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Correcting a magnetic field map through the alignment of tracks

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Abstract

A magnetic spectrometer is made of different track detectors at some nominal positions, immersed in a magnetic field. In addition to direct measurements, their actual positions are usually refined *a posteriori* through an alignment procedure exploiting the fact that tracks at different positions with different directions have different sensitivities to translations and rotations of the detectors. The sensitivity of the trajectories to variations of the field suggests the possibility to obtain an estimation of small discrepancies of the actual field map from the nominal one, exploiting the fact that tracks with different momenta undergo different deviations. Here a basis of field corrections is built with polynomials of increasing degree, satisfying the Maxwell equations, and the dependence of the trajectories on these corrections is linearized. Then the coefficients of the corrections are included as free parameters in an alignment procedure, in addition to the usual geometrical degrees of freedom. Due to the degeneracy between field scale and momentum scale, the corrected field map needs to be globally normalized, either by using reliable measurements at some positions, or by tuning the equivalent mass of pairs of particles from identified decays. A simple model of a forward magnetic spectrometer is built to evaluate in a realistic context the efficiency of the method in terms of systematic and statistical uncertainties. Possible generalizations are discussed.

Keywords: magnetic field, tracking, alignment, correction

1. Introduction

Most colliders or fixed target experiments include a magnetic spectrometer, made of several tracking devices inserted within or around a magnetic field, which provides a measurement of the particle momenta. A precise knowledge
5 of the positions of the various subdetectors and of the field map is needed to achieve the best possible precision on the physical quantities of interest for charged particles, especially momenta and impact parameters. Hardware devices provide external measurements, but a software alignment procedure is usually operated to adjust the geometrical description, using samples of tracks
10 of charged particles going through the spectrometer. In this study we examine the possibility to go further and to exploit the sensitivity of the trajectories to a distortion of the magnetic field with respect to the nominal map, by including additional degrees of freedom in the alignment parameters. The underlying idea is the following: the standard alignment procedures use samples of tracks
15 with different positions and directions to give constraints on the geometry of the detector, considered as a set of rigid bodies; in addition, using tracks with different momenta gives constraints on the field, because the variations of curvature cannot be completely absorbed in rotations and translations of the subdetectors.

20 As an example we consider a simplified description of the spectrometer: an upstream and a downstream detector, each one measuring several points on the tracks, surrounding a magnetic region, as illustrated in Fig.1. Both detectors are supposed to be internally aligned, that is, the only geometric degrees of freedom introduced in the algorithm describe their relative position. Such a model gives
25 a realistic estimation of the precision that can be expected in real conditions, in combination with the more complete geometric alignment procedures which have been developed in real configurations, especially to cope with so called “weak modes” (for example using kinematical and/or vertex constraints as exploited in [1]).

30 In Sect.2 we give a general description of the alignment procedure and the

Figure 1: Layout of the model of spectrometer. The field lines are produced by two magnetic dipoles (see Sect.4); for convenience they are represented in yz projection, while the bended trajectories are drawn in xz projection.

subsequent field normalization. In Sect.3 we discuss the notion of *field corrections* and we present two ways of building a basis of polynomial functions of x, y, z satisfying the Maxwell equations, with definite parities in x, y, z . In Sect.4 we apply the formalism to the simple model of forward spectrometer
35 with the layout defined above. In that case, the geometric parameters are the relative position and orientation of the second one with respect to the first one (3 translations+3 rotations), and the magnetic ones are the coefficients of the linear combination of corrections. In Sect.6 we evaluate the systematic errors (due to the limited number of correcting terms) and the statistical ones (going
40 as $1/\sqrt{N}$, N being the number of tracks used). We show that the normalization may be achieved using kinematical constraints (here: the decay of neutral particles), and we discuss some possible residual biases. In Sect.7 we sketch out possible extensions of the formalism to actual spectrometer layouts.

The aim of this paper is to point out the possibility to obtain corrections, not
45 to optimize the procedure in a given real detector configuration. For example, the sensitivity of trajectories to field modifications is intrinsically more complex than the effect of geometrical displacements, and evaluating them requires much more computations. In this study, we use systematically the Runge-Kutta method at order 4, with a step length of 20 cm along the z axis, which takes
50 about 20 microseconds per track following on a MacBook Pro (2.6 GHz Intel Core); some simplifications may be obtained, for example through parametrized extrapolations, but the optimization is specific to each experiment, and no general estimation of the computing load can be done. In any case, correcting the field map is not supposed to be repeated as often as making a geometrical
55 alignment.

2. Principle of the procedure

Within a linear approximation around a reference trajectory (that may be achieved after iterations), the least squares fit of a trajectory provides a χ_{min}^2 which is a quadratic function of the measurements. This remains true when

60 using a Kalman Filter [5]. The principle of the alignment procedure is to introduce free parameters (corrections supposed to be small) which affect the measurements, and to adjust them in order to minimize the sum χ_{glob}^2 of the χ_{min}^2 of a large sample of tracks. If the effect of these corrections on the measurements may be linearized, χ_{glob}^2 is a quadratic function of both the measurements and
65 the correction parameters, and its minimization provides values through a set of linear equations. From this point of view, the magnetic corrections will be handled in the same way as the geometric ones, except that the dependence of measurements on such corrections is more complex.

However, it is clear that the field cannot be unambiguously defined by such a
70 procedure, whatever the level of correction: a multiplication of the field components by a uniform factor is exactly compensated by the same factor applied to all momenta of the particles in the sample. As a consequence, although there is no degeneracy in the alignment fit, the result needs to be normalized. This may be done by using reliable and precise external measurement at some point(s), or
75 kinematical constraints on the sample of tracks itself, for example by adjusting invariant masses of pairs of particles from identified decays.

3. Description of field corrections

3.1. General considerations on different options

Dealing with corrections refers usually to a *hierarchy* (successive orders, producing effects of decreasing order of magnitude). Intuitively it should refer to
80 the amplitude of the modifications on position and direction for particles going through the field region, which are at first order integrals of the field components over the trajectories, divided by the signed momentum. From this point of view, if we assume that the deviation of the real field from the nominal one may be
85 Taylor-expanded in powers of the coordinates, a natural choice is polynomials of x, y, z , ordered by degree. It was shown in [2] that triplets of polynomials for B_x, B_y, B_z can be selected to obey the Maxwell equations.

An advantage of polynomials is that the choice of the origin of coordinates is arbitrary, as far as one considers the space generated by *all* polynomials up to a given degree. If there is *a priori* a symmetry expressed as parities in x , y , z , or several coordinates, the space may be restricted to polynomials obeying this symmetry.

Polynomials suffer a drawback: they diverge for large values of the coordinates. But for our purpose, they do not need to be extrapolated outside the region actually covered by the tracks: our aim is to obtain effective corrections on the possible trajectories, not an accurate map over the whole field region.

In some configurations, the combinations proposed in [3] (products of trigonometric and hyperbolic functions) may be useful: for example $\cos(\alpha x) \cos(\beta y) \cosh(\gamma z)$ (or any function obtained by replacing some 'cos' by 'sin') is harmonic if $\alpha^2 + \beta^2 = \gamma^2$. Such functions diverge strongly at large $|z|$, so they should be applied into a well delimited domain in z .

