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Abstract: We study Heterotic supergravity on Hyper-Kähler manifolds in the presence of non-trivial warping and three form flux with Abelian bundles in the large charge limit. We find exact, regular solutions for multi-centered Gibbons-Hawking spaces and Atiyah-Hitchin manifolds. In the case of Atiyah-Hitchin, regularity requires that the circle at infinity is of the same order as the instanton number, which is taken to be large. Alternatively there may be a non-trivial density of smeared five branes at the bolt.

Keywords: Superstring Vacua, Superstrings and Heterotic Strings, Flux compactifications, Black Holes in String Theory

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1 Introduction

Heterotic flux backgrounds are interesting models of string backgrounds. There is no in principle impediment from using the R-NS string worldsheet formalism although compact models with minimal or no supersymmetry remain challenging to construct.

In this work we study local Heterotic flux backgrounds on Hyper-Kähler four manifolds: in particular the Gibbons-Hawking spaces [1] and the Atiyah-Hitchin manifold [2]. Key to our configurations is that the gauge fields are Abelian and we take the large charge limit such that $\mathrm{Tr}F \wedge F$ dominates $\mathrm{Tr}R \wedge R$ in the Bianchi identity. This large-charge limit has been previously studied on the Eguchi-Hanson space [3] and the conifold [4, 5] and this type of limit is familiar from the large-charge supergravity limit crucial to the development of holography [6] in type II and M-theory.

Our strategy is to first compute explicit solutions to the Hermitian-Yang-Mills equations on Hyper-Kähler spaces and then backreact them on the geometry. This backreaction affects only the conformal mode of the metric but generates a non-trivial three-form flux.

According to Aspinwall [7] we are studying *Goldilocks* theories with just the right amount of supersymmetry; perhaps then not surprisingly we solve the supergravity background exactly. For the Atiyah-Hitchin background it is nonetheless somewhat impressive that we can both solve Hermitian-Yang-Mills exactly and integrate analytically the resulting Poisson equation for the backreaction of this instanton.

The instanton we use on the Atiyah-Hitchin manifold is well known from a classic duality paper by Ashoke Sen [8]. Perhaps the central result of our work is that the Heterotic backreaction of this instanton can be made regular. This is somewhat non-trivial since the negative mass of Atiyah-Hitchin induces a negative warp factor thus violating the desired signature of space-time. We circumvent this in two ways: first by allowing the asymptotic circle to be large and secondly by including smeared five-brane sources.

We also study the presence of electric H-flux and fundamental strings.The electric flux modifies the BPS equations in a straightforward way and for each solution with magnetic H-flux, the electric flux can be added through a harmonic function on the Hyper-Kähler manifold. We analyze limits in which we recover AdS_3 geometries but these reduce to the known $\mathrm{AdS}_3 \times S^3 \times HK_4$.

Upon completing this work we were made aware that our BPS equations have turned up in five dimensional supergravity. The local equations we study can essentially be found in [9, 10] but with very different global and regularity requirements. This is not surprising since we can dimensionally reduce our solutions on $\mathbb{R}^5$ to get solutions of ungauged five dimensional supergravity. In addition, these equations also turn up in type II supergravity for T^2 fibrations over Hyper-Kähler spaces and the type II analogue of the Gibbons-Hawking solutions we find have been analyzed in [11]. It is straightforward to convert our solutions on the Atiyah-Hitchin manifold to such type II backgrounds.

2 Hyper-Kähler heterotic backgrounds

The primary backgrounds we consider are of the form $\mathbb{R}^{1,5} \times HK_4$ where HK_4 is a warped Hyper-Kähler-four manifold. We consider a non-trivial three-form flux $\mathcal{H}_{(3)}$, dilaton Φ and Heterotic gauge field F. The background metric ansatz is:

$$\mathrm{d}s^2_{10} = \mathrm{d}s^2_{1,5} + H\,\mathrm{d}s^2_4 \tag{2.1}$$

where $\mathrm{d}s^2_4$ is an Hyper-Kähler metric on a four-manifold HK_4 and H a conformal factor. The BPS equations are fairly standard:

$$e^{2\phi} = H \tag{2.2a}$$
$$\mathcal{H}_{(3)} = - *_4 \mathrm{d}H \tag{2.2b}$$
$$J_a \llcorner F = 0, \qquad a = 1,2,3 \tag{2.2c}$$

where the $*_4$ is the Hodge dual w.r.t. the Hyper-Kähler metric on HK_4 and J_a are the three Kähler forms.

A major difficulty in finding explicit solutions of Heterotic supergravity with non-trivial three-form flux is to satisfy the Bianchi identity at the appropriate order in α'. Following

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earlier works by some of the authors [3–5], our strategy will be to work in a large (fivebrane) charge limit, ensuring that the contribution of the Tr$R \wedge R$ term is subdominant and can be consistently neglected. The Heterotic Bianchi identity simplifies to

$$\mathrm{d}\mathcal{H}_{(3)} = \alpha' \mathrm{Tr}\, F \wedge F \tag{2.3}$$

implying from the three-form ansatz (2.2b) that:

$$\mathrm{d} *_4 \mathrm{d}H = -\alpha' \mathrm{Tr}\, F \wedge F\,. \tag{2.4}$$

2.1 Principal torus bundles and type IIA/IIB solutions

We consider a more general class of backgrounds, that can be viewed as local models of the principal torus bundles over wrapped K3 surfaces introduced in [12] and discussed in many works including [13–16]. They generalize the solutions based on Eguchi-Hanson space presented in [17]. The general ansatz for such principal two-torus bundle $T^2 \hookrightarrow M_6 \xrightarrow{\pi} HK_4$ over a Hyper-Kähler four-manifold is of the form

$$\mathrm{d}s^2_{10} = \mathrm{d}s^2_{1,3} + H\,\mathrm{d}s^2_4 + \frac{U_2}{T_2}|dx + Tdy + \pi^\star \alpha|^2\,, \tag{2.5}$$

where α is a connection one-form on HK_4 such that $\vartheta = dx + Tdy + \pi^\star\alpha$ is a globally defined one-form on M_6 with

$$\frac{1}{2\pi}\mathrm{d}\vartheta = \pi^\star \varpi\,, \quad \varpi = \varpi_1 + T\varpi_2\,, \quad \varpi_i \in H^2(HK_4, \mathbb{Z})\,, \tag{2.6}$$

and by supersymmetry

$$J^a \wedge \varpi = 0\,, \quad a = 1,2,3\,. \tag{2.7}$$

The expression for the three-form becomes then

$$\mathcal{H}_{(3)} \;=\; - *_4 \mathrm{d}H - \frac{\alpha' U_2}{T_2}\mathrm{Re}\,\left(*_4 \mathrm{d}\vartheta \wedge \bar{\vartheta}\right)\,. \tag{2.8}$$

By an appropriate choice of $\varpi \in H^2(HK_4, \mathbb{Z})$ one can find solutions with $\mathrm{d}\mathcal{H}_{(3)} = 0$, which can also be obtained as supersymmetric solutions of type IIA or type IIB supergravity with NS-NS fluxes, as was discussed in [3] and [18].

