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Upscaling of spectral induced polarization response using random tube networks

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SUMMARY

In order to upscale the induced polarization (IP) response of porous media, from the pore scale to the sample scale, we implement a procedure to compute the macroscopic complex resistivity response of random tube networks. A network is made of a 2-D square-meshed grid of connected tubes, which obey to a given tube radius distribution. In a simplified approach, the electrical impedance of each tube follows a local Pelton resistivity model, with identical resistivity, chargeability and Cole–Cole exponent values for all the tubes—only the time constant varies, as it depends on the radius of each tube and on a diffusion coefficient also identical for all the tubes. By solving the conservation law for the electrical charge, the macroscopic IP response of the network is obtained. We fit successfully the macroscopic complex resistivity also by a Pelton resistivity model. Simulations on uncorrelated and correlated networks, for which the tube radius distribution is so that the decimal logarithm of the radius is normally distributed, evidence that the local and macroscopic model parameters are the same, except the Cole–Cole exponent: its macroscopic value diminishes with increasing heterogeneity (i.e. with increasing standard deviation of the radius distribution), compared to its local value. The methodology is also applied to six siliciclastic rock samples, for which the pore radius distributions from mercury porosimetry are available. These samples exhibit the same behaviour as synthetic media, that is, the macroscopic Cole–Cole exponent is always lower than the local one. As a conclusion, the pore network method seems to be a promising tool for studying the upscaling of the IP response of porous media.

Key words: Electrical properties; Hydrogeophysics; Numerical modelling.

1 INTRODUCTION

Induced polarization (IP) method is now commonly used in a broad range of geophysical applications, for instance ore or hydrocarbon prospecting (e.g. Veeken *et al.* 2009), location and delineation of contaminant plumes (e.g. Mewafy *et al.* 2013; Johansson *et al.* 2015; Ntarlagiannis *et al.* 2016), monitoring of biodegradation processes in contaminated area (e.g. Flores Orozco *et al.* 2011; Abdel Aal *et al.* 2014) and even hydrological characterization of the subsurface (e.g. Attwa & Günther 2013).

A recently developed version of the method—the spectral induced polarization (SIP) method—consists in injecting a sinusoidal current I in a rock sample (in laboratory conditions) or in the subsurface (in field conditions) by means of two current injection electrodes, and in measuring the resulting voltage difference ΔV

by means of at least two measurement electrodes. Doing so for different frequencies, one obtains the spectrum of the complex resistivity of the medium ρ^* . In the complex domain,

$$\rho^*(\omega) = K \frac{\Delta V^*(\omega)}{I^*(\omega)} = K \frac{|\Delta V^*(\omega)|}{|I^*(\omega)|} e^{-i\varphi(\omega)} = \rho(\omega) e^{-i\varphi(\omega)}, \quad (1)$$

where superscript $*$ refers to complex quantities, K is a geometrical constant linked among others to the location of the electrodes, ω is the pulsation frequency, ρ is the resistivity amplitude, φ is the phase and $i = (-1)^{\frac{1}{2}}$ is the pure imaginary number. Amplitude and phase spectra are related to the ability of the material to store electrical charges reversibly.

Understanding large-scale measurements (field and rock samples) requires the knowledge of the polarization processes that occur at the pore scale, together with the development of appropriate

upscaling procedures. At frequencies lower than 100 Hz, and in absence of metallic particles (which act as strong capacitors when submitted to a low-frequency current, e.g. Wong 1979; Misra *et al.* 2016a,b), the main polarization mechanisms are (1) the polarization of the Stern layer at the interface between the mineral grains and the saturating fluid (e.g. Rosen & Saville 1991; Revil 2012, 2013), and (2) the membrane polarization, which is controlled by the differential diffusion of cations and anions through small pores (e.g. Marshall & Madden 1959; Bückner & Hördt 2013a). Many models have been proposed to formalize these two phenomena at the pore scale (e.g. Marshall & Madden 1959; Titov *et al.* 2002; Revil & Florsch 2010; Bückner & Hördt 2013a,b). In addition, a broad variety of semi-empirical models have been published to reproduce the observed IP response of rock samples (e.g. Pelton *et al.* 1978; Dias 2000; Weller *et al.* 2013; Weller & Slater 2015) in the quasi-static approximation (below 1000 Hz). Upscaling methods used to link the models at the pore scale and the models at the sample scale are usually based on effective medium approximation (e.g. Dukhin & Shilov 2002)—for instance, the differential effective medium approach for suspensions of grains in a background electrolyte. For heterogeneous media, the issue of upscaling is still open.

In this paper, we use the method of random pore networks to address this question of upscaling. In other words, based on a known complex impedance function at the pore scale, we model the complex impedance (or complex resistivity) function of the macroscopic assemblage of pores. Note that the same procedure could be applied to a suspension of spheres or spheroidal grains (with their electrical double layer) in a background electrolyte as long as the complex impedance function of the basic element is known. Pore networks have been extensively used to study the permeability and the formation factor of rocks (e.g. Kirkpatrick 1973; Koplik 1981; David *et al.* 1990; David 1993; Bernabé *et al.* 2003), as well as the relationship between electrical conductivity and permeability (Bernabé & Revil 1995), their transport properties (e.g. Bruderer & Bernabé 2001; Bernabé *et al.* 2016) and even their self-potential response (Bernabé 1998; Jackson 2010). Briefly, the method consists in designing 2-D or 3-D networks of connected tubes, for a given connectivity (linked to the topology, i.e. to the type of mesh: triangular, square, hexagonal and so on), and to attribute to the tubes a given distribution of radii (and/or lengths), similar to the pore radius distribution that can be observed in rocks. The two end face of the network are then submitted to appropriate boundaries conditions, and the conservation law (for hydraulic flow or electrical current) is applied to each node to which tubes are connected. This yields to a linear system of field equations enabling the potential (hydraulic or electrical) to be solved, from which the hydraulic flow or electrical current in the tubes are inferred, and therefore the macroscopic fluxes at the end faces of the macroscopic body. From these fluxes, the parameter of interest, such as the permeability or the formation factor, can be deduced like in a real laboratory experiment. Even though using the network method obviously assumes strong simplifications, in particular concerning the topology of the simulated medium, it has some advantages. The method is very simple. It presents the possibility to carry out a huge number of simulations on different network realizations for a given law of radius distribution, and thus to obtain statistics. Finally, the influence of the topology and the shape of the radius distribution can be extensively studied (e.g. Bernabé & Bruderer 1998; Bernabé *et al.* 2003), which is almost impossible with real rocks. In this way, these conceptual tools permit general laws to be evidenced (e.g. Bernabé *et al.* 2010, 2011).

