Homogenization of the brush problem
with a source term in L^1

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Abstract

We consider a domain which has the form of a brush in $3D$ or the form of a comb in $2D$, i.e. an open set which is composed of cylindrical vertical teeth distributed over a fixed basis. All the teeth have a similar fixed height; their cross sections can vary from one teeth to another one and are not supposed to be smooth; moreover the teeth can be adjacent, i.e. they can share parts of their boundaries. The diameter of every tooth is supposed to be less than or equal to ε, and the asymptotic volume fraction of the teeth (as ε tends to zero) is supposed to be bounded from below away from zero, but no periodicity is assumed on the distribution of the teeth.

In this domain we study the asymptotic behavior (as ε tends to zero) of the solution of a second order uniformly elliptic equation with a zeroth order term which is bounded from below away from zero, when the homogeneous Neumann boundary condition is satisfied on the whole of the boundary.

First, we revisit the problem where the source term belongs to L^2. This is a classical problem, but our homogenization result takes place in a geometry which is more general that the ones which have been considered before. Moreover we prove a corrector result which is new.

Then, we study the case where the source term belongs to L^1. Working in the framework of renormalized solutions and introducing a definition of renormalized solutions for degenerate elliptic equations where only the vertical derivative is involved (such a definition is new), we identify the limit problem and prove a corrector result.

Keywords: Homogenization, oscillating boundaries, Neumann brush (and comb) problems, renormalized solutions, degenerated elliptic equations.

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