With the two options described above, the correction has to be restricted to a well defined region, and boundary conditions cannot be imposed. As a consequence, the corrections introduce discontinuities on the edges of this region, and a stepwise computation of trajectories (e.g. the Runge-Kutta method) needs to set a step exactly on the boundary to avoid numerical errors that would spoil the intrinsic precision of the method.

On the contrary, one can try to use over the whole space functions that go to zero at large distance, as real magnetic fields should do. For example, any distortion can be considered as a superposition of fields generated by magnetic dipoles (or higher multipoles); these fields satisfy the Maxwell equations by construction. A hierarchical basis of corrections may consist of 1,2,... dipoles, whose positions and magnitudes are free parameters. However, without a prior knowledge of the source of the discrepancy, many trials may be needed to find appropriate starting values. We do not explore here this option.

An alternative solution could be a Fourier expansion in trigonometric functions which remain finite over any domain. However a combinations of such functions cannot obey Maxwell equations, so they have to be used as effective

approximations (moreover they cannot obey boundary conditions). Adjusting
120 a field map by “physically impossible” terms may be a good empirical way to
correct the trajectories for field discrepancies that need not to be explicitly
known. An expansion at order n in x, y, z requires *a priori* $O(n^3)$ free param-
eters for each field component. We do not develop this approach further, but it
is clear that the formalism proposed below may be applied in a straightforward
125 way to any type of corrections, as long as the linear dependence of the deviation
to the free parameters remains valid.

3.2. Building harmonic polynomials

In a region without magnetic properties and without electric currents, the
field satisfies the Maxwell equations:

$$\nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \times \mathbf{B} = \mathbf{0}$$

Equivalently, $\mathbf{B}$ is the gradient of a pseudoscalar function Φ such that $\Delta\Phi = 0$
(*harmonic* function). So, finding triplets of polynomials of degree n to describe
 $B_x(x, y, z)$, $B_y(x, y, z)$ and $B_z(x, y, z)$ amounts to find harmonic polynomials
 $\Phi(x, y, z)$ of degree $n + 1$. This was done in [2], with explicit formulae. In
practice one can just build recursively homogeneous polynomials of degree d ,
written as

$$\sum_{i+j+k=d} C_{i,j,k} x^i y^j z^k$$

satisfying the relations:

$$(i+1)(i+2)C_{i+2,j,k} + (j+1)(j+2)C_{i,j+2,k} + (k+1)(k+2)C_{i,j,k+2} = 0 \quad (1)$$

For example this formula may be used to obtain the terms in z^{k+2} as functions of
130 the terms in z^k , that is, recursively, all terms starting from the ones with $k = 0$
and $k = 1$. Doing so, $d + 1$ polynomials are built starting from x^d , $x^{d-1}y$,
 $x^{d-2}y^2, \dots$, y^d , and d polynomials starting from $x^{d-1}z$, $x^{d-2}yz$, $x^{d-3}y^2z, \dots$,
 $y^{d-1}z$, deriving for each of them the coefficients of terms with a power of z
greater than 1; the result is listed in Table 1 up to $d = 5$. Such a method pro-
135 vides automatically families of defined parity in x, y and z , amongst the $2d + 1$

homogeneous independent harmonic polynomials of degree d . This property will be transmitted to the three components of the gradient: if Φ has parity $\Pi_x \Pi_y \Pi_z$ and $\mathbf{B} = \nabla \Phi$, then B_x has parity $(-\Pi_x) \Pi_y \Pi_z$, B_y has parity $\Pi_x (-\Pi_y) \Pi_z$ and B_z has parity $\Pi_x \Pi_y (-\Pi_z)$. So, if a symmetry of the field is guaranteed, the
140 corrections may be easily restricted to polynomials satisfying this symmetry. It is interesting to note that the space of harmonic polynomials of degree 1 to $d+1$ has dimension $(d+1)(d+3)$, while taking all combinations of B_x, B_y, B_z as polynomials of degree $\leq d$ would require $3 \times (d+1)(d+2)(d+3)/6$ coefficients: the constraint of Maxwell equations reduces the number of degrees of freedom
145 by a factor $1 + d/2$.

There is another way to obtain homogeneous harmonic polynomials, using the 3D functions based on spherical harmonics: $r^\ell Y_{\ell m}(\theta, \varphi)$, translated in cartesian coordinates, is an harmonic polynomial of x, y, z of degree ℓ . Following this remark, one can exploit the operator which increases or decreases the value of m for a given ℓ . Let us consider the operator D^- on complex functions:

$$D^- = (x - iy) \frac{\partial}{\partial z} - z \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right)$$

and translate this formula into an operation on a pair of real functions F and G : $D^-(F + iG) = F^- + iG^-$ with

$$\begin{aligned} F^- &= x \frac{\partial F}{\partial z} + y \frac{\partial G}{\partial z} - z \left(\frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} \right) \\ G^- &= x \frac{\partial G}{\partial z} - y \frac{\partial F}{\partial z} - z \left(\frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \right) \end{aligned}$$

This gives another recursive method to build a basis of harmonic polynomials, taking the real and imaginary parts of the complex polynomials, starting from:

$$r^\ell Y_\ell(\theta, \varphi) \propto (x + iy)^\ell = P_\ell(x, y, z) + iQ_\ell(x, y, z)$$

Here the operation D^- preserves the global parity $(-1)^\ell$, and reverses the parity in z . Applying k times the operation D^- to $(x + iy)^\ell$, we obtain the polynomials P_ℓ^{k-} and Q_ℓ^{k-} , which have definite parities in x, y, z , according to the following table:

parities of ℓ, k	++	+-	-+	--
parities in x, y, z of P_ℓ^{k-}	+++	-+-	-++	+-+
parities in x, y, z of Q_ℓ^{k-}	--+	+-+	+-+	---

Applying ℓ times the operator D^- to $r^\ell Y_{\ell\ell}$ gives $r^\ell Y_{\ell 0}$ which is real, so $Q_\ell^{\ell-} = 0$: the algorithm generates $\ell+1$ polynomials of type P and ℓ polynomials of type Q . The first ones are listed in Table 2. It is clear that for any degree and any combination of parities in x, y, z , these polynomials generate the same space of functions as the previous ones. It has been tested that the result of the fitting procedure described in Sect.5 is independent of the choice of the basis, within the rounding errors.

4. Modelling a simplified spectrometer

4.1. Track detectors

We take here a simplified example of a forward spectrometer, as shown in Fig.1; this configuration is similar to the layout of LHCb at CERN [4], but the method aims be applied to any configuration. The beam is along z axis. The trajectories may be defined by two functions $x(z)$ and $y(z)$, and the *state* of the trajectory on a given z -plane is defined as a 5-vector $S = (x, y, t_x, t_y, q/p)$ with $t_x = dx/dz$, $t_y = dy/dz$, p being the momentum and $q = \pm 1$ the sign of the charge. Both the upstream and the downstream detectors measure x or y , or a combination of x and y , or both coordinates, at several positions in z . The field region is essentially the interval between the upstream and the downstream detector; we ignore the magnetic effects outside this region (more precisely, we restrict the field corrections to this region). In the following, up to the end, the units are implicitly m for coordinates and lengths, T for magnetic field, T.m³ for magnetic moments.

