrational spectroscopy investiga-
tions from the bulk phase sulfate (Rudolph et al., 1999).
As a consequence of these observations, we assume that
the fractionation measured in Veeramani et al. (2015) ex-
periments could be roughly approximated to represent is-
otopic fractionation between minerals and Zn(H$_2$O)$_6^{2+}$ com-
plex. The value $\varepsilon = -0.30\%$ (respectively $\varepsilon = +0.18\%$)
would represent thus an average fractionation factor be-
tween sphalerite (respectively hydrozincite) and Zn hexa-
aquo complex.

In order to calculate the $\beta$-factor of Zn(H$_2$O)$_6^{2+}$ com-
plex using the same numerical methods as those used for
Zn-bearing minerals, a large cubic cell of 20 Å size is made
and Martyna-Tuckerman correction (Martyna and Tuck-
erman, 1999) is enabled to avoid interaction of the charged
Zn aqueous complex with its periodic images. Zn-O bond
lengths and vibrational frequencies of this complex are
given in Table S2 and S3 of Supplementary Information.
In $\beta$ $^{66}$Zn/$^{64}$Zn at 22°C for Zn(H$_2$O)$_6^{2+}$ is calculated to
be 3.03%. This value calculated with plane-wave basis
set is in good agreement with Black et al. (2011) value of
3.11%, which is calculated with B3LYP functionals and
the largest localised basis set of the study (aug-cc-pVDZ)
at 22°C. Sphalerite and hydrozincite $\ln \beta$ $^{66}$Zn/$^{64}$Zn val-
ues at 22°C are 2.40% and 3.19%. Thus, equilibrium fraction-
tion factors predict sphalerite and hydrozincite to be
enriched in light and heavy isotopes with respect to aque-
ous Zn, respectively (Figure 7). In $\beta$ differences between
mineral and solution, which are $-0.63\%$ for sphalerite-
solution and $+0.17\%$ for hydrozincite-solution, are consis-
tent with the experimental values, which are $-0.3\%$ and
$+0.18\%$ respectively. It should be noted that with in-
creasing NaHCO$_3$ concentration in ZnCl$_2$ solution during
hydrozincite precipitation, the formation of Zn carbonate
complexes could be expected. These complexes display
$\beta$-factor values higher than Zn hexaquo complex one ac-
cording to Fujii et al. (2014). As a consequence, including
such complexes in Zn speciation modelling in hydrozincite
precipitation experiment would result in the mineral be-
ing either less enriched in heavy isotopes than experimen-
tally measured or even, ligther than the solution, which
would be in contradiction with the experiment. In the
same way, the addition of Na$_2$S$_2$ in ZnSO$_4$ solution dur-
ing sphalerite precipitation would lead to the formation of
Zn hydrogenosulfides complexes. These complexes display
$\beta$-factors lower than Zn hexaquo complex one. The con-
ideration of these complexes in solution would result in a
lower aqueous Zn $\beta$-factor value than 3.03% calculated for
Zn hexaquo complex. In $\beta$ difference between sphalerite
and solution could then decrease and potentially tend to-
wards the experimental value of $\varepsilon = -0.3\%$ (Figure 7).
4.3. Geochemical implications for natural samples

