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Analysis of the acoustic/hydrodynamic separation in an aeroacoustic 2D plane mixing layer flow

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A compressible two-dimensional spatially developing plane mixing layer flow is simulated to investigate the acoustic/hydrodynamic flow scale separation. Once both the acoustic and the hydrodynamic pressure fields are clearly identified in the flow, a novel methodology is implemented in order to separate these two contributions. This method relies on a coupling of two complementary tools: Fast Fourier Transform and Proper Orthogonal Decomposition. The first step consists in applying the FFT to the available space-time pressure field. Then in the wavenumber-frequency domain, a 2D area is extracted. This area corresponds to the acoustic region where the signal is propagated at the speed of sound. However, the extracted low-frequency, low-wavenumber part contains also a hydrodynamic part which is more energetic than the acoustic part. Therefore, the second step proposes to perform a POD decomposition of the FFT filtered pressure field in order to extract the low energetic acoustic pressure field embedded in the aeroacoustic pressure field. The method is validated by analyzing the properties of the acoustic field which has the correct spectral content, the best correlation with the far acoustic pressure field and a nearly gaussian distribution.

Nomenclature

f	Frequency	δ_{ω_0}	Initial vorticity thickness
k_x	Streamwise wavenumber	δ_ω	Local vorticity thickness
c	Speed of sound	τ	Time delay
c_h	Apparent horizontal speed of sound	$\Delta U = U_2 - U_1$	Velocity difference
U_1	Low stream velocity	$U = (U_2 - U_1) / 2$	Convection velocity
U_2	High stream velocity	$S_t = f\delta_{\omega_0} / \Delta U$	Strouhal number
POD	Proper Orthogonal Decomposition	$(.)_{acou}$	Acoustic fluctuation
FFT	Fast Fourier Transform	$(.)_{hydro}$	Hydrodynamic fluctuation

I. Introduction

Aeroacoustic flow field is of multi-scales nature and generally needs to be decomposed for educing either the mechanisms at the origin of the noise or the acoustic part embedded in the flow, as for instance, in a turbulent jet.¹⁻³ Also, the investigation of the aeroacoustic wall pressure field impacting an elastic structure that radiates some noise, needs to consider which wall pressure component (acoustic and/or hydrodynamic) contributes mainly to the noise.^{4,5} The separation of the acoustic and hydrodynamic components of an aeroacoustic flow field remains a crucial scientific challenge.

A first distinction between the two components which needs to be made, concerns their propagation velocities. The hydrodynamic part is convected by the flow while the acoustic part propagates at the speed of sound. Thus, a Fourier filtering technique can be used to separate both components by taking into account the different phase velocities.^{2,4} However, such a filtering procedure may not be appropriate in the low frequency-low wavenumber domain.^{4,5} Indeed, the wavenumber resolution and the frequency discretization are generally not sufficient to properly distinguish both components in the low wavenumber-low frequency domain. Furthermore, the Fourier filtering approach forgets any temporally localized acoustic event in the signals, which could limit such filtering application. To remediate to such

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a limitation, the wavelet technique was also tested and validated using an experimental jet noise database.^{3,6,7} This technique allows for the extraction of the pseudo-sound component by using a proper thresholding procedure for selecting the associated 'acoustic' wavelet coefficients.

The second difference to be emphasized between the two components is in their energy content. Indeed, the acoustic part is of very low energy content compared to the energy level of the hydrodynamic component. Thus, the Proper Orthogonal Decomposition (POD) which is based on a energy optimality decomposition has been tested for such a separation.^{5,8} It was observed that the first POD modes acts as a filtering technique in the wavenumber space (that contains the main part of the signal energy) but can not properly separate both components especially in the low frequency-low wavenumber domain.⁸

In order to tackle down the above limits, an original coupling of both POD and FFT filtering methods allowing the acoustic-hydrodynamic separation in the whole wavenumber-frequency domain is proposed. The aim of the present research is then to evaluate the effectiveness of the Fourier and POD mathematical tools to effectively extract the acoustic component embedded in an aeroacoustic flow-field.