Figure 1. 2-D square network of connected tubes. Each tube can be used to represent a pore. The electrical potential is imposed at the nodes in entry ($y = 1$) and exit ($y = N_y$). There are no outflowing currents on the lateral faces ($x = 1$ and $x = N_x$). The radii of the tubes follow a given distribution, which can be for instance derived from mercury porosimetry or nuclear magnetic resonance.

2 METHODOLOGY

2.1 Tube network

We consider a simulated porous material with a pore network composed of a set of connected capillaries. More precisely, we consider 2-D square networks (Fig. 1), made of a set of $N_x \cdot N_y$ nodes (N_x in the x -direction and N_y in the y -direction). These nodes are connected to each other by $2 N_x \cdot N_y - N_x - N_y$ tubes representing the pores. Note that we use 2-D networks and not 3-D ones to allow larger network size, but the methodology is extendable to 3-D. The network is submitted to given boundary conditions such as insulating boundaries on some sides and with an imposed voltage on others like in a real experiment. The radii of the tubes obey to a pore size distribution and they are randomly chosen on this (normalized) probability distribution. An example of a non-correlated random network is shown in Fig. 2, together with the associated ‘experimental’ radius distribution (Fig. 2a). In laboratory experiments, these experimental radius distributions can be determined either from mercury porosimetry (using the capillary pressure curve, e.g. Ritter & Drake 1945) or from nuclear magnetic resonance measurements using the distribution of the relaxation times (e.g. Gallegos *et al.* 1987).

The principle of a simulation consists in solving the (complex valued) electrical potential at the nodes, to deduce the electrical current through the tubes, from which the macroscopic IP response of the network can be inferred. In our case, the tubes are characterized by a complex impedance function, which is supposed to be known. The central questions that we will address are: (1) how to upscale the problem to get the complex impedance of the network and (2) can we describe the macroscopic impedance function from the local impedance function and from the properties of the network such as the pore size distribution.

2.2 System for the complex electrical potential

Kirchhoff law (Kirchhoff 1845) for the complex electrical current, applied at each node (x, y) inside the internal domain of the network

Figure 2. Example of random uncorrelated media. ‘Experimental’ distribution (a) of the tube radii (the decimal logarithm of the pore tube radius distribution is normally distributed, with a mean radius of 10 μm and a standard deviation of 0.5) associated with the network (100×100 nodes, 19 800 tubes) shown in (b).

(Fig. 1), writes:

$$\forall (x, y) \in [2, \dots, N_x - 1] \times [2, \dots, N_y - 1]$$

$$J_{(x,y-1) \rightarrow (x,y)}^* + J_{(x-1,y) \rightarrow (x,y)}^* + J_{(x+1,y) \rightarrow (x,y)}^* + J_{(x,y+1) \rightarrow (x,y)}^* = 0 \quad (2)$$

where $J_{p \rightarrow q}^*$ is the electrical current inside the tube relying node p to node q . The local constitutive equation (Ohm’s law) is

$$J_{p \rightarrow q}^* = \sigma_{p \rightarrow q}^* (V_p^* - V_q^*), \quad (3)$$

where $\sigma_{p \rightarrow q}^*$ is the complex conductivity of the tube (including not only the fluid, but also the effect of the embedding mineral) and V_p^* (respectively V_q^*) is the complex electrical potential at node p (respectively q). Considering eq. (3), eq. (2) becomes:

$$\begin{aligned} \sigma_{(x,y-1) \rightarrow (x,y)}^* V_{x,y-1}^* + \sigma_{(x-1,y) \rightarrow (x,y)}^* V_{x-1,y}^* + \sigma_{x,y}^* V_{x,y}^* \\ + \sigma_{(x+1,y) \rightarrow (x,y)}^* V_{x+1,y}^* + \sigma_{(x,y+1) \rightarrow (x,y)}^* V_{x,y+1}^* = 0, \end{aligned} \quad (4)$$

where

$$\begin{aligned} \Sigma_{x,y}^* = -(\sigma_{(x,y-1) \rightarrow (x,y)}^* + \sigma_{(x-1,y) \rightarrow (x,y)}^* + \sigma_{(x+1,y) \rightarrow (x,y)}^* \\ + \sigma_{(x,y+1) \rightarrow (x,y)}^*). \end{aligned} \quad (5)$$

The boundary conditions applied at the entry (i.e. the nodes for which $y = 1$) and at the exit (i.e. the nodes for which $y = N_y$) are (see Fig. 1):

$$\forall x \in [1, \dots, N_x] \quad V_{x,1}^* = V_0 e^{i\omega t} \quad (6a)$$

$$\forall x \in [1, \dots, N_x] \quad V_{x,N_y}^* = 0 \quad (6b)$$

where V_0 is the amplitude (real number), ω is the pulsation frequency and t stands for time. On the lateral faces, there are no outward fluxes:

$$\forall y \in [2, \dots, N_y - 1] \quad \begin{cases} J_{(1,y-1) \rightarrow (1,y)}^* + J_{(2,y) \rightarrow (1,y)}^* \\ \quad + J_{(1,y+1) \rightarrow (1,y)}^* = 0 \\ J_{(N_x,y-1) \rightarrow (N_x,y)}^* + J_{(N_x-1,y) \rightarrow (N_x,y)}^* \\ \quad + J_{(N_x,y+1) \rightarrow (N_x,y)}^* = 0 \end{cases}, \quad (7)$$

that is:

$$\begin{aligned} \sigma_{(1,y-1) \rightarrow (1,y)}^* V_{1,y-1}^* + \Sigma_{1,y}^* V_{1,y}^* + \sigma_{(2,y) \rightarrow (1,y)}^* V_{2,y}^* \\ + \sigma_{(1,y+1) \rightarrow (1,y)}^* V_{1,y+1}^* = 0, \end{aligned} \quad (8a)$$

with

$$\Sigma_{1,y}^* = -(\sigma_{(1,y-1) \rightarrow (1,y)}^* + \sigma_{(2,y) \rightarrow (1,y)}^* + \sigma_{(1,y+1) \rightarrow (1,y)}^*), \quad (8b)$$

and

$$\begin{aligned} \sigma_{(N_x,y-1) \rightarrow (N_x,y)}^* V_{N_x,y-1}^* + \sigma_{(N_x-1,y) \rightarrow (N_x,y)}^* V_{N_x-1,y}^* + \Sigma_{N_x,y}^* V_{N_x,y}^* \\ + \sigma_{(N_x,y+1) \rightarrow (N_x,y)}^* V_{N_x,y+1}^* = 0, \end{aligned} \quad (8c)$$

with

$$\Sigma_{N_x,y}^* = -(\sigma_{(N_x,y-1) \rightarrow (N_x,y)}^* + \sigma_{(N_x-1,y) \rightarrow (N_x,y)}^* + \sigma_{(N_x,y+1) \rightarrow (N_x,y)}^*). \quad (8d)$$