3 Gibbons-Hawking: ALE and ALF

We can solve explicitly (2.2) and (2.4) for the multicentered Gibbons-Hawking ALE and ALF spaces, that we denote collectively by M_{GH}. The corresponding Hyper-Kähler metrics are given by:[1]

$$\mathrm{d}s^2_4 = V(x)^{-1}(\mathrm{d}\tau + \omega)^2 + V(x)\mathrm{d}x \cdot \mathrm{d}x\,, \tag{3.1a}$$

$$\mathrm{d}V = *_3 \mathrm{d}\omega\,, \tag{3.1b}$$

$$V = \epsilon + 2m \sum_{i=1}^{k} \frac{1}{|\mathbf{x} - \mathbf{x}_i|}\,, \tag{3.1c}$$

[1] As usual $*_3$ is the Hodge dual on $\mathbb{R}^3$.

where $\epsilon = 0$ gives the ALE (multi Eguchi-Hanson) series and $\epsilon = 1$ the ALF (multi Taub-NUT) series. The periodicity of τ is determined by expanding around a pole of $V(\mathbf{x})$ to be:

$$\tau \sim \tau + 8\pi m\,, \tag{3.2}$$

and the triplet of Kähler forms is given by

$$J_a = (\mathrm{d}\tau + \omega) \wedge \mathrm{d}x^a - V *_3 \mathrm{d}x^a \quad , \; a = 1, 2, 3\,. \tag{3.3}$$

We will consider heterotic supergravity solutions for warped ALE or ALF spaces supported by Abelian gauge bundles. To explicitly write the gauge fields we denote

$$V_i = \frac{2m}{|\mathbf{x} - \mathbf{x}_i|}\,, \qquad \mathrm{d}\omega_i = *_3 \mathrm{d}V_i\,; \tag{3.4}$$

Then a representative of the topologically non-trivial gauge fields is locally given by

$$A_i = \omega_i - \frac{V_i}{V}(\mathrm{d}\tau + \omega)\,. \tag{3.5}$$

We note that

$$\sum_{i=1}^{k} A_i = \epsilon \frac{\mathrm{d}\tau + \omega}{V} - \mathrm{d}\tau \tag{3.6}$$

is topologically trivial since $V^{-1}(\mathrm{d}\tau + \omega)$ is globally defined. Thus there are $(k-1)$ non-trivial gauge fields, in agreement with the $(k-1)$ non-trivial two-cycles. The corresponding field strengths are

$$F_i = \mathrm{d}A_i = V *_3 \mathrm{d}\left[\frac{V_i}{V}\right] - \mathrm{d}\left[\frac{V_i}{V}\right] \wedge (\mathrm{d}\tau + \omega)\,. \tag{3.7}$$

It is straightforward to see that F_i are anti-self dual and solve Hermitian Yang-Mills[2]

$$*_4 F_i = -F_i\,, \qquad J_a \wedge F_j = 0\,. \tag{3.8}$$

For the Bianchi identity (2.4) we compute

$$F_i \wedge F_j = -2V *_3 d\left[\frac{V_i}{V}\right] \wedge d\left[\frac{V_j}{V}\right] \wedge (d\tau + \omega) \tag{3.9}$$

$$d *_4 d\left[\frac{V_i V_j}{V}\right] = -F_i \wedge F_j \tag{3.10}$$

so that if we take

$$F = \frac{1}{4m}\sum_{i=1}^{k} dA_i\, \mathbf{q}_i \cdot \mathcal{T}\,, \tag{3.11}$$

[2]For a one form α on $\mathbb{R}^3$ we have $*_4\alpha = -(d\tau + \omega) \wedge *_3\alpha$. In particular this means that a function which is invariant under the U(1) generated by ∂_τ is harmonic on the Gibbons-Hawking space iff it is harmonic on $\mathbb{R}^3$.

where $\mathcal{T} \in \mathrm{U}(1)^{16}$ is in the Cartan subalgebra of $E_8 \times E_8$ or SO(32) and $\mathbf{q}_i$ the corresponding charge vectors, and solve (2.4), we get the general solution:[3]

$$H = \delta + h(\mathbf{x}) + \frac{\alpha'}{8m^2 V} \sum_{i,j=1}^{k} V_i V_j \mathbf{q}_i \cdot \mathbf{q}_j \,. \tag{3.12}$$

Here $\delta = 0, 1$ is an integration constant (not to be confused with ϵ a similar integration constant in the Gibbons-Hawking warp factor V) and $h(\mathbf{x})$ is any harmonic function on M_{GH}. Taking $h(\mathbf{x})$ to be invariant under ∂_τ we have

$$h(\mathbf{x}) = \frac{1}{m} \sum_\alpha \frac{q_\alpha}{|\mathbf{x} - \mathbf{x}_\alpha|} \tag{3.13}$$

corresponding to mobile neutral five-brane sources inserted at $\mathbf{x}_\alpha$.

In appendix A we show how the two center solution is related to the Eguchi-Hanson solution that was discussed in particular in [3].

3.1 Five-brane and magnetic charges

At infinity we can compute the five-brane charge using (2.2b) and (3.12). We have

$$\mathcal{H}_{(3)} = (\mathrm{d}\tau + \omega) \wedge *_3 \mathrm{d}H \tag{3.14}$$

and so[4]

$$\mathrm{d}H = -\frac{\alpha'}{4mkr^2} \left(\sum_{i,j=1}^{k} \mathbf{q}_i \cdot \mathbf{q}_j \right) \mathrm{d}r - \frac{1}{m} \sum_\alpha \frac{q_\alpha}{r^2} + \dots \tag{3.15a}$$

$$\mathcal{H}_{(3)} = \left[-\frac{\alpha'}{4mk} \sum_{i,j=1}^{k} \mathbf{q}_i \cdot \mathbf{q}_j - \frac{1}{m} \sum_\alpha q_\alpha \right] (\mathrm{d}\tau + \omega) \wedge \Omega_2 + \dots \tag{3.15b}$$

and the Maxwell five-brane charge is

$$\mathcal{Q}_M = \frac{1}{4\pi^2 \alpha'} \int_{S^3/\mathbb{Z}_k} \mathcal{H}_{(3)} = -\frac{2}{k^2} \sum_{i,j=1}^{k} \mathbf{q}_i \cdot \mathbf{q}_j - \sum_\alpha \frac{8 q_\alpha}{k} \,. \tag{3.16}$$