As a preliminary investigation, a canonical flow configuration is considered: the two-dimensional spatially developing isothermal compressible plane mixing layer. The numerical simulation of this case including the acoustic far field allows then a direct investigation of the effectiveness of the mathematical post-processing tools.

II. Numerical solver and flow parameters

In the present investigation, a 2D spatially developing plane mixing-layer is considered in a cartesian frame of reference (Oxy , where x corresponds to the streamwise propagation direction). This flow is obtained from the mixing of two parallel streams ($U_2 = 0.5c$ and $U_1 = 0.25c$ where c is the speed of sound, and the indices 1 and 2 refer to the bottom and top flows respectively). These flow parameters are similar as the ones considered in previous studies.⁹

The compressible mixing layer has been simulated using a 2D DNS numerical solver previously developed.¹⁰ This code solves the pseudo-characteristic formulation of the 2D Navier-Stokes equations.¹¹ Such a formulation allows an easier prescription of the outflow and inflow conditions.¹⁰ To enforce both numerical stability and accuracy, a fourth-order accurate upwind DRP scheme is used to evaluate all spatial derivatives except near the computational domain boundaries.¹⁰ Equations are integrated in time using a third-order TVD Runge-Kutta scheme.

Instantaneous inflow conditions are generated at the left boundary of the computational domain ($x = 0$). Based on the pseudo characteristic formulation, only the flow derivatives have to be imposed. Thus, a gaussian white noise of very low amplitude (10^{-3}) is imposed to the inflow streamwise velocity component. This inflow condition differs from those generally based on forced disturbances.^{9,12,13} It allows to reproduce the random initial mixing flow issuing from the merging of the developing lower and upper flows around the splitter plate separating both streams. With such an inflow condition, a transient region is observed, where the first instabilities are developed. The initial vorticity thickness δ_{ω_0} (at $x = 0$) is further considered as a characteristic scale.

Figure 1 displays an instantaneous fluctuating acoustic pressure field (far field) superimposed onto the vorticity field (near field). The colormap (figure 1-top) corresponds to the acoustic pressure levels. In this figure (bottom), a zoom view of the instantaneous vorticity field and of the associated instantaneous fluctuating hydrodynamic pressure field is also represented. These graphs emphasize the large differences in the energy levels of the hydrodynamic and far acoustic pressure fields. The end of the transient region is located at $x = 42\delta_{\omega_0}$.

The computational domain size under investigation (excluding sponge regions) is $(L_x, L_y) = ([0 : 294\delta_{\omega_0}], [-126\delta_{\omega_0} : 126\delta_{\omega_0}])$. This physical domain is discretized on a regular cartesian mesh of $(N_x \times N_y) = (2100 \times 1800)$ points.

Once the initial conditions are entirely evacuated, instantaneous flow variables are stored at some specific lines mentioned in figure 1:

- in the far acoustic field, $y = y_{a1} = -121\delta_{\omega_0}$ and $y = y_{a2} = 121\delta_{\omega_0}$ with $x > 42\delta_{\omega_0}$.
- along a y -line: $y = y_0 = -9\delta_{\omega_0}$, located in the near field ($42\delta_{\omega_0} < x < 125\delta_{\omega_0}$) and crossing the large scale flow structures ($x > 200\delta_{\omega_0}$).
- along the mixing layer flow, $y = y_{ml1}$ and $y = y_{ml2}$, with $x > 42\delta_{\omega_0}$. These last two lines correspond to the locations of $\pm\delta_{\omega}(x)/2$ along the x axis, $\delta_{\omega}(x)$ being the local vorticity thickness at the position x .

During the simulation, 16001 instantaneous pressure fields are stored every two time numerical steps, along each line described before. Such a time duration corresponds to the fluid passage of more than 35 vortex roll-up periods in the section $x = 240\delta_{\omega_0}$ and more than 90 vortex roll-up periods in the section $x = 90\delta_{\omega_0}$.

III. Spectral analysis of the flow field

To elucidate the spectral content of the mixing layer flow as well as the acoustic flow field, a spectral analysis is conducted. In the following, each spectrum is represented in dB scale with the conventional reference pressure of 2×10^{-5} Pa. All the numerical power spectral densities shown in the following are calculated using overlapping blocks coupled with a Hann windowing.