Note here that contrary to the classical way of proceeding for SIP measurements, the electrical potential is imposed at the boundaries, not the electrical current. Eqs (4), (6a), (6b), (8a) and (8c), together with eqs (5), (8b) and (8d), form a linear system of equations with $N_x \cdot N_y$ unknowns $V_{x,y}^*$. For simplification, we divide all the $V_{x,y}^*$ in the equations by $V_0 e^{i\omega t}$, defining the real numbers $f_{x,y}$ and $\phi_{x,y}$ by:

$$\frac{V_{x,y}^*}{V_0 e^{i\omega t}} = f_{x,y} + i\phi_{x,y}. \quad (9)$$

Similarly, we introduce the real numbers $g_{p \rightarrow q}$ and $\gamma_{p \rightarrow q}$ as

$$\sigma_{p \rightarrow q}^* = g_{p \rightarrow q} + i\gamma_{p \rightarrow q}. \quad (10)$$

Doing so, the system can take the form:

$$\begin{bmatrix} \mathbf{G} & -\mathbf{\Gamma} \\ \mathbf{\Gamma} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{z} \\ \boldsymbol{\zeta} \end{bmatrix}, \quad (11)$$

where matrices $\mathbf{G}$ and $\mathbf{\Gamma}$ and vectors $\mathbf{z}$ and $\boldsymbol{\zeta}$ are real.

System (11) is solved using the QR decomposition (qr function in Matlab) to obtain $\mathbf{f}$ and $\boldsymbol{\theta}$. Then, knowing $V_{x,y}^*/V_0 e^{i\omega t}$ at each nodes (from eq. 9), it is possible to compute the complex electrical current in each tube using eq. (3).

2.3 Amplitude and phase spectra of the network

At the macroscopic scale, the total current I^* through the network is related to the applied macroscopic potential gradient ΔV^* by Ohm’s law $I^* = -\sigma_{\text{bulk}}^* \Delta V^*$, where σ_{bulk}^* is the bulk conductivity of the network. As a result, the conductivity amplitude α for a given

frequency pulsation ω (i.e. the wavelength of the imposed sinusoidal electrical potential at the boundaries) is given by:

$$\begin{cases} \alpha_{\text{in}}(\omega) = \left| \frac{J_{\text{in}}^*}{\Delta V^*} \right| = \left| \frac{J_{\text{in}}^{\text{total}}}{V_0 e^{i\omega t}} \right| = \left| \frac{1}{V_0 e^{i\omega t}} \sum_{x=1}^{N_x} J_{(x,1) \rightarrow (x,2)} \right| \\ \alpha_{\text{out}}(\omega) = \left| \frac{J_{\text{out}}^*}{\Delta V^*} \right| = \left| \frac{J_{\text{out}}^{\text{total}}}{V_0 e^{i\omega t}} \right| = \left| \frac{1}{V_0 e^{i\omega t}} \sum_{x=1}^{N_x} J_{(x,N_y-1) \rightarrow (x,N_y)} \right| \end{cases}, \quad (12)$$

and the phase shift φ by:

$$\begin{cases} \varphi_{\text{in}}(\omega) = \arg\left(-\frac{J_{\text{in}}^*}{\Delta V^*}\right) = \arg(V_0 e^{i\omega t}) - \arg(J_{\text{in}}^{\text{total}}) \\ \quad = -\arg\left(\sum_{x=1}^{N_x} J_{(x,1) \rightarrow (x,2)}\right) \\ \varphi_{\text{out}}(\omega) = \arg\left(-\frac{J_{\text{out}}^*}{\Delta V^*}\right) = \arg(V_0 e^{i\omega t}) - \arg(J_{\text{out}}^{\text{total}}) \\ \quad = -\arg\left(\sum_{x=1}^{N_x} J_{(x,N_y-1) \rightarrow (x,N_y)}\right) \end{cases}. \quad (13)$$

As the total complex current is conservative in the low-frequency limit of the Maxwell equations, in which we work, we have $\alpha_{\text{in}} = \alpha_{\text{out}}$ and $\varphi_{\text{in}} = \varphi_{\text{out}}$. In practice, the computations produce values having a relative difference lower than 1×10^{-10} per cent.

2.4 Local model for the complex resistivity

For the complex resistivity of each tube linking node p to node q , denoted $\rho_{p \rightarrow q}^*$, we arbitrarily used a ‘local’ Pelton resistivity model (Pelton *et al.* 1978; see also Tarasov & Titov 2013):

$$\rho_{p \rightarrow q}^* = \rho_{0,l} \left(1 - m_l \left(1 - \frac{1}{1 + (i\omega\tau_{p \rightarrow q})^{c_l}} \right) \right) = \frac{1}{\sigma_{p \rightarrow q}^*}. \quad (14)$$

Here $\rho_{0,l}$ corresponds to the DC resistivity (in Ohm·m), m_l is the chargeability (in $\text{V} \cdot \text{V}^{-1}$ —dimensionless) equal to $(\rho_{0,l} - \rho_{\infty,l})/\rho_{0,l}$, where $\rho_{\infty,l}$ is the limit of $\rho_{p \rightarrow q}^*$ for infinite frequency, $\tau_{p \rightarrow q}$ is the time constant (in s) and c the so-called Cole–Cole exponent (dimensionless) that is comprised between 0 and 1. The Pelton model, a derivation from the Cole–Cole model, was originally designed to explain macroscopic IP spectra. However, it has been demonstrated to be a good model to represent the polarization of single grains or pores (Leroy *et al.* 2008; Niu & Revil 2016).

