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# Exploring the effect of the pore size distribution on the streaming potential generation in saturated porous media, insight from pore network simulations

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1                   **Exploring the effect of the pore size distribution on the**  
2                   **streaming potential generation in saturated porous media,**  
3                   **insight from pore network simulations**

4                   **Damien Jougnot <sup>1</sup>, Aida Mendieta <sup>1</sup>, Philippe Leroy <sup>2</sup>, Alexis Mainault <sup>1</sup>**

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7                   **Key Points:**

- 8                   • We simulate streaming potentials for 2D networks with different pore size distribu-
- 9                   tions
- 10                  • The pore size distribution has a very restricted influence on electrokinetic coupling
- 11                  coefficients
- 12                  • A recent effective excess charge density model accounts for all the pore size distri-
- 13                  butions

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## Abstract

Understanding streaming potential generation in porous media is of high interest for hydrological and reservoir studies as it allows to relate water fluxes to measurable electrical potential distributions. This streaming potential generation results from an electrokinetic coupling due to the presence of an electrical double layer developing at the interface between minerals and pore water. Therefore, the pore sizes of the porous medium are expected to play an important role in the streaming potential generation. In this work we use 2D pore network simulations to study the effect of the pore size distribution upon this electrokinetic mechanism. Our simulations under well-controlled conditions allow a detailed study of the influence of a large range of permeabilities (from  $10^{-16}$  to  $10^{-10}$  m<sup>2</sup>) for different ionic concentrations (from  $10^{-4}$  to 1 mol L<sup>-1</sup>). We then use and compare two different approaches that have been used over the last decades to model and interpret the generation of the streaming potential: the classical coupling coefficient or the effective excess charge density, which has been defined recently. Our results show that the four pore size distributions tested in the present work have a restricted influence on the coupling coefficient for ionic concentration smaller than  $10^{-3}$  mol L<sup>-1</sup> while it completely drives the behaviour of the effective excess charge density over orders of magnitude. Then, we use these simulation results to test an analytical model based on a fractal pore size distributions [Guarracino and Jougnot, 2018]. We show that this model predicts well the effective excess charge density for all the tested pore size distribution within its intrinsic limitations, that is, for a thin double layer compared to the pore size.

## 1 Introduction

Self-Potential (SP) is one of the oldest geophysical methods [Fox, 1830] and consists in measuring the naturally occurring electrical field at the surface of or within geological media. The SP signal results from the superposition of multiple sources coming from contributions of two main processes: the electrokinetic (EK) contribution (i.e., related to water flux) and the electrochemical contributions (i.e., related to ionic concentration, thermal gradient, or redox gradient). In this work we focus on SP signals generated by electrokinetic phenomena: the so-called streaming potential. Details on the possible contributions to the SP signal can be found in Revil and Jardani [2013] or Jouniaux *et al.* [2009], among other references.

45 The streaming potential has been the subject of numerous scientific studies over  
 46 the last two centuries [since *Quincke*, 1859] and involved in many applications: from oil  
 47 and gas reservoir exploration to more recent critical zone studies [e.g., *Revil et al.*, 1999a;  
 48 *Jougnot et al.*, 2015]. In geological media, minerals and organic matter exhibit a charged  
 49 surface (usually negative) that is compensated by an excess of charges in the pore water  
 50 distributed in the so-called electrical double layer (EDL) surrounding these grains [e.g.  
 51 *Hunter*, 1981]. These charges can be dragged by a water flow, generating a charge separa-  
 52 tion that in turn generates an electrical current and a resulting electrical potential dis-  
 53 tribution. Given the difficulty of directly measuring the water flow in geological media,  
 54 relating this measurable electrical potential distribution to the water flux is therefore of in-  
 55 terest for many reservoir or environmental applications [e.g., *Jouniaux et al.*, 2009; *Revil*  
 56 *and Jardani*, 2013].