Figure 1. Top: Instantaneous vorticity field superimposed onto instantaneous fluctuating pressure in the far field. The colormap (expressed in Pa) is associated with the acoustic pressure. Bottom: Zoom view of the instantaneous vorticity field (left) and the associated fluctuating pressure field (right). These figures give details about the y -lines along with instantaneous flow variables are stored.

A. Frequency spectra in the mixing layer flow

Figure 2 presents the frequency spectra computed at selected streamwise x_{ml1} -positions along the y_{ml1} line. These spectra are represented as a function of the Strouhal number $S_t = f\delta_{\omega_0}/\Delta U$ where ΔU is the velocity difference $U_2 - U_1$. The spectra for the first locations exhibit a frequency peak corresponding to a Strouhal number of 0.08. The energy content of the spectra is more and more broadband further downstream in the flow where a low frequency content is observed. These spectra are similar to those previously observed in a similar 2D plane mixing layer flow¹⁴ where turbulent boundary layers developing around the splitter plate were used as inflow conditions.

B. Wavenumber-frequency spectra in the far field

It was first checked that the pressure intensity decays as $1/r$ (r being the radius distance from the mixing layer center) in the far acoustic field. This confirms the far-field acoustic behavior. The sound radiation is studied by computed the wavenumber frequency spectra of the far acoustic pressure field. Figure 3 ((a) and (b)) displays the spectrum in the (k_x, f) space at the lines $y = y_{a1}$ and $y = y_{a2}$, denoted $\hat{p}(k_x, f, y_{a1})$ and $\hat{p}(k_x, f, y_{a2})$ respectively. These spectra are represented as a function of $k_x\delta_{\omega_0}$ (X-axis) and the Strouhal number S_t (Y-axis). Figure 4 ((a) and (b)) also represents two associated wavenumber spectra for selected Strouhal numbers: $S_t = 0.12$ and $S_t = 0.61$.

Such a representation allows for the determination of the horizontal apparent speed of sound, $c_h = 2\pi f_h/k_h$, with k_h the wavenumber where the wavenumber frequency spectrum reaches its maximum at the given f_h frequency. Then, by taking into account the convective velocity (U_1 or U_2 as a function of region under interest), one retrieves the projection of the speed of sound onto the horizontal line. As an illustration, figures 4-(a) and (b) show that the spectrum maxima (black circles) are obtained for $((S_t, k_x\delta_{\omega_0}) = (0.122, 0.085))$ and $((S_t, k_x\delta_{\omega_0}) = (0.122, 0.095))$ respectively. These values correspond to an horizontal apparent speed of sound of 747m.s^{-1} and 670m.s^{-1} . Taking into account the low (U_1) and high (U_2) velocity streams, these apparent sound velocities correspond to an emission angle of 48° in the upper part and 68° in the lower part. Similar emission angles are obtained from wavenumber spectra for $S_t = 0.61$ (green circles in figure 4-(a) and (b)). This enables to determine a global view of the main

Figure 2. Frequency spectrum along the mixing layer flow y_{ml1} .

emission angle of the acoustic waves emitted in the flow. In figures 3 (a) and (b), the area delimited by the two dotted lines corresponds to pressure waves propagated at the speed of sound. These waves are associated with the aeroacoustic sources emitted in the mixing layer flow.

C. Wavenumber-frequency spectra in the near field

Figure 3 ((c) and (d)) represents the wavenumber frequency spectra obtained from pressure fields stored along the y_0 and y_{ml1} lines (see figure 1) respectively. The associated wavenumber spectra obtained at two Strouhal numbers are also given in figure 4 ((c) and (d)). First, the hydrodynamic pressure field is of higher energy than the energy of the far acoustic pressure field. Second, we clearly identify the hydrodynamic component which corresponds to the spectral peaks and represented with some black and green square symbols. The convective velocities related to the spectral peaks are: i) 105m.s^{-1} and 88m.s^{-1} in the mid-lower stream part (figure 3 (c)) for $S_t = 0.12$ and $S_t = 0.61$ respectively.

