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Coalitional Games for Joint Co-Tier and Cross-Tier Cooperative Spectrum Sharing in Dense Heterogeneous Networks

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ABSTRACT With the dense deployment of small cells in the next generation of mobile networks, the users from different tiers suffer from high downlink interferences. In this paper, we propose a game theoretic approach for joint co-tier and cross-tier collaboration in heterogeneous networks and analyze the relevance of the proposed scheme. First, we propose a coalition structure game with a weighted Owen value as imputation, where the small-cell base stations (SBSs) and their connecting macrocell user equipments (MUEs) form *a priori* union. We prove that the proposed framework optimizes the users profit. As an additional global benefit, the SBSs are encouraged to host the harmed public users in their vicinity. Second, we propose a canonical game with a weighted solidarity value as imputation to allow cooperation among SBSs and MUEs when they fail to connect to nearby SBSs. We prove that the weak players are protected in this scheme and that a high degree of fairness is provided in the game. We compare through extensive simulations the proposed frameworks with state-of-the-art resource allocation solutions, access modes, and legacy game-theoretic approaches. We show that the proposed framework obtains the best performances for the MUEs and small-cells user equipments in terms of throughput and fairness. Throughput gain is in order of 40% even reaching 50% for both types of users.

INDEX TERMS Small-cells, heterogeneous networks, resource allocation, interference mitigation, game theory, cooperative games.

I. INTRODUCTION

One of the main challenges of the fifth generation mobile networks is responding to the exponentially-increasing demand for higher data capacity and data rates. This involves a greater spectrum in low and high bands and more antennas, as well as the deployment of more small-cells overlaying the existing macrocells. Indeed, operators are looking to offload traffic from their Macrocell Base Stations (MBSs) as they anticipate data traffic to grow by 1000 times by 2018. Accordingly, the dense deployment of small-cells is a crucial part of addressing this growth. These small-cells are connected to the backhaul network via optical fibre or DSL and allow not only higher spectrum efficiency but also greater overall network capacity. There are several additional advantages, such as improved indoor coverage, reduced costs and power consumption along with a higher Quality of Service (QoS) satisfaction [2]. However, the coexistence of users and base stations from both tiers comes with several challenges.

In order for capacity to increase in tandem with the addition of small-cells, a rigorous interference and resource management has to be planned. An approach that eliminates cross-tier interference is the split-spectrum policy in which the small-cell tier uses a dedicated bandwidth distinct from the macro-tier [3]–[5]. The drawback of this approach is inefficiency in terms of spectrum reuse. On the other hand, the co-channel deployment approach allows both macrocells and small-cells to access to the entire spectrum resource, though cooperation among the different tiers is required. Two types of interferences induced by the co-channel deployment can seriously degrade the performances of the network, e.g., the cross-tier interference (from the MBS to the SBSs) and co-tier interference (from SBSs to SBSs). In order to

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mitigate these interferences several decentralized solutions have been proposed [6]–[9]. On one hand, the non-cooperative approaches have been widely studied and are characterized by the independent decisions of the players who aim to improve their own performances. To solve non-cooperative games, the most widely-used concept is the well-known Nash equilibrium [10]. Non-cooperative game theory has been considered for resource allocation [11]–[14] and power control [15], [16]. A non-cooperative evolutionary game based on stochastic geometry analysis for small-cells resource allocation is presented in [13]. On the other hand, with the need for self-organizing, decentralized and autonomous networks, cooperative approaches have emerged as a key solution for the success of dense heterogeneous networks.

According to cooperative approach, game theory is an essential tool in helping the various entities make decisions in two-tier networks [17]. A cooperative game can occur in a group (i.e., a coalition) in which the players share information and try to attempt to negotiate the attainment of common objectives. Coalitional games can be classified into two categories [18]: canonical games and coalition structure games, both will be studied in this paper. In a canonical game, all players aim to form and stabilize a grand coalition. The value of the coalition is then divided among the players such that none of the players has any incentive to leave the grand coalition. In coalition structure games, players are rational in forming coalitions or a priori unions in order to maximize profit.

In [19] and [20], the authors proposed a canonical and a coalitional game model, respectively, as a technique for co-tier interferences mitigation among cooperative small-cells. In [21], the authors propose a bankruptcy game approach for resource allocation in cooperative networks. However, according to these three frameworks, a split-spectrum approach is adopted, and hence the cross-tier cooperation is not investigated. A theoretical game-based cognitive radio resource management approach is proposed in [22] and [23]. In these studies, a coalition game is developed for use in subchannel allocation in situations where cognitive small-cells act as secondary users and have a higher priority than MUEs but the collaboration between the two tiers is not investigated; in these cases, the priority applied can result in a deterioration of the macro-tier performances. In [24], the authors consider the cross-tier cooperation among SUEs and MUEs in order to alleviate the downlink interference. It allows the MUEs to explore nearby small-cells by cooperating with the SUEs, which act as relays; however, in these cases the closed-access mode is adopted which prevents the MBS from offloading its high data traffic to the small-cell tier.

A hybrid access mode is a promising solution as it allows a public user suffering from downlink interferences from a nearby SBS to connect to this small-cell in order to process its demands. However, only a limited amount of the small-cell resources is available to all users, while the rest is operated within a closed subscribed group (CSG) manner [25]. Very few papers have investigated the collaboration between the harmed MUEs and neighbouring SBSs in instances when the first fails to connect to the small-cell. Yet another problem not fully investigated is when the cooperative games involve hybrid or open-access small-cells. Several questions arise: How can the cooperation between users and base stations from different tiers be modelled? How can the small-cell tier properly process the MUEs demands, while managing the cooperative resource allocation in a fair and strategic manner? How can the SBSs be encouraged to serve public users without degrading their own performances? How can the different bargaining power levels be managed in a game?

Such questions are essential to the successful dense deployment of small-cells; our proposed cooperative model attempts to provide solutions to these issues. The main contribution of this paper is to propose a new cooperative-game framework for co-tier and cross-tier interference mitigation and resource management under an open-access mode of small-cells, which allows the MBS to offload its data traffic to the dense small-cell tier. When MUEs are served by a nearby SBS, we propose forming a union of the related SBS and MUEs in a given game in order to attribute a reasonable profit to the hosted MUEs while rewarding the SBS for actively participating in the interference mitigation process. Accordingly, when the union SBS-MUEs is formed, its members commit themselves to bargaining with the others as a unit. Any gain obtained by the players of the unions are then shared according to a coalition structure solution (i.e. Weighted Owen). When the harmed MUE fails to connect with a nearby SBS, it participates in a cooperative game with its interferers in order to split the available resources following a solidarity-based imputation scheme (see Fig.1 and Fig.2). Our key contributions are summarized in the following:

1) We create a new collaborative framework based on two different game theoretic approaches in which the end user benefit is quantified in terms of throughput and fairness for both MUEs and SUEs of the system.
2) We address the resource allocation problem when MUEs and SUEs coexist in a small-cell coverage area using a $\mathcal{CS}$ game based on the formation of unions when SBSs host public users in their neighbourhood.
   - We prove that the proposed $\mathcal{CS}$ game under a Weighted Owen imputation value optimizes the profit of the MUEs and SBSs which participate in the unions. A direct consequence is that the SBSs are encouraged to host the harmed public users in their coverage area as the users joining forces get a better profit than bargaining individually.
3) We address the co-tier and cross-tier resource allocation problem when MUEs are not hosted but interfered by neighbouring SBSs using a canonical game approach.
   - We propose a new algorithm for the computation of the canonical game imputation (i.e. a Weighted Solidarity value). We prove that it protects the

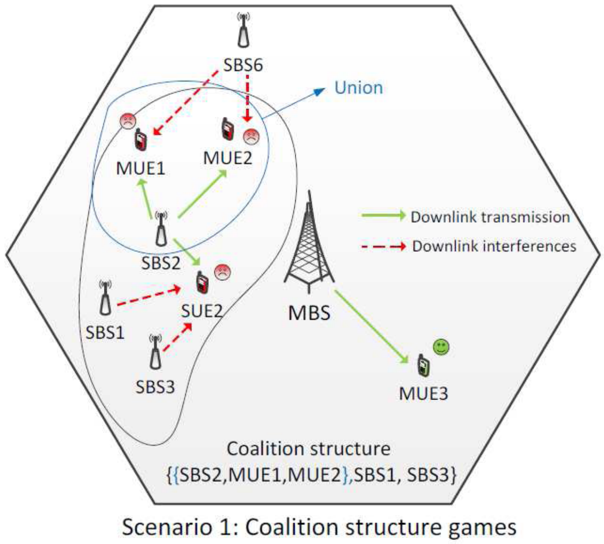

FIGURE 1. An illustration of a scenario leading to the proposed coalition formation and coalition structure game.