57 For more than a century, the classical approach to quantitatively relate the electrical  
 58 potential field to the water flux (or to a hydraulic pressure field) has been achieved by the  
 59 use of the EK coupling coefficient,  $C_{EK}$  (V Pa<sup>-1</sup>),

$$C_{EK} = \left. \frac{\partial V}{\partial P} \right|_{\mathbf{J}=\vec{0}}, \quad (1)$$

60 where  $V$  is the electrical potential (V) and is  $P$  the water pressure (Pa), in the assump-  
 61 tions that the system is under a quasi-static equilibrium and that no external current  $\mathbf{J}$   
 62 is injected into the medium. *Helmholtz* [1879] and *von Smoluchowski* [1903] proposed  
 63 the so-called Helmholtz-Smoluchowski (HS) equation to determine  $C_{EK}$  from a limited  
 64 amount of parameters:

$$C_{EK}^{HS} = \frac{\varepsilon_w \zeta}{\eta_w \sigma_w}, \quad (2)$$

65 where  $\varepsilon_w$ ,  $\sigma_w$ , and  $\eta_w$  are the dielectric permittivity (F m<sup>-1</sup>), the electrical conductivity  
 66 (S m<sup>-1</sup>), and the dynamic viscosity (Pa s) of the pore water, respectively. The  $\zeta$ -potential,  
 67  $\zeta$  (V), corresponds to the electrical potential at the shear plane in the EDL, which is the  
 68 plane separating mobile and immobile water molecules [e.g. *Hunter*, 1981; *Leroy et al.*,  
 69 2012; *Li et al.*, 2016, Fig. 1]. The HS equation has been successfully used to predict stream-  
 70 ing potential measurements in geological media [e.g., *Jouniaux and Pozzi*, 1995a; *Pengra*  
 71 *et al.*, 1999]. It is interesting to note that the HS equation seems completely independent  
 72 from the pore space geometry of the medium. However, there is a strong assumption in  
 73 this model: the surface conductivity of the grains,  $\sigma_s$  (S m<sup>-1</sup>), must be negligible com-  
 74 pared to the pore water conductivity, that is  $\sigma_s \ll \sigma_w$ . When this is not the case, alterna-

75 tive formulas have been proposed by several researchers [e.g., *Morgan et al.*, 1989; *Revil*  
76 *et al.*, 1999b; *Glover and Déry*, 2010], taking into account surface conductivity and mak-  
77 ing some assumptions on the pore space geometry.

78 More recently, an alternative approach to quantify the streaming potential generation  
79 has been proposed, focusing on the excess charge effectively dragged by the water flow.  
80 To the best of the authors knowledge, the first occurrence of this approach in the literature  
81 in english is in *Kormiltsev et al.* [1998] and was later independently found by *Revil and*  
82 *Leroy* [2004]. This parameter is an alternative to the coupling coefficient and can easily be  
83 related to it by re-writing the water flow and electrical current equations [see *Kormiltsev*  
84 *et al.*, 1998, for the first derivation]

$$85 \quad C_{EK} = -\frac{\hat{Q}_v k}{\sigma \eta_w}, \quad (3)$$

86 where  $\sigma$  and  $k$  are the electrical conductivity ( $\text{S m}^{-1}$ ) and permeability ( $\text{m}^2$ ) of the medium,  
87 respectively. Following the formalism of *Revil* and co-authors, we call  $\hat{Q}_v$  the effective ex-  
88 cess charge density ( $\text{C m}^{-3}$ ). Note that it is called  $\alpha$  in *Kormiltsev et al.* [1998].

89 Several studies have shown empirical evidence to prove that the effective excess  
90 charge density depends on the permeability of the porous media [*Titov et al.*, 2002; *Jar-*  
91 *dani et al.*, 2007; *Bolève et al.*, 2012], indicating that this parameter is strongly influenced  
92 by the petrophysical properties of the considered geological medium. It has been shown  
93 that the pore water chemistry, both the composition and the ionic concentration, also have  
94 a significant effect on  $\hat{Q}_v$  [e.g., *Jougnot et al.*, 2012, 2015; *Cherubini et al.*, 2018].

95 Recently, *Guarracino and Jougnot* [2018] proposed an analytical model directly re-  
96 lating  $\hat{Q}_v$  to the permeability, porosity, pore water chemistry (through the ionic concentra-  
97 tion), and the  $\zeta$ -potential. This closed-form equation was derived with the assumptions of  
98 a simple binary symmetric pore water electrolyte and pore radii much larger than the dif-  
99 fuse layer thickness. In order to achieve the derivation of this analytical solution, the au-  
100 thors based their approach on the use of tortuous capillaries and a fractal pore size distri-  
101 bution. Interestingly, the pore size distribution does not directly appear in the closed-form  
102 equation. *Guarracino and Jougnot* [2018]’s model performs very well with different SP  
103 datasets from laboratory measurements [*Pengra et al.*, 1999; *Glover and Déry*, 2010]. Note  
104 that *Soldi et al.* [2019] propose an extension of this model to partially saturated conditions.