ii) 124m.s^{-1} and 120m.s^{-1} along the mixing layer flow (figure 3 (d)) for $S_t = 0.12$ and $S_t = 0.61$ respectively. These values are in the interval $[U_1 : U_c]$, with $U_c = (U_1 + U_2)/2$ the convection velocity. This confirms that these spectral peaks correspond to the convective flow. Conversely, the acoustic peak (indicated with black and green circles in figure 4) is less pronounced than in previous analysis and even it is not visible when regarding the mixing layer spectrum along the y_{ml1} line (figures 3-(d) and 4-(d)). Indeed due to the high energy level of the hydrodynamic part, the acoustic contribution is entirely hidden in the flow field without any clear identification thanks to the FFT analysis.

IV. Hydrodynamic and Acoustic FFT-POD-based separation

In this section, the separation of the hydrodynamic and acoustic pressure fields embedded in the mixing layer flow is presented. First the extraction of the far acoustic pressure field is done. This acoustic pressure will be used to perform some future causality analyses allowing the validation of the separation methodology applied to the near-field aeroacoustic pressure. Second, the FFT-POD based separation methodology is detailed. Then, the application of this methodology is presented. Finally, the acoustic component extracted from the aeroacoustic pressure field stored along the mixing layer flow is statistically analyzed confirming the acoustic nature of this component.

A. FFT filtering of the far acoustic field

As a first step, the acoustic pressure field in the far field is extracted. For this, a wavenumber-frequency FFT filtering is performed (see figure 3 - (b)). Due to the symmetrical property of the frequency axis in the (f, k_x) domain, only the positive frequencies are retained to explain the methodology. Based on this spectral representation, a mask is created containing the flow field propagated at high velocities. Note that these velocities are not exactly the speed of sound added or subtracted to the fluid stream velocity U_1 . Indeed, they correspond to the apparent horizontal velocities detected along the line. To avoid any confusion for the rest of the paper, we denoted c_h the apparent horizontal velocity related to the propagated acoustic waves.

This mask corresponds then to the area delimited with the two dotted lines. Outside this area, the (f, k_x) spectrum is put to zero value. An inverse FFT is finally performed to retrieve the acoustic pressure field (denoted $p_{acou}(y_{a1}, x_{a1}, t)$) of the signal in the physical domain.

B. FFT-POD based separation methodology

In the following, the separation will be presented from pressure fields stored along the line $y_i = y_0$ and $y_i = y_{ml1}$. Then as stated before, 16001 instantaneous pressure fields are available on a regular mesh of 1800 points along the x -direction. The first 300 points localized in the transient flow region are not considered. Let us assume an

Figure 3. Wavenumber frequency spectrum computed along selected y -lines. (a): upper stream part, y_{a2} ; (b): lower stream part y_{a1} . (c): mid-lower stream part, y_0 ; (d): along the mixing layer flow y_{ml1} .

aeroacoustic pressure field denoted $p(y_i, x, t)$ containing both acoustic and hydrodynamic components. A similar procedure is performed and the steps involved in the *FFT – POD* filtering methodology are the following:

1. Apply a 2D Fourier Transform to compute the wavenumber frequency spectrum of the pressure field denoted $\hat{p}(y_i, k_x, f)$,
2. Extract a 2D zone in the $(k_x - f)$ spectral domain corresponding to the 'acoustic' region. This region is defined by a broadband signal traveling at least with the horizontal acoustic velocity, $c_h = 2\pi f/k_x$. The spectrum is then multiplied with the following filter: $H_c(f, k_x) = 1$ if $2\pi f/k_x > c_h$ and equal to 0 otherwise. This filter will keep all propagating acoustic waves whose propagation velocity is higher or equal to the velocity c_h . That leads to obtain the acoustic spectra: $\hat{p}_c(y_i, k_x, f) = \hat{p}(y_i, k_x, f) \times H_c(f, k_x)$. The remainder spectrum denoted $\hat{p}_r(y_i, k_x, f)$ is equal to $\hat{p}(y_i, k_x, f) - \hat{p}_c(y_i, k_x, f)$.

