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E. Blokhina,[1] D. Galayko,[2] P. Harte,[1] P. Basset,[3] and O. Feely[1]

[1]*University College Dublin, Belfield, Dublin 4, Ireland*
[2]*UPMC–Sorbonne Universités, 4, place Jussieu, 75005 Paris, France*
[3]*Université Paris-Est, ESYCOM, ESIEE Paris, BP 99–2 Bd Blaise Pascal, 93162 Noisy-le-Grand, Cedex, France*

Based on the formal analysis of a resonant electrostatic vibration energy harvester operating in constant-charge mode with a gap-closing transducer, we show that the system displays universal behaviour patterns. In this paper, we treat the harvester as a nonlinear forced oscillator and bound the area of control parameters where the system displays regular harmonic oscillations allowing the conditioning circuit to operate in the most effective mode. Before the system exhibits irregular behaviour, there exists a universal optimal value of normalised converted power regardless of the system design and control parameters. © *2012 American Institute of Physics.*
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Conversion of mechanical energy to electrical energy at the microscale level has become the subject of a growing area of research[1,2] since it promises autonomous energy supply for miniature wireless sensors and mobile devices.[3,4] Among the existing mechanical energy conversion techniques, electrostatic transducers are particularly suitable and compatible with microtechnologies.[5]

Resonant vibration energy harvesters (VEHs) employ high-quality resonators that can convert vibration energy in a narrow frequency band. Although for a large number of applications, the sources of vibration energy can be described as wideband and nonlinear techniques are required for effective energy harvesting,[6–9] resonant harvesters are still the fundamental "building" block of energy scavenging systems. Wideband systems are often obtained by a modification of existing resonant VEHs prototypes. For this reason, a deep understanding of the dynamics of resonant VEHs is necessary for their design and the design of wideband VEHs based on them. These systems are very complex and raise many analytical and conceptual difficulties.

Although many studies address capacitive (electrostatic) conversion of mechanical energy, nonlinear analysis of electrostatic vibration energy harvester (e-VEH) dynamics in the coupled electromechanical mode is still weakly developed. However, only this analysis provides the information about converted energy for given parameters of the system. References 10–12 presented extended dynamical analysis of a very common configuration of resonant e-VEHs based on a gap-closing transducer operating in constant-charge mode.[1] Reference 11 proposed a straightforward analytical description of the regular and desirable behaviour of the system, whose characteristics (the amplitude of oscillations, the converted power, etc.) are calculated from parameters of the system (a resonator coupled with a conditioning circuit) and from the environmental parameters (the magnitude and frequency of external vibrations). In addition, Refs. 10 and 12 highlighted the existence of irregular and undesirable regimes of such e-VEHs.

The study presented in this paper uses the results of Refs. 10–12 to predict the maximum power that can be converted by an e-VEH with the common gap-closing transducer. It was found that e-VEHs utilising the gap-closing transducer and operating in the constant-charge triangular QV (charge-voltage) cycle[1] have a constant universal factor Π_{max} relating the maximum convertible power to the product of the basic system parameters. This study provides the values of the system parameters that guarantee the optimal value of converted power. This result shows that there is a common behaviour pattern in the system. From the standpoint of an e-VEH designer, it allows one to find an optimal transducer geometry for a given resonator and adapt the geometry of the system to the operating condition.

A resonant e-VEH consists of a high-Q resonator, a variable capacitor (transducer) C_{tran}, and a conditioning circuit that implements the constant-charge energy conversion cycle.[1] The conditioning circuit discharges the transducer to zero when the transducer capacitance is at a local minimum and charges it to a charge Q_0 when its capacitance is at a local maximum. The energy conversion is achieved when the transducer capacitance decreases keeping its charge Q_0 constant. During this process, mechanical energy is converted into electrical energy, and the transducer acts as a damper in the mechanical domain. A detailed description of the conversion cycle and a schematic view of the system can be found in Ref. 13.