105 Pore network simulations can be used as a numerical tool to predict the electroki-  
106 netic coupling coefficient, and consequently the effective excess charge density, for dif-

ferent pore size distributions. *Bernabé* [1998] proposed a pioneer work to model streaming potential in heterogeneous media. Based on this work, further investigations on coupling effects in charged media in 2 or 3D have been performed [e.g., *Brovelli and Casiani*, 2010; *Obliger et al.*, 2014; *Zhang et al.*, 2015], mainly to evaluate the impact of the electrokinetic coupling on the permeability in microporous media.

In this work, we use a pore network numerical code based on the works of *Bernabé* [1998] and *Maineult et al.* [2018]. It allows for the prediction of the coupling coefficient, permeability, and formation factor of a 2D pore network with well-controlled pore size distributions, and therefore the effective excess charge density from Eq. 3. After presenting the theoretical framework for the electrokinetic phenomena and the numerical method that we implemented, we will (1) study the effect of the pore size distribution on the streaming potential generation and (2) check for the applicability of the *Guarracino and Jougnot* [2018] analytical model for the prediction of the effective excess charge density obtained for different pore size distributions.

## 2 Theory of streaming current generation

### 2.1 Governing equations

Streaming current generation in geological media can be described by the following macroscopic governing equations [e.g., *Sill*, 1983]:

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_s, \quad (4)$$

$$\nabla \cdot \mathbf{J} = 0, \quad (5)$$

where  $\mathbf{J}$  is the total current density ( $\text{A m}^{-2}$ ),  $\mathbf{E} = -\nabla V$  is the electrical field ( $\text{V m}^{-1}$ ), and  $\mathbf{J}_s$  is the source current density ( $\text{A m}^{-2}$ ). In the absence of external current, that is when no current is injected into the medium, combining Eqs. (4) and (5) yields,

$$\nabla \cdot (\sigma \nabla V) = \nabla \cdot \mathbf{J}_s. \quad (6)$$

When considering only EK processes in the SP signals, the source current density (i.e., streaming current density) can then be expressed as,

$$\mathbf{J}_s = \sigma C_{EK} \nabla (P - \rho_w g z), \quad (7)$$

where  $\rho_w$  is the water density ( $\text{kg m}^{-3}$ ),  $g$  is the gravitational acceleration ( $\text{m s}^{-2}$ ), and  $z$  is the elevation (m). We call Eq. (7) the coupling coefficient approach.

133 As described in *Kormiltsev et al.* [1998], combining Eq. 3 and Darcy's equation  
 134 [*Darcy*, 1856], we obtain the Darcy velocity:

$$\mathbf{u} = -\frac{k}{\eta_w} \nabla (P - \rho_w g z). \quad (8)$$

135 Including Eq. 8 in Eq. 7, one can obtain the streaming current density from the effective  
 136 excess charge approach,

$$\mathbf{J}_s = \hat{Q}_v \mathbf{u}. \quad (9)$$

137 Combining Eqs. 6 and 9 allows relating the streaming potential distribution to the  
 138 Darcy velocity, a variable of uttermost interest in hydrology or reservoir studies, through  
 139 the medium conductivity and effective excess charge density:

$$\nabla \cdot (\sigma \nabla V) = \nabla \cdot (\hat{Q}_v \mathbf{u}). \quad (10)$$