Note that for such FFT filtering application, a value slightly inferior to this apparent sound velocity, c_h is retained to avoid any disagreement.

3. Apply a 2D inverse Fourier transform to each filtered signal, providing $p_c(y_i, x, t)$ and $p_r(y_i, x, t)$ pressure fields respectively.
4. Perform a Proper Orthogonal Decomposition of the resulted $p_c(y_i, x, t)$:

$$p_c(y_i, x, t) = \sum_{n=1}^{N_{mod}} a^{(n)}(t) \Phi^{(n)}(y_i, x)$$

where $\Phi^{(n)}$ is the n^{th} POD eigenfunction of the two-point pressure correlation tensor, $R_{p_c p_c}(x, x') = \overline{p_c(y_i, x, t) p_c(y_i, x', t)}$ where an overline indicates the time average operator. $a^{(n)}$ is the associated POD temporal coefficient of projection and N_{mod} is the total number of POD modes.

The application of the POD is recommended due to its optimality in terms of energy. In the present context, the low frequency hydrodynamic component is of higher energy than the low frequency acoustic component.

5. Discriminate the low frequency-low wavenumber energetic hydrodynamic component from the pressure field p_c by projecting this pressure field onto the first N' energetic POD modes:

$$p_c(y_i, x, t) = \sum_{n=1}^{N'-1} a^{(n)}(t) \Phi^{(n)}(y_i, x) + \sum_{n=N'}^{N_{mod}} a^{(n)}(t) \Phi^{(n)}(y_i, x) \quad (1)$$

Figure 4. Wavenumber spectrum extracted for two specific S_t values. (a): upper stream part, y_{a2} ; (b): lower stream part y_{a1} . (c): mid-lower stream part, y_0 ; (d): along the mixing layer flow y_{ml1} .

In this sense, the 'true' acoustic pressure becomes:

$$p_{acou}(y_i, x, t) = \sum_{n=N'}^{N''} a^{(n)}(t) \Phi^{(n)}(y_i, x) \quad (2)$$

Note that N'' is not necessary equal to N_{mod} as it will be seen later, the high order POD modes do not contribute to the acoustic part and they are more related to the background numerical noise justifying that $N'' < N_{mod}$.

Moreover, the hydrodynamic pressure field is:

$$p_{hydro}(y_i, x, t) = p_r(y_i, x, t) + \sum_{n=1}^{N'-1} a^{(n)}(t) \Phi^{(n)}(y_i, x). \quad (3)$$

6. Determine the numbers (N' , N'') by analyzing the properties of the resulted acoustic pressure field, especially thanks to a causality analysis with the far acoustic pressure field. With such a validation, the resulted acoustic pressure field will be the pressure field that has both the highest correlation with the far acoustic field and the required spectral content.

C. Application of the FFT-POD based separation methodology

STEPS 1-3

The first step based on the FFT application was described in section §III. Then, an area delimited with dotted lines (see figures 3 and 4) in the (f, k_x) space is retained. This area is related to events that propagate at high velocities. In each case, the H_c filtering is defined (delimited with dotted lines). However, as it could be seen in figure 3 ((c) and (d)) also in figure 4 ((c) and (d)), such a filter may also extract a part of the low wavenumber-low frequency hydrodynamic component. This is in agreement with previous works where such FFT filtering avoids any well separation of the two components in this area.

After applying this FFT filtering procedure, a 2D inverse Fourier transform is performed leading to obtained pressure fields, $p_c(y_i, x, t)$ and $p_r(y_i, x, t)$.

STEP 4: POD APPLICATION

POD is a multivariate statistical technique that has been applied widely in many fields like in turbulence due to its effectiveness in revealing coherent structures embedded in the turbulent flow. Indeed, POD acts as a filtering of the energetic flow scales of the turbulent flow. Here an original application is proposed where POD is used to extract the energetic part of the pressure field $p_c(y_i, x, t)$ that is associated with the low wavenumber-low frequency hydrodynamic part.