In Ref. 11, a normalised equation in the form of a nonlinear driven oscillator was obtained to model the system. The normalised displacement y of the resonator is described by

$$y'' + 2\beta y' + y = \alpha \cos(\Omega \tau + \theta_0) + f_t(y, y'), \qquad (1)$$

where the prime denotes the derivative with respect to dimensionless time τ. Nonlinearity of the oscillator is introduced by the force $f_t(y, y')$ generated in the transducer

$$f_t(y, y') = \begin{cases} \dfrac{\nu_0}{(1 - y_{max})}, & y' < 0 \\ 0 & y' > 0 \end{cases}. \qquad (2)$$

To obtain this equation, the following normalised variables were introduced: displacement $y = x/d$ obtained from the dimensional displacement x, time $\tau = \omega_0 t$, dissipation $\beta = b/(2m\omega_0)$, external frequency $\Omega = \omega_{ext}/\omega_0 = 1 + \sigma$, where σ is a small mismatch in the two frequencies, $\alpha = A_{ext}/(d\omega_0^2)$ and $\nu_0 = W_0/(d^2 m\omega_0^2)$. Here, m is the mass of the resonator, b is the damping factor, $\omega_0 = \sqrt{k/m}$ is the natural frequency, k is the spring constant, d is the rest gap (i.e., the gap in the absence of all forces), A_{ext} is the external acceleration amplitude, ω_{ext} is the external frequency, ϑ_0 is the initial phase of the external force, and W_0 is the energy that the conditioning circuit provides to the transducer at the beginning of each energy conversion cycle (at each local maximum of the transducer capacitance C_{tran} according to the QV cycle implementation, see Ref. [10]).

Now we select a minimal set of design and operating parameters required to define the system dynamics. We divide them into three groups: environmental parameters, fixed parameters, and design parameters. The environmental parameters are defined by possible applications of the system. They are the acceleration amplitude A_{ext} (which can vary) and the frequency of external vibrations $\omega_{ext} \approx \omega_0$ (fixed for a given e-VEH configuration as follows from the narrow band hypothesis).

The fixed parameters are S (the area of the transducer), m, and k. The area S and the mass m are directly related to the maximum amount of convertible energy, and generally must be maximized. Their limits are defined by the size of the harvesting system (typically, by the maximal volume of 1 cm³ for wireless sensor[5]). The spring constant k is linked to ω_0 and m and is not independent. The design parameters are those that a designer is free to choose in order to obtain particular system characteristics. They are W_0 and d. Note that for the same e-VEH, the designer can freely choose the value of W_0. In contrast to W_0, d is fixed during the fabrication of a particular harvester.

Note that for a particular harvester, we classify W_0 and A_{ext} as the *control parameters* of the dynamical system in the form of Eq. (1) as they can vary for a given e-VEH and directly affect the behaviour of the system. Typical parameters used in the numerical simulations are listed in Table I.

The desired mode of the e-VEH operating regime is steady-state harmonic oscillations. Indeed, the control circuit works most effectively when only one maximum of a waveform is detected per cycle of oscillations.[14,15] Since the conditioning circuit requires power to run switches, ideally switches should be opened and closed once in a oscillation cycle. In the case when the system displays irregular oscillations, the

harvester may not convert any effective power given the power required to supply the work of the system.

The harmonic steady-state solution of Eq. (1) obtained from the multiple scales method[16] has the form[11]

$$y_0(\tau) = y_{av,0} + a_0 \cos((1+\sigma)\tau + \theta_0 - \psi_0), \qquad (3)$$

where a_0, $y_{av,0}$, and ψ_0 are the steady-state amplitude, average displacement (constant shift), and phase of oscillations. The amplitude a_0 and the phase ψ_0 are found from the equations

$$\frac{\alpha^2}{4} = \left(\beta a_0 + \frac{b_1(y_{m,0})}{2}\right)^2 + \left(a_0\sigma + \frac{a_1(y_{m,0})}{2}\right)^2 \qquad (4)$$