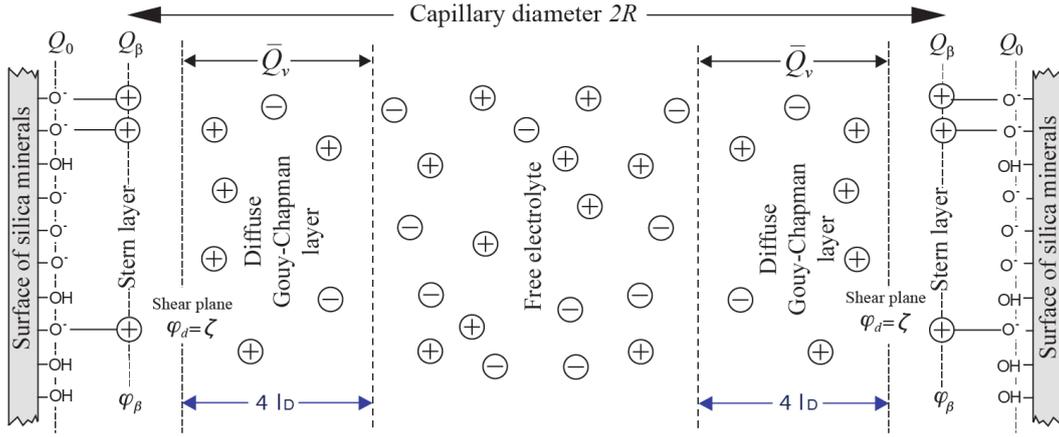
## 140 2.2 Electrochemical properties

141 Most geological materials have a solid matrix made of components with charged  
 142 surfaces (mostly minerals but also organic matter) in contact with water due to the hydrox-  
 143 idation of the surface sites and ion substitutions in the crystal [*Hiemstra and Van Riems-*  
 144 *dijk*, 2006; *Leroy et al.*, 2013, 2015; *Li et al.*, 2016]. An EDL is formed at the pore surface  
 145 to compensate the surface charge as the system "solid matrix plus pore water" must sat-  
 146 isfy the electroneutrality principle [e.g., *Hunter*, 1981; *Leroy and Revil*, 2004]. As shown  
 147 in Fig. 1, the surface charge  $Q_0$  ( $\text{C m}^{-2}$ ) is counterbalanced by charges in the EDL of  
 148 the pore water: (1) by charges adsorbed in the compact Stern layer  $Q_\beta$  (often considered  
 149 to have a negligible thickness, therefore expressed in  $\text{C m}^{-2}$ ) and (2) by a distribution of  
 150 charges in the diffuse layer  $\bar{Q}_v$  ( $\text{C m}^{-3}$ ). This yields

$$\frac{S_{sw}}{V_w} (Q_0 + Q_\beta) + \bar{Q}_v = 0, \quad (11)$$

151 where  $S_{sw}$  is the surface of the solid in contact with water ( $\text{m}^2$ ) and  $V_w$  is the pore water  
 152 volume ( $\text{m}^3$ ). The term  $\bar{Q}_v$  is called the excess charge density in the diffuse layer. We  
 153 call co-ions and counter-ions the ions with the same and the opposite sign of the surface  
 154 charge density, respectively. In typical silica rocks, under typical environmental conditions,  
 155 surfaces are usually negatively charged; the co-ions and counter-ions are therefore anions  
 156 and cations, respectively [e.g., *Sverjensky*, 2006].

159 The distribution of ions in the diffuse layer depends on the distribution of the micro-  
 160 scopic (or local) electrical potential in the pores,  $\psi$  (V), which follows the Poisson equa-



157 **Figure 1.** Scheme of the electrical double layer at the surface of silica minerals in contact with water for a  
 158 given capillary radius  $R$ .  $l_D$  correspond to the Debye length (Eq. 18).

161 tion:

$$\nabla^2 \psi = -\frac{\bar{Q}_v}{\varepsilon_w} \quad (12)$$

162 where  $\varepsilon_w$  is the dielectric permittivity of the pore water ( $\text{F m}^{-1}$ ). We consider that the  
 163 bulk pore water (i.e., the part of the electrolyte free from the effects of the charged sur-  
 164 faces) is an electrolyte composed of  $M$  ionic species  $i$  with a bulk concentration  $C_i^w$  (mol  
 165  $\text{m}^{-3}$ ). The excess charge density in the diffuse layer is supposed to follow a Boltzmann  
 166 distribution yielding:

$$\bar{Q}_v(r) = N_A \sum_{i=1}^M q_i C_i^w \exp\left(-\frac{q_i \psi(r)}{k_B T}\right) \quad (13)$$

167 where  $r$  is the distance from the shear plane (m) (that is the pore wall as we neglect the  
 168 Stern layer thickness),  $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$  is the Avogadro's number,  $k_B = 1.381 \times$   
 169  $10^{-23} \text{ J K}^{-1}$  is the Boltzmann constant,  $T$  is the absolute temperature (K), and  $q_i = \pm z_i e_0$   
 170 is the ion charge (C) which depends on its valency,  $z_i$ , and the elementary charge,  $e_0 =$   
 171  $1.602 \times 10^{-19} \text{ C}$ . Note that the extension of the diffuse layer corresponding to the fraction  
 172 of the pore space in which the excess charge density is not negligible, can be approxi-  
 173 mated by a thickness equal to  $4l_D$  (Fig. 1).