Amongst the minerals studied here, some are more relevant than others in environmental geochemistry. Sphalerite isotope composition in natural samples has been extensively measured as it is the primary ore mineral (Maréchal, 1998; Maréchal et al., 1999; Mason et al., 2005; Wilkinson et al., 2005). Maréchal (1998) studied also isotopic composition of Zn in secondary minerals such as the carbonate minerals smithsonite and hydrozincite that could form through supergene alteration of sphalerite. Four smithsonite samples from Laurion (Greece), Wiesloch (Germany), Gabilan (Spain) and Raibl (Italy) were measured by Maréchal (1998). In these locations, smithsonite $\delta^{66}Zn$ are 0.16%, 0.33%, −0.06% and 0.69% respectively, while sphalerite $\delta^{66}Zn$ are 0.02%, 0.57%, 0.00% and 0.44% respectively. Average smithsonite and sphalerite $\delta^{66}Zn$ are 0.3±0.3% and 0.1±0.2% respectively. In view of these values, Zn isotope fractionation between the two minerals are not notably different. In the study of Maréchal (1998), one sample of hydrozincite from Riedbodeneck (Germany) is also found to be isotopically identical to a sphalerite sample from the same location. Assuming that Zn carbonates are formed by supergene alteration of sphalerite in these locations, the significant theoretical fractionation between Zn($\text{H}_2\text{O}$)$_2^{2+}$ complex and Zn carbonates could suggest that Zn is quantitatively reincorporated in precipitating secondary carbonates. The precipitation of Zn carbonate in low temperature environment is attested by, e.g., Jacquet et al. (2008), who have found hydrozincite in heavily Zn polluted carbonated soils. The study of Pašava et al. (2014) investigated zinc isotope composition in minerals at La Florida mine (Basque Cantabrian Basin, Spain), where very intense supergene alteration leads to the formation of hydrozincite, smithsonite and hemimorphite. Six sphalerite and two hydrozincite samples are measured and average $\delta^{66}Zn$ values of −0.3±0.4% and 0.3% are found respectively. In this case, the different $\delta^{66}Zn$ values measured for the two minerals attests that only a fraction of Zn liberated by the dissolution of sphalerite has been reincorporated in secondary carbonates. The isotopic enrichment of hydrozincite is consistent with a positive theoretical fractionation factor calculated between this phase and aqueous Zn. This is also in agreement with the experimental work of Veeramani et al. (2015). It should be noted that calculated $\beta$-factors values are for perfect minerals assuming stoichiometric Zn sites. In natural samples, crystalline defects, such as cationic vacancies or chemical impurities, can modify isotopic fractionation values. In addition, during supergene processes at low temperature, other phenomena such as Rayleigh or kinetic fractionation could also occur.

Zn isotope fractionation is also relevant to investigate high temperature geochemical processes. Terrestrial igneous rocks display an average $\delta^{66}Zn$ value of 0.28±0.05 (Chen et al., 2013). Wang et al. (2016) suggested a possible Zn isotope fractionation during peridotites melting as they measured $\delta^{66}Zn$ value of 0.17±0.06% in non-metasomatised peridotites, which is $\sim$ 0.1% lighter than primitive basalts. In peridotites, spinels are isotopically heavier than olivines and pyroxenes by $\sim$ 0.1%. However, the two spinel minerals modelled in this study, franklinite and gahnite, display significantly different isotopic properties depending on the nature of first cation neighbours of Zn. In peridotites, spinels usually display a chemical composition with Mg, Fe, Al and Cr as major cations (Eggins et al., 1998) i.e., notably more complex than that of the end-members investigated in the present study. Accordingly, further studies would be required to thoroughly understand the dependence of Zn fractionation on the spinel chemistry to interpret the data on natural samples.

5. Conclusion

Equilibrium Zn isotope fractionation factors are calculated from first-principles in minerals covering a wide range of Zn crystallographic environments. Good agreement between calculated and experimental structural and vibrational properties is a guarantee of reliable theoretical isotopic fractionation factors. $\beta$-factors are calculated based either on the full-frequency or on the force constant calculations. Amongst the studied minerals, smithsonite and sphalerite display the lowest $\beta$-factors with $\ln \beta^{66}Zn/^{64}Zn$ of 2.30% and 2.40% at 22°C, respectively. Galnate is the isotopically heaviest mineral with $\ln \beta^{66}Zn/^{64}Zn$ of 4.18% at 22°C. The range of isotopic fractionation is smaller than 2% at 22°C, which is a consequence of small variability in Zn atoms first-neighbours (either S or O) and its single oxidation state (+2). Nonetheless, the closed d-shell of this element allows a variable coordination number (4, 5, 6). The investigation of the theoretical isotopic fractionation shows that it is mainly driven by the Zn-first neighbour bond strength. Bond length and charge on atoms involved in the bonding affect the bond strength. The smallest bond strength and largest bond lengths are observed for octahedrally coordinated Zn and correspond to the smallest $\beta$-factor values and vice-versa. The equilibrium Zn isotope fractionation factors calculated in this study for Zn-bearing minerals along with already existing ones for solvated Zn species (Fujii et al., 2010, 2011, 2014; Black et al., 2011) provides a basis to interpret the isotopic variations observed in nature.

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Ghose, S., 1964. The crystal structure of hydrozincite, Zn5(OH)6(CO3)2· Acta Crystallogr. 17 (8), 1051–1057.


Hill, R. J., 1976. The crystal structure and infrared properties of polyhedron 26 (17), 4955–4962.


Zinc isotope variation in hydrothermal systems: Preliminary evidence from the irish midlands ore field. Econ. Geol. 100 (3), 583–590.