In eq. (14), $\tau_{p \rightarrow q}$ is linked to the radius of the tube $r_{p \rightarrow q}$ by:

$$\tau_{p \rightarrow q} = \frac{r_{p \rightarrow q}^2}{2D_l}, \quad (15)$$

D_l being the diffusion coefficient of the counterions in the Stern layer (Revil *et al.* 2012; Niu & Revil 2016). Eq. (15) is valid for a sequence of at least two tubes in membrane polarization models (e.g. Titov *et al.* 2002; Bückner & Hördt 2013a,b), and also in the realm of the dynamic Stern layer model (Niu & Revil 2016). It is important to underline here that the values $\rho_{0,l}$, m_l , c_l and D_l are the same for all the tubes. We recognize that the fact that $\rho_{0,l}$ and m_l are the same for all tubes is a strong hypothesis: in reality, pores can present variations of these parameters, depending for example on the content of coating clays. This approach was adopted to simplify the problem, in order to have only one varying parameter (i.e. the radius).

2.5 Upscaling summary

To summary, the direct upscaling procedure for a given frequency consists in:

- (1) Attributing a complex resistivity to each tube of the network.

(2) Obtaining the electrical potential at each node by solving the conservation laws.

(3) Computing the resulting total electrical current inflowing or outflowing from the network.

(4) Deducing the amplitude and phase value from this current.

3 APPLICATION TO SYNTHETIC PORE RADIUS DISTRIBUTIONS

3.1 Principle

For a given tube radius distribution, a random network is generated. To have a sufficiently representative medium (i.e. larger than the elementary representative volume, see end of Section 3.2), we use a 100×100 mesh (i.e. 19 800 tubes). Then the macroscopic (i.e. the bulk) IP response of this network, in terms of the resistivity (or, equivalently, conductivity) amplitude and phase, is computed, for a given set of frequencies $f = \omega/(2\pi)$. Note that IP measurements are typically performed in the frequency range from 1 mHz to 20 kHz, but here we restrain the upper boundary to 1 kHz.

For all the tests reported in this section, we choose the ‘local’ values $\rho_{0,l} = 100 \text{ Ohm} \cdot \text{m}$, $m_l = 0.1 \text{ V} \cdot \text{V}^{-1}$, $c_l = 0.5$ and $\log_{10}(D_l) = -11$ in eqs (14) and (15), based on preliminary tests carried out on real samples (see Section 4). We choose the local value $c_l = 0.5$ based on the recent works by Revil *et al.* (2014a) and Niu & Revil (2016); this is representative of a diffusive process (elementary Warburg model, see Revil *et al.* 2014a). Note that in real rocks, $\rho_{0,l}$ and m_l are expected to vary from pore to pore (e.g. depending on the content of coating clays; moreover, $\rho_{0,l}$ should be linked to the radius of the pore: the surface conductivity, and thus the total conductivity, increases with decreasing radius for instance). We choose them as constant here only to discuss the upscaling effect associated with the pore size distribution, therefore simplifying the physics of the problem.

Finally, we fit the macroscopic IP response of the network also by Pelton resistivity model, that is, the bulk resistivity ρ_{bulk}^* writes:

$$\rho_{\text{bulk}}^* = \rho_{0,m} \left(1 - m_m \left(1 - \frac{1}{1 + (i\omega\tau_m)^{c_m}} \right) \right) = \frac{1}{\sigma_{\text{bulk}}^*}, \quad (16)$$

since, as discussed in several papers (Revil *et al.* 2014a,b, 2015; Niu & Revil 2016), clean sandstones as well as clay-coated sandstones would exhibit such a behaviour.

We search for the macroscopic parameters $\rho_{0,m}$, m_m , c_m and $\log_{10}(\tau_m)$ producing a Pelton resistivity model that fits the best the computed macroscopic IP response—what we call ‘macroscopic optimization’. We use simulated annealing (Aarts & Van Laarhoven 1985) as optimization procedure. A description of the algorithm can be found in Mainault (2016)—but without the monotony test. The variation ranges are $[80 \text{ } 120] \text{ Ohm} \cdot \text{m}$ for $\rho_{0,m}$, $[0 \text{ } 1]$ for m_m and c_m , and $[-3 \text{ } 3]$ for $\log_{10}(\tau_m)$. This first optimization is then followed by a simplex optimization (Nelder & Mead 1965) to refine the solution (function `fminsearch` in Matlab). Finally, we estimate the macroscopic parameter D_m using

$$D_m = \frac{r_{\text{peak}}^2}{2\tau_m}, \quad (17)$$

where r_{peak} is the radius associated with the maximum of the radius distribution.

Figure 3. IP response of random uncorrelated media (mean radius $10 \mu\text{m}$, standard deviation of 0, 0.5 and 1, respectively), for the local parameters $\rho_{0,l} = 100 \text{ Ohm}\cdot\text{m}$, $m_l = 0.1 \text{ V}\cdot\text{V}^{-1}$, $\log_{10}(D_l) = -11$ and $c_l = 0.5$. The dots correspond to the computed response of the networks ('synthetic data'), the circles to the predictions resulting from the macroscopic optimization of the synthetic data. Note that the smaller dots correspond to synthetic data that were not taken into account for the optimization.

3.2 Monomodal radius distribution, uncorrelated media

We first study uncorrelated media (see Fig. 2b for an example). The radii r are distributed in such a way that their decimal logarithms are normally distributed, that is

$$P(\log_{10}(r) \leq X) = \frac{1}{2} + \frac{1}{2} \text{erf}\left(\frac{X - \log_{10}(r_{\text{peak}})}{\text{SD}\sqrt{2}}\right), \quad (18)$$

where SD is the standard deviation; we set r_{peak} equal to $10 \mu\text{m}$ (see Fig. 2a for an example). In order to study the effect of the increasing heterogeneity of the radius distribution (i.e. its broadening), we start from the Dirac distribution (i.e. the standard deviation equal to 0, corresponding to a homogeneous medium with all radii set to $10 \mu\text{m}$), and then increase the standard deviation until 1.

Three examples of computed spectra are shown in Fig. 3 (dots), for SD = 0, 0.5 (corresponding to the network shown in Fig. 2) and 1. The amplitudes increase from around 10.3 to 11.2 mS m^{-1} (Fig. 3a). The phase reaches a maximum value of about 22 mrad at 31.6 mHz for the Dirac distribution. Increasing standard deviation lowers the value of the phase peak and broadens the spectrum, but the peak frequency remains the same (Fig. 3b).

The predictions of the macroscopic optimization explain very well the computed IP responses (circles in Fig. 3), meaning that the macroscopic IP response of the networks obeys a Pelton resistivity model effectively (eq. 16). The estimated macroscopic parameters resulting from the macroscopic optimization are given in Table 1. Note that $\rho_{0,m}$ is equal to 99, not $100 \text{ Ohm}\cdot\text{m}$. This results from the size of the network: smaller networks produce values lower than 99, whereas larger ones produce values tending towards 100 (for instance, 10×10 networks give $90 \text{ Ohm}\cdot\text{m}$, 25×25 96, and 50

Table 1. Macroscopic model parameters for the random uncorrelated media.