174 The excess charge density which is effectively displaced by the water flow is called  
 175 effective or dynamic excess charge, depending on the authors, and symbolized as  $\hat{Q}_v$  or  
 176  $\bar{Q}_v^{eff}$  ( $\text{C m}^{-3}$ ). It has to be distinguished from the other excess charge densities contained  
 177 in the pore space [see the discussion in *Revil, 2017*]. The total excess charge density  $Q_v$

178 (C m<sup>-3</sup>), which includes all the charges of the EDL, is given by:

$$Q_v = \frac{S_{sw}}{V_w} (Q_\beta) + \bar{Q}_v = \rho_s \left( \frac{1 - \phi}{\phi} \right) e_0 N_A CEC, \quad (14)$$

179 where  $CEC$  is the cationic exchange capacity (meq kg<sup>-1</sup>),  $\phi$  is the porosity, and  $\rho_s$  is the  
 180 solid grain density (kg m<sup>-3</sup>). Note that the CEC of hydroxide minerals such as quartz  
 181 strongly depends on the pH and salinity [Leroy *et al.*, 2013]. As discussed in Jougnot *et al.*  
 182 [2012], the excess charge density of the diffuse layer  $\bar{Q}_v$  (Fig. 1) is usually considerably  
 183 smaller than the total excess charge density  $Q_v$  and larger than the effective excess charge  
 184 density  $\hat{Q}_v$  :

$$\hat{Q}_v \ll \bar{Q}_v \ll Q_v. \quad (15)$$

185 This is due to the fact that the effective excess charge density is weighted by the pore wa-  
 186 ter velocity distribution through the pore (Fig. 10a). This concept is described in detail in  
 187 Jougnot *et al.* [2012] and called "flux-averaging" in opposition to the "volume-averaging"  
 188 up-scaling technique described in Revil *et al.* [2007].

### 189 2.3 Electrokinetic coupling at the pore scale

190 Following the capillary-based approaches proposed by Jackson [2008, 2010] and  
 191 Linde [2009], Jougnot *et al.* [2012] consider the porous medium as a bundle of capillaries  
 192 to develop the flux-averaging up-scaling procedure. The effective excess charge density  
 193  $\hat{Q}_v^R$  dragged by the water flow in a single tube of radius  $R$  (m) is defined by:

$$\hat{Q}_v^R = \frac{\int_{r=0}^R \bar{Q}_v(r) v(r) dr}{\int_{r=0}^R v(r) dr}, \quad (16)$$

194 where  $v(r)$  is the pore water velocity across the capillary (m s<sup>-1</sup>).

195 In order to propose an analytical solution for Eq. (16), Guarracino and Jougnot  
 196 [2018] consider the Debye-Hückel approximation, an usual way to derive analytically  
 197 the distribution of the local electrical potential [e.g., Jougnot *et al.*, 2012, 2015; Guar-  
 198 racino and Jougnot, 2018; Soldi *et al.*, 2019]. This approximation is an accurate solu-  
 199 tion of the Poisson-Boltzmann equation (Eq. 12) for low local electrical potentials, i.e.,  
 200  $|\zeta| \ll (k_B T)/|q_i| \approx 25$  mV (for  $T = 298$  K) and monovalent ions. The microscopic elec-  
 201 trical potential distribution in the diffuse layer of a NaCl pore water solution can then be  
 202 expressed as,

$$\psi(r) = \zeta \exp\left(-\frac{r}{l_D}\right), \quad (17)$$

203 where  $l_D$  is the Debye length (m) defined as,

$$l_D = \sqrt{\frac{\varepsilon_w k_B T}{2e_0^2 C^w N_A}}. \quad (18)$$

204 Note that this is a solution obtained for a flat surface [e.g., *Hunter*, 1981]. Nevertheless,  
 205 it can be used for large pores, that is for a small curvature compared to the diffuse layer  
 206 thickness [see discussion in *Jougnot et al.*, 2012; *Thanh*, 2018]. For a NaCl solution, Eq.  
 207 (13) becomes,

$$\bar{Q}_v(r) = N_A e_0 C_{NaCl}^w \left[ e^{-\frac{e_0 \psi(r)}{k_B T}} - e^{\frac{e_0 \psi(r)}{k_B T}} \right]. \quad (19)$$