$$\frac{\alpha}{2}\sin\psi_0 = \beta a_0 + \frac{b_1}{2}, \quad \frac{\alpha}{2}\cos\psi_0 = -a_0\sigma - \frac{a_1}{2},$$

where $y_{m,0} = y_{av,0} + a_0$ is the maximum displacement. We have used the Fourier series for $f_t(\tau) = f_0 + a_1\cos\theta(\tau) + b_1\sin\theta(\tau)$ for the transducer force, where $\theta(\tau) = [(1+\sigma)\tau + \theta_0 - \psi_0]$ and the functions f_0, a_1, and b_1 are the standard coefficients of the Fourier series but depend on the amplitude a_0 and the average displacement $y_{av,0}$. Note that due to the zero harmonic in the Fourier series, the oscillation (3) has the constant shift $y_{av,0} = f_0$.

For the gap closing transducer, the coefficients are

$$f_0 = \frac{\nu_0}{2(1 - y_m)}, \quad a_1 = 0, \quad b_1 = \frac{2\nu_0}{\pi(1 - y_m)}. \qquad (5)$$

In Refs. [10] and [12], it has been shown that increasing A_{ext} when all other parameters are fixed leads to irregular motion of the resonator and therefore to an improper operating mode of the harvester. It was shown that for a given W_0, there exists an optimum (maximum) A_{ext} at which the behaviour is regular, and which guarantees maximum power conversion at a given parameter set.

The plane of the control parameters (W_0, A_{ext}) shown in Fig. 1 summarises the results. Line 1 (blue) shows the necessary conditions required to start steady-state harmonic oscillations in the resonator. Line 2 (black) shows the doubling bifurcation and bounds the area of steady-state oscillations with period 2. Finally, the system displays irregular oscillations or unstable above line 3.
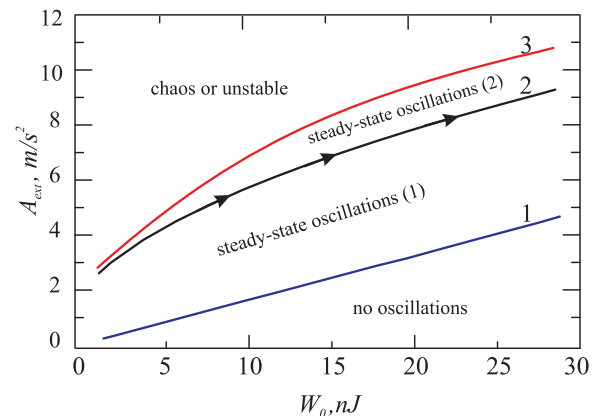
TABLE I. Parameters of the system.

m	200×10^{-6} kg
b	1.41×10^{-3} Nsm^{-1}
k	$80 - 300$ Nm^{-1}
d	$10 - 200 \times 10^{-6}$ m
S	10×10^{-4} m²
W_0	$1 - 320 \times 10^{-9}$ J
A_{ext}	$0.2 - 32$ ms^{-2}

FIG. 1. Plane of parameter (W_0, A_{ext}) where the different areas correspond to different regimes displayed by the system. Line 1 is the onset (necessary condition) of steady-state oscillations, line 2 is the doubling bifurcation, and line 3 is transition to irregular motion and chaos.

The optimal regime of the harvester is reached if it operates around the *optimal* line, i.e., around line 2 in the plane of control parameters that correspond to the period doubling bifurcation. Indeed, if one fixes energy W_0 and starts increasing A_{ext}, slicing the plane of parameters in Fig. 1 in the vertical direction, power converted by the harvester will only increase until line 2 is reached.