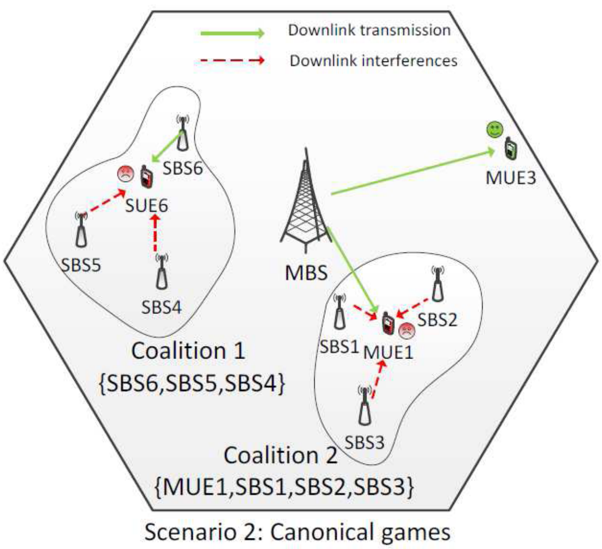

FIGURE 2. An illustration of a scenario leading to the proposed coalition formation and canonical game.

weak players when their power of bargaining is low and provides a higher degree of fairness. Additionally, it does not allow any user to obtain more than its claim and discourages players from asking for higher demands.

In the next section of this paper, we present the problem formulation and motivations, followed by the SBSs and MUEs interference set formation process in Section III. Section IV presents the cooperative game framework and the identified imputation values. Finally, to validate the effectiveness of our proposed cooperative game approach, we present the simulation results in Section V.

II. PROBLEM FORMULATION AND MOTIVATIONS

In cooperative games, users are given the opportunity to collaborate in order to split the available resources. With the development of self-organizing and decentralized small-cells, the latter should be capable of managing not only the interferences they induce to each other but also the interferences induced to the neighbouring MUEs. To do so, it is essential to incorporate a cross-tier interference collaborative mitigation scheme into the existing co-tier models. The SUEs can be easily represented by their SBSs that participate in the game and redistribute the payoff among their CSG users. However, as we want our model to be distributed, we need to incorporate the MUEs into the game as players. Indeed, when MUEs are interfered with one or several SBSs, they compete with the latter for the same resources in a co-channel deployment. Hence, they can form a coalition and bargain their resources with the interfering SBSs. A second plausible scenario might involve the MUE connecting with a nearby hybrid or open-access SBS: if the SBS represents the MUE in the game, it might unfairly split the resources among its own CSG and public users. The concept that players have a ''right to talk'' in a game, has been introduced in [26] and it is one that we also wish to be extended to MUEs of the system. Indeed, in a resource allocation game, the SBSs express their demands in terms of a number of tiles (i.e. resources) and participate in a game with the other agents in order to split the available resources. When the nearby MUEs are attached to a given SBS, their demands are ''absorbed'' by the paired SBS (added to the initial demand of the SBS CGS users). However, the reward might be unfairly redistributed by the hosting SBS among the users of both types (subscribers or public users).

The coalition structure will give us an essential model in which players need to organize themselves into groups for the purpose of sharing the network's resources. This is a great opportunity for heterogeneous networks in which the neighbourhood small-cells concept described earlier in which the small-cells are encouraged to allow the access to public users in their neighbourhood. It is no longer appropriate to consider the SBSs as single players. Indeed, the SBS plays the game for the purpose of reallocating its payoff to its own users but the MUEs must have the chance to participate in the negotiation process too, in order to not be cheated by the SBSs who might prioritize their own subscriber users. This is facilitated by the introduction of a coalition structure into the game which consists of a partition of players into a union. This can be considered analogous to a family that has to take into account all its members before making decisions that impact them. Such an approach also creates a better bargaining situation, as the other party has to convince jointly all the members. Some imputation values, like the Owen value [27] ignore the power of the unions and attribute the same weight to the unions as to single players. The imputation value of our model is therefore superior because it gives a greater weight to the unions while allowing entities from both tiers to participate in the spectrum sharing. In this way, a single game allows us to fairly manage inter-union collaboration as well as intra-union resource bargaining. In turn, this also facilitates the achievement of an overall consensus, in which each user has a ''right to talk'' in the game.

III. SYSTEM MODEL

We consider the downlink of an Orthogonal Freqency Division Multiple Access (OFDMA) macrocell network overlaid by N SBSs and K MUEs. The SBSs reuse the entire bandwidth allocated to the underlying macrocell. Let $\mathcal{F} = \{F_1, .., F_n, .., F_N\}$ be the set of SBSs and $\mathcal{M} = \{M_1, .., M_n, .., M_K\}$, the set of MUEs in a given macrocell. For their downlink transmission, SBSs might cause co-tier interference to the neighbouring SBSs and cross-tier interference to the surrounding MUEs as we consider a spectrum-sharing approach. In non-cooperative networks, the users of both tiers consider the downlink interferences of all surrounding SBSs. Here, under the cooperative approach, SBSs and MUEs of the system collaborate in order to share the available resources and mitigate the co-tier and cross-tier interferences. Each SBS and MUE of the system determines its interfering set in the downlink in order to form cooperative sets as depicted in Fig.1 and Fig.2.

A. INTERFERENCE SET DETECTION

Each user SUE within a given small-cell F_n boundary, calculates the ratio of the received signal from F_n to the signal received from all surrounding macrocells and from the surrounding SBSs [19]. To determine the interference set of the small-cell of interest, we need to consider only the interference induced by one SBS at a time. The downlink signal-to-interference-plus-noise ratio (SINR) achieved by a SUE y_n associated with small-cell F_n on a particular tile k when interfered by small-cell $F_{n'}$ is given by [19]:

$$\gamma^k_{y_n,F_n} = \frac{P^k_{F_n} G^k_{y_n,F_n}}{\sum\limits_{m \in M} P^k_m h^k_{y_n,m} G^k_{y_n,m} + P^k_{F_{n'}} G^k_{y_n,F_{n'}} + \sigma^2} \tag{1}$$

where $G^k_{y_n,F_n}$ and $G^k_{y_n,m}$ represent the channel gains from SBS F_n and MBS m to SUE y_n, respectively, in small-cell F_n on tile k, σ^2 the noise power and M the set of surrounding MBSs. Let I^f_n be the interference set of F_n composed of F_n and SBSs causing interferences to its users. If the SINR $\gamma^k_{y_n,F_n}$ achieved by FUE F_n is inferior to a certain SINR threshold δ_f, then the SBS $F_{n'}$ is considered to be an interferer of F_n and joins its interference set I^f_n. We proceed this way for each SBS in the network until all SBSs have formed their interference set or remained alone if they are not interfered by any neighbouring SBSs.

In the same manner, each MUE calculates the ratio of the received signal from its corresponding MBS to the signal received from all surrounding small-cells and macrocells. Similarly, we take into account only the interference induced by one SBS at a time to determine if it is an interferer of the MUE M_n. The SINR achieved by a MUE M_n associated with macrocell m on a particular tile k when interfered by the small-cell F_n can be written as:

$$\gamma^k_{M_n,m} = \frac{P^k_m G^k_{M_n,m} h^k_{M_n,m}}{\sum\limits_{m' \in M'} P^k_{m'} h^k_{M_n,m'} G^k_{M_n,m'} + P^k_{F_n} G^k_{M_n,F_n} + \sigma^2} \tag{2}$$

where $h^k_{M_n,m}$ is the exponentially-distributed channel fading power gain associated with tile k. $G^k_{M_n,m}$ and $G^k_{M_n,F_n}$ represent the path loss associated with k from a MBS m and SBS F_n to a MUE M_n, respectively, in macrocell m. Let I^m_n be the interference set of M_n composed of M_n and SBSs interfering with M_n. Here, if the SINR $\gamma^k_{M_n,m}$ achieved by MUE M_n is under to a certain SINR threshold δ_m, the SBS F_n is considered to be an interferer of M_n and joins the interference set I^m_n.

Within the interference set I^m_n, if the MUE M_n is located in the coverage area of a SBS F_n, and if the MUE receives a better SINR from this SBS than from its serving MBS it always tries to connect to the corresponding SBS. If the MUE succeeds in connecting to a nearby SBS, the two are assumed to be paired in the sequel. These pairs will be used in the next section to build the $\mathcal{CS}$ games. Once we have determined the interference set for each SBS and MUE of the system, we sort these interference sets in descending order, firstly according to their cardinality and secondly according to the overall demand of the interference set.