209 Then the exponential terms of Eq. (19) are approximated by a four-term Taylor series:

$$e^{\pm \frac{e_0 \psi(r)}{k_B T}} = 1 \pm \frac{e_0 \psi(r)}{k_B T} + \frac{1}{2} \left( \frac{e_0 \psi(r)}{k_B T} \right)^2 \pm \frac{1}{6} \left( \frac{e_0 \psi(r)}{k_B T} \right)^3. \quad (20)$$

211 Substituting Eq. (20) in Eq. (19) and solving (16) considering a Poiseuille flow, it yields:

$$\begin{aligned} \hat{Q}_v^R = & -\frac{8N_A e_0^2 C_{NaCl}^w \zeta}{k_B T (R/l_D)^4} \left\{ 6 - e^{-\frac{R}{l_D}} \left[ \left( \frac{R}{l_D} \right)^3 + 3 \left( \frac{R}{l_D} \right)^2 + 6 \left( \frac{R}{l_D} \right) + 6 \right] \right\} \\ & + \frac{24N_A e_0^2 C_{NaCl}^w \zeta}{k_B T (R/l_D)^3} \left\{ 2 - e^{-\frac{R}{l_D}} \left[ \left( \frac{R}{l_D} \right)^2 + 2 \left( \frac{R}{l_D} \right) + 2 \right] \right\} \\ & - \frac{16N_A e_0^2 C_{NaCl}^w \zeta}{k_B T (R/l_D)^2} \left\{ 1 - e^{-\frac{R}{l_D}} \left[ \left( \frac{R}{l_D} \right) + 1 \right] \right\} \\ & - \frac{4N_A e_0^4 C_{NaCl}^w \zeta^3}{3(k_B T)^3 (3R/l_D)^4} \left\{ 6 - e^{-\frac{3R}{l_D}} \left[ \left( \frac{3R}{l_D} \right)^3 + 3 \left( \frac{3R}{l_D} \right)^2 + 6 \left( \frac{3R}{l_D} \right) + 6 \right] \right\} \\ & + \frac{4N_A e_0^4 C_{NaCl}^w \zeta^3}{(k_B T)^3 (3R/l_D)^3} \left\{ 2 - e^{-\frac{3R}{l_D}} \left[ \left( \frac{3R}{l_D} \right)^2 + 2 \left( \frac{3R}{l_D} \right) + 2 \right] \right\} \\ & - \frac{8N_A e_0^4 C_{NaCl}^w \zeta^3}{3(k_B T)^3 (3R/l_D)^2} \left\{ 1 - e^{-\frac{3R}{l_D}} \left[ \left( \frac{3R}{l_D} \right) + 1 \right] \right\}. \end{aligned} \quad (21)$$

213 Considering the thin double layer assumption  $l_D \ll R$ , *Guarracino and Jougnot* [2018]  
 214 simplify Eq. 21 to obtain the following analytical solution to predict the effective excess  
 215 charge in a single capillary with a radius  $R$ ,

$$\hat{Q}_v^R = \frac{8N_A e_0 C_{NaCl}^w}{(R/l_D)^2} \left[ -2 \frac{e_0 \zeta}{k_B T} - \left( \frac{e_0 \zeta}{3k_B T} \right)^3 \right]. \quad (22)$$

216 This solution is considered valid for  $R > 5l_D$ , see discussion in *Guarracino and Jougnot*  
 217 [2018] (their Fig. 2) and in *Thanh* [2018]. Note that the rather simple Eq. (22) is influ-  
 218 enced both by geometry ( $R$ ), interface ( $\zeta$ ,  $l_D$ ), and chemical properties ( $C_{NaCl}^w$ ).