One can also define a Page charge, which is quantized, as:

$$\begin{aligned} \mathcal{Q}_P &= \frac{1}{4\pi^2 \alpha'} \int_{S^3/\mathbb{Z}_k} \left(\mathcal{H}_{(3)} - A \wedge F \right) \\ &= -\sum_\alpha \frac{8 q_\alpha}{k} \in \mathbb{Z} \,. \end{aligned} \tag{3.17}$$

[3] we have chosen to work with Hermitian gauge fields, normalized as $\mathrm{Tr}\, \mathcal{T}_\alpha \mathcal{T}_\beta = 2\delta_{\alpha\beta}$.

[4] The volume form of a three-sphere is

$$\begin{aligned} ds^2_{S^3} &= \frac{1}{4} \left[\frac{1}{4m^2} (d\tau + \omega)^2 + d\Omega_2^2 \right] \\ 2\pi^2 &= \int \mathrm{vol}(S^3) = \frac{1}{8} \int \frac{1}{2m} (d\tau + \omega) \wedge \Omega_2 \,. \end{aligned}$$

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Magnetic charges. We take a basis of two cycles to be Δ_i where the poles of the Δ_i are at x_i and x_{i+1}. Then the matrix of magnetic charges associated with the Abelian gauge bundle are:

$$\begin{aligned} q_{i,j} = \frac{1}{2\pi}\int_{\Delta_j} F_i &= \frac{\mathbf{q}_i\cdot\mathcal{T}}{8\pi m}\left(\int \mathrm{d}\tau\right)\int_{x_j}^{x_{j+1}} \mathrm{d}\left[\frac{V_i}{V}\right] \\ &= \mathbf{q}_i\cdot\mathcal{T}\left[\delta_{j+1,i}-\delta_{j,i}\right] . \end{aligned} \tag{3.18}$$

3.2 Partial blow-down limits and fivebranes

The function $V(\mathbf{x})$ has k-poles, now suppose that k' of these poles are co-incident, which correspond to a partial blow-down limit of the ALE or ALF space. In the present situation some of the Abelian instantons (3.7) become point-like as the corresponding two-cycles shrink and we expect heterotic five-brane to appear. We now check that in the region around such a pole we obtain the near horizon of five-brane solution of Callan, Harvey and Strominger [19, 20] where the three-sphere is orbifolded by $\mathbb{Z}_{k'}$.

For simplicity we set $\mathbf{x}_j = 0$ for $j = 1, \dots k'$, and in the neighborhood of this pole the functions H and V behaves like

$$H \overset{r\to 0^+}{\simeq} \frac{1}{r^2}\frac{\alpha'}{2k'm}Q_5 \quad , \qquad V \overset{r\to 0^+}{\simeq} \frac{4mk'}{r^2} \tag{3.19}$$

hence the solution approaches

$$\mathrm{d}s^2_{10} \overset{r\to 0^+}{\simeq} \mathrm{d}s^2_{1,5} + 2\alpha' Q_5\left[\frac{\mathrm{d}r^2}{r^2} + \frac{r^2}{4}\left(\sigma_1^2+\sigma_3^2+\left(\frac{\sigma_3}{2k'm}\right)^2\right)\right] \tag{3.20a}$$

$$\mathcal{H}_{(3)} \overset{r\to 0^+}{\simeq} \frac{\alpha' Q_5}{2}\sigma_1\wedge\sigma_2\wedge\frac{\sigma_3}{2k'm} \tag{3.20b}$$

where the five-brane charge is given by:

$$Q_5 = \sum_{i,j=1}^{k'} \mathbf{q}_i\cdot\mathbf{q}_j\,. \tag{3.21}$$

3.3 Double scaling limit

For the two-center Eguchi-Hanson solution ($k = 2$) there exists an interesting double scaling limit [3], defined as:

$$g_s \to 0\,, \qquad \lambda := \frac{g_s\sqrt{\alpha'}}{a} \text{ fixed and finite}\,, \tag{3.22}$$

where a is the distance between the two centers. This limit decouples the asymptotically locally Euclidian region, and λ becomes the effective coupling constant of the interacting string theory.

In the spherical coordinates reviewed in appendix A the metric of the solution becomes

$$\mathrm{d}s^2 = \mathrm{d}s_{1,5} + \frac{\alpha' Q_5}{2}\left[\frac{\mathrm{d}r^2}{r^2(1-\frac{a^4}{r^2})} + \frac{1}{4}\left(1-\frac{a^4}{r^2}\right)\sigma_3^2 + \mathrm{d}\Omega_2^2\right] \tag{3.23}$$

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and the corresponding heterotic string theory admits an exactly solvable worldsheet CFT. This space has an asymptotic linear dilaton hence admits a holographic description as a little string theory [21] but, unlike the CHS background, is given by a smooth solution of heterotic supergravity.

A double scaling limit can be described in principle for arbitrary k. Let us define $\mathbf{x}_i = a\mathbf{y}_i$ where the coordinates $\mathbf{y}_i$ are dimension-less and a is a common scale factor. The double-scaling limit can be then described exactly as before by eq. (3.22); in practice the double scaling limit amounts to setting $\delta \to 0$ in (3.12).

It would be interesting to check if one could derive a worldsheet CFT for the double scaled solutions when $k > 2$, in particular whenever the centers are arranged following a simple pattern, for instance a homogeneous distribution on a circle.

4 Atiyah-Hitchin

The Atiyah-Hitchin space M_{AH} is a four-dimensional smooth manifold with an explicit Hyper-Kähler metric which at long distances approximates Taub-NUT with a negative mass parameter. The original work is [2, 22] and an interesting simplification was given in [23]. Our notation will follow a more recent work [10] where M_{AH} was used as a potential Euclidean Hyper-Kähler base manifold for five dimensional supergravity solutions. In [10] regularity required the absence of closed time-like curves and this effectively excluded physical solutions whereas for our computations the non-trivial regularity conditions are essentially just positivity of the warp factor and we will find regular solutions.