IV. CROSS-TIER MACROCELL-SMALLCELLS COOPERATION AS COALITIONAL GAMES

A. PROPOSED GAME THEORY APPROACH

In the presence of dense small-cell deployment within urban environments, the overall demand in the shared spectrum often exceeds the number of available tiles $|Z|$ [21]. Assuming that in the same interference set, the SBSs and MUEs can share information about their demands, we formulate the problem of co-tier and cross-tier interference mitigation and resource allocation as a cooperative game. To solve the problem mentioned above, we propose the following approach. When a MUE connects to a SBS, the latter does not absorb the demand of the connecting public user. Instead, the MUE is considered as a player, even if the demand of the MUE will be processed by the paired SBS. The MUE will join the SBS in a union, known as a priori union. If the MUE fails to connect to the nearby SBS, the MUE acts as a single player and participates in a resource allocation game with the nearby SBSs. In such a case, the demand of the MUE will be processed by the MBS. Such a game is also played when only SBSs are involved in an interference set (co-tier model).

Cooperative games involve a set of players in a system, who seek to form cooperative groups (i.e interference sets) in order to improve their performances. The aim of the proposed cooperative approach is: 1) to form an interference set in order to reduce co-tier and cross-tier downlink interferences, 2) to find a binding agreement among the agents of the same set to split the available resources.

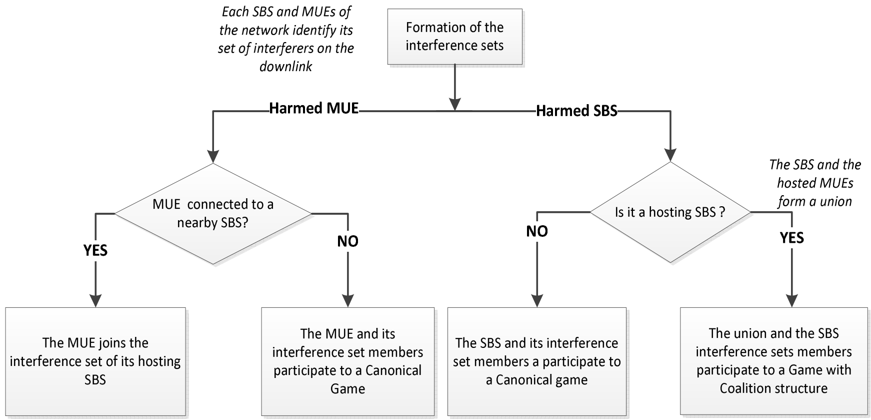

FIGURE 3. **General flow chart describing the formation of the coalition structures and the classification of the two games.**

We follow the steps described in Figure 3. The first step is the interference set detection developed in section III.A. The next step is a game iteration following the order in the sorted list of detected interference sets. The type of the game depends upon the type of the harmed agent in the formed interference set (MUE or SBS). If a SBS is not hosting any MUE or if a harmed MUE fails to connect to a nearby SBS, it participates in a canonical game with its interference set members. However, if the SBS is hosting a MUE, they form a union and a $\mathcal{CS}$ game is played between this union and the interference set of the paired SBS. Both type of games are defined in the sequel.

B. DEFINITION AND FORMULATION OF THE GAME

As mentioned before, in the presence of dense small-cell deployment within urban environments, the overall demand in the shared spectrum often exceeds the number of available tiles $|Z|$ [21]. Assuming that the SBSs and MUEs belonging to the same interference set I_n can share informations about their respective demands and allocations, the resource allocation problem can be formulated as a cooperative game with transferable utility. Let I_n denotes the interference set of the current game iteration. It corresponds either to I_n^f or I_n^m but we omit this differentiation for simplicity.

Definition 1: A cooperative game with transferable utility (TU-game) is a pair $(\mathcal{N}, v)$ where $\mathcal{N}$ is a non-empty set and $v : 2^{\mathcal{P}} \rightarrow \mathbb{R}$ a characteristic function defined on the power set of $\mathcal{N}$ satisfying $v(\emptyset) = 0$.

We consider $\mathcal{N} \equiv \mathcal{I}_n$ and we define the worth $v(S)$ associated to each coalition $S \subseteq \mathcal{N}$ the amount of available resources not claimed by its complement nor already allocated to players of $\mathcal{N}$ in a precedent iteration of the game.

$$v(S) = max\{0, |Z| - \sum_{j \in \mathcal{N} \setminus S} d(j) - \sum_{j' \in \mathcal{C}} p(j')\}, \forall S \subseteq \mathcal{N} \setminus \{\emptyset\} \quad (3)$$

with $\mathcal{C} \subseteq \mathcal{N}$ being the set of players of $\mathcal{I}_n$ that have already participated to the game in a previous iteration, $p(j')$ being the number of resources allocated obtained by the users of $\mathcal{C}$ and $d(j)$ the number of tiles claimed by the complement of the coalition S. Indeed, a SBS or MUE can be part of several interference sets, and following the order of the interference sets described above (i.e the largest and most demanding sets first), one agent might have already played and received its payoff in a previous interference set.

C. NON-EMPTINESS OF THE CORE AND STABILITY OF THE GRAND COALITION

We assume that the grand coalition $\mathcal{N}$ will be formed, it is then necessary to explain why it is stable. Hence, as the core of a canonical game is directly related to the grand coalition's stability we need first to prove that the core is non-empty for the considered game in (3). It has been proven in [28] that convex games have non-empty core, hence ensuring the stability of the grand coalition.

A TU game is convex if and only if:

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \forall S, T \subseteq \mathcal{N} \quad (4)$$

Yet, as the characteristic function in (3) corresponds to the function of a bankruptcy game, the proof of the convexity of a bankruptcy games as the one considered here in (3) can be found in [29]. This convexity property also implies that the game is superadditive and supermodular [30], hence satisfying the following inequality: $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)\ \forall S \subset T \subset \mathcal{N} \setminus \{i\}$. This inequality implies that the marginal contribution of a player to a coalition is larger than its marginal contribution to another smaller coalition, hence ensuring the stability of the grand coalition.

For the $\mathcal{CS}$ game proposed in the subsection D, we consider the same characteristic function defined in (3). The union formed and the coalition structure incorporated into the game act only as an additional element which influences how the worth of the grand coalition is split among its members.

Once we assumed that the players of a game form the so-called grand coalition; the problem then is to agree on how to share the received profit $v(\mathcal{N})$ among the players in an interference set. The output of this decision is called the imputation of the game $x = (x_1, x_2, \ldots, x_n)$ which is a payoff vector where player i receives x_i.

In the next section, we discuss the possible imputation values satisfying efficiency ($\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N})$) that we can use for the proposed games. We will explain which imputation values are the most appropriate for these games if fairness and stability are the most desired properties of the payoff.

D. THE PROPOSED CANONICAL GAME

As depicted in Fig.3, upon each significant change in demands or network topology, the SBSs and the MUEs of the system determine their set of interferences following the method detailed in Section III. If the MUE fails to connect to a nearby SBS, it participates in a canonical game with the members of its interference set. Similarly, if a SBS is not hosting any MUE, it participates in a canonical game with the other agents of its coalition. Aforementioned, we assumed that the SBSs and MUEs of each interference set agree to form the grand coalition. We will develop here how the payoff of the grand coalition $v(\mathcal{N})$ will be split among these players.

1) IMPUTATION VALUE FOR THE CANONICAL GAME

Let us first define the most well-known solution, the Shapley value $Sh(\mathcal{N}, v)$ as [30]:

$$Sh_i(\mathcal{N}, v) = \sum_{S \subseteq \mathcal{N}: i \in S} \frac{(|\mathcal{N}| - |S|)!(|S|-1)!}{|\mathcal{N}|!} \Delta^i(v, S) \; \forall i \in \mathcal{N} \tag{5}$$

$\Delta^i(v, S)$ is the marginal contribution of a player i in coalition S defined as $\Delta^i(v, S) = v(S \cup i) - v(S)$.

The drawback of this solution is that it essentially considers the productivity of the players. In other words, the stronger a player is, the higher its payoff in the game. Yet, the strength of a player in the proposed game is determined by its demand, and therefore low-demand users might be affected by this imputation value. With both types of players in the game, the SBSs generally have more bargaining power in the game as they collect the demands of their several SUEs, while the MUE acts as a single player weighing only its own demand. In any case, it is important to avoid rewarding the players who have higher demands so that those who would claim more resources in order to gain a stronger position within the game are prevented from doing so.

a: SOLIDARITY VALUE

We want to apply a solution concept that incorporates some degree of solidarity so as to protect the MUEs from powerful SBSs and also to protect other weak SBSs within a given interference set. An appealing solution is the Solidarity value $Sl(\mathcal{N}, v)$ which takes into account the principles of productivity and redistribution, expressed as [31]:

$$Sl_i(\mathcal{N}, v) = \sum_{S \subseteq \mathcal{N}: i \in S} \frac{(|\mathcal{N}| - |S|)!(|S|-1)!}{|\mathcal{N}|!} \Delta^{av}(v, S) \; \forall i \in \mathcal{N} \tag{6}$$

with $\Delta^{av}(v, S) = \frac{1}{S} \sum_{i \in S} \Delta^i(v, S)$. Productivity is taken into account as the marginal contribution $\Delta^i(v, S)$ appears in the calculation. This value also shows some redistribution; not only is the player's marginal contribution considered, but so too is that of the players in a given coalition. In this way, the weak players of the game are protected.