## 219 2.4 Electrokinetic coupling at the REV scale

220 In order to study the streaming potential generation in natural geological media, a  
 221 second upscaling procedure has to be performed to go from  $\hat{Q}_v^R$  to the effective excess  
 222 charge density at the Representative Elementary Volume (REV) scale,  $\hat{Q}_v^{REV}$ . The flux-  
 223 averaging approach proposed by *Jougnot et al.* [2012] yields,

$$\hat{Q}_v^{REV} = \frac{\int_{R_{min}}^{R_{max}} \hat{Q}_v^R v^R f_D dR}{\int_{R_{min}}^{R_{max}} v^R f_D dR}, \quad (23)$$

224 where  $v^R$  is the average pore water velocity ( $\text{m s}^{-1}$ ) in capillaries having a radius  $R$ , and  
 225  $f_D$  is the capillary size distribution. Eq. 23 holds for any capillary size distribution. *Joug-*  
 226 *not et al.* [2012] propose to determine  $f_D$  from the hydrodynamic curves of the considered  
 227 porous medium. This can be accomplished by two approaches: one based on the water  
 228 retention curve  $f_D^{WR}$ , the other based on the relative permeability curve  $f_D^{RP}$ . Both ap-  
 229 proaches require numerical simulation.

230 *Guarracino and Jougnot* [2018] recently proposed an analytical approach to deter-  
 231 mine  $\hat{Q}_v^{REV}$  at the REV scale considering a fractal pore size distribution under water satu-  
 232 rated conditions. They solve Eq. 23 with  $\hat{Q}_v^R$  from Eq. 22. Their analytical developments,  
 233 based on the Debye-Hückel approximation, yield the following rather simple formula,

$$\hat{Q}_v^{REV} = N_A e_0 C^w l_D^2 \left[ -2 \frac{e_0 \zeta}{k_B T} - \left( \frac{e_0 \zeta}{3 k_B T} \right)^3 \right] \frac{1}{\tau^2} \frac{\phi}{k}. \quad (24)$$

234 where  $\tau$  is the dimensionless hydraulic tortuosity of the medium. The above equation pre-  
 235 dictes the effective excess charge density in terms of both macroscopic hydraulic paramete-  
 236 rers (porosity, permeability, and tortuosity) and parameters of chemical or interfacial na-  
 237 ture (ionic concentration,  $\zeta$ -potential and Debye length). One can see that the fractal pore  
 238 size distribution does not explicitly appear in Eq. 24, as it is included in the porosity and  
 239 permeability terms. Indeed, when developing the analytical solution presented above (Eq.  
 240 24), all the information related to the pore space geometry (e.g., the fractal pore size dis-  
 241 tribution) was included in the definition of porosity and permeability [see *Guarracino and*  
 242 *Jougnot*, 2018, for more details on the model development]. This model has been recently  
 243 extended to partially saturated conditions by *Soldi et al.* [2019]. Note that *Thanh* [2018]  
 244 proposed an expression similar to Eq. 24 but only valid for a single capillary radius in-  
 245 stead of a distribution of radii.

246 While the *Guarracino and Jougnot* [2018] analytical solution proposes an explicit  
 247 link between  $\hat{Q}_v$  and the medium's permeability, numerous previous studies have shown  
 248 an empirical relationship between these two parameters before [e.g., *Titov et al.*, 2002; *Jar-*  
 249 *dani et al.*, 2007; *Bolève et al.*, 2012; *Cherubini et al.*, 2018]. Among these works, *Jardani*  
 250 *et al.* [2007] propose the following empirical relationship

$$\log_{10}(\hat{Q}_v^{REV}) = A_1 + A_2 \log_{10}(k), \quad (25)$$

251 where  $A_1 = -9.2349$  and  $A_2 = -0.8219$  are constant values obtained by fitting Eq. 25  
 252 to a large set of experimental data that includes various lithologies and ionic concentra-  
 253 tions. It has been widely used for SP [e.g. *Jardani and Revil*, 2009; *Linde et al.*, 2011;

254 *Soueid Ahmed et al., 2014; Roubinet et al., 2016*] and seismoelectrics [e.g. *Jougnot et al.,*  
 255 *2013; Revil et al., 2015; Monachesi et al., 2015*] applications.

### 256 **3 Streaming potential modeling in a 2D pore network**

257 The present section describes the pore network model that we developed and used  
 258 to simulate the streaming potential generation in synthetic porous media. We first describe  
 259 the electrokinetic coupling at the capillary scale and then how the up-scaling is performed  
 260 in 2D pore networks with different pore size distributions. Note that the simulations are  
 261 based on the classical coupling coefficient approach (Eq. 7) and that the effective excess  
 262 charge density is obtained from the numerical simulation results and Eq. 3.