Standard deviation	$\rho_{0,m}$	m_m	$\log_{10}(D_m)$	c_m
0.00	99.0000	0.10000	-11.0000	0.50000
0.05	99.0034	0.10005	-11.0006	0.49820
0.10	99.0130	0.10019	-11.0018	0.49291
0.25	99.0641	0.10099	-11.0008	0.46110
0.50	99.2000	0.10323	-11.0127	0.38427
0.75	99.2723	0.10496	-11.0228	0.31472
1.00	99.3292	0.10618	-11.0215	0.25974

Figure 4. Relative differences between the macroscopic model parameters and the local model parameters (i.e. $(X_m - X_l)/X_l$, where X is the considered parameter, m standing for 'macroscopic' and l for 'local') for random uncorrelated media, with respect to the standard deviation of the radius distribution. Local parameters are $\rho_{0,l} = 100 \text{ Ohm}\cdot\text{m}$, $m_l = 0.1 \text{ V}\cdot\text{V}^{-1}$, $\log_{10}(D_l) = -11$ and $c_l = 0.5$. Only the parameter c_m varies significantly (up to -50 per cent).

$\times 50$ 98). However, networks larger than 100×100 nodes have huge computational time and can even cause memory overflow on a standard personal computer. When considering the increase of the standard deviation SD, we observe that $\rho_{0,m}$, m_m , and $\log_{10}(D_m)$ are very stable, with low relative difference (Fig. 4), and are very close to $\rho_{0,l}$, m_l and $\log_{10}(D_l)$. Interestingly, c_m is very close to c_l for very low standard deviations ($\text{SD} < 0.1$), but diminishes significantly with increasing the network heterogeneity (up to -50 per cent of relative difference for $\text{SD} = 1$).

3.3 Monomodal radius distribution, correlated media

We then proceed similarly for correlated networks, which are more representative of real porous media. Indeed, in real media, the succession of pores is not random but obeys to some correlation length scales. We design our correlated network using an isotropic 2-D Von Karman correlation function defined by the standard deviation, σ , the Hurst exponent, ν , whose value is comprised between 0 and 1, and an isotropic characteristic length L_c . The Hurst exponent is directly related to the fractal dimension of the medium (e.g. Turcotte 1997). The 2-D power spectrum of the pore distribution may be usually normalized (Holliger & Levander 1992) as:

$$P(k_x, k_y) = \frac{\Gamma(\nu + 1)}{\Gamma(\nu)} \frac{4\pi\sigma^2 L_c^2}{(1 + k^2)^{\nu+1}}, \quad (19)$$

with

$$k^2 = L_c^2 (k_x^2 + k_y^2). \quad (20)$$

Figure 5. Example of random correlated media (100×100 nodes, 19 800 tubes). The decimal logarithm is normally distributed, with a mean radius of $10 \mu\text{m}$ and a standard deviation of 0.5, a Hurst exponent of 0.9 and a characteristic scale L_c of 1 (a) and 10 (b).

Table 2. Macroscopic model parameters for the random correlated media (Hurst exponent 0.9 and characteristic scale 1).

Standard deviation	$\rho_{0,m}$	m_m	$\log_{10}(D_m)$	c_m
0.0000	99.0000	0.10000	-11.0000	0.50000
0.0501	99.0034	0.10005	-11.0005	0.49819
0.1025	99.0139	0.10020	-11.0016	0.49258
0.2510	99.0600	0.10095	-11.0085	0.46122
0.4942	99.2025	0.10320	-11.0283	0.38583
0.7542	99.2746	0.10487	-11.0325	0.31443
1.0038	99.4226	0.10692	-11.0785	0.26005

Table 3. Macroscopic model parameters for the random correlated media (Hurst exponent 0.9 and characteristic scale 10).

Standard deviation	$\rho_{0,m}$	m_m	$\log_{10}(D_m)$	c_m
0.0000	99.0000	0.10000	-11.0000	0.50000
0.0510	99.0032	0.10005	-11.0001	0.49813
0.1079	99.0151	0.10022	-11.0012	0.49179
0.2489	99.0604	0.10096	-11.0074	0.46151
0.4944	99.2981	0.10388	-11.0679	0.38305
0.8096	99.2408	0.10413	-11.0497	0.30667
1.0299	99.5149	0.10554	-11.2612	0.26588

Here, k_x and k_y are the wavenumber components and Γ is the gamma function. A random mesh of ‘virtual’ radius values, with a size of $N_x \cdot N_y$, was then generated in the spectral domain and transformed in the spatial domain by a 2-D inverse Fourier transform of the square root of the power spectrum multiplied by a random phase between $-\pi$ and π (Bohlen 2002). Finally, the value of the radius of the tube between node p and q is taken as the mean of the values of the ‘virtual’ radii at node p and q , respectively. At the end, we check that the peak radius of the so-obtained tube radius distribution, as well as its standard deviation SD, are effectively equal to the desired peak radius value and standard deviation.

Two examples of correlated networks, with a radius distribution having a peak radius of $10 \mu\text{m}$ and a standard deviation of 0.5, are shown in Fig. 5. The Hurst exponent is equal to 0.9. The characteristic length size L_c defines the size of the aggregates ($L_c = 1$ in Fig. 5a and 10 in Fig. 5b). The results of the macroscopic optimization (Tables 2 and 3) yield to the same conclusions as before, that is to say $\rho_{0,m}$, m_m and $\log_{10}(D_m)$ do not vary significantly with increasing standard deviation. At the opposite, the values of c_m decrease exactly as for the uncorrelated network cases (Fig. 6).

3.4 Bimodal radius distribution, uncorrelated media

Finally, we study uncorrelated media having a radius distribution with two peaks (similar, for instance, to rocks exhibiting a double porosity, such as some limestones, or sandstones with high clay

Figure 6. Evolution of the macroscopic model parameter c_m with respect to the standard deviation, for the random uncorrelated and random correlated media. Local parameters are $\rho_{0,l} = 100 \text{ Ohm}\cdot\text{m}$, $m_l = 0.1 \text{ V}\cdot\text{V}^{-1}$, $\log_{10}(D_l) = -11$ and $c_l = 0.5$.

content). We use the following cumulative probability:

$$P(\log_{10}(r) \leq X) = \frac{1}{2} + \frac{1}{4} \operatorname{erf} \left(\frac{X - \log_{10}(r_{\text{peak},1})}{\text{SD}\sqrt{2}} \right) + \frac{1}{4} \operatorname{erf} \left(\frac{X - \log_{10}(r_{\text{peak},2})}{\text{SD}\sqrt{2}} \right). \quad (21)$$

Note that we give the same weight to the two peaks in order to restrain the number of varying parameters.