In real detectors, the position and the direction of the particles extrapolated back to the origin is essentially determined by the upstream region, which includes a vertex detector. So the only parameter which is sensitive to the field is the momentum, which is determined by the combination of the upstream and

Figure 2: .

the downstream detectors. In this study they are supposed to provide at $z_u = 2$ and $z_d = 8$ uncorrelated measurements of x , y , t_x , t_y with gaussian errors as indicated in Fig.2, without a significant information on q/p . Moreover, to account

180 indicated in Fig.2, without a significant information on q/p . Moreover, to account for multiple scattering, a term equal to $10^{-4}/p(\text{GeV}/c)$ is added quadratically to the errors on t_x and t_y in both detectors (this corresponds to about 5% of radiation length). The sensitive region is defined by $|x/z| < 0.25$, $|y/z| < 0.25$ for the upstream detector, $|x/z| < 0.3$, $|y/z| < 0.25$ for the downstream one.

185 The relevant characteristics of the spectrometer are displayed on Fig.2.

4.2. Magnetic field

The nominal field is produced by two magnetic dipoles in the plane $z = z_0 = (z_u + z_d)/2$, in the y direction at 4 m above and below the xz plane,

each one with a magnetic moment 15 T.m^3 , such that the field in the central
190 region around the z axis has mainly a B_y component with a maximal value of
1 T, and an integral value of about 4 T.m along the z axis. In addition to this
nominal field, we consider a distortion with an amplitude below $\sim 10 \text{ mT}$ in the
region covered by the tracks. In practice, this distortion will be produced by
one or several additional dipole(s), at a distance of at least a few meters from
195 the acceptance region. We present results for two options:

1. **1-dip**: one dipole at position $(x = 1, y = 5, z = 4)$ with a magnetic
moment $\mathbf{M} = (0.1, -0.1, 0.3)$
2. **3-dip**: the previous dipole, a second one at $(2, -3, 11)$ with $\mathbf{M} = (0.1, -0.1, 0.3)$
and a third one at $(-3, 2, 0)$ with $\mathbf{M} = (0.2, -0.3, 0.1)$

200 We define a fiducial volume defined by $z_u < z < z_d, |x/z| < 0.3, |y/z| < 0.25$,
which contains most of the trajectories of the particles of physical interest. We
have checked that for any trajectory with $p > 2 \text{ GeV}/c$ contained within this
volume, the distortion of position and direction produced by the polynomials
found in Sect. 3 depend quasi linearly on the field modification in the range up
205 to a few 10 mT .

4.3. Sample of test trajectories

We consider here a sample of particles produced in a collider at the origin
(primary interaction) or at a displaced point (secondary interactions in mat-
ter) and going forward, crossing both detectors. In addition, we require the
210 trajectory to be fully contained within the fiducial region.

To avoid random fluctuations without generating a huge sample requiring
very heavy computations, we define a reduced *working sample* of trajectories
going in straight line from the origin to the upstream detector, in $N_x \times N_y$
directions t_x and t_y equally spaced into the interval $(-0.25, 0.25)$, with N_p
215 values of the momentum $(N_p/k)p_{min}$, and both signs of the charge. Secondary
tracks have a slightly different distribution in phase space, so they are expected
to carry a complementary information: we add a similar sample starting from

Figure 3: Relative error on momentum, obtained from Eq.2, which is equivalent to a Kalman Filter applied to tracks in the simplified detector model.

a displaced origin along the z axis, at $z = z_u/2$. The relative precision on the momentum achieved in this sample is of the order of 0.4% (see Fig.3).

220 Once the computations are done over the working sample, one can obtain a good simulation of any realistic distribution in the phase space through weighting methods, which reduce greatly the amount of computations needed. The regular distribution in $1/p$ corresponds roughly to a continuous distribution of density proportional to $1/p^2$; in addition we apply a weight depending on the
225 transverse momentum p_t as $\exp(-p_t/p_0)$. Our standard sample is built with $N_x = N_y = 10$, $N_p = 20$, $p_{min} = 3$ GeV/c and $p_0 = 0.5$ GeV/c. We have checked that larger values of N_x, N_y, N_p do not change significantly the results. To check the impact of taking a discrete and symmetric working sample instead of a random one, we have compared once the results obtained with a regular
230 sample built with $N_x = N_y = 50$, $N_p = 50$, giving a total weight about 50000, to the same sample where the values of p , t_x and t_y have been randomized by gaussian distributions (width 1% for p , 0.01 for t_x and t_y): the statistical errors differ by typically 10% or less. From that we conclude that the estimations using regular samples would be quite reliable for actual samples of 10^6 particles
235 or more.

With this approach we can obtain also an easy separation between the *sta-*

240 *tistical* errors, which scale as $1/\sqrt{N}$ (N being the number of tracks actually used
 to fit the corrections), and the *systematic* ones, which would be obtained in the
 ideal limit of infinite N . The statistical ones are given by the fitting formalism
 itself (see below). The systematic ones are due to the difference between the
 true field and the best correction at a given degree: they are obtained by ap-
 plying the fitting procedure to tracks without measurement errors, to mimic an
 infinite sample of tracks with independent measurement errors.

5. Formalism of alignment

245 5.1. Principle

We want to obtain a good approximation of the actual field $\mathbf{B}_{\text{nominal}} +$
 $\mathbf{B}_{\text{distort}}$, except for a scaling factor, by $\mathbf{B}_{\text{nominal}} + \sum_i \alpha_i \mathbf{B}_i^{\text{corr}}$ where the $\mathbf{B}_i^{\text{corr}}$ are
 the triplets of polynomials found in Sect. 3. In the absence of information on the
 distortion $\mathbf{B}_{\text{distort}}$, it is natural to introduce first corrections of low degree, and
 250 then higher degrees up to a satisfactory approximation: even if the distortion
 has a complex spatial structure, its effect on the trajectories is integrated over
 a long range, and in most cases it may be reproduced efficiently by a smoothed
 map.

It is important to note that, in the general case where $\mathbf{B}_{\text{nominal}}$ is not an
 255 exact combination of the corrections $\mathbf{B}_i^{\text{corr}}$, the result of the fit cannot be exactly
 proportional to $\mathbf{B}_{\text{nominal}}$, so the degeneracy of the scale is automatically solved
 by the procedure described below. In other terms, the field scale does not need
 to be defined *a priori*, it is a byproduct of the fitting procedure. However, the
 solution may need a normalization factor to better approximate the true field
 260 map. The nominal field used in our working example is not reducible to any
 polynomial; the (unlikely) case where it is nearly polynomial is examined in
 Sect.6.3.

There are correlations between magnetic distortions and detector displace-
 ments (this will be quantitatively established in the following). So we have to
 265 introduce geometric degrees of freedom in the procedure as well as the magnetic

corrections. Here we consider both detectors as solid bodies, without internal degrees of freedom (internal alignment is supposed to be already achieved), and we introduce $n_g = 6$ *geometric* parameters to define their relative position: three for translations $\mathcal{T}_x, \mathcal{T}_y, \mathcal{T}_z$ along x, y, z axes, and three for elementary rotations $\mathcal{R}_x, \mathcal{R}_y, \mathcal{R}_z$ around these axes. By convention we apply these operations to the downstream detector, and we take the point $(0, 0, z_d)$ as origin for the rotations.