The metric is

$$ds^2_{AH} = \frac{1}{4}a_1^2 a_2^2 a_3^2 d\eta^2 + \frac{1}{4}a_1^2\sigma_1^2 + \frac{1}{4}a_2^2\sigma_2^2 + \frac{1}{4}a_3^2\sigma_3^2 \tag{4.1}$$

with the SU(2) invariant one-forms satifsying $d\sigma_i = \frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k$ given by

$$\begin{aligned}
\sigma_1 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi \\
\sigma_2 &= \sin\psi d\theta - \cos\psi \sin\theta d\phi \\
\sigma_3 &= d\psi + \cos\theta d\phi
\end{aligned}$$

and the a_i are subject to the following system of ODE's:

$$\frac{\dot{a_1}}{a_1} = \frac{1}{2}\left[(a_2 - a_3)^2 - a_1^2\right] \tag{4.2}$$

and cyclic permutations (dot is the derivative with respect to η). One defines new functions quadratic in the a_i:

$$w_1 = a_2 a_3\,, \quad w_2 = a_1 a_3\,, \quad w_3 = a_1 a_2 \tag{4.3}$$

and then the system of ODE's is then

$$(w_1 + w_2)' = -2\frac{w_1 w_2}{u^2} \tag{4.4}$$

$$(w_2 + w_3)' = -2\frac{w_2 w_3}{u^2} \tag{4.5}$$

$$(w_3 + w_1)' = -2\frac{w_3 w_1}{u^2} \tag{4.6}$$

(prime is derivative with respect to θ) with solution

$$w_1 = -uu' - \frac{1}{2}u^2 \csc\theta \tag{4.7}$$

$$w_2 = -uu' + \frac{1}{2}u^2 \cot\theta \tag{4.8}$$

$$w_3 = -uu' + \frac{1}{2}u^2 \csc\theta\,, \tag{4.9}$$

where

$$u = \frac{1}{\pi}\sqrt{\sin\theta}K\left(\sin^2\frac{\theta}{2}\right) \tag{4.10}$$

and η is given in terms of θ through

$$u^2 d\eta = d\theta\,, \qquad \eta = -\int_\theta^\pi \frac{d\theta}{u^2}\,. \tag{4.11}$$

For our gauge field ansatz we need an anti-self dual two form on M_{AH}, this is then guaranteed to solve Hermitian Yang-Mills without the need to construct the explicit Hyper-Kähler structure.[5] In a classic paper on dualities [8], Sen gave an integral expression for exactly such an anti-self dual, harmonic two-form on M_{AH} but the appearance of this two-form dates back to the works [25–27]. Interestingly, from the work [10] we have the closed-form expression of this two-form

$$\Omega = h\left(a_1^2 dr\wedge\sigma_1 - \sigma_2\wedge\sigma_3\right)\,, \tag{4.12}$$

$$h = \frac{u^2}{w_1 \sin\frac{\theta}{2}}\,. \tag{4.13}$$

In [10] they consider self-dual forms but with a small modification of the frames this is made anti-self dual. More precisely our choice of frames is

$$e_0 = \frac{a_1a_2a_3}{2}d\eta\,, \qquad e_i = \frac{a_i}{2}\sigma_i\,, \tag{4.14}$$

whereas in [10] an additional minus sign in e_0 was used. So we have locally

$$\Omega = -d(h\,\sigma_1)\,. \tag{4.15}$$

In fact one can construct a triplet of anti-self dual forms Ω_- and a triplet of self-dual forms Ω_+ in a similar manner:

$$\Omega_{i-} = -d(h_i\sigma_i)\,, \qquad \Omega_{i+} = d(h_i^{-1}\sigma_i) \tag{4.16}$$

with

$$h_1 = \frac{u^2}{w_1 \sin\frac{\theta}{2}}\,, \quad h_2 = \frac{u^2}{w_2}\,, \quad h_3 = \frac{u^2}{w_3\cos\frac{\theta}{2}}\,, \tag{4.17}$$

however only $\Omega_{1-} = \Omega$ is normalizable. Given that there is a single non-trivial two-cycle in M_{AH} one might be pleased to know that this normalizable form is dual to this two-cycle but there was no guarantee that the dual two-form would have an SU(2) invariant representative.

[5] One could in principle write down the Hyper-Kähler structure using the results of [24] or by computing the Killing spinors.

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4.1 Bianchi identity

We take our gauge field to be

$$F = \Omega\, \mathbf{q} \cdot \mathcal{H} \tag{4.18}$$

where $\mathcal{H} \in \mathrm{U}(1)^{16}$ is in the Cartan subalgebra of $E_8 \times E_8$ or SO(32) and $\mathbf{q}$ the corresponding charge vector. The three-form flux is

$$\mathcal{H}_{(3)} = -\frac{H'}{4}\sigma^1 \wedge \sigma^2 \wedge \sigma^3 \tag{4.19}$$

and the Bianchi identity is

$$d *_4 dH = -2q^2 \Omega \wedge \Omega\,, \tag{4.20}$$

where $q^2 = \mathbf{q} \cdot \mathbf{q}$.

Quite remarkably, one can integrate this Poisson equation explicitly

$$H = h_0 + h_1 \eta + \frac{2q^2}{w_1} \tag{4.21}$$

where $\{h_0, h_1\}$ are constant coefficients of the s-wave harmonic functions on M_{AH}. The last term is manifestly negative definite for the whole region $0 \le \theta \le \pi$ but we will see that one can compensate for this by a choice of harmonic function and obtain a positive definite warpfactor.

4.2 Regularity

The regularity of M_{AH} has been previously studied in detail, we repeat it here to help determine regularity of our warp factor.

In the region $\theta \sim \pi$, we define a radial co-ordinate $r = -\log\cos\frac{\theta}{2}$ and using

$$K = r + \log(4) + \dots \tag{4.22}$$

$$u = \frac{\sqrt{2}}{\pi} r e^{-r/2} + \dots \tag{4.23}$$

$$w_1 = -\frac{r}{\pi^2} \tag{4.24}$$

we find that the metric is

$$ds^2_{AH} = dr^2 + r^2(\sigma_1^2 + \sigma_2^2) + \sigma_3^2 + \dots \tag{4.25}$$

and

$$\frac{1}{w_1} = -\frac{\pi^2}{r} + \mathcal{O}(r^{-2}) \tag{4.26}$$

$$\eta = -\frac{\pi^2}{r} + \frac{\pi \log(4)}{r^2} + \mathcal{O}(r^{-3}) \tag{4.27}$$

so that the asymptotic expansion of the warp factor is

$$H = h_0 - \frac{h_1 + 2q^2}{r} + \dots \tag{4.28}$$

In the region $\theta \sim 0$, we define a new radial variable $\rho = \frac{\theta^2}{64}$ and the metric is