Examples of studied distributions, for $r_{\text{peak},1} = 10 \mu\text{m}$ and $r_{\text{peak},2} = 0.1 \mu\text{m}$, are shown in Fig. 7, and associated IP spectra in Fig. 8. For the double Dirac distribution (SD = 0, Fig. 7a), we

Figure 7. Radius distributions for the random uncorrelated media with two peaks (mean radii 0.1 and 10 μm with weight 1/2 each, standard deviation of 0, 0.5 and 1, respectively). For the standard deviation 1, the two peaks overlap, explaining the observed unique peak in the IP response (see Fig. 8).

Figure 8. IP response of random uncorrelated media, with a two-peak radius distribution (mean radii 0.1 and 10 μm with weight 1/2 each, standard deviation of 0, 0.5 and 1, respectively). Local parameters are $\rho_{0,l} = 100 \text{ Ohm}\cdot\text{m}$, $m_l = 0.1 \text{ V}\cdot\text{V}^{-1}$, $\log_{10}(D_l) = -11$ and $c_l = 0.5$. The dots correspond to the computed response of the networks ('synthetic data'), the circles to the predictions resulting from the macroscopic optimization of the synthetic data. The associated radius distributions are shown in Fig. 7.

can observe two inflexion points in the amplitude (black dots in Fig. 8a), and two peaks in the phase spectra (black dots in Fig. 8b). As evidenced by the response of the distribution with $SD = 0.5$ (Fig. 7a), the increase in heterogeneity results in less marked inflexion points (dark grey dots in Fig. 8a) and in a decrease of the amplitude of the two-phase peaks (dark grey dots in Fig. 8b). When the two peaks of the radius distribution overlap (for instance for $SD = 1$, Fig. 7c), the phase spectrum presents only one peak, but significantly broadened (light grey dots in Fig. 8b).

To find the macroscopic parameters of the networks, we fitted the macroscopic response with a dual Pelton resistivity model (e.g. Kemna *et al.* 2000):

$$\rho_{\text{bulk}}^* = \frac{1}{\sigma_{\text{bulk}}^*} = \frac{\rho_{0,m,1}}{2} \left(1 - m_{m,1} \left(1 - \frac{1}{1 + (i\omega\tau_{m,1})^{c_{m,1}}} \right) \right) + \frac{\rho_{0,m,2}}{2} \left(1 - m_{m,2} \left(1 - \frac{1}{1 + (i\omega\tau_{m,2})^{c_{m,2}}} \right) \right), \quad (22)$$

where subscript 1 (respectively 2) corresponds to parameters associated with $r_{\text{peak},1}$ (respectively $r_{\text{peak},2}$). The optimization procedure is the same as before. The predictions inferred from the estimated macroscopic parameters (circles in Fig. 8) fit very well the computed IP responses, meaning that the assumption of the dual Pelton resistivity model (eq. 22) was correct. The estimated macroscopic parameters (Table 4) have the same behaviour as observed for one-peak distributions, that is, only $c_{m,1}$ and $c_{m,2}$ varies significantly, decreasing with the increasing standard deviation of the tube radius distribution, as soon as SD is larger than 0.1.

4 APPLICATION TO REAL SAMPLES

4.1 Estimation of the parameters

To study synthetic radius distributions (Section 3), we impose the local parameters of the Pelton complex resistivity ($\rho_{0,l}$, m_l , c_l and D_l), then compute the macroscopic response of the network, and finally fit this macroscopic response with a Pelton resistivity model to obtain the macroscopic parameters of the network ($\rho_{0,m}$, m_m , c_m and D_m). The procedure is slightly different for real samples, as the local parameters are here unknown.

To estimate the local parameters, we first construct a network (25×25 nodes and 1200 tubes) whose radius distribution obeys the measured pore access radius distribution inferred from mercury porosimetry (an example of such a distribution is shown in Fig. 9). Using the simulated annealing, we explore the parameter space to find $\rho_{0,l}$, m_l , c_l and D_l to be used in eqs (14) and (15) in order to obtain a macroscopic IP response of the network as close as possible to the measured IP response. Since a single network realization, especially on such small networks, have no statistical significance, we then explore again the parameter space for 100 network realizations with the simplex method, using as starting point the four local parameters resulting from the simulated annealing optimization on the initial network. An example of results from such a procedure is shown in Fig. 10: the crosses represent the measured data and the black lines represent the computed responses of the 100 network realizations. Each realization explains rather well the observations. For the estimated values of the local parameters $\rho_{0,l}$, m_l , c_l and D_l , we use their mean values over the 100 realizations. Histograms of these local parameter values (Fig. 11) show that they are not very dispersed, and rather well determined. Finally, to obtain the macroscopic parameters $\rho_{0,m}$, m_m , c_m and D_m , we fit the measured IP response with a Pelton resistivity model as before (Fig. 12, see Section 3.1).

4.2 Results

We use the data for the samples studied by Revil *et al.* (2014a), that is, six siliciclastic (sandstones and mudstones) samples from the Great Divide Basin (Wyoming, USA), denoted S436, S439, S490, S493, S498 and S499. The petrophysical properties of these materials are extensively described in Revil *et al.* (2014a) and will

Table 4. Macroscopic model parameters for the random uncorrelated media with two peaks in the radius distribution. Subscript 1 corresponds to the 10 μm peak, and subscript 2 to the 0.1 μm peak.

Standard deviation	$\rho_{0,m,1}$	$\rho_{0,m,2}$	$m_{m,1}$	$m_{m,2}$	$\log_{10}(D_{m,1})$	$\log_{10}(D_{m,2})$	$c_{m,1}$	$c_{m,2}$
0.00	99.6590	98.3413	0.10282	0.09715	−10.9773	−10.9769	0.49989	0.49991
0.05	98.3659	99.6393	0.10268	0.09741	−10.9770	−10.9802	0.49788	0.49811
0.10	99.1972	98.8224	0.10142	0.09879	−10.9801	−10.9797	0.49194	0.49291
0.25	97.8918	100.2166	0.10646	0.09522	−10.9735	−10.9748	0.45626	0.45793
0.50	99.5091	98.8213	0.10284	0.10485	−11.0293	−11.0115	0.37106	0.35970
0.75	99.6946	98.7478	0.09298	0.11975	−11.1166	−11.1103	0.30604	0.27599

Figure 9. Pore radius distribution for core sample S498, inferred from mercury porosimetry measurements.