Let $E[\delta_{\gamma_i}(\mathcal{N}, v)]$ be the average gain of player i. This value refers to the expected variation in the payoff of player i assuming that each player of $\mathcal{N}$ has the same opportunity to leave the game .

Axiom 1: An imputation value satisfies the equal average gains if $\forall$ $(\mathcal{N}, v)$*and* $\forall\{i, j\} \subseteq \mathcal{N}$, $E[\delta_{\gamma_i}(\mathcal{N}, v)] = E[\delta_{\gamma_j}(\mathcal{N}, v)]$.

It has been proven in Theorem 3 [32] that the solidarity value satisfies the equal average gains axiom. This provides two important assets for wireless communications systems. First, it incorporates a major sense of fairness into the game due to the equal average gain described above. Secondly, the agents of the network are not motivated by claiming more than what they really need to process their calls, thereby avoiding the cheating behaviour of some players who might ask for more resources in order to increase their bargaining power. However, this solution has an important limit, which is expressed in the remark below.

Remark 1: A player might receive more than their claim when the solidarity value is used as imputation, thereby violating the satiation axiom defined below.

Axiom 2: After applying an imputation value to a game $(\mathcal{N}, v)$*, no player of a bankruptcy game should receive more than their claim. Therefore, if* $\sum_{i \in \mathcal{N}} d_i \geqslant Z$; $x_i \leq d_i$, $\forall i \in \mathcal{N}$.

Note that the game under study, defined in (3), is a bankruptcy game, since the amount of resources available for each interference set is below the total demand of the members of the set. However, when the solidarity imputation value is applied in order to share the amount of available resources, some claimants may receive more than they claim, which violates the above satiation axiom. This is based on the fact that the solidarity value contributes to the expected average marginal contribution of a player. Here, productive players cede some parts of their marginal contributions to the weaker members, which reflects a sort of social sympathy and is a desired property of the solidarity value. Thus, the variance of the payoff distribution has been reduced. When a user with a low demand is part of an interference set containing several powerful users, the satiation axiom is difficult to satisfy as several of them will contribute to the marginal contribution of the weak members.

b: PROPOSED WEIGHTED SOLIDARITY VALUE

When the problem specified in Remark 1 is encountered, we need to apply a solution with the same degree of social empathy but that protects the system from attributing a profit exceeding the claim of the users. Indeed, in this case the solidarity value outperforms its initial role of protecting weak users and actually allocates a bigger payoff to them than it should. We propose a weighted solidarity value when the payoff obtained by the solidarity value is higher than a player's claim. The weights of the weighted solidarity value allow us to decrease the power of some users who receive an excessive amount of resources in order to limit the maximum of their profit to their demands. We propose to compute the appropriate weights in order to satisfy the satiation axiom for every player of the game. To do so, we need first to identify the users for whom the payoff obtained by the solidarity value is greater than the demand. If there is only one user of the interference set violating the satiation axiom, the weight applied to this user is the ratio of the demand by the payoff obtained from the solidarity value. This is explained by the fact that the non-weighted form of the Solidarity value simply uses a weight vector equal to one. Otherwise, if there are more than two users in a given interference set violating the satiation axiom, we need to compute the optimal weight vector to remedy this violation. To do so, we solve an equation in which the number of variables is equal to the number of users violating the satiation rule. Let $\mathcal{J} \subseteq \mathcal{N}$ be the partition of users for whom the payoff obtained by the solidarity value exceeds their demands, and let $d = (d_1, \ldots, d_n)$ be the vector of demands where d_i stands for the demand of user $i forall i \in \mathcal{N}$. Let $w = (w_1, \ldots, w_n)$ be the vector of weights where w_i is the weight of the player $i \in \mathcal{N}$ and $\sum_{i\in\mathcal{N}} \frac{w_i}{|\mathcal{N}|} = 1$.

We need to solve the linear system for all $i \in \mathcal{J}$ of the form:

$$w_i = \frac{\frac{d_i}{|\mathcal{J}|} * \left(|\mathcal{N}| + \sum_{k\in\mathcal{J}-\{i\}} w_k - |\mathcal{J}| \right)}{Sl_i(\mathcal{N}, v)} \quad \forall i \in \mathcal{J} \tag{7}$$

We can then update the vector of weights with the set of solutions obtained from the resolution of the above set of equations $(w_i)^*_{i\in\mathcal{J}}$. We express the final weight vector to be used for the weighted solidarity value as:

$$w_i = \begin{cases} 1 & \text{if } i \in \mathcal{N}\backslash\{\mathcal{J}\} \\ (w_i)^* & \text{otherwise} \end{cases} \tag{8}$$

Algorithm 1 summarizes the different steps to compute the proposed weighted solidarity value.

Axiom 3: The weighted solidarity value with the computed weights satisfies the satiation rule.

This is explained simply by the fact that the weight vector that has been applied does not allow any user to obtain more than its claim.

E. THE PROPOSED COALITION STRUCTURE GAME

We recall first the motivations behind the $\mathcal{CS}$ game. After the MUEs have identified their set of interferers, a harmed MUE leaves its interference set and joins that of a hosting SBS, if it can connect to it. Note that two types of SBSs have been identified: the hosting SBS which corresponds to the small-cell permitting access to one or more MUEs; and the non-hosting SBSs that do not have any connected MUE. In the latter case, the non-hosting SBS participates in a canonical game with its interference set members, as discussed in the previous section. On the other hand, when a SBS type is hosting, it participates with its interference set members in a cooperative resource allocation game in which the SBS and its hosted MUEs (one or several) form a priori union. The resulting coalition structure is incorporated into the game.

Algorithm 1 Calculate the Imputation $(x_i)_{i\in\mathcal{N}}$ of $(\mathcal{N}, v)$

Initialize $(w_i) = 1\ \forall i \in \mathcal{N}$ {The weight vector elements are all set to one} and initialize $\mathcal{J} = \{\emptyset\}$
$x_i \leftarrow Sl_i(\mathcal{N}, v)\ \forall i \in \mathcal{N}$ {We compute the imputation value with the Solidarity Value}
for all $i \in \mathcal{N}$ **do**
  if $x_i > d_i$ **then**
    $\mathcal{J} := \mathcal{J} \cup i$ {If the payoff obtained by the solidarity value exceeds the demand of user i, i joins the subset $\mathcal{J}$ of satiation axiom violating users}
  end if
end for
if $\mathcal{J} \neq \{\emptyset\}$ **then**
  Go to **procedure** {We need to compute the weighted solidarity value $\forall i \in \mathcal{N}$}
else
Output: x_i {The satiation axiom has not been violated by any user of $\mathcal{N}$}
end if
Procedure: $Sl_i^{w_i}$ {Procedure to compute the weighted solidarity value of users $i \in \mathcal{J}$}
Input: $(Sl_i(\mathcal{N}, v), d_i)\forall i \in \mathcal{J}$
Find the solutions of the set of linear equations (7)
$w_i = (w_i)^* \forall i \in \mathcal{J}$ and $w_i = 1\ \forall i \in \mathcal{N} \setminus \{\mathcal{J}\}$ {We update the vector of weights with the weights resulting from the resolution of the linear equations as in (8)}
$x_i^* \leftarrow Sl_i^{w_i}\ \forall i \in \mathcal{N}$ {We compute the imputation value with the weighted Solidarity Value}
Output: $x_i \leftarrow x_i^*\ \forall i \in \mathcal{N}$

Definition 2: If $P = \{P_1, P_2, \ldots, P_m\}$ is a partition of $\mathcal{I}_n$ that satisfies $\cup_{1\leq j\leq m} P_j = \mathcal{N}$ and $P_i \bigcap P_j = \emptyset$ if $i \neq j$ then P is a coalition structure over $\mathcal{N}$. The sets $P_j \in P$ are the unions of the coalition structure.

Let $P(\mathcal{N})$ be the set of all coalition structures over $\mathcal{N} \equiv I_n$. We will denote the game $(\mathcal{N}, v)$ with the coalition structure $P \in P(\mathcal{N})$ as $(P, \mathcal{N}, v)$ [32]. In the proposed game, a MUE always tries first to connect to the closest SBS of the formed interference set.

If connected, the hosting SBS F_1 and its n' connecting MUEs form a union $\{F_1, M_1, \ldots, M_{n'}\}$. The coalition structure obtained from the incorporation of this union can be expressed as: $P = \{F_1, M_1, \ldots, M_{n'}\}, \{F_2\}, \{F_3\}, \ldots \{F_n\}\}$, where $\{F_1, M_1, \ldots, M_{n'}\}$ is a partition of $\mathcal{I}_n^f$ formed by the related SBS-MUEs; every other player is a singleton.