The POD application leads then to express the pressure field, $p_c(y_i, x, t)$, as follows: $\sum_{n=1}^{N_{mod}} a^{(n)}(t) \Phi^{(n)}(y_i, x)$. Tables 1 and 2 give the energy mode convergence of the POD eigenvalues computed from the pressure fields $p_c(y_0, x, t)$

Figure 5. Time correlation $R(\tau, x_{ml1} = 100\delta_{\omega_0}, x_{a1})$ as a function of $\tau \Delta U / \delta_{\omega_0}$ (X-axis) and x_{a1} (Y-axis). (a) based on the original pressure field. (b) based on the extracted hydrodynamic pressure field p_{hydro} . (c) based on the extracted acoustic pressure field p_{acou} .

and $p_c(y_{ml1}, x, t)$ respectively. The first 4 POD modes contain 90% and 76% of the whole energy of the pressure fields $p_c(y_0, x, t)$ and $p_c(y_{ml1}, x, t)$ respectively. The energy repartition confirms that the $p_c(y_0, x, t)$ pressure field is more coherent than the $p_c(y_{ml1}, x, t)$ pressure field. Indeed, along the line y_0 the low-frequency low-wavenumber component is mainly due to the large scale vortex developing at the end of the numerical domain. Conversely, along the mixing layer (y_{ml1} line), the low-frequency low-wavenumber part contains a lot of energetic flow scales as the spectral analysis in the mixing layer flow emphasizes previously (see figure 2).

Table 1. POD mode energy convergence computed from pressure field stored along y_0 line, expressed as a percentage energy value contained in the first N' modes (second line) and in the N'^{th} mode (third line).

Modes N'	1	2	3	4	5	10	30
$\sum_{n=1}^{N'} \lambda^{(n)} / \sum_{n=1}^{N_{mod}} \lambda^{(n)}$	58.1	79.4	86.2	89.7	92.6	98.3	99.989
$\lambda^{(N')} / \sum_{n=1}^{N_{mod}} \lambda^{(n)}$	58.1	21.3	6.8	3.5	2.9	0.45	0.00002

As the extraction of acoustic pressure fields seems to be more complicated for the pressure field $p_c(y_{ml1}, x, t)$ than $p_c(y_0, x, t)$, in the following we are going to present only the acoustic-hydrodynamic separation in the near field along the mixing layer flow. Indeed, along the line y_{ml1} , the acoustic part is entirely embedded in the whole aeroacoustic pressure field and can not be easily extracted. In this case, the extraction of the acoustic part is a great challenging task.

STEPS 5 AND 6

The purpose of the POD application consists then in determining the POD modes that can represent the acoustic

Figure 6. Time correlation $R(\tau, x_{ml1} = 100\delta_{\omega_0}, x_{a1} = 156\delta_{\omega_0})$ as a function of $\tau\Delta U/\delta_{\omega_0}$, computed from the original pressure field, the hydrodynamic pressure field (see equation 3 with $N' = 3$) and from the acoustic pressure field (see equation 2 with $(N', N'') = (3, 8)$).

Figure 7. Time correlation $R(\tau, x_{ml1} = 100\delta_{\omega_0}, x_{a1} = 156\delta_{\omega_0})$ as a function of $\tau\Delta U/\delta_{\omega_0}$. (a) computed from p_{acou} pressure field using different N' values (see equation 2, with $N'' = 8$). (b) computed from p_{acou} pressure field using $(N', N'') = (3, 8)$ and $(N', N'') = (3, 50)$.

pressure field contained in the FFT filtering pressure field $p_c(y_{ml1}, x, t)$. For such an investigation, $p_{acou}(y_i, x, t) = \sum_{n=N'}^{N''} a^{(n)}(t)\Phi^{(n)}(y_i, x)$ is computed for all possible combinations of (N', N'') numbers. To determine the optimal numbers, it is proposed to establish a direct link between the far acoustic pressure field $p_{acou}(y_1, x, t)$ and the extracted acoustic pressure field in the mixing layer flow. The following space-time correlation is then computed

$$R_{acou}^{N', N''}(x_{ml1}, x_{a1}, \tau) = \overline{p_{acou}(y_{ml1}, x_{ml1}, t)p_{acou}(y_{a1}, x_{a1}, t + \tau)} \quad (4)$$

Then by analyzing $R^{N', N''}(x_{ml1}, x_{a1}, \tau)$ values obtained at some specific locations x_{ml1} and by sweeping each possible retarded time, the determination of (N', N'') is done when the highest correlation is obtained. In the present case, it is observed that such a highest correlation is obtained for $N' = 3$ and $N'' = 8$.