Figure 10. Measured IP response of core sample S498 (red crosses and dots), and the predictions resulting from the local optimization of these data (grey lines), carried out on 100 uncorrelated networks (25×25 nodes, 1200 tubes). Note that the data figured by dots were not taken into account for inversion.

not be repeated here. Their pore size distributions are characterized from mercury porosimetry.

The local and macroscopic parameters are compared in Table 5 and in Fig. 13. It can be seen that $\rho_{0,l}$ and $\rho_{0,m}$ are very close (Fig. 13a), and the same for m_l and m_m (Fig. 13b). This behaviour is in agreement with what we observe on synthetic distributions. The relation between $\log_{10}(D_l)$ and $\log_{10}(D_m)$ is more dispersed (Fig. 13c), and does not follow the 1:1 trend. However, the absolute relative variations (computed as $2|(a - b)/(a + b)|$) do not exceed 10 per cent (mean 5 per cent). Finally, c_m is always smaller than c_l ,

as for the synthetic distributions, with absolute relative variations up to 48 per cent (mean 28 per cent, Fig. 13d).

5 DISCUSSION

The very simple methodology of random network produces coherent results, as evidenced by the response of the simple Dirac distribution corresponding to a homogeneous medium (Fig. 3). Under the assumption that the resistivity $\rho_{0,l}$, the chargeability m_l , the Cole–Cole exponent c_l and the diffusion coefficient D_l are the same for all the tubes (i.e. the only varying parameter is the tube radius), the macroscopic parameters (i.e. $\rho_{0,m}$, m_m , c_m and D_m) are the same as the local ones (i.e. $\rho_{0,l}$, m_l , c_l and D_l), as expected, meaning that the upscaling procedure is coherent and successful. Therefore, it appears that the random network methodology can be used as an efficient tool adequate for studying the upscaling of the IP problem, from the pore scale to the sample scale.

We observe, still under the assumption stated above, that using a Pelton resistivity model for the complex resistivity of the pores generates a Pelton resistivity model at the macroscopic scale, as shown in Fig. 3 (or a dual Pelton resistivity model in the case of bimodal radius distribution, Fig. 8). The macroscopic resistivity $\rho_{0,m}$, chargeability m_m and diffusion coefficient D_m are very close to the local resistivity $\rho_{0,l}$, chargeability m_l and diffusion coefficient D_l , respectively (note here again that the local resistivity is not those of the fluid in the tube, but also includes the effects of the surrounding mineral). On the contrary, the macroscopic Cole–Cole exponent c_m decreases with increasing heterogeneity, with relative difference $(c_m - c_l)/c_l$ up to −48 per cent for the standard deviation value equal to 1 when c_l is equal to 0.5 (Fig. 4). We also computed the evolution of c_m in function of the standard deviation SD for two supplemental cases: $c_l = 0.25$ and $c_l = 0.75$ (Fig. 14a). In the first case, the decrease is less marked, with maximal relative difference up to −25 per cent, whereas in the second case the decrease is more important, with maximal relative difference up to −62 per cent (Fig. 14b). It means that the upscaling properties of the macroscopic Cole–Cole exponent depend also on the absolute value of the local Cole–Cole exponent, and not only on the heterogeneity of the porous medium.

This observation of a lower macroscopic value of the Cole–Cole exponent, made on synthetic pore radius distribution, is also valid for the few experimental measurements reported here (Fig. 13). From the upscaling point of view, and again under the assumptions we make, it could imply that the resistivity and chargeability values determined from measurements performed at the sample scale are representative of the values that they have at the pore scale. Note that the heterogeneity generates a strong variation in radius, and thus in the time constant τ (eq. 15). As the τ distribution is directly linked to the Cole–Cole exponent c (Cole & Cole 1941), this explains why c varies so much with increasing heterogeneity. On the contrary, the local resistivity and chargeability were constant over the whole tube

Figure 11. Distributions of the values of the local model parameters, resulting from the 100 local optimizations of the data for core sample S498.

Figure 12. Measured IP response of core sample S498 (dots), and the predictions resulting from the macroscopic optimization of these data (circles). Note that the data figured by the smaller dots were not taken into account for the optimization.

network, resulting in very small variations at the macroscopic scale. Finally, we also observe that the macroscopic diffusion coefficient D_m is systematically higher than the local coefficient D_l (Fig. 13c), but the absolute relative variation as well as the number of samples are too small to conclude on the generalization of this observation. It may suggest that the macroscopic value of the time constant τ_m is slightly decreased compared to the average local value τ_l during the upscaling process.

Note also that the macroscopic parameters exhibit exactly the same behaviour for 25×25 and 50×50 networks (except the absolute value of the macroscopic resistivity, see Section 3.2). In

particular, c_m presents the same characteristics, that is, an identical decreasing rate with increasing standard deviation; but the results are much more erratic for 10×10 networks. It means that the elementary representative volume is larger than (or equal to) 25×25 and that the way c_m is decreasing does not change with the increasing size of the network. In other words, the upscaling properties of the Cole–Cole exponent seem not to depend on the considered scale, provided that the considered volume is large enough.

Concerning the influence of the correlation distance of the radius distribution, it seems that the spatial repartition of the tube radii does not play a significant role on the evolution of the macroscopic parameters (still under the assumption stated above). The evolution of the Cole–Cole exponent c_l is indeed the same in correlated and uncorrelated media (Fig. 4). The heterogeneity of the radius distribution is the most important parameter influencing the macroscopic IP response.

Introducing local distributions for the resistivity, chargeability and Cole–Cole exponent based on realistic scenarios constitutes the basis for future works. We preliminarily tested three simple cases. In the first one, the local resistivity $\rho_{0,loc}$ varies linearly with the varying radius, following $\rho_{0,loc} = 100 + 50 (\log_{10}(r) - \langle \log_{10}(r) \rangle)$ with $\rho_{0,loc}$ in Ohm·m and r in μm (Fig. 15a); as the radius decreases, the local resistivity diminishes since the stronger effect of the surface conductivity increases the pore conductivity. In the second one, the local chargeability m_{loc} increases linearly with the decreasing radius, following $m_{loc} = 0.1 + 0.05 (\log_{10}(r) - \log_{10}(r))$ with m_{loc} in $\text{V} \cdot \text{V}^{-1}$ and r in μm (Fig. 15b). Finally, the third case is the combination of the first and the second case. We observe that when the local resistivity varies, the conductivity amplitude spectrum (Fig. 16a) is shifted towards the higher conductivities, whereas the phase spectrum (Fig. 16b) does not change. When the local chargeability varies, the amplitude spectrum is slightly shifted towards lower conductivities at lower frequencies; the phase spectra is shifted toward the higher frequencies, meaning that the macroscopic time relaxation constant has decreased (Table 6). When both vary, we observe a combination of the two previous effects. Again,

Table 5. Local (subscript « l ») and macroscopic (subscript « m ») model parameters for the samples.