Hence, we need to find an appropriate coalitional value to split the resources among and within the unions.

Specifically, we propose that in a coalitional structure game the hosting SBS and its interference set members first play a quotient game (i.e game among the unions) in which the union acts as a single player. Next, they play an internal game (a game within a union among the members of the union) to split what the SBS-MUE union has obtained. Let (M, v_P) be the quotient game induced by the CS game $(P, \mathcal{N}, v)$, considering the unions of P as players. Furthermore, let (P_k, v_k) be the internal game taking place among the players within each union. Earlier, we defined a value as a function that assigns to each game $(\mathcal{N}, v)$ a vector $(x)_{i\in\mathcal{N}}$ representing the amount that each player i in $\mathcal{N}$ expects to get in the game. Similarly here, a coalitional value is a function that assigns a vector of worth to each game with coalition structure $(P, \mathcal{N}, v)$. One of the most important coalitional values is the Owen value [27]. The Owen value applies the Shapley value at both levels, among the unions and within the unions.

1) THE WEIGHTED OWEN VALUE AS THE IMPUTATION VALUE OF THE $\mathcal{CS}$ GAME

We restrict our attention here to coalitional values satisfying the efficiency property; yet, the grand coalition is formed and the coalition structure described above is incorporated in the game, hence influencing the way the amount obtained by the grand coalition is shared among its members. Note that the agents of a $\mathcal{CS}$ game play first a quotient game (i.e, a game among the unions) where the union acts as a unit, followed by an internal game (a game within a union among the players of the union) to split what the union has obtained. We will separately define the values applied to compute the payoff of the player in the quotient game from those applied in the internal game. The coalitional value applied to this type of game is the weighted Owen value [33]. The weighted Owen value takes the size of each union into account. Indeed, the use of a symmetric imputation value would be unjustified as the players are groups of agents in the proposed model. The size of the unions depends on the number of MUEs hosted by the SBSs in the system. An obvious candidate for the quotient game is the weighted Shapley value by which users are weighted by the size of the unions they stand for. The inter-coalitions and intra-coalitions bargaining processes and the corresponding imputation values are now described.

a: THE QUOTIENT GAME

When a SBS and its connecting MUEs form a partition and compete with the other members of the harmed SBS interference set as a unit, a situation in which coalitions have different sizes develops. It seems reasonable to assign a size-aware weight to each coalition. Lets define the reduced game (M, v_P) corresponding to the quotient game induced by the CS game $(P, \mathcal{N}, v)$, considering the unions of P as players. Here, $M = (1, 2, \ldots, m)$, with m representing the number of unions in the game and $v_P(K) := v(\cup_{i\in K} P_i)$ for all $K \subseteq M$. In the quotient game, the profits are divided among unions following the weighted Shapley value. The weighted Owen value computes the weights from the given coalition structure, the weights being proportional to the size of the coalition. Hence $w_u = \frac{|P_u|}{|\mathcal{N}|}$ $\forall P_u \in P$, having $\sum_{u\in M} w_u = 1$. The unanimity games will allow us to define the weighted Shapley value, which describes how a coalition splits one unit between its members: for all $K \subseteq \mathcal{N}$, the unanimity game of the coalition K, $(\mathcal{N}, u_K))$, is defined by:

$$u_K(S) = \begin{cases} 1 & \text{if } S \supseteq K \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Recall that the unanimity game is only used to help us define the used values.

We can then define the weighted Shapley value for each unanimity game $(\mathcal{N}, u_K)$ as [33]:

$$Sh_i^w(\mathcal{N}, u_K) = \begin{cases} \frac{w_i}{\sum_{j\in K} w_j} & \text{for } i \in K \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Where $w = (w_i)_{i\in\mathcal{N}}$ is a vector of positive weights. The coalition splits the payoff among its members proportionally according to their weight.

As the SBS and MUE form a union and the other players are singletons in an interference set, the union has more weight in the game, and therefore gets a larger profit than if the MUE and SBS were acting as singletons. Hence, we protect the MUE and we reward the collaborating SBS by allowing the harmed public users to connect.

Remark 2: As the weighted Shapley value satisfies efficiency in the internal game (P,v), it follows that the weighted Owen value satisfies the quotient game property. We can then state that the profit of each coalition corresponds to its weighted Shapley value in the game among coalitions, with weights given by its size. Then

$$\forall P_u \in P, \sum_{i\in P_u} \varphi_i^{\mathcal{N}} = Sh_i^{wM}(v\backslash P) \tag{11}$$

Proposition 1: When the estate is shared with the weighted Owen value, a player of both types (e.g., MUE or SBS) gets a better profit by joining forces than they do bargaining themselves.

Proof: Let P_u and P_v represent two coalitions belonging to P, and let $P^{u+v} = (P\backslash\{P_u, P_v\}) \cup \{P_u \cup P_v\}$. We will say that a coalitional value $\mho$ is joint monotonic if [26]:

$$\sum_{i\in P_u \bigcup P_v} \mho_i^N(P) \leqslant \sum_{i\in P_u \bigcup P_v} \mho_i^N(P^{u+v}) \tag{12}$$

This means that if P_u and P_v join forces, they win a better profit than they would acting as singletons. It has been proven in [26] that the weighted Owen value is joint-monotonic in convex games (Proposition 3-1). Our game under study in (3) corresponds to a bankruptcy game, it has been proven it is convex in [29]. The proof of the joint monotonicity of the weighted Owen value in convex games concludes the proof of Proposition 1. □

Algorithm 2 Proposed Algorithm for Cooperative Downlink Cross-Tier Interferences Mitigation and Resource Management

Initial State: Deployment of the SBSs and MUEs in the system and each agent express its demand in term of number of tiles.

Phase I:Interference set detection

a) Based on the minimum required SINR the SBSs and MUEs of the system determine their set of interferences I_n^f and I_n^m $\forall f \in F$ and $\forall m \in K$. If the SINR received by a harmed MUE from a nearby SBS is higher than the one received by the MBS, the MUE tries to connect to the corresponding SBS.

b) The interference sets are sorted according first to cardinality, then to the overall demand in a descending order.

Phase II: The game iteration

repeat For each interference set I_n^m following the settled order

if the harmed MUE succeeded in connecting to a nearby SBS **then**

a) The MUE leaves its respective interference set and joins the one of its hosting SBS.

b) The agents of the interference set who have not participated to a game in a precedent iteration form a coalition structure $\mathcal{N}$ with the formed a priori union and all the other SBSs as singletons

c) A $\mathcal{CS}$ game (P, n, v) is played (i.e a quotient game and an internal game)

d) Every player $i \in \mathcal{N}$ receives the payoff x_i from the Weighted Owen value

else

a) The MUEs and their interfering SBS who have not played to a game in a precedent iteration form the grand coalition and participate in a canonical game

b) The players receive their payoff from the Solidarity value or the weighted Solidarity value

end if

until all players of the system have played

Phase III: Resource allocation

The MUEs and SUEs receive from their serving base station (MBS or SBS) the resources obtained from the game

b: THE INTERNAL GAME

Regarding the internal game, the weighted Owen value attributes the final payoff to the users of each union by splitting the worth gained by the quotient game with the Shapley value. The solution value for the internal game (P_k, v_k) is the Shapley value with $v_k(S) = Sh_k^w(M, v_{P|S})$. That is, $\varphi_i(P, \mathcal{N}, v) = Sh_i(P, v_k)$. We remind here that the internal game will take place among the hosting SBS and its hosted MUEs. We note that we use the Shapley value in the internal game to avoid penalizing the hosting SBS. Indeed, if a solidarity-based value is applied in the internal game, the unique SBS if its demand is high, will have to participate to the marginal contribution of the several MUEs in its union. This will result in lower SBS performances. On the other hand, as a Weighted Owen value has been applied in the quotient game, a substantial amount of resources have been obtained by the union, and as there is only one SBS per union, the MUEs will still have a significant payoff. As it is important to reward the hosting SBSs while attributing a reasonable amount of resource to the hosted MUES in each union, the Shapley value is the most appropriate in this case.

TABLE 1. Numerical values.