#### 263 **3.1 Coupled transport equations in a single capillary**

264 The pore network simulations consider the electrokinetic coupling occurring in cap-  
 265 illaries (i.e., pores). Our numerical simulations are based on the numerical framework of  
 266 *Bernabé* [1998], where the magnitudes of the hydraulic,  $Q$  ( $\text{m}^3 \text{s}^{-1}$ ), and electrical,  $J$  ( $\text{A}$   
 267  $\text{s}^{-1}$ ), fluxes in a single capillary of radius  $R$  (m) and length  $l$  (m) are given by the follow-  
 268 ing equations:

$$\left\{ \begin{array}{l} Q = -\frac{\pi R^4}{8\eta_w} \frac{(P_u - P_d)}{l} + \frac{\pi \epsilon_w R^2 \zeta}{\eta_w} \left( 1 - \frac{2}{R^2 \zeta} \int_0^R r \psi(r) dr \right) \frac{(V_u - V_d)}{l} \\ J = \frac{\pi \epsilon_w R^2 \zeta}{\eta_w} \left( 1 - \frac{2}{R^2 \zeta} \int_0^R r \psi(r) dr \right) \frac{(P_u - P_d)}{l} \\ \quad - \left[ \frac{2\pi \epsilon_w^2}{\eta_w} \int_0^R r \left( \frac{d\psi(r)}{dr} \right)^2 dr + 2\pi \sigma_w \int_0^R r \cosh \left( \frac{ze\psi(r)}{k_B T} \right) dr \right] \frac{(V_u - V_d)}{l} \end{array} \right. , \quad (26)$$

269 where  $P$  is the hydraulic pressure,  $V$  is the electrical potential and where the subscripts  $u$   
 270 and  $d$  are for the up and down water pressure and electrical potential values, respectively.  
 271 This set of equations is a fully coupled system taking into account the classical Poiseuille  
 272 flow, Ohm's law, and both the electrofiltration (i.e., a water displacement generating an  
 273 electrical field) and the electroosmotic (i.e., an electrical field generating a water displace-  
 274 ment) couplings [e.g., *Nourbehecht, 1963*]. Eq. 26 can be condensed into,

$$\left\{ \begin{array}{l} Ql = -\gamma^h (P_u - P_d) + \gamma^c (V_u - V_d) \\ Jl = \gamma^c (P_u - P_d) + \gamma^e (V_u - V_d) \end{array} \right. , \quad (27)$$

275 where  $\gamma^h$  is the modified hydraulic conductance (in  $\text{m}^4 \text{Pa}^{-1} \text{s}^{-1}$ ),  $\gamma^e$  is the modified elec-  
 276 trical conductance (in S m), and  $\gamma^c$  is the modified coupling conductance (in  $\text{m}^4 \text{V}^{-1} \text{s}^{-1}$ ).  
 277 Note that the capillaries are submitted to a gradient of water pressure in steady-state con-  
 278 ditions and that generates, in turn, an electrical potential gradient.

279 Given the importance of the local electrical potential,  $\psi$ , in the above equations,  
 280 we use the code proposed by *Leroy and Mainault* [2018] to solve the general Poisson-  
 281 Boltzmann equation in each cylindrical pore at a given ionic concentration.

282 In the simulations, the  $\zeta$ -potential depends on the ionic concentration in the bulk  
 283 pore water and is determined by the following relationship [*Pride and Morgan*, 1991]:

$$\zeta(C^w) = a + b \log_{10}(C^w), \quad (28)$$

284 where  $a$  and  $b$  are fitting parameters. For this study we use the parameter values obtained  
 285 by *Jaafar et al.* [2009] for NaCl brine:  $a=-6.43$  mV and  $b=20.85$  mV for silicate materials.  
 286 Note that *Cherubini et al.* [2018] propose different values of  $a$  and  $b$  for carbonates based  
 287 on experimental streaming potential measurements.

288 The electrical conductivity of the water also depends on the ionic concentration. In  
 289 our simulation, we consider the *Sen and Goode* [1992] empirical model:

$$\sigma_w(C^w, T) = (a_1 + a_2T + a_3T^2) C^w - \left( \frac{a_4 + a_5T}{1 - a_6\sqrt{C^w}} \right), \quad (29)$$