Sample	$\rho_{0,l}$	$\rho_{0,m}$	m_l	m_m	$\log_{10}(D_l)$	$\log_{10}(D_m)$	c_l	c_m
S436	51.0687	50.1435	0.23084	0.24139	-11.8476	-10.7534	0.35182	0.25442
S439	9.6419	9.2862	0.034705	0.031959	-10.9826	-10.6858	0.765	0.48392
S490	35.6619	34.5903	0.17737	0.16454	-10.8596	-10.3923	0.48814	0.37589
S493	9.8835	9.5018	0.068381	0.059576	-10.8382	-10.4972	0.42828	0.37044
S498	15.4836	14.9913	0.092097	0.090523	-11.1282	-10.4985	0.53548	0.37002
S499	45.4526	43.9791	0.21947	0.19424	-10.648	-10.2643	0.49969	0.43049

Figure 13. Comparison of the macroscopic and local model parameters: (a) resistivity ρ_0 , (b) chargeability m , (c) diffusion coefficient D and (d) Cole-Cole exponent c .

these simple examples evidenced the interest of the tube network for the study of the IP upscaling.

We can note here than the macroscopic spectra can also be inverted in terms of complex conductivity model, using:

$$\sigma_{\text{bulk}}^* = \sigma_{0,m} \left(1 + \frac{m'_m}{1 - m'_m} \left(1 - \frac{1}{1 + (i\omega\tau'_m)^{c'_m}} \right) \right) \quad (23)$$

(see for instance Tarasov & Titov 2013) instead of eq. (16). We should have $\sigma_{0,m} = \rho_{0,m}^{-1}$, $m'_m = m_m$, $c'_m = c_m$ and:

$$\tau'_m = (1 - m_m)^{1/c} \tau_m \quad (24)$$

(Florsch *et al.* 2010; Tarasov & Titov 2013). For instance, when inverting the third case using the Pelton resistivity model, we obtained $\rho_{0,m} = 95.65 \text{ Ohm}\cdot\text{m}$, $m_m = 0.104 \text{ V}\cdot\text{V}^{-1}$, $c_m = 0.389$ and $\tau_m = 2.9231$ (Table 6). Using the complex conductivity model, we get $1/\sigma_{0,m} = 95.65 \text{ Ohm}\cdot\text{m}$, $m'_m = 0.104 \text{ V}\cdot\text{V}^{-1}$, $c'_m = 0.389$

and $\tau'_m = 2.2054$. Eq. (24) is therefore verified, proving again the coherence of our approach.

Using the Pelton resistivity model for the complex resistivity at the pore scale may appear restrictive, since it somehow removes the question of the physical phenomena occurring at the pore scale. We would answer that we used the Pelton resistivity model exactly for this reason: we wanted to simplify the pore scale model and focus on the upscaling issue only. Further works obviously require the implementation of more physical responses at the pore scale (such as models by Titov *et al.* 2002; Revil & Florsch 2010; Bückner & Hördt 2013a,b), as well as the study of the influence of the pore geometry—that is, is this upscaling property still valid if pores are more complicated than simple tubes? An extension to 3-D networks is also possible to produce networks more representative of real core samples. Finally, the methodology we use opens other research directions, such as the study of the effect of anisotropy, which can be easily implemented in correlated networks, the extension to grain

Figure 14. Evolution of the macroscopic model parameter c_m with respect to the standard deviation for the (a) random uncorrelated media, and (b) associated relative difference $(c_m - c_l)/c_l$, for $c_l = 0.25, 0.5$ and 0.75 , respectively. Other local parameters are $\rho_{0,l} = 100 \text{ Ohm}\cdot\text{m}$, $m_l = 0.1 \text{ V}\cdot\text{V}^{-1}$ and $\log_{10}(D_l) = -11$.

assemblages instead of pore assemblages, and even the study of drainage and imbibition with local models taking into account the effect of the saturation.

6 CONCLUSIONS

We implement the tube network methodology to study the upscaling of IP response from the pore scale (the local scale) to the sample scale (the macroscopic scale) on a very simplified system. Using a Pelton resistivity model for the complex resistivity of the tubes, with identical resistivity, chargeability, Cole–Cole exponent and diffusion coefficient values for all the tubes but a different time constant that depends on the radius, we show that the macroscopic response also obeys a Pelton resistivity model, with the same resistivity, chargeability and diffusion coefficient values, but with a different Cole–Cole exponent. The macroscopic value of this last parameter decreases strongly with the increasing heterogeneity of the pore radius distribution, for correlated and uncorrelated media. The same behaviour was observed on six sandstone samples.

We demonstrate the usefulness of the method for the study of the upscaling of IP response in the simplified case of a local Pelton resistivity model. Further works will require the implementation of more physical models for the IP properties at the pore scale, in order

Figure 15. Evolution of the (a) local resistivity and (b) local chargeability versus tube radius chosen for a more realistic scenario of porous medium.

Figure 16. Macroscopic IP spectra obtained in the case of variable radius (black dots), variable radius and local resistivity (blue dots), variable radius and local chargeability (orange dots), and variable radius, local resistivity and local chargeability (red dots), using the relationships described in Fig. 15 for the case of the 100×100 random uncorrelated network with the decimal logarithm of the pore tube radius distribution normally distributed, with a mean radius of $10 \text{ }\mu\text{m}$ and a standard deviation of 0.5 (shown in Fig. 2b).

Table 6. Macroscopic model parameters for the spectra shown in Fig. 16, deduced by inversion.

Case	$\rho_{0,m}$	m_m	$\log_{10}(\tau_m)$	c_m
Varying radius	99.2	0.103	0.712	0.384
Varying radius and resistivity	95.6	0.103	0.716	0.387
Varying radius and chargeability	99.2	0.104	0.452	0.385
Varying radius, resistivity and chargeability	95.7	0.104	0.466	0.389

to characterize the upscaling of the Stern layer polarization and the membrane polarization.

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