Some representative results are displayed in the following figures. Figure 5 displays the correlation $R^{3,8}(x_{ml1} = 100\delta_{\omega_0}, x_{a1}, \tau)$ as a function of x_{a1} (far acoustic field) and the time delay, τ . It is then observed that the correlation reaches its maximum for $x_{a1} \approx 156\delta_{\omega_0}$. The correlation between the hydrodynamic flow and the far acoustic pressure field denoted $R_{hydro}(x_{ml1}, x_{a1}, \tau) = p_{hydro}(y_{ml1}, x_{ml1} = 100\delta_{\omega_0}, t)p_{acou}(y_{a1}, x_{a1}, t + \tau)$ is also represented for com-

Table 2. POD mode energy convergence computed from pressure field stored along y_{ml1} line, expressed as a percentage energy value contained in the first N' modes (second line) and in the N'^{th} mode (third line).

Modes N'	1	2	3	4	5	10	30
$\sum_{n=1}^{N'} \lambda^{(n)} / \sum_{n=1}^{N_{mod}} \lambda^{(n)}$	37.4	58.1	69.4	76.2	82.0	94.72	99.69
$\lambda^{(N')} / \sum_{n=1}^{N_{mod}} \lambda^{(n)}$	37.4	20.7	11.3	6.8	5.8	1.22	0.03

Figure 8. Time correlation $R(\tau, x_{ml1} = 177\delta_{\omega_0}, x_{a1})$ as a function of $\tau\Delta U/\delta_{\omega_0}$ (X-axis) and x_{a1} (Y-axis). (a) based on the extracted hydrodynamic pressure field p_{hydro} . (b) based on the extracted acoustic pressure field p_{acou} .

Figure 9. Skewness (black line) and flatness (dotted line) computed from (a) original and (b) acoustic pressure fields.

parison. The same correlation is also computed from the original aeroacoustic pressure field, denoted R_{orig} .

First, the correlation computed from the original pressure field $p_{hydro}(y_{ml1}) + p_{acou}(y_{ml1})$ and the far acoustic pressure field is quasi identical to the one obtained from the hydrodynamic flow field. As expected we found a large correlation level between both acoustic pressure fields whereas the correlation between hydrodynamic and far field pressures is negligible. The time correlations computed at the following two points: $(x_{ml1}, x_{a1}) = (100\delta_{\omega_0}, 156\delta_{\omega_0})$ are plotted in figure 6. Moreover, figure 7 illustrates the effect of POD mode numbers (N' , N'') (see equations 2 and 3) on the time correlations. This figure displays the time correlation $R_{acou}^{N', N''}(\tau)$ computed at the same following two points: $(x_{ml1}, x_{a1}) = (100\delta_{\omega_0}, 156\delta_{\omega_0})$ and based on different values of N' and N'' . Then, for a fixed N'' value, figure 7-(a) shows that the first two POD modes do not contribute to the acoustic part of the extracted pressure field. As expected, the first POD energetic modes seem to be rather associated with the hydrodynamic pressure field. On the other hand, for a fixed $N' = 3$ value, the $R_{acou}^{N'=3, N''}(\tau)$ time correlation are computed for numerous values of N'' . For N'' varying from 50 to N_{mod} , no difference is observed. The maximum of correlation is obtained for $N'' = 8$. This indicates that high order POD modes are background numerical noise.

Knowing the locations $(y_{ml1}, x_{ml1} = 100\delta_{\omega_0})$ and $(y_{a1}, x_{a1} = 156\delta_{\omega_0})$ where the time correlation is maximal, the associated emission angle can be computed and is equal to $\theta_1 = 64^\circ$. In this case, this related emission angle is in relative good agreement with previous analysis (see §IV-B) and published works.^{14,15} Moreover, by analyzing the time delay ($\tau\Delta U/\delta_{\omega_0} \approx 28.6$), the acoustic wave velocity can be computed ($\approx 376\text{m.s}^{-1}$). This last value corresponds to the speed of sound (c) added to the low stream velocity projected onto the acoustic wave direction ($U_1 \cos(\theta_1) = 36\text{m.s}^{-1}$).