We now study the power converted by the e-VEH, which is one of the major characteristics of the system. The goal of this analysis is to find the optimal pair of parameters (A_{ext}, W_0) globally optimising converted power. In constant-charge mode, converted power is expressed as

$$P = W_0 \left(\frac{C_{max}}{C_{min}} - 1 \right) f_{ext}, \tag{6}$$

where C_{max} and C_{min} are the capacitances corresponding to the maximal and minimal displacements in one cycle and f_{ext} is the vibration frequency. In our case, this frequency defines the frequency of the variation of the transducer capacitance.

For normalisation purposes, we introduce a quantity,

$$P_0 = d^2 m \omega_0^2 f_{ext}, \tag{7}$$

that has the dimension of power and is a constant for a particular design of the device. Now let us rearrange expression (6) using the definition of the normalised coefficient $\nu_0 = W_0/(d^2 m \omega_0^2)$ and the normalised power $\Pi = P/P_0$

$$\Pi = \frac{P}{P_0} = \nu_0 \left(\frac{1 - y_{min}}{1 - y_{max}} - 1 \right), \tag{8}$$

where we used the expressions $C_{max} = C_0/(1 - y_{max})$ and $C_{min} = C_0/(1 - y_{min})$. The capacitance at rest state is $C_0 = \varepsilon_0 S/d$, where ε_0 is the vacuum permittivity. The normalised power Π is a dimensionless quantity that shows what energy is converted by the devices with respect to power P_0.

Our aim is to study this normalised power Π as function of the control parameters A_{ext}, restricting W_0 to the values that belong only to the optimal curve. Therefore, in order to determine whether a peak value exists among the values of Π, we will study $\Pi(A_{ext,opt})$ by moving along the optimal line as is shown by the arrows in Fig. 1.

The normalised power Π as a function of $A_{ext,opt}$ is shown in Fig. 2 for the spring constant $k = 80$ N/m (note that the role of this parameter is to define the natural frequency of the resonator). The plot contains three different lines that correspond to three rest gaps of the transducer ($d = 15$, $d = 20$, and $d = 25$ μm). The solid lines are obtained by employing a theory developed in Refs. 11 and 12, while the circles represent the data that is obtained solely from numerical simulations and is given for a comparison with the theory. In numerical simulations, the bifurcation points, y_{max} and y_{min}, for formula (6) were obtained by solving the original nonlinear equation (1).

Each line displays a peak of the normalised power Π at certain A_{max}. This peak has the same value $\Pi_{max} \approx 0.324$ regardless of the gap d. Only three values of d are presented in the plot, however, in simulations we obtain the same peak for a wide range of $d = 10$–200 μm, and selected results
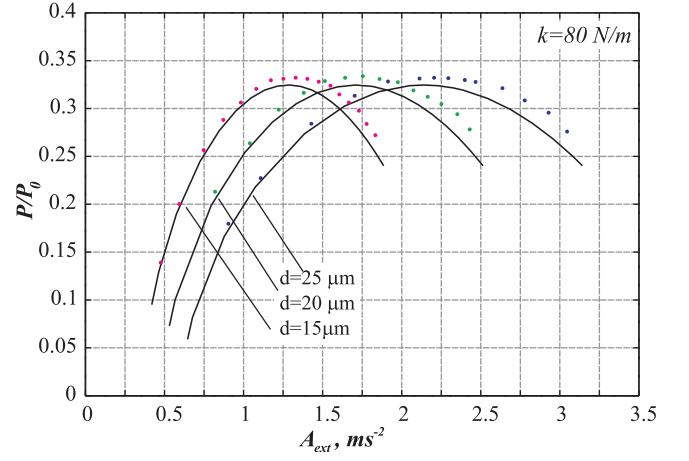

FIG. 2. Normalised power Π as a function of the acceleration amplitude $A_{ext,opt}$ belonging to the optimal line. The spring constant is $k = 80$ N/m and the three different lines correspond to different rest gaps $d = 15$, $d = 20$, and $d = 25$ μm. The circles show the normalised power calculated from the numerical simulations for comparison with theoretical results.

are given in Table II. For each d, the table gives the maximum Π and corresponding parameters: optimal (W_0, A_{ext}), displacement range and maximal voltage V_{max} across the transducer. The latter is an important parameter since it indicates the voltage that the conditioning circuit must support, which is in practice limited to 100 V.