$$ds^2_{AH} = d\rho^2 + 4\rho^2\sigma_1^2 + \frac{1}{16}(\sigma_2^2 + \sigma_3^2) + \dots \tag{4.29}$$

with

$$\frac{1}{w_1} = -4 + 32\rho^2 + \dots \tag{4.30}$$

$$\eta = \log \rho^2 + \dots \tag{4.31}$$

so that the IR expansion of the warp factor is

$$H = (h_0 - 8q^2) + 64q^2\rho^2 + \dots \tag{4.32}$$

From these expansions we see that with

$$h_0 > 8q^2 \,, \qquad h_1 = 0 \tag{4.33}$$

we have a positive warp factor which is regular everywhere. We define a rescaled radial coordinate near $\theta \sim \pi$ to be $\widehat{r} = h_0^{1/2} r$ and $\widehat{\rho} = h_0^{1/2}\rho$ near $\theta = 0$ so that

$$\theta \sim \pi : \qquad ds_{10} = ds^2_{1,5} + d\widehat{r}^2 + \widehat{r}^2(\sigma_1^2 + \sigma_2^2) + h_0\sigma_3^2 + \dots \tag{4.34}$$

$$\theta \sim 0 : \qquad ds^2_{10} = ds^2_{1,5} + d\widehat{\rho}^2 + 4\widehat{\rho}^2\sigma_1^2 + \frac{h_0}{16}(\sigma_2^2 + \sigma_3^2) + \dots \tag{4.35}$$

and see that the cost of a positive warp factor is that both the circle at infinity and the two-sphere at the bolt are large.

It is also important that the $\mathrm{Tr} R_- \wedge R_-$ term in the Bianchi identity remains small compared to $\mathrm{Tr} F \wedge F$. From explicit computations we find that the only possible divergences in $\mathrm{Tr} R_- \wedge R_-$ appear through the warp factor as[6] $\{H'/H \,, H''/H\}$ which by tuning h_0 can be made sufficiently small with respect to $\mathrm{Tr} F \wedge F$. This confirms that our large charge approximation remains valid and these warped Atiyah-Hitchin solutions are good Heterotic backgrounds at leading order.

Alternatively we can obtain a positive warp factor through[7]

$$h_0 = 1, \qquad h_1 < -2q^2 \,. \tag{4.36}$$

This corresponds to smearing neutral five-branes on the S^2 at $\theta = 0$. Note that due to this smearing, at the IR ($\theta = 0$) the harmonic function parameterized by h_1 scales like a source in $\mathbb{R}^2$. In the UV ($\theta = \pi$), due to the finite circle, the harmonic function scales like $\frac{1}{r}$ which is that of a source in $\mathbb{R}^3$ not $\mathbb{R}^4$. The solution is of course singular for the usual reason that smeared branes are singular but this is of a good type and is resolved in string theory.

[6] An explicit computation using the Chern connection can be found in [14] and agrees with our conclusion here.

[7] The value of h_0 could be chosen to be another non-zero number $\mathcal{O}(q^0)$.

4.3 Five-brane charge

Computing the five-brane charge requires understanding some global features of M_{AH}. From (4.25) and (4.29) we see that there are two inequivalent, emergent U(1) symmetries in the UV and IR, which are broken in the bulk. From [23] we know that a regular manifold requires the periodicities to be

$$0 \leq \psi \leq 2\pi , \quad 0 \leq \theta \leq \pi , \quad 0 \leq \phi \leq 2\pi \tag{4.37}$$

as well as that the free $\mathbb{Z}_2$ symmetry

$$I_1 : \quad \theta \to \pi - \theta , \quad \phi \to \pi + \phi , \quad \psi \to -\psi \tag{4.38}$$

is enforced. The horizontal space in the UV is thus $\mathbb{RP}^3/I_1$.

Using (4.28) we have

$$\mathcal{H}_{(3)} = \left[(h_1 + 2q^2) + \ldots \right] \wedge \sigma^1 \wedge \sigma^2 \wedge \sigma^3 \tag{4.39}$$

compute the Maxwell five-brane charge to be

$$\mathcal{Q}_M = \frac{1}{4\pi\alpha'} \int_{\mathbb{RP}^3/I_1} \mathcal{H}_{(3)} = h_1 + 2q^2 \, . \tag{4.40}$$

This is not required to be quantized. The Page charge is defined as in (3.17) and we find

$$\mathcal{Q}_P = h_1 \tag{4.41}$$

which must be integral.

4.4 Gauge field charge

The gauge field charge is computed using (4.12) and (4.18) and the IR expansion

$$h = -2 + \ldots \tag{4.42}$$

Under the symmetry (4.38), the bolt remains a two sphere[8] whose volume is 4π. We find

$$\begin{aligned} \frac{1}{2\pi} \int_{S^2} F &= 2\mathbf{q} \cdot \mathcal{H} \frac{1}{2\pi} \int_{S^2} \sigma_2 \wedge \sigma_3 \\ &= 4\mathbf{q} \cdot \mathcal{H} \in \mathbb{Z} \, . \end{aligned} \tag{4.43}$$

5 Fundamental string sources and AdS$_3$ solutions

Heterotic backgrounds with an $\mathbb{R}^{1,1}$ factor allow for the inclusion of F1-strings along $\mathbb{R}^{1,1}$ in addition to the magnetic five-branes. The electric source of three form flux induces a non-trivial warp factor and allows for AdS$_3$ solutions. To include these fundamental strings,

[8] As explained in [23] there is an additional, optional $\mathbb{Z}_2$ symmetry usually denoted I_3 which would convert the bolt into an $\mathbb{RP}^3$.

we first consider internal eight-manifolds X_8 and then specialize the internal manifold to be a product of Hyper-Kähler manifolds.