Macrocell dimensions	$400m * 400m$
Small-cell coverage area	$40m$
SINR threshold for MUEs (δ_m) (SUEs (δ_f))	$15dB(20dB)$
Thermal noise density	$-174dBm/Hz$
Carrier frequency	$2GHz$
Maximum transmitted power at the MBS (SBS)	$40W(40mW)$
Number of MUEs per macrocell	50
Bandwidth reservation of prioritized SBSs	$\delta = 80\%$
Height of the MBS (SBS)	$25m(10m)$
Number of tiles allocated per cell (both tiers)	100 tiles
Demand in term of tiles per user (SUE or MUE)	$1 - 25$ tiles
Number of users per small-cell	4
Number of users per macrocell	100

V. SIMULATION RESULTS AND ANALYSIS

In this section, we present the simulation results of our proposed cooperative game approach. We assume that a macrocell is overlaid by 200 small-cells in a spectrum-sharing network. We simulated several scenarios of varying user demand and location within the network. The simulation parameters are summarized in Table 1. Based on the SINR, the agents determine their interfering sets. We consider the pathloss model of Winner II calculated in dB as: $G_{i,j}^k(d) = 44.9 - 6.55\ log_{10}(h_{BS})\ log_{10}(d) + 34.46 + 5.83\ log_{10}(h_{BS}) + 23\ log_{10}(f_c/5) + n_{ij}W_{i,j}$, d being the distance between a user i and a base station of either type j, h_{BS} the height of the base station, f_c the carrier frequency. Also, n_{ij} denotes the number of walls and $W_{i,j} = 5dB$ denotes the wall loss. Note that for communications from a SBS to an indoor SUE attached to an other SBS, $n_{ij} = 2$, for all other cases $n_{ij} = 1$. The SBSs collect the demands of their users and the MUEs of the system express their demands in terms of number of tiles. We assume that the users of the system have equal priority; the proposed framework can be easily adapted to different degrees of priority in a future study. We ran 500 simulations, allowing us to reach a confidence level close to 100%.

A. COMPUTATIONAL COMPLEXITY

First, we will discuss the computational complexity of the proposed framework. The complexity of the different coalitional values can be compared for the canonical game. Recalling that the Shapley value is obtained with $O(2^n)$ operations with n the number of players in an interference set (maximum 12 in our simulations) [34]. For the $\mathcal{CS}$ game, the computational complexity of the Owen and weighted Owen value are similar. These values are the average of all marginal contributions of i in all orderings of the players that preserve the grouping of the players into unions. Hence, they need $O(A * 2^k + 2^A)$ operations to be computed, with A the number of unions and k the maximum number of agents in a union. In Equation 6, we can see that the computational complexity of the solidarity value is similar to the Shapley value, which is $O(2^n)$. However the computation time of the Solidarity value is slightly higher but the difference is negligible. This is explained in Equation 6, as to compute the Solidarity value we do not only need to compute the marginal contribution of a player but also the marginal contribution of the other players in a given coalition. In terms of average computation time, the solidarity values needs 0.23 seconds to be computed in the symmetric version and 0.27 seconds for the weighted version. For the coalition structure values, 0.028 seconds are needed for the Owen value and 0.032 seconds for the Weighted Owen value. These lower computational time of the $\mathcal{CS}$ game are justified by the partnership in this type of games that allows us to treat a union of partners as an individual, hence reducing the size of the game. At the same time, the size of the game is essential to the computational complexity in game theory which permits a lower computational time. The computational complexity of the solidarity value is marginally higher, but still polynomial. The empirical tests performed on the executing times of each method show that the variances are very small (executing times are nearly similar). This is an expected result as we are calculating polynomial formulas in each case studied here. However, the computational complexity of the centralized approach is much higher and has an average computation time of dozens of seconds. Note that the centralized approach simulated in this paper refers to the Centralized-Dynamic Frequency Planning [35].

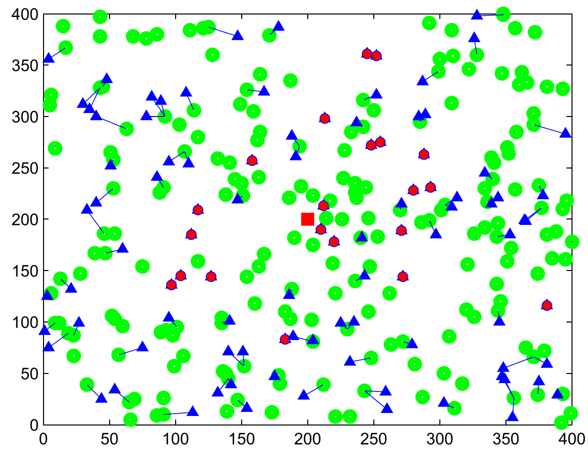

FIGURE 4. **A snapshot of a dense small-cell networks. The SBSs are modelled by a Poisson process represented by green points. The center red square represents the MBS, the blue triangles represent the MUES: those with a red point in the centre are served by the MBS; those with a blue point are served by the SBS offering the best SINR. The blue lines represent the link between a SBS and its hosted MUE.**

B. COMPARISON WITH OTHER SCHEMES OF THE ART

In Figure 4, we presented a snapshot of a dense heterogeneous network in which 200 small-cells overlay a macrocell with 100 MUEs. This shows that the MUEs located in the center area of the cell are better served by the MBS as the required SINR is reached. However, in the cell edge most of the MUEs fail to reach their required SINR when connected to the MBS, and therefore need to be served by the SBS offering the best signal. We notice from the density of the network that in both cases cooperative spectrum access is needed as the users from both tiers compete for the same resources under the co-channel deployment. In Figure 5, we compare the F-ALOHA [36] and the centralized approach [35] to the proposed game theory model. Our proposed model shows better performances in all cases for dense (100 SBSs) and very dense (200 SBSs) network configurations. In high-density cases, with our model we have 50% of users obtain a throughput of more than 90% compared to only 30% with the centralized approach.

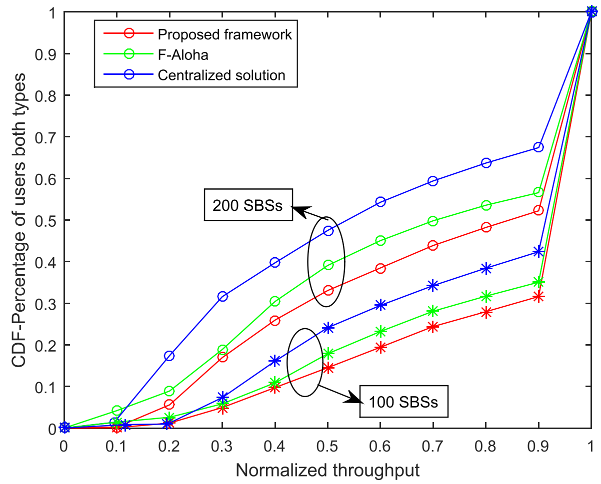

FIGURE 5. **Throughput Cumulative Distribution Function for users of both types: Comparison of the centralized approach, the F-Aloha method and the proposed framework.**

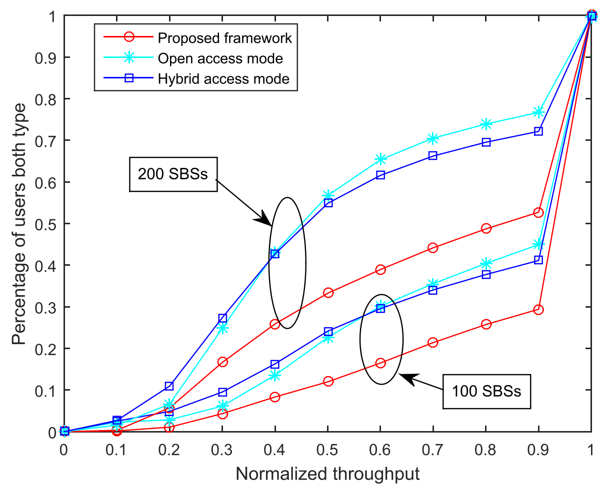

FIGURE 6. **Throughput Cumulative Distribution Function for users of both types: Comparison of the proposed framework with the open-access mode and the traditional hybrid-access mode $\delta = 0.8$.**

C. COMPARISON WITH OTHER TYPES OF ACCESS

In Figure 6, we compare our model to a traditional open-access mode and hybrid prioritized access. In the former case, the SBSs of the system collect the demands of the connected users of both types (CSG users and public users) and participate to a resource allocation game with the neighbouring SBSs. The MUEs that are served by a SBS do not participate in the canonical game. The SBSs redistributes the obtained payoff proportionally among their users of both types. Under the hybrid-access model, the SBSs similarly collect the demand of both type of users but prioritize their CSG users during the redistribution stage; in fact, 80% of resources are reserved for the CSG users. We can observe that the median throughput is always higher for our framework and specially for the high density case as it is equal to 0.9, meaning that 50% of the users have a throughput of 0.9 or more while the median throughput is only equal to 0.45 for the other access modes. Our framework outperforms these two access modes for the following reason: it incorporates the coalition structure, and so the number of users in the resulting union impacts the payoff distribution. This proves that it is not only sufficient to have more bargaining power (a larger demand), but also numerical superiority in order to reap a better reward, which our model achieves. Moreover, the hybrid access heavily penalizes the MUEs in the system, as the hybrid SBSs will take advantage of the connection of the MUEs to obtain more bargaining power in the game and then unfairly redistribute the resources.