The important result is that the same pattern is displayed for different k. The same peak value $\Pi_{max} \approx 0.324$ is also reached for $k = 150$ N/m and $k = 300$ N/m and we conclude that it is a universal constant for this system. A set of three curves corresponding to various d for $k = 300$ N/m are shown in Fig. 3. The maximum power that can be converted by the harvester is

$$P_{max} = \Pi_{max} \times P_0. \tag{9}$$

Let us compare this value with the limit obtained in Ref. 17. In the latter reference, the maximum possible convertible power (the limit power) with a resonator of the mass m, submitted to a sinusoidal external acceleration with the amplitude and the frequency A_{ext} and ω_{ext} and employing an ideal electromechanical transducer is given by

TABLE II. Peak values of the normalised power Π versus the rest gap d and corresponding values of the other system parameters.

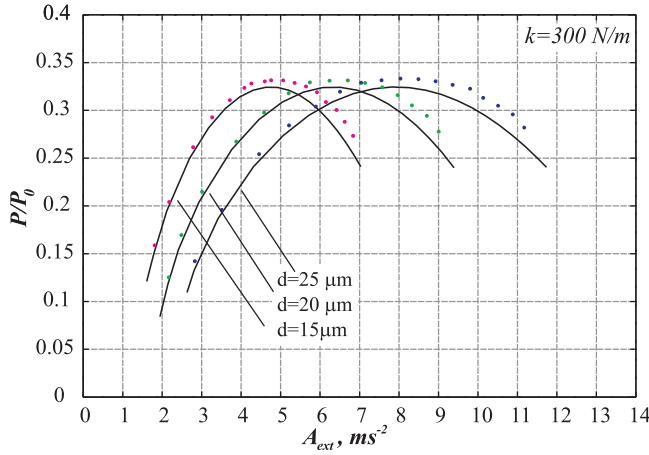| d [μm]	Π_{max}	W_0 [nJ]	A_{ext} [ms^{-2}]	y_{min}	y_{max}	V_{max} [V]
			$k = 80$ N/m			
20	0.325	3.5	1.71	-0.339	0.662	14.59
40	0.324	13.5	3.36	-0.351	0.668	28.79
60	0.324	31.5	5.12	-0.340	0.662	43.78
80	0.324	55.5	6.80	-0.342	0.664	58.18
100	0.324	88.5	8.58	-0.335	0.661	73.29
			$k = 300$ N/m			
20	0.324	13.1	6.33	-0.339	0.662	28.26
40	0.324	52.5	12.65	-0.339	0.662	56.52
60	0.324	122.5	19.31	-0.327	0.656	85.95
80	0.324	210	25.31	-0.339	0.662	113.06
100	0.324	320	31.26	-0.347	0.666	139.99

FIG. 3. Normalised power Π as a function of the acceleration amplitude $A_{ext,opt}$ belonging to the optimal line. The spring constant is $k = 300$ N/m.

$$P_{lim} = \frac{1}{2}\omega_{ext} m X_{lim} A_{ext}, \tag{10}$$

where X_{lim} gives the maximum range of the resonator motion $-X_{lim} < x(t) < X_{lim}$. For gap-closing transducers, X_{lim} cannot exceed the gap d, it is generally assumed for an estimation that $X_{lim} = d$. Expressing P_{max} through P_{lim}, one obtains that

$$P_{max} = \frac{\Pi_{max}}{\pi}\frac{d}{X_{lim}} \times \frac{d\omega_0^2}{A_{ext}} P_{lim}. \tag{11}$$

In order to give a numerical estimation, one should calculate the ratio $d\omega_0^2/A_{ext}$ when the maximum Π_{max} is reached. In particular, from Table II one can see that this ratio is constant $\approx 4.72 \pm 0.06$ for all k and d. Substituting Π_{max} and $d\omega_0^2/A_{ext}$ into formula (11), we obtain that $P_{max} \approx 0.49 P_{lim}$. This is the absolute upper limit of the power that a resonant e-VEH with the gap-closing transducer can convert for optimal values of the control parameters.