5.2. Building the chi2 of a track

In order to shorten the expressions we introduce the following notation for a $(n \times n)$ symmetric matrix W and a n -vector S :

$$W[S] = S^T W S = \sum_{i,j} W_{ij} S_i S_j$$

In the same way, if A is a $(n \times p)$ matrix, we note $W[A] = A^T W A$. If S_u^m is the state measured in the upstream detector (as the result of a local fit within this detector), a local χ^2 may be written as a function of the local state $S(z_u)$:

$$\chi_u^2(S) = \chi_{u,\min}^2 + W_u[(S(z_u) - S_u^m)]$$

where W_u is the *weight matrix* of the local parameters. This expression may be used even if the local measurements do not define completely S_u . For example, in our model, the local fit provides x, y, t_x, t_y with uncorrelated errors, without information on q/p , so we can write:

$$W_u = \text{diag}(1/\sigma_x^2, 1/\sigma_y^2, 1/\sigma_{t_x}^2, 1/\sigma_{t_y}^2, 0)$$

so that $\chi_u^2(S)$ does not depend on q/p . In the following, we omit the constant term $\chi_{u,\min}^2$.

In most spectrometers the *linear approximation* around a reference trajectory may be used (after iteration if needed), that is, for small deviations from the reference state S^{ref} , the difference $\delta S_d = S_d - S_d^{\text{ref}}$ depends linearly on $\delta S_u = S_u - S_u^{\text{ref}}$ when extrapolating the trajectory from z_u to z_d ; this is expressed using the jacobian matrix $D_{u \rightarrow d}$: $\delta S_d = D_{u \rightarrow d} \delta S_u$. The jacobian matrix is computed

280 by numerical differentiation for each track of the working sample. In order to simplify the notations, we write S for δS and D for $D_{u \rightarrow d}$ in the following.

The expression of $\chi_u^2(S)$ may be rewritten in terms of states at z_d :

$$\chi_u^2(S(z_d)) = W_u[D^{-1}S(z_d) - S_u^m] = W_u'[S(z_d) - (S_u^m)']$$

with $W_u' = (D^{-1})^T W_u D^{-1}$ and $(S_u^m)' = DS_u^m$.

In the same way the measurement in the downstream detector gives a local χ^2 :

$$\chi_d^2(S(z_d)) = W_d[S(z_d) - S_d^m]$$

The measurements in the two detectors (including the multiple scattering errors) are independent, so we can build a global χ^2 by just adding the two contributions; replacing $S(z_d)$ by S for convenience, we obtain:

$$\chi^2(S) = W_u'[S - (S_u^m)'] + W_d[S - S_d^m] \quad (2)$$

Now we introduce the matrix of *sensitivity* A of the measured states on the vector α of correction parameters, to be evaluated for each track of the sample:

$$(S_d)_{\text{corr}} = S_d + A\alpha$$

The downstream measurement S_d^m is sensitive only to translations and rotations, according to the following table (we omit the fifth component q/p , which has no sensitivity to the corrections):

	x	y	t_x	t_y
T_x	-1	0	0	0
T_y	0	-1	0	0
T_z	t_x	t_y	0	0
R_x	$-yt_x$	$-yt_y$	$-t_x t_y$	$-1 - t_y^2$
R_y	xt_x	xt_y	$1 + t_x^2$	$t_x t_y$
R_z	$-y$	x	$-t_y$	t_x

290 The propagated downstream measurement $(S_u^m)'$ is sensitive only to magnetic corrections. The coefficients of sensitivity are computed numerically, by

applying for each correction additional terms to B_x, B_y, B_z . To obtain values within a few mT in the whole correcting region, we multiply by 10^{-3} the expressions in Table 1, x, y, z being replaced by $x/3, y/3, (z-5)/3$: these reduced coordinates are chosen such that their absolute value is less than 1 in the whole region, so that the absolute value of the polynomials never exceeds a few mT.

Finally the χ^2 of the track is a quadratic function of the components of the local state S at z_d and the correction parameters (with superscript g for the geometrical ones, b for the magnetic ones):

$$\chi^2(S) = W'_u[S + A^b\alpha^b - (S_u^m)'] + W_d[S + A^g\alpha^g - S_d^m] \quad (3)$$

5.3. Minimization of a global chi2

Here we use a procedure similar to the so called *Millepede* method [6]¹. For a sample of N measured tracks we have a global χ^2 to be minimized, as a sum over the tracks; when using a weighted sample, all W matrices in the following expressions are implicitly multiplied by the statistical weight associated to the track k ; the sum of the weights is normalized to N .

$$\chi_{\text{glob}}^2 = \sum_k ((W'_u)_k[S_k + A_k^b\alpha^b - (S_u^m)'_k] + (W_d)_k[S_k + A_k^g\alpha^g - (S_d^m)_k])$$

The free parameters are the n_g components of α^g , the n_b components of α^b and the $5N$ components of the track states S_k . Canceling the derivatives of χ_{glob}^2 gives two global systems of respectively n_b and n_g linear equations:

$$\begin{aligned} \left(\sum_k (W'_u)_k [A_k^b] \right) \alpha^b + \sum_k (A_k^b)^T (W'_u)_k S_k &= \sum_k (A_k^b)^T (W'_u)_k (S_u^m)'_k \\ \left(\sum_k (W_d)_k [A_k^g] \right) \alpha^g + \sum_k (A_k^g)^T (W_d)_k S_k &= \sum_k (A_k^g)^T (W_d)_k (S_d^m)_k \end{aligned}$$

and one local system of 5 equations for each track:

$$(W'_u)_k A_k^b \alpha^b + (W_d)_k A_k^g \alpha^g + ((W'_u)_k + (W_d)_k) S_k = (W'_u)_k (S_u^m)'_k + (W_d)_k (S_d^m)_k$$

¹Actually this way of reducing a problem with a few “global” parameters and many “local” ones was introduced earlier in [7] to fit a vertex with many tracks.