D. COMPARISON WITH OTHER COOPERATIVE GAME SOLUTIONS

We want to show the superiority of our two-level models. Therefore, it is necessary to compare them with several other game-theoretic models. First, we compare our framework to a coalitional game without a coalition structure, which means the unions are not taken into consideration and the payoff is distributed as if every player of a given interference set is a singleton. We divide the estate using the most common value, the Shapley value. The second model of comparison used the proposed coalitional structure game model while applying the Owen value. Therefore, the size of the unions are not taken into account and have the same weight equal to that of the single players in the interference set. It is essential to compare our model to these two approaches, as it shows that the incorporated formation of the union is necessary, as is the application of the imputation value. In Figure 7, we present these performances for both types of users in the system. We can observe our framework outperforms the two other approaches: the median throughput is always higher for our framework and specially for the high density case as it is equal to 0.9, meaning that 50% of the users have a throughput of 0.9 or more while the median throughput is only equal to 0.5 for the other cooperative game models. In Figures 8 and 9, we analyze separately the performances of the MUEs and the SUEs in the system to show that we have not penalized one type of player for the benefit of another. Clearly, all the MUEs and the SUEs have consistently achieved better performances in the system. In Figure 8, we can observe that our model allows 88% of the MUEs with a throughput higher than 0.9 compared to 58% with the centralized approach 62% with the Shapley value and 65% with the Owen value. In Figure 9, we can observe that the median throughput is always higher for our framework :in the high density case the median throughput is equal to 0.55 while it is only equal

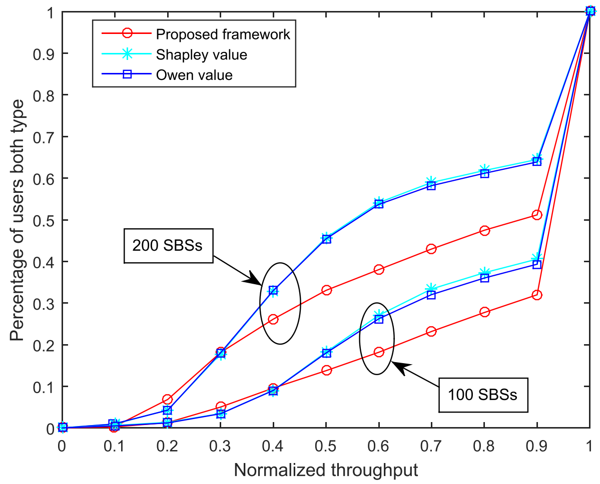

FIGURE 7. Throughput Cumulative Distribution Function for users of both types: Comparison of the proposed framework with other coalitional games.

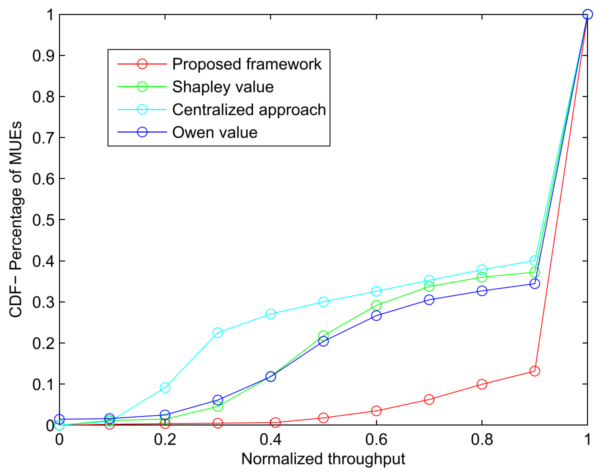

FIGURE 8. Throughput Cumulative Distribution Function for all MUEs of the system: Comparison of the centralized approach,the shapley value, the owen value and the proposed framework.

to 0.45 for the other models, in the low density case the median throughput is equal to 0.85 for our model and approximatively 0.75 for the three other frameworks. We also want to demonstrate that the users participating in a cooperative game that are part of a union have an advantage bargaining as a union than they do acting as singletons. Therefore, in Figures 10 and 11, we isolate the users within unions and analyze their performance in contrast to that which would be achieved if the same users participated in the game independently. It is essential to show again that the proposed framework allows us to protect the harmed MUEs while rewarding the hosting SBSs who participate actively in the interference mitigation. As depicted in these figures, both types of players show better performances at all levels, although SUEs show a slightly lower performances in high throughput. This is explained by the fact that the other models strongly penalize

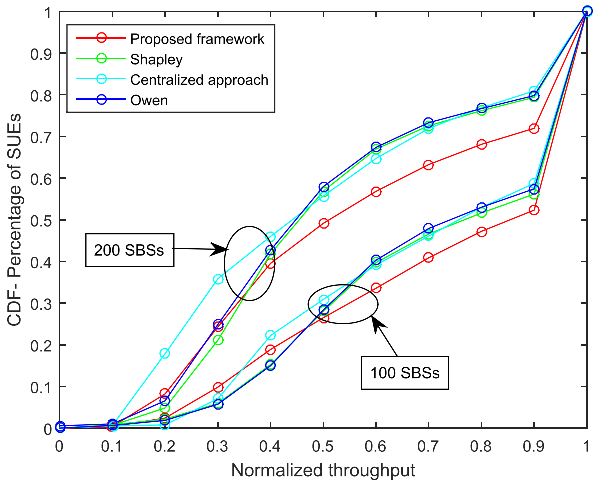

FIGURE 9. Throughput Cumulative Distribution Function for all SUEs of the system: Comparison of the centralized approach,the shapley value, the owen value and the proposed framework.

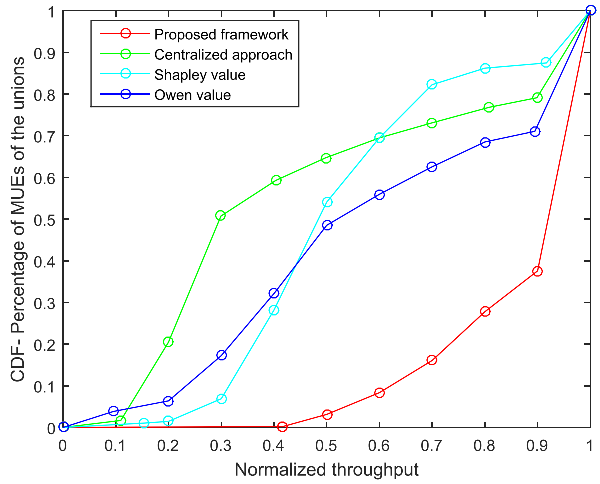

FIGURE 11. Throughput Cumulative Distribution Function for MUEs inside a union only: Comparison of the centralized approach, the Shapley value, the Owen value and the proposed framework.

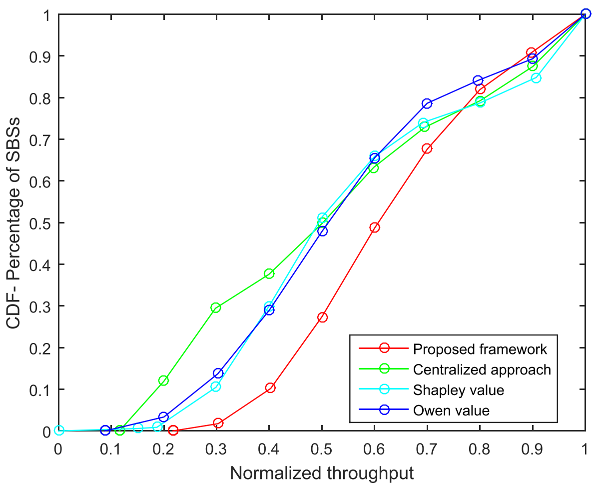

FIGURE 10. Throughput Cumulative Distribution Function for SBSs inside a union only: Comparison of the centralized approach, the Shapley value, the Owen value and the proposed framework.

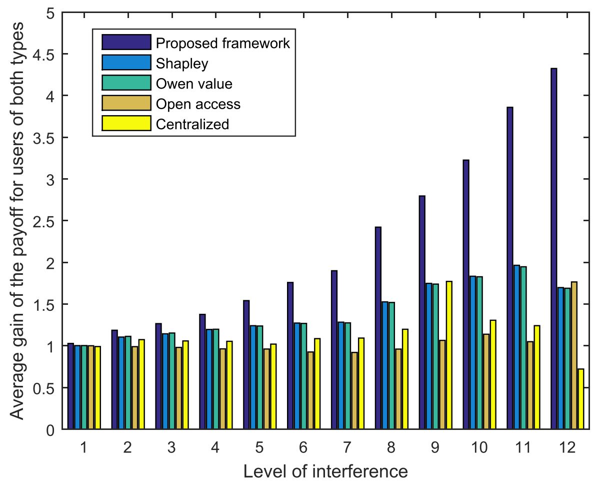

FIGURE 12. Average gain of the payoff as a function of the interference degree with the hybrid access as basis of comparison ($\delta = 0.8$).

the MUEs in the higher throughput as they have more bargaining power, and can therefore allow SBSs to negotiate a larger profit.