The metric and three form are

$$ds^2_{10} = e^{2A} ds^2_{1,1} + ds^2_8 \tag{5.1}$$

$$H^{(3)} = \mathrm{vol}_2 \wedge h^{(1)} + h^{(3)} \tag{5.2}$$

where $\mathrm{vol}_2 = e^{2A} dx^0 \wedge dx^1$. Then we find that the BPS equations are a slight embellishment of those found in [28]:

$$\Psi \lrcorner d\Psi = d(\phi - A) \tag{5.3}$$

$$H_{(3)} = h_{(1)} \wedge \mathrm{vol}_2 + h_{(3)} \tag{5.4}$$

$$h_{(1)} = -2dA \tag{5.5}$$

$$h_{(3)} = *_8 e^{2(\phi - A)} d\left(e^{2(A-\phi)}\Psi\right) \tag{5.6}$$

where Ψ is the Spin(8) structure on M_8:[9]

$$\begin{aligned}\Psi =\;& e^{1234} + e^{1256} + e^{1278} + e^{3456} + e^{3478} + e^{5678} \\ &+ e^{1357} - e^{1368} - e^{1458} - e^{1467} - e^{2358} - e^{2367} - e^{2457} + e^{2468} .\end{aligned} \tag{5.8}$$

We must supplement the BPS equations with the Bianchi identity (2.3) and then due to the non-trivial warp-factor A, one must also impose the three form flux equation of motion:

$$0 = d\left(e^{-2\phi} *_{10} H_{(3)}\right) \qquad \Rightarrow \qquad \begin{cases} 0 = d\left(e^{-2\phi} *_8 h_{(1)}\right) \\ 0 = d\left(e^{2(A-\phi)} *_8 h_{(3)}\right) \end{cases} . \tag{5.9}$$

5.1 Product of Hyper-Kähler manifolds

Our solutions with string and five-brane charges have a natural splitting of the internal eight manifold into a product of Hyper-Kähler manifolds[10]

$$ds^2_{10} = e^{2A} ds^2_{\mathbb{R}^{1,1}} + ds^2_{M_1} + e^{2B} ds^2_{M_2} \tag{5.10}$$

where M_i are both HyperKahler four manifolds, whose triplet of Kähler forms we denote

$$\{J_i, \mathrm{Re}\,\Omega_i, \mathrm{Im}\,\Omega_i\} . \tag{5.11}$$

The functions A, B depend only on the co-ordinates y_i of M_2. The Spin(8) structure is given by

$$\Psi = \frac{1}{2}\left(J_1 \wedge J_1 + 2e^{2B} J_1 \wedge J_2 + e^{4B} J_2 \wedge J_2 + e^{2B}(\Omega_1 \wedge \Omega_2 + \overline{\Omega}_1 \wedge \overline{\Omega}_2)\right) . \tag{5.12}$$

[9] We note that with canonical holomorphic frames $E_i = e_{2i-1} + ie_{2i}$ such that $ds^2_8 = E_i \otimes \overline{E}_i$ the SU(4) structure is $J = \frac{1}{2i} E_i \wedge \overline{E}_i$, $\Omega = E_1 \wedge E_2 \wedge E_3 \wedge E_4$ and

$$\Psi = \frac{1}{2}\left(J \wedge J + \Omega + \overline{\Omega}\right) . \tag{5.7}$$

[10] One might consider an additional warp factor in front of $ds^2_{M_1}$ however from [29, 30] we know that this must be constant.

We find the BPS conditions, Bianchi identity and equations of motion give[11]

$$\phi = A + B \tag{5.13}$$

$$h_{(3)} = -*_{M_2} d\, e^{2B} \tag{5.14}$$

$$d *_{M_2} d\, e^{2B} = -\frac{1}{2}\alpha' \mathrm{Tr} F \wedge F \tag{5.15}$$

$$0 = d *_{M_2} d\, e^{-2A} \tag{5.16}$$

$$0 = J_2 \llcorner F \tag{5.17}$$

so we see that the only additional pieces of data from the equations in section 2 is that e^{2A} is harmonic on M_2 and the dilaton receives a shift proportional to A. For the Atiyah-Hitchin manifold we can smear F1-strings on the S^2 bolt in much the same way as we have described for smearing 5-branes on the bolt around (4.36), that is by

$$e^{2A} \sim \eta\,. \tag{5.18}$$

We will now be somewhat more explicit for the Gibbons-Hawking spaces.

5.2 AdS$_3$ from Gibbons-Hawking

When M_2 is a Gibbons-Hawking space, the U(1) invariant harmonic functions are

$$e^{-2A} = 1 + \sum_r \frac{\widehat{q}_r}{|\mathbf{x} - \mathbf{x}_r|} \tag{5.19}$$

corresponding to strings placed along $\mathbb{R}^{1,1}$ and at fixed points of ∂_τ on M_2.

If in addition we choose to place these strings at poles of V we recover AdS$_3$ geometries near such a pole. We put k' poles of V as well as the strings at $\mathbf{x}_r = \mathbf{x}_i = 0$ then in the vicinity of x_i we have

$$e^{2A} = \frac{r}{\widehat{q}_0}, \dots \qquad e^{2B} = \frac{1}{r}\frac{\alpha'}{4m} Q_5 + \dots\,, \qquad V = \frac{2mk'}{r} + \dots \tag{5.20}$$

$$\tag{5.21}$$

so that

$$ds^2_{10} = \frac{r}{\widehat{q}_0} ds^2_{1,1} + + ds^2_{M_1} + 2\alpha' k'^2 Q_5 \left[\frac{1}{4}\frac{dr^2}{r^2} + ds^2_{S^3/\mathbb{Z}_k}\right] \tag{5.22}$$

$$= 2\alpha' k'^2 Q_5 \left[ds^2_{AdS_3} + ds^2_{S^3/\mathbb{Z}_k}\right] + ds^2_{M_1} \tag{5.23}$$

$$e^{2\phi} = \frac{\alpha' Q_5}{4m\widehat{q}_0} \tag{5.24}$$

where $r = \rho^2$. The F1-charge is given as usual by

$$Q_1 = \frac{4m\widehat{q}_0 \mathrm{vol}(M_1)}{\alpha'^3}\,. \tag{5.25}$$

The gauge field vanishes in this limit and the background is sourced by three-form flux.

[11] Note that is $E = e^B \widetilde{E}$ is an 8d frame $*_8 E \wedge J_1 \wedge J_1 = 2 *_8 E \wedge \mathrm{vol}_{M_1} = 2e^{3B} *_{M_2} \widetilde{E} = 2e^{2B} *_{M_2} E$.

6 Conclusions

The key aspect of our solutions with Abelian gauge bundles is that we have taken a large charge limit and consistently suppressed the $\mathrm{Tr}\, R \wedge R$ term in the Bianchi identity, which is subdominant at leading order in the expansion in $\frac{1}{q^2}$. We have shown how this large charge limit can lead to exact supersymmetric flux backgrounds. and it is particularly interesting the Atiyah-Hitchin manifold can provide a regular background. This configuration requires some ingenuity to counteract the negative mass and result in a background of the correct signature. This Atiyah-Hitchin based solution is distinctly different from those based on Gibbons-Hawking; while the latter can be viewed as marginal deformations of the orbifold of the CHS solutions the finite two-cycle in the Atiyah-Hitchin manifold cannot be blown down. As such we do not have a worldsheet theory from which we can imagine obtaining this as the background geometry.