Since the definition of X_{lim} is somewhat arbitrary, P_{lim} should be seen as a figure of merit for a given harvester geometry rather than an achievable power limit. In our case, as is shown by Table II, the actual range of the resonator motion is $2X_{lim} = d(y_{max} - y_{min}) \approx d$. Based on that, we assume that $X_{lim} = d/2$ in Eq. (11) and obtain a more realistic estimation maximal power: $P_{max} \approx 0.98 P_{lim}$. This result

indicates that the harvester converts almost maximal possible power when operating in the optimal regime.

The presented study provides designers with a simple and comprehensive tool necessary to optimise the configuration of e-VEHs of the considered type. It is remarkable that the limit of the regular behaviour is strongly linked with the fundamental upper limit of the power convertible with a given resonator. The results suggest a use of smart adaptive conditioning circuit able to dynamically track the optimal operating mode.

[1]S. Meninger, J. O. Mur-Miranda, R. Amirtharajah, A. Chandrakasan, and J. H. Lang, IEEE Trans. Very Large Scale Integr. (VLSI) Syst. **9**, 64 (2001).
[2]S. P. Beeby, M. J. Tudor, and N. M. White, Meas. Sci. Technol. **17**, R175 (2006).
[3]E. Torres and G. Rincón-Mora, IEEE Trans. Circuits Syst., I: Regul. Pap. **56**, 1938 (2009).
[4]Y. Naruse, N. Matsubara, K. Mabuchi, M. Izumi, and S. Suzuki, J. Micromech. Microeng. **19**, 094002 (2009).
[5]P. D. Mitcheson, E. M. Yeatman, G. K. Rao, A. S. Holmes, and T. C. Green, Proc. IEEE **96**, 1457 (2008).
[6]L. Gammaitoni, I. Neri, and H. Vocca, Appl. Phys. Lett. **94**, 164102 (2009).
[7]F. Cottone, H. Vocca, and L. Gammaitoni, Phys. Rev. Lett. **102**, 080601 (2009).
[8]A. Erturk, J. Hoffmann, and D. J. Inman, Appl. Phys. Lett. **94**, 254102 (2009).
[9]L. Gammaitoni, Contemp. Phys. **53**, 119 (2012).
[10]D. Galayko, R. Guillemet, A. Dudka, and P. Basset, in *Proceedings of International Conference on Solid-State Sensors, Actuators and Microsystems* (TRANSDUCERS) (IEEE, 2011), p. 2382.
[11]E. Blokhina, D. Galayko, P. Basset, and O. Feely, "Steady-state oscillations in resonant electrostatic vibration energy harvesters," IEEE Trans. Circuits Syst., I: Regul. Pap. (in press).
[12]E. Blokhina, D. Galayko, R. Wade, P. Basset, and O. Feely, in *Proceedings of IEEE International Symposium on Circuits and Systems* (IEEE, 2012), p. 397.
[13]D. Galayko and P. Basset, IEEE Trans. Circuits Syst., I: Regul. Pap. **58**, 299 (2011).
[14]B. C. Yen and J. H. Lang, IEEE Trans. Circuits Syst., I: Regul. Pap. **53**, 288 (2006).
[15]J. Miranda, Ph.D. dissertation, MIT, 2003.
[16]A. Nayfeh, *Introduction to Perturbation Techniques* (Wiley, 1993).
[17]P. Basset, D. Galayko, A. M. Paracha, F. Marty, A. Dudka, and T. Bourouina, J. Micromech. Microeng. **19**, 115025 (2009).