Here α^b and α^g are grouped into the vector α with $n_t = n_b + n_g$ components. We can define a $(n_t \times n_t)$ matrix $\mathcal{D}$, a n_t -vector $\mathcal{U}$ and a $(n_t \times 5)$ matrix $\mathcal{E}_k$ for each track:

$$\mathcal{D} = \begin{pmatrix} \sum_k (W'_u)_k [A_k^b] & 0 \\ 0 & \sum_k (W_d)_k [A_k^g] \end{pmatrix} \quad \mathcal{E}_k = \begin{pmatrix} (W'_u)_k A_k^b \\ (W_d)_k A_k^g \end{pmatrix}$$

$$\mathcal{U} = \begin{pmatrix} \sum_k (A_k^b)^T (W'_u)_k (S_u^m)'_k \\ \sum_k (A_k^g)^T (W_d)_k (S_d^m)_k \end{pmatrix}$$

With the notations:

$$\mathcal{F}_k = (W'_u)_k + (W_d)_k \quad \mathcal{V}_k = (W'_u)_k (S_u^m)'_k + (W_d)_k (S_d^m)_k$$

the global systems may be merged into one:

$$\mathcal{D} \alpha + \sum_k \mathcal{E}_k^T S_k = \mathcal{U} \quad (4)$$

and the local systems written as:

$$\mathcal{E}_k \alpha + \mathcal{F}_k S_k = \mathcal{V}_k \quad \rightarrow \quad S_k = \mathcal{F}_k^{-1} (\mathcal{V}_k - \mathcal{E}_k^T \alpha) \quad (5)$$

Substituting S_k by this expression for each k in the global system, we obtain a system of n_t linear equations which provides the fitted alignment coefficients

310 (α^b, α^g) :

$$(\mathcal{D} - \sum_k \mathcal{E}_k \mathcal{F}_k^{-1} \mathcal{E}_k^T) \alpha = \mathcal{U} - \sum_k \mathcal{E}_k \mathcal{F}_k^{-1} \mathcal{V}_k \quad (6)$$

The covariance matrix of α is:

$$\mathcal{C} = (\mathcal{D} - \sum_k \mathcal{E}_k \mathcal{F}_k^{-1} \mathcal{E}_k^T)^{-1}$$

As expected from the fact that the nominal field is not polynomial, the matrix $\mathcal{D} - \sum_k \mathcal{E}_k \mathcal{F}_k^{-1} \mathcal{E}_k^T$ is not singular and Eq.6 has a unique solution. Within the linear approximation, no iteration is needed. In the case where an iteration is really necessary (that is, if the nominal field is very different from the true one), it can be done with the same algorithm, by including the first order corrections into $\mathbf{B}_{\text{nominal}}$, and recomputing the dependence of the trajectories on the corrections. The field normalization can be postponed until the fit is satisfactory.

315

5.4. Combined fit with direct field measurements

The formalism above can be extended to include a set of direct independent measurements of the field components at some points $\mathbf{r}_k = (x_k, y_k, z_k)$ with an error σ_k^b , by introducing additional terms in the global χ^2 :

$$\chi_{\text{direct}}^2 = \sum_k (\mathbf{B}_{\text{fit}}(\mathbf{r}_k) - \mathbf{B}_{\text{meas}}(\mathbf{r}_k))^2 / (\sigma_k^b)^2$$

with

$$\mathbf{B}_{\text{fit}}(\mathbf{r}_k) = \mathbf{B}_{\text{nominal}}(\mathbf{r}_k) + \sum_{i=1}^{n_b} \alpha_i \mathbf{B}_i^{\text{corr}}(\mathbf{r}_k)$$

Minimizing the global χ^2 gives a system of equation similar to Eq.6, with additional terms to $\mathcal{D}$ and $\mathcal{U}$ for $1 \leq i, j \leq n_b$:

$$\mathcal{D}_{ij} \rightarrow \mathcal{D}_{ij} + \sum_k \mathbf{B}_i^{\text{corr}}(\mathbf{r}_k) \cdot \mathbf{B}_j^{\text{corr}}(\mathbf{r}_k) / (\sigma_k^b)^2$$

$$\mathcal{U}_i \rightarrow \mathcal{U}_i + \sum_k \mathbf{B}_i^{\text{corr}}(\mathbf{r}_k) \cdot (\mathbf{B}_{\text{meas}}(\mathbf{r}_k) - \mathbf{B}_{\text{nominal}}(\mathbf{r}_k)) / (\sigma_k^b)^2$$

320 These expressions have straightforward extensions if only one or two field components are measured, or if the measurement errors of are different, or even correlated.

If the measurements are precise and unbiased, they reduce strongly the possible bias of the global field scale.

325 6. Results on errors and field normalization

6.1. Correlations

The correlations between the different alignment parameters (translations, rotations and magnetic corrections) are plotted in Fig.4, for a fit using the polynomials derived from spherical harmonics up to degree 3 (as listed in Table 2), with an origin at (0,0,5) (middle point of the magnetic region). Due to the symmetry in x and y of the framework (nominal field + working sample), correlations appear only between corrections which have the same parity in x and y : this explains the empty diagonals in each rectangular subdivision of the

plot. There are correlations between geometric and magnetic corrections, so they should be fitted together. These correlations may be understood through geometrical considerations. For example, with $B_y > 0$, the positive/negative particles acquire in average a direction with a negative/positive x component: in both cases, an increase of B_z “pushes” them towards $y > 0$, producing a correlation with a translation $\mathcal{T}_y$.

Large correlations between magnetic corrections result in an increase of errors when increasing the degree of correction, for a given sample of tracks. On the other hand, moderate correlations between the geometric parameters and the magnetic ones preserve the level of precision on the former ones. This will be described in more detail in the following.

Weak modes appear in purely geometric alignment procedures with a magnetic field. The “curvature bias” that could be associated to the shearing of the downstream detector (as studied in [1]) does not occur here with our assumption of a perfect internal alignment (and anyway in our field configuration it would not be an absolutely weak mode when using tracks of different directions and momenta). Some purely geometric parameters are correlated in our model: $(\mathcal{T}_x, \mathcal{R}_y)$ and $(\mathcal{T}_y, \mathcal{R}_x)$. This may be understood through the divergence of trajectories from the z axis in the sample: rotating positively the downstream plane around the x axis results in decreasing/increasing $|y|$ on the upper/lower side, that is, in average, a negative translation in y ; with a correlation about 0.6 between $(\mathcal{T}_x$ and $\mathcal{R}_y)$, this cannot be considered as a weak mode. A similar effect exists for a rotation around the y axis, but in that case, with a field along y , the degree of freedom given to q/p for each track allows the global fit for a “tunable” translation in x : it can easier compensate this rotation by a global translation in x , and the correlation $(\mathcal{T}_y, \mathcal{R}_x)$ is much stronger ($\simeq 0.99$), so there is a combination of $(\mathcal{T}_y$ and $\mathcal{R}_x)$ which is a weak mode. In real conditions most of the available tracks are divergent, so this correlation has to be compensated by a large statistics or external constraints. We discuss in Sect.7 the possible combination of a large number of geometric degrees of freedom with magnetic corrections.

Figure 4: Correlations between the geometric and magnetic parameters of the global fit over the working sample of tracks. Open squares correspond to negative values.

365 6.2. Systematic and statistical errors

To summarize the systematic errors on the corrected field map, we show in Fig.5 the differences between the true field (including the distortion) and the ideally corrected one (with infinite statistics), at different degrees. A trajectory is not sensitive to the field component parallel to it, so it makes sense to plot
 370 these differences along straight lines from the origin, defined by the slopes t_x, t_y , in terms of the longitudinal component $B_l = (t_x B_x + t_y B_y + B_z) / \sqrt{1 + t_x^2 + t_y^2}$ and the transverse one $B_t = \sqrt{B_x^2 + B_y^2 + B_z^2 - B_l^2}$. Only low momenta are sensitive to the longitudinal component, so the alignment procedure is supposed to constrain mainly the transverse ones. On Fig.5 we distinguish 9 *central* lines
 375 ($t_x = -0.1, 0, 0.1$, $t_y = -0.1, 0, 0.1$) from 16 *peripheral* ones ($t_x = \pm 0.2$ or $t_y = \pm 0.2$). As expected, the precision is improved with increasing degree. It is interesting to note that the longitudinal component is practically as precise as the transverse one: this is an indirect consequence of the constraint of Maxwell equations.