In these backgrounds, the gauge fields are completely solved for by using the Hermitian Yang-Mills equations which then provide a source for the three-form flux. It is conceivable that non-Abelian bundles could be constructed such that $\mathrm{Tr}\, F \wedge F$ dominates $\mathrm{Tr}\, R \wedge R$ everywhere.[12] Since the Kronheimer-Nakajima construction [33] gives a solution of all instantons on ALE Gibbons-Hawking spaces, one could possibly even construct such instantons, however most instantons will provide a source the Bianchi identity whose solution is a general function of four variables and thus unsolvable. A particularly neat class of instantons is based on the 't Hooft ansatz [34]:

$$\begin{aligned} A_0 &= \frac{1}{2}\vec{G} \cdot \vec{\sigma}\,, \qquad\qquad \vec{A} = \frac{1}{2}\left[\vec{\omega}(\vec{G}\cdot\vec{\sigma}) - V(\vec{G}\times\vec{\sigma})\right] \\ \vec{G} &= -V^{-1}\vec{\nabla}\log f \end{aligned} \tag{6.1}$$

with f harmonic on $\mathbb{R}^3$. For finite action, the centers of f are constrained to lie at the poles of V. These instantons can have large $\mathrm{Tr}\, F \wedge F$ in the limit of large number of poles of f but $\mathrm{Tr}\, R \wedge R$ will not be suppressed.

There are numerous directions for progress on the worldsheet description of these backgrounds. The elliptic genus for type II on ALE spaces has been computed recently [35] based on general developments in this field [36] and we expect to be able to provide a similar solution for these Heterotic models or the type II flux backgrounds of section 2.1. It would also be interesting to provide an exactly solvable worldsheet model of the near-horizon region of the multi-center Gibbons-Hawking backgrounds, generalizing the gauged WZW model of the two-centered solution.

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[12] Interesting five dimensional solutions with non-Abelian gauge fields have appeared recently [31] and the lift to the Heterotic string has been discussed [32]. However it is not clear to us how these solutions will solve the exact Bianchi identity.

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A Eguchi-Hanson

When $k = 2$ and $\epsilon = 0$, the explicit co-ordinate transformation is known [37] from the Gibbons-Hawking space to the Eguchi-Hanson space [38]. In Cartesian co-ordinates the two center ALE Gibbons-Hawking space has

$$\omega = \left[\frac{z - a^2/8}{\sqrt{x^2 + y^2 + (z - a^2/8)^2}} + \frac{z + a^2/8}{\sqrt{x^2 + y^2 + (z + a^2/8)^2}}\right] d\left(\tan^{-1}\frac{y}{x}\right) \tag{A.1}$$

$$V = \frac{1}{\sqrt{x^2 + y^2 + (z - a^2/8)^2}} + \frac{1}{\sqrt{x^2 + y^2 + (z + a^2/8)^2}}\,. \tag{A.2}$$

Following [37] we have ($a \le r$):

$$\begin{aligned} x &= \frac{a^2}{8}\sqrt{\frac{r^4}{a^4} - 1}\sin\theta\cos\psi \\ y &= \frac{a^2}{8}\sqrt{\frac{r^4}{a^4} - 1}\sin\theta\sin\psi \\ z &= \frac{1}{8}r^2\cos\theta \end{aligned}$$

so that

$$\begin{aligned} V &= \frac{16}{a^2}\frac{\frac{r^2}{a^2}}{\frac{r^4}{a^4} - \cos^2\theta}\,, \\ \omega &= \frac{2\cos\theta(\frac{r^2}{a^2} - 1)}{\frac{r^2}{a^2} - \cos\theta}d\psi\,. \end{aligned} \tag{A.3}$$

As an example, we write explicitly the solution for Heterotic five-branes on Eguchi-Hanson with additional F1-strings.[13]

$$ds^2_{M_2} = \frac{dr^2}{f^2} + \frac{r^2}{4}\left[\sigma_1^2 + \sigma_2^2 + f^2\sigma_3^2\right] \tag{A.4}$$

$$f^2 = 1 - \frac{a^4}{r^4} \tag{A.5}$$

$$h_{(3)} = 2f^2r^3(e^{2B})'\sigma_1\wedge\sigma_2\wedge\sigma_3$$

$$F = d\left(\frac{a^2}{r^2}\eta\right) \tag{A.6}$$

$$e^{2B} = 1 + \frac{8\alpha' q^2}{r^2} + \frac{Q_5}{8a^2}\log\left[\frac{r^2/a^2 - 1}{r^2/a^2 + 1}\right] \tag{A.7}$$

[13] One can take M_1 to be T^4 or $K3$ with the Ricci-flat metric.

$$e^{-2A} = 1 + \frac{Q_1}{a^2} \log\left[\frac{r^2/a^2 - 1}{r^2/a^2 + 1}\right] \tag{A.8}$$

$$e^{2\Phi} = e^{2(A+B)} \tag{A.9}$$

In addition to the Heterotic five-branes which resolve the singularity, there are Q_1 mobile F1-strings and Q_5 NS5-branes smeared on the blown-up S^2. Due the smearing of the strings, the near horizon limit has a log-singularity at $r \sim a$ in the warp factor e^{2A} and thus there is no enhancement to AdS_3. In the blow-down limit $a \to 0$ where the Eguchi-Hanson space becomes $\mathbb{C}^2/\mathbb{Z}_2$, the gauge field vanishes and we get the $\mathbb{Z}_2$ orbifold of the usual F1-NS5-solution, the near-horizon limit is $\mathrm{AdS}_3 \times S^3/\mathbb{Z}_2 \times M_1$.

References

[1] G.W. Gibbons and S.W. Hawking, *Gravitational Multi-Instantons*, *Phys. Lett.* **B 78** (1978) 430 [INSPIRE].

[2] M.F. Atiyah and N.J. Hitchin, *Low-Energy Scattering of Nonabelian Monopoles*, *Phys. Lett.* **A 107** (1985) 21 [INSPIRE].

[3] L. Carlevaro, D. Israël and P.M. Petropoulos, *Double-Scaling Limit of Heterotic Bundles and Dynamical Deformation in CFT*, *Nucl. Phys.* **B 827** (2010) 503 [arXiv:0812.3391] [INSPIRE].

[4] L. Carlevaro and D. Israël, *Heterotic Resolved Conifolds with Torsion, from Supergravity to CFT*, *JHEP* **01** (2010) 083 [arXiv:0910.3190] [INSPIRE].

[5] N. Halmagyi, D. Israël and E.E. Svanes, *The Abelian Heterotic Conifold*, *JHEP* **07** (2016) 029 [arXiv:1601.07561] [INSPIRE].