E. IMPACT OF INTERFERENCE DEGREE AND USER DEMANDS

Here, we assess how the allocated resources are affected by demand volume and the interference degree of the network. Figure 12 investigates the impact of the interference degree on the performances of the proposed model. The interference degree corresponds to the cardinality of the interference set. In this case, we are evaluating the gain of payoff using the traditiona hybrid access model as a basis. We can see that the proposed model consistently outperforms the other frameworks, reaching up to 300% of gain in very high interference levels. This improvement is justified by the use of the unions because in very dense interference sets the cardinality of the unions is taken into account and the user can obtain an adequate reward. This is added to the solidarity value applied in the canonical games, which protects the weak players that generally suffer from unfair payoff distribution in high interference degrees in the other models.

Figure 13 shows the performances in terms of user demands. Clearly, our model performs better at all levels of demand, although the gain is less important at high user demands. Note that the centralized approach achieves slightly better performances in the highest level of demand. This can be interpreted as an opportunity, since from a network-management standpoint, the users should be discouraged from requesting high demands. In Figure 14, we summarize the two types of games that can occur under the proposed cooperative model. We compute the number of games that take place at each iteration. Note that an iteration

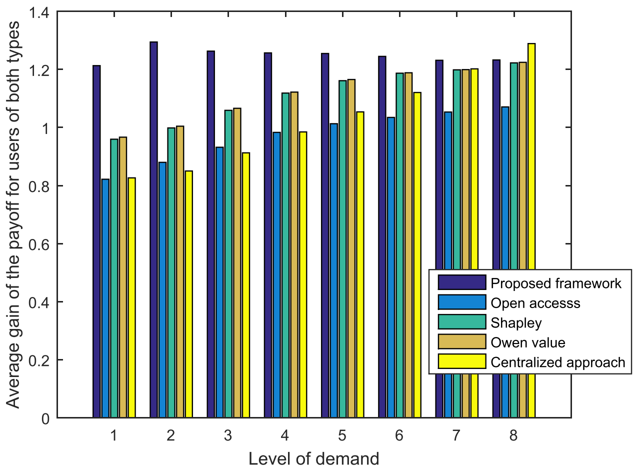

FIGURE 13. Average gain of the payoff as a function of the level of demand with the hybrid access as basis of comparison ($\delta = 0.8$).

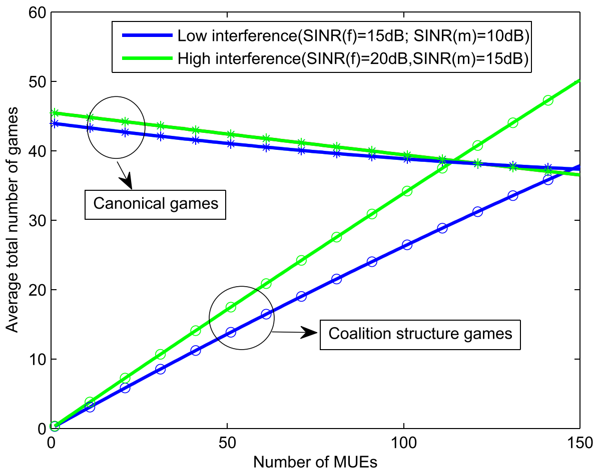

FIGURE 14. Number of canonical games and $\mathcal{CS}$ games played at each iteration of the system for two levels of interferences.

occurs whenever the topology of the network or the demand expressed by the users change significantly, and results on the participation in the resource allocation game of every player in the network. We have run the simulations for two levels of interference (required SINR) and noticed that the number of canonical games is not affected by the level of interference. In fact, as the deployment of small-cells is very dense, the number of canonical games that occur is slightly affected by the interference level. We notice that this number decreases with the growth of MUEs population. Naturally, as the number of MUEs in the system increases, they are more likely to connect to nearby SBSs and to participate in $\mathcal{CS}$ games instead.

F. PERFORMANCE ANALYSIS OF THE PROPOSED WEIGHTED SOLIDARITY VALUE

In Figure 15, we present the results of the proposed weighted solidarity value. Recall that this value has been proposed in

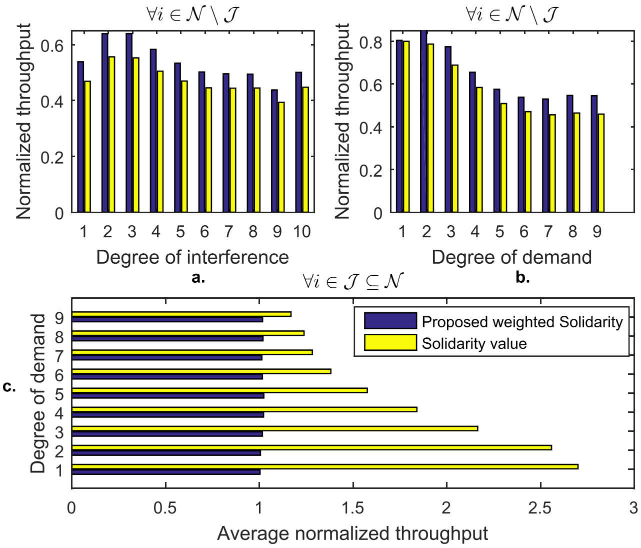

FIGURE 15. Throughput Cumulative Distribution Function for the users of a system participating to a canonical game when the satiation axiom is violated: Comparison of the proposed weighted solidarity value (with the computed weights in algorithm 1) and the Solidarity value.

order to alleviate the problem of satiation violation of the solidarity value. For the sake of analysis, we isolated for each interference set $\mathcal{I}_n$ the users belonging to the subset $\mathcal{J}$ (players obtaining more than what they claimed with the solidarity value) from the other players of the same interference set. We isolate the games in which the satiation rule has been violated and compare the performance of our model with the traditional solidarity value. The expected result is that no player obtains more that it claims, accordingly the normalized throughput should not exceed 1. In Figure 15-c, we notice that the satiation violation for the solidarity value as the average normalized throughput reaches up to 270 %. This naturally affects the others players of the game because this excess of resources is not redistributed among players. We also notice from this figure, that in our proposed model all the users with excess of payoff in the solidarity value have now obtained the maximum of their demands and the excess is redistributed to the other members of the system. This is further shown in Figures 15-a and 15-b, where the players belonging to $\mathcal{N} \setminus \mathcal{J}$ benefit from this scheme since they reach a higher throughput on the different degrees of interference and demand.

G. PERFORMANCE EVALUATION IN TERMS OF FAIRNESS

Finally, in Table 2, we present the results of the fairness evaluation for each scheme presented in the previous results.

TABLE 2. Mean fairness index.

Proposed model	Shapley value	Owen value	Open access	Hybrid prioritized access
0.8256	0.7973	0.7967	0.7680	0.7591

The Jain's fairness index is defined as [37]:

$$Fairness = (\sum_{i=1}^{N}(x_i/(d_i))^2/(N\sum_{i=1}^{N}(x_i/(d_i)^2) \quad (13)$$

where x_i indicates the allocated resources to user i. We can notice that our proposed model gives the highest fairness, thanks to the combination of two game-theoretic approaches adapted to each situation. First, the proposed framework gives the right to play to every user of the network such that no user is penalized by a representing entity in the game (i.e a prioritized SBS). It also allows to protect the weak players according to the equal average gains in the proposed Weighted Solidarity value. Finally, the priority given to unions ensures that bigger profits are allocated to groups composed of multiple users instead of being monopolized by singletons, hence achieving a higher fairness among users.

VI. CONCLUSION

In this paper, we have proposed a novel framework of cross-tier cooperation among SBSs and MUEs that offers a significant improvement in performance for users from both tiers. This framework also provides more fairness to the game through an adaptive and solider game theoretic model. It allows the public users to connect to nearby SBSs and thereby offload the traffic of the macrocells, while rewarding this desired cross-tier collaboration. Weak players in the system whose demand is lesser are protected under this model. It also permits every single public user to participate in the resource allocation game and not being penalized when SBSs are prioritized. Compared to several alternative solutions and access modes, we showed that our proposed approach achieves better performance in terms of throughput and fairness for both types of users (MUEs and SUEs). Future work will extend the proposed model to a QoS and mobility-aware framework. The model could also be enhanced by adding an admission control policy allowing to block users who fail to obtain the minimum requirements in the proposed framework, and redistribute the retrieved resources among the accepted users. Different levels of power transmission, as well as various traffic classes and priorities could also be investigated.

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