380 By convention, the statistical errors are computed from a weighted sample equivalent to 10^6 tracks. As expected, they increase with the degree of the correction, essentially because of large correlations between some parameters. Fig.6 shows a comparison between the statistical errors and the systematic ones, for corrections of degree 0 to 6. Here the error is defined as the quadratic average
 385 of the deviation of the transverse component of the field along the trajectories of the working sample. In our configuration, the errors with 10^6 tracks have a similar order of magnitude at degree 4; with 10^9 tracks, a precision much better than 1 mT could be achieved with a correction at degree 6, giving a systematic error on momentum of a few 10^{-4} , much below the contribution of measurement
 390 errors.

The systematic and statistical errors on the geometrical parameters are shown in Fig.7 (some systematic errors are outside the bounds of the figure). These parameters are not strongly correlated to the magnetic ones, so the statistical errors do not increase rapidly with the degree of correction; in any case,

Figure 5: Deviations from true field (with the 1-dip distortion) along straight lines from origin. Dotted: nominal field without correction; solid: with correction. From top to bottom: correction up to degree 2, 4, 6.

Figure 6: Precision of the transverse component of the corrected field as a function of the degree. Solid symbols: systematic errors (disk for 1-dip, square for 3-dip); open circle: statistical error with a sample of 10^6 tracks.

395 they are small compared to the measurement precision in the downstream de-
 tector. The systematic errors decrease, as expected, with increasing degree of
 correction, but they are not negligible at intermediate degrees. This may induce
 systematic biases on the curvature, hence on q/p ; this effect will be discussed
 in Sect.6.4.

400 6.3. Normalization of the field

As mentioned above, the fitting procedure is not sensitive to a global scal-
 ing factor of the field. A possible way to determine the normalization from
 reconstructed invariant masses is illustrated in Fig.8. Here we assume that the
 direction of the particles at their production point is precisely defined by the up-
 stream detector, and is not affected by modifications of the field: so to compute
 405 the bias on the invariant mass, we take the true directions at the vertex and we
 account only for variations of the fitted momenta. In the ultrarelativistic limit,
 for a mother particle of mass M giving two daughter particles of the same mass
 $m \ll M$, the invariant mass is scaled by the same factor as the momenta: this
 410 is what we observe for the decay $J/\psi \rightarrow \mu^+ \mu^-$. For $K^0 \rightarrow \pi^+ \pi^-$, the factor is
 reduced and depends on the decay angle. In both cases, if an average scaling
 factor is computed from the fitted momenta of the working sample and then
 applied to the daughter particles of the decays, the bias on the invariant mass
 vanishes. Conversely, with real data, this factor may be tuned to suppress the
 415 bias in samples of decays.

Let us point out that the scaling correction is small because the distortion is
 small compared to the nominal field, but it is not expected to go systematically
 to zero for a fit of large degree. In particular, if $\mathbf{B}_{\text{nominal}}$ is nearly described
 by polynomials of degree d , a fit at degree d is not well constrained because
 420 a solution $\mathbf{B}_{\text{nominal}} + \delta\mathbf{B}$ is equivalent, for any scaling factor λ , to $\mathbf{B}_{\text{nominal}} +$
 $\delta\mathbf{B}_\lambda$ where $\delta\mathbf{B}_\lambda = (\lambda - 1)\mathbf{B}_{\text{nominal}} + \lambda\delta\mathbf{B}$ is a approximately a polynomial
 of degree d . In practice, applying the procedure to nominal fields which are
 nearly polynomial may result in large fitted corrections, such that the linear
 approximation is no longer valid. A possible remedy is to exploit the freedom

Figure 7: Errors on the geometric alignment parameters (translations and rotations) as a function of the degree. Top: systematic errors in 1-dip and 3-dip configurations; bottom: statistical errors.

Figure 8: Normalization of the field using invariant masses (here in the case of a fit of degree 4 on the 1-dip configuration). Top: bias on momenta (dotted: without field correction; solid: with field correction, before normalization; dashed: after scaling). Next: zoom on momentum bias after field correction. Bottom: bias on reconstructed invariant mass of decaying objects (solid: with field correction, before normalization; dashed: after field scaling).

425 on the global scale to set a constraint which prevents the fitted correction to
be large. For example, in our configuration, removing one of the first three
components of α^b from the list of free parameters is equivalent to constraining
one component of $\delta\mathbf{B}$ to vanish at the central point (0,0,5), so that the fitted
correction has the same order of magnitude as the actual distortion. Then the
430 normalization may be applied in the same way as described above.

6.4. Residual bias on curvature

At a given degree, we have seen that the residual distortion of the field is
associated to a bias on the free geometrical parameters. As a result, there may
be a systematic bias on the signed curvature. In our configuration the central
435 tracks undergo essentially a field along y : then a shift in x or a rotation around
 y axis of the downstream detector results in a systematic shift of q/p . This bias
is shown in Fig.9 for the tracks of the working sample with $t_x^2 + t_y^2 < 0.01$, with
a correction at degree 2 or 3 (at higher degrees, the average value of the bias is
small compared to the dispersion).

440 For decaying neutral particles, such an effect produces a bias on the recon-
structed invariant mass, depending on the decay angle in the mother frame.
An example is shown in Fig.10, for decays of J/ψ and K_S^0 of large momentum,
emitted at $t_x, t_y = \pm 0.02, \pm 0.06, \pm 0.1$, with 10 values of the decay angle θ^* in
the mother frame, averaging over the azimuthal angle, for a fit at degree 2 in
445 the 3-dip configuration. These biases cannot be directly converted into field
corrections, but they can provide *a posteriori* hints for momentum corrections
through a shift in q/p ; if the statistics of decays is large enough, corrections
depending on the direction may be set, at least in the central region for high
momentum tracks. For low momenta, a bias on q/p has less impact on invariant
450 masses.

6.5. Combination with direct measurements

Direct measurements may help to reduce the systematic errors from fits at
low degree, or statistical errors at high degree. To evaluate the order of magni-
tude of the gain that can be expected, we introduce unbiased measurements of

Figure 9: Systematic bias on q/p on the central tracks of the working sample. Circles: 1-dip configuration; squares: 3-dip configuration. Solid: degree 3; open: degree 2. An artificial horizontal shift has been introduced between series of points to distinguish the points corresponding to degree 3, which are by chance at the same level.

Figure 10: Systematic bias on invariant masses of decaying particles of momentum P , in the 3-dip configuration, at degree 2. Circles: $P = 50$ GeV/c; squares: $P = 30$ GeV/c. Solid: K_S^0 ; open: J/ψ .