[6] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[7] P.S. Aspinwall, *Compactification, geometry and duality: $N = 2$*, hep-th/0001001 [INSPIRE].

[8] A. Sen, *Dyon - monopole bound states, selfdual harmonic forms on the multi - monopole moduli space and* $\mathrm{SL}(2,\mathbb{Z})$ *invariance in string theory*, *Phys. Lett.* **B 329** (1994) 217 [hep-th/9402032] [INSPIRE].

[9] I. Bena and N.P. Warner, *Bubbling supertubes and foaming black holes*, *Phys. Rev.* **D 74** (2006) 066001 [hep-th/0505166] [INSPIRE].

[10] I. Bena, N. Bobev and N.P. Warner, *Bubbles on Manifolds with a* U*(1) Isometry*, *JHEP* **08** (2007) 004 [arXiv:0705.3641] [INSPIRE].

[11] R. Minasian, M. Petrini and A. Zaffaroni, *New families of interpolating type IIB backgrounds*, *JHEP* **04** (2010) 080 [arXiv:0907.5147] [INSPIRE].

[12] K. Dasgupta, G. Rajesh and S. Sethi, *M theory, orientifolds and G-flux*, *JHEP* **08** (1999) 023 [hep-th/9908088] [INSPIRE].

JHEP08(2017)138

[13] E. Goldstein and S. Prokushkin, *Geometric model for complex nonKähler manifolds with* SU(3) *structure*, *Commun. Math. Phys.* **251** (2004) 65 [hep-th/0212307] [INSPIRE].

[14] J.-X. Fu and S.-T. Yau, *The theory of superstring with flux on non-Kähler manifolds and the complex Monge-Ampere equation*, *J. Diff. Geom.* **78** (2008) 369 [hep-th/0604063] [INSPIRE].

[15] K. Becker, M. Becker, J.-X. Fu, L.-S. Tseng and S.-T. Yau, *Anomaly cancellation and smooth non-Kähler solutions in heterotic string theory*, *Nucl. Phys.* **B 751** (2006) 108 [hep-th/0604137] [INSPIRE].

[16] K. Becker and S. Sethi, *Torsional Heterotic Geometries*, *Nucl. Phys.* **B 820** (2009) 1 [arXiv:0903.3769] [INSPIRE].

[17] J.-X. Fu, L.-S. Tseng and S.-T. Yau, *Local Heterotic Torsional Models*, *Commun. Math. Phys.* **289** (2009) 1151 [arXiv:0806.2392] [INSPIRE].

[18] D. Martelli and J. Sparks, *Non-Kähler heterotic rotations*, *Adv. Theor. Math. Phys.* **15** (2011) 131 [arXiv:1010.4031] [INSPIRE].

[19] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Worldbrane actions for string solitons*, *Nucl. Phys.* **B 367** (1991) 60 [INSPIRE].

[20] C.G. Callan Jr., J.A. Harvey and A. Strominger, *Supersymmetric string solitons*, hep-th/9112030 [INSPIRE].

[21] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS five-branes and holography*, *JHEP* **10** (1998) 004 [hep-th/9808149] [INSPIRE].

[22] M.F. Atiyah and N.J. Hitchin, *Low-energy scattering of nonAbelian magnetic monopoles*, *Phil. Trans. Roy. Soc. Lond.* **A 315** (1985) 459 [INSPIRE].

[23] G.W. Gibbons and N.S. Manton, *Classical and Quantum Dynamics of BPS Monopoles*, *Nucl. Phys.* **B 274** (1986) 183 [INSPIRE].

[24] D. Olivier, *Complex coordinates and Kähler potential for the Atiyah-Hitchin metric*, *Gen. Rel. Grav.* **23** (1991) 1349 [INSPIRE].

[25] N.S. Manton, *Monopole and Skyrmion Bound States*, *Phys. Lett.* **B 198** (1987) 226 [INSPIRE].

[26] G.W. Gibbons and P.J. Ruback, *The Hidden Symmetries of Multicenter Metrics*, *Commun. Math. Phys.* **115** (1988) 267 [INSPIRE].

[27] N.S. Manton and B.J. Schroers, *Bundles over moduli spaces and the quantization of BPS monopoles*, *Annals Phys.* **225** (1993) 290 [INSPIRE].

[28] J.P. Gauntlett, D. Martelli and D. Waldram, *Superstrings with intrinsic torsion*, *Phys. Rev.* **D 69** (2004) 086002 [hep-th/0302158] [INSPIRE].

[29] A. Strominger, *Superstrings with Torsion*, *Nucl. Phys.* **B 274** (1986) 253 [INSPIRE].

[30] C.M. Hull, *Compactifications of the Heterotic Superstring*, *Phys. Lett.* **B 178** (1986) 357 [INSPIRE].

[31] P. Meessen, T. Ortín and P.F. Ramirez, *Non-Abelian, supersymmetric black holes and strings in 5 dimensions*, *JHEP* **03** (2016) 112 [arXiv:1512.07131] [INSPIRE].

[32] P.A. Cano, P. Meessen, T. Ortín and P.F. Ramirez, *Non-Abelian black holes in string theory*, arXiv:1704.01134 [INSPIRE].

JHEP08(2017)138

[33] P.B. Kronheimer and H. Nakajima, *Yang-mills instantons on ale gravitational instantons*, *Math. Ann.* **288** (1990) 263.

[34] H. Boutaleb-Joutei, A. Chakrabarti and A. Comtet, *Gauge Field Configurations in Curved Space-times. 4. Selfdual* SU(2) *Fields in Multicenter Spaces*, *Phys. Rev.* **D 21** (1980) 2280 [INSPIRE].

[35] J.A. Harvey, S. Lee and S. Murthy, *Elliptic genera of ALE and ALF manifolds from gauged linear σ-models*, *JHEP* **02** (2015) 110 [arXiv:1406.6342] [INSPIRE].

[36] F. Benini, R. Eager, K. Hori and Y. Tachikawa, *Elliptic Genera of 2d $\mathcal{N} = 2$ Gauge Theories*, *Commun. Math. Phys.* **333** (2015) 1241 [arXiv:1308.4896] [INSPIRE].

[37] M.K. Prasad, *Equivalence of Eguchi-Hanson metric to two-center Gibbons-Hawking metric*, *Phys. Lett.* **B 83** (1979) 310 [INSPIRE].

[38] T. Eguchi and A.J. Hanson, *Asymptotically Flat Selfdual Solutions to Euclidean Gravity*, *Phys. Lett.* **B 74** (1978) 249 [INSPIRE].

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