Another example of the time correlation representation is provided in figure 8. The correlations maps R_{hydro} and R_{acou} are represented as a function of x_{a1} and τ for the reference $(y_{ml1}, x_{ml1} = 177\delta_{\omega_0})$ point. The time correlation R_{orig} based on the original pressure field is similar to the one computed from the hydrodynamic pressure field. In this

Figure 10. Superimposition of the original, hydrodynamic and acoustic pressure field. a) at a fixed instant, (b) at a fixed x -position, top to bottom, $x = 100\delta_{\omega_0}$, $x = 170\delta_{\omega_0}$ and $x = 240\delta_{\omega_0}$.

case, the time correlation meets its maximum value for $(x_{a1}, \tau \Delta U / \delta_{\omega_0}) = (257\delta_{\omega_0}, 29.8)$. These values correspond to an acoustic wave propagated at an angle of $\theta_2 = 55^\circ$ with a velocity of $389 \approx c + U_1 \cos(\theta_2)$.

ANALYSIS OF THE PROPERTIES OF THE EXTRACTED ACOUSTIC PRESSURE FIELD

An investigation of the statistical properties of the resulted acoustic and hydrodynamic pressure fields is done by computing the skewness and flatness coefficients. Figure 9 presents these coefficients for the two components. It is observed that the acoustic pressure field approaches a Gaussian distribution with a skewness and flatness coefficients of 0 and 3 respectively. Conversely, a Gaussian distribution is not retrieved for the hydrodynamic pressure field.

Finally, an illustration of the instantaneous flow decomposition is given in figure 10 where an original instantaneous pressure field is superimposed onto its acoustic and hydrodynamic components. In this figure, the original pressure field is superimposed onto the hydrodynamic pressure field and the acoustic pressure field. The instantaneous pressure fields are represented at a fixed instant (along the streamwise direction, graph (a)) and also at some three fixed x -locations (as a function of time, three graphs (b)). The hydrodynamic part of the pressure field is quasi similar to the original signal.

V. Conclusion

Based on a numerical simulation of a canonical flow configuration *i.e* a two-dimensional compressible spatially developing plane mixing layer flow, aeroacoustic pressure fields are extracted at the boundaries of the mixing layer flow. After demonstrating that these pressure fields contains both an hydrodynamic and an acoustic part, we implemented a novel methodology aiming at extracted the acoustic component embedded in the aeroacoustic pressure field. Based

on the knowledge of the properties (propagation velocities, energy content) of both the hydrodynamic and the acoustic pressure fields, an efficient coupling of two complementary mathematical tools (Fourier Transform and Proper Orthogonal Decomposition) is performed. First, the FFT is applied to the aeroacoustic pressure field in order to filter in the wavenumber frequency domain the component of the pressure that convects at the speed of sound. Such a filtering can not be efficient in the low-frequency low-wavenumber domain where both acoustic and hydrodynamic components are present. As the hydrodynamic part is of higher energy content than the acoustic pressure field, it is proposed to decompose the FFT filtered pressure field thanks to the POD application. The first energetic POD modes are then removed. Then by projecting the filtered pressure field onto some selected POD modes, the acoustic pressure field can be isolated. The POD mode selection has been done and validated by performing a causality analysis between the extracted acoustic near field and the far acoustic pressure field. It is demonstrated that the correlation between hydrodynamic pressure field and the far acoustic pressure field is negligible. Conversely, the FFT-POD filtered pressure field (acoustic component) is quite well correlated with the far acoustic pressure field. This is confirmed by computing the speed of associated acoustic waves which corresponds to the speed of sound. Finally, the acoustic field extracted from the near aeroacoustic pressure field has the following properties: i) the correct spectral content, ii) the best correlation with the far acoustic pressure field and iii) a nearly gaussian distribution.

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