455 B_x, B_y, B_z at 8 points in the central region ($x = \pm 1, y = \pm 1, z = 5 \pm 1$) with the same error σ^b . The results for the average statistical error on the transverse field (as defined in Sect.6.2) are given in the following table for a sample of 10^6 tracks:

degree	3	4	5	6
no direct measurement	0.198	0.319	0.895	1.420
$\sigma^b = 1$ mT	0.178	0.264	0.404	0.547
$\sigma^b = 0.5$ mT	0.148	0.209	0.293	0.380

460 The improvement is quite significant for degrees 5 and 6. However, if very large samples of tracks are available, a few direct measurements are not really useful from a statistical point of view. It could be better to use them as a check of consistency.

7. Possible extensions for real spectrometers

465 7.1. Complex subdetectors

In a full description of the spectrometer, S_u^m is the state obtained from a forward Kalman Filter (increasing z) including all detectors up to z_u , W_u being the inverse of its covariance matrix. In the same way, the downstream state is obtained from a backward Kalman Filter down to z_d . If the amount material
470 between z_u and z_d is not negligible the extrapolation can include an energy loss and a additional contribution to the multiple scattering error. The rest of the formalism is unchanged.

7.2. Detectors immersed in the field region

If the field is not negligible within the detectors, the sensitivity of each
475 track portion to the field corrections has to be evaluated, and included in Eq.3, through a contribution in $A^b \alpha^b$ in the first term, and another such contribution to be added to $A^g \alpha^g$ in the second term.

If really needed, the field region could be split in several parts, with independent correction coefficients. In such a case, the upstream and downstream
480 states could be sensitive to separate subsets of correction parameters.

If several detectors are immersed in the region where a significant field correction can be expected, it may be better to consider only one field region, to express the $\chi_{\min}^2$ of each track as a quadratic function of the measurements and to rewrite it as a quadratic function of the alignment parameters, using the sensitivities of each measurement to the corrections: in that way, one has to find the minimum of the sum $\sum \chi_{\min}^2$ over to sample of tracks, which is itself a quadratic function of the parameters.

7.3. Combination with many geometric degrees of freedom

There is no general method to perform a full magnetic+geometric alignment. A reasonable strategy could be to reserve for rare occurrences the magnetic corrections, which require large samples of tracks and heavy computations, assuming that the field is enough stable to ensure that the residual errors between two corrections are acceptable for the physics analyses. The frequent geometric alignments (including mechanical and kinematical constraints according to the state of the art, see for example [1], [8]) would be done with a fixed field map as usual. To “align” the field, an iterative procedure would be applied: fit the corrections with a reduced subset of geometric parameters (or even none), rescale the field, redo the complete geometric alignment, and so on. In any case, the procedure has to be tried and optimized within the framework of each experimental setup.

8. Summary and discussion

A formalism has been proposed to evaluate a correction to a magnetic field map within a spectrometer, using tracks made of two segments bracketting the region where the correction is wanted. The principle is similar to the usual method for a geometrical alignment (minimize a global χ^2 by adding degrees of freedom for displacements of detector elements), except that the new degrees of freedom correspond to a collection of elementary corrections of the field map $\mathbf{B}_i^{\text{corr}}(x, y, z)$. The method may be applied without iteration if the modifications of the trajectories depend linearly on the amplitudes of corrections, for

510 any combination of them; this condition is fulfilled if the discrepancy between
the nominal field and the actual one is small in relative value. Contrary to the
geometrical alignment, the dependences have no simple expressions: in most
field configurations, they should be computed by numerical differentiation for
each track of the sample, or by a dedicated parametrization. So, the procedure
515 is more time consuming than an usual geometrical alignment.

Here we have used a basis of corrections obeying the Maxwell equations by
construction, to reduce the number of adjustable parameters. In the absence of
prior knowledge of the shape of the distortion to be corrected for, polynomials
520 provide a neutral approach with the interesting possibility of increasing pro-
gressively the degree until the gain in χ^2 is counterbalanced by the increase of
the statistical errors on the fitted parameters; the balance depends on the size
of available sample of tracks and the computing power dedicated to this task.
Moreover one can build a basis with elements having definite symmetries. In
525 practice, one has to define a correction region covering the largest possible frac-
tion of the acceptance, and constrained by tracking detectors: these conditions
are fulfilled in spectrometers which make an optimal use of the bending power
of the magnetic field.

Different corrections may be more or less correlated, because they produce
530 similar effects on the trajectories of the sample. In addition there is a fun-
damental degeneracy of the problem: a global factor applied to all momenta
compensates exactly the same factor applied to the field. This degeneracy does
not affect the fitting procedure if the nominal map is not polynomial, but the
fitted field may have a biased scale. The scaling factor may be determined,
535 either by kinematical constraints (for example the mass of decaying objects), or
by a direct precise measurement of the field at one or a few points. Comparing
the normalization obtained by different constraints is a good way to validate the
procedure and the choice of the collection of corrections. Correlations also ex-
ist between field corrections and geometrical displacements. As a consequence,
540 the “alignment” of the magnetic field affects geometrical parameters as well;

some of them may be directly included in the procedure, other ones may be refitted after the field correction, in an iterative magnetic/geometric procedure. The best strategy has to be tuned in the particular conditions of the experiment.

545 This formalism was applied to a simplified example of a spectrometer made of two dipoles producing a nearly uniform field between an upstream and a downstream detector, each one measuring the position and the direction of one track element. It may be applied as such if these detectors have also some sensitivity to the momentum, provided the field correction region is restricted
550 to the gap between them; if not, it is possible to modify it by introducing additional terms in the dependences introduced in Sect.5.

To study the performance of the algorithm, a distortion of the field was introduced at the level of a few 10^{-3} in relative value. Increasing the degree of the polynomials results in a good convergence of the correction towards the
555 actual distortion. Various checks were done to evaluate the evolution of statistical errors with the degree of the polynomials, to validate the use of invariant masses to fix the global scale, and to study residual systematic effects for a given degree, for example, a systematic bias on the momentum.

In this implementation, the distortion of the field was a relatively smooth
560 function of the position in the region of interest. A more chaotic distortion would in principle need higher degrees to be described in detail; however what matters is the *effective* deviation integrated over the trajectories, which may be well reproduced with a lower degree. Further tests would be needed to confirm this assumption for specific configurations. Another limitation of this study is
565 the simplified simulation of the detectors. Hopefully it is supposed to contain the most important features of a spectrometer (including the effect of multiple scattering), especially for the momentum estimation; for example, introducing extended upstream and/or downstream detectors, with some marginal sensitivity to the curvature, would not change significantly the precision on momentum.

570

As a general rule, the systematic errors of the correction decrease with the

degree of the polynomials. However, it is probable that, as in our example, the statistical errors in an actual setup grow quasi exponentially with this degree. In principle, colliders may accumulate huge sample of tracks within short times,
575 but the computation load is roughly proportional to both the size of the sample and the number of free parameters ($d(d+2)$ up to degree d). So, the degree has to be tuned to only reduce the distortions of physical quantities to an acceptable level compared to other sources of errors. On the other hand, the tools to compute the perturbation of the trajectories by magnetic corrections
580 have to be carefully optimized.

Some attention should be paid to the composition of the training sample of tracks to be used to fit the correction. Particles of low momentum are more curved by the field, so they offer a good sensitivity; however they are more scattered in the material, so the trajectory has larger errors. In any case, un-
585 ambiguous tracks seen on both sides of the field region provide information. Tracks coming from secondary interactions in the material may be also useful if they are well reconstructed. However electrons and positrons undergoing a moderate radiative energy losses may pass the quality cuts and bias the curvature estimation: for a given configuration of the spectrometer, a specific study
590 is needed to determine whether a clean identification is needed.

The formalism may be extended to more complex configurations (forward or barrel), divided into several regions with independent corrections, embedding more than two subdetectors, with different samples of tracks covering the whole domain or a part of it.

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References

- [1] J. Amoraal et al., Nucl. Inst and Meth. A 712 (2013) 48

- 600 [2] H. Wind, J. of Computational Physics, Vol. 2, Issue 3 (1968) 274
- [3] H. Wind, Nucl. Inst and Meth., 84 (1970) 117
- [4] LHCb Collaboration, JINST 3 (2008) S08005
- [5] W. Hulsbergen, Nucl. Inst and Meth. A 600 (2009) 471
- [6] V. Blobel, Nucl. Inst and Meth. A 566 (2006) 5
- 605 [7] P. Billoir et al., Nucl. Inst and Meth. A 241 (1985) 115
- [8] LHCb Collaboration, Phys. Lett. B 708 (2012) 241
- LHCb Collaboration, Phys. Rev. Lett. 110 182001 (2013)

Appendix: Maxwell-compatible polynomials

Coefficients of polynomials for the pseudo-potential $\Phi(x, y, z)$ and the com-
 610 ponents of a field $\mathbf{B} = \nabla\Phi$ in a magnetically empty region, obtained by the
 two methods given in Sect.3.2. For the highest degrees, the expressions have
 been scaled by a numerical factor (the same for B_x, B_y, B_z) to simplify the
 writing. For each degree, the two bases of polynomials are mathematically
 equivalent (they generate the same subspace); the first method seems to give
 615 shorter expressions in general, so it is more efficient if the dependences have to
 be computed for a very large sample of tracks.

Φ	B_x	B_y	B_z
x	1	0	0
y	0	1	0
z	0	0	1
$x^2 - z^2$	x	0	$-z$
$y^2 - z^2$	0	y	$-z$
xz	z	0	x
yz	0	z	y
xy	y	x	0
$z(3x^2 - z^2)$	$2xz$	0	$x^2 - z^2$
$x(3z^2 - x^2)$	$z^2 - x^2$	0	$2xz$
$z(3y^2 - z^2)$	0	$2yz$	$y^2 - z^2$
$y(3z^2 - y^2)$	0	$z^2 - y^2$	$2yz$
$y(x^2 - z^2)$	$2xy$	$x^2 - z^2$	$-2yz$
$x(y^2 - z^2)$	$y^2 - z^2$	$2xy$	$-2xz$
xyz	yz	xz	xy
$x^4 - 6x^2z^2 + z^4$	$x(x^2 - 3z^2)$	0	$z(z^2 - 3x^2)$
$y^4 - 6y^2z^2 + z^4$	0	$y(y^2 - 3z^2)$	$z(z^2 - 3y^2)$
$xz(z^2 - x^2)$	$z(z^2 - 3x^2)$	0	$-x(x^2 - 3z^2)$
$3(x^2y^2 - x^2z^2 - y^2z^2) + z^4$	$3x(y^2 - z^2)$	$3y(x^2 - z^2)$	$z(2z^2 - 3x^2 - 3y^2)$
$zy(z^2 - y^2)$	0	$z(z^2 - 3y^2)$	$-y(y^2 - 3z^2)$
$yz(3x^2 - z^2)$	$6xyz$	$z(3x^2 - z^2)$	$3y(x^2 - z^2)$
$xz(3y^2 - z^2)$	$z(3y^2 - z^2)$	$6xyz$	$3x(y^2 - z^2)$
$xy(x^2 - 3z^2)$	$3y(x^2 - z^2)$	$x(x^2 - 3z^2)$	$-6xyz$
$xy(y^2 - 3z^2)$	$y(y^2 - 3z^2)$	$3x(y^2 - z^2)$	$-6xyz$

Table 1: Harmonic polynomials constructed by using Eq.1 for the pseudo-potential, and components of the gradient.

term	Φ	B_x	B_y	B_z
P_1	x	1	0	0
Q_1	y	0	1	0
P_1^-	z	0	0	1
P_2	$x^2 - y^2$	x	$-y$	0
Q_2	xy	y	x	0
P_2^-	xz	z	0	x
Q_2^-	yz	0	z	y
P_2^{2-}	$2z^2 - x^2 - y^2$	x	y	$-2z$
P_3	$x^3 - 3xy^2$	$x^2 - y^2$	$-2xy$	0
Q_3	$3x^2y - y^3$	$2xy$	$x^2 - y^2$	0
P_3^-	$z(x^2 - y^2)$	$2xz$	$-2yz$	$x^2 - y^2$
Q_3^-	xyz	yz	xz	xy
P_3^{2-}	$x(x^2 + y^2 - 4z^2)$	$3x^2 + y^2 - 4z^2$	$2xy$	$-8xz$
Q_3^{2-}	$y(x^2 + y^2 - 4z^2)$	$2xy$	$x^2 + 3y^2 - 4z^2$	$-8yz$
P_3^{3-}	$z(3x^2 + 3y^2 - 2z^2)$	$2xz$	$2yz$	$x^2 + y^2 - 2z^2$
P_4	$x^4 - 6x^2y^2 + y^4$	$x(x^2 - 3y^2)$	$y(y^2 - 3x^2)$	0
Q_4	$xy(x^2 - y^2)$	$y(3x^2 - y^2)$	$y(x^2 - 3y^2)$	0
P_4^-	$xz(x^2 - 3y^2)$	$3z(x^2 - y^2)$	$-6xyz$	$x(x^2 - 3y^2)$
Q_4^-	$yz(3x^2 - y^2)$	$6xyz$	$3z(x^2 - y^2)$	$y(3x^2 - y^2)$
P_4^{2-}	$y^4 - x^4 + 6z^2(x^2 - y^2)$	$x(3z^2 - x^2)$	$y(y^2 - 3z^2)$	$3z(x^2 - y^2)$
Q_4^{2-}	$xy(-x^2 - y^2 + 6z^2)$	$y(6z^2 - 3x^2 - y^2)$	$x(6z^2 - x^2 - 3y^2)$	$12xyz$
P_4^{3-}	$zx(3x^2 + 3y^2 - 4z^2)$	$z(9x^2 + 3y^2 - 4z^2)$	$6xyz$	$3x(x^2 + y^2 - 4z^2)$
Q_4^{3-}	$zy(3x^2 + 3y^2 - 4z^2)$	$6xyz$	$z(3x^2 + 9y^2 - 4z^2)$	$3y(x^2 + y^2 - 4z^2)$
P_4^{4-}	$3(x^4 + 3x^2y^2 + y^4)$	$3x(x^2 + y^2 - 4z^2)$	$3y(x^2 + y^2 - 4z^2)$	$4z(2z^2 - 3x^2 - 3y^2)$
	$-24z^2(x^2 + y^2) + 8z^4$			

Table 2: Polynomials deduced from the real and imaginary parts of the spherical harmonics, and components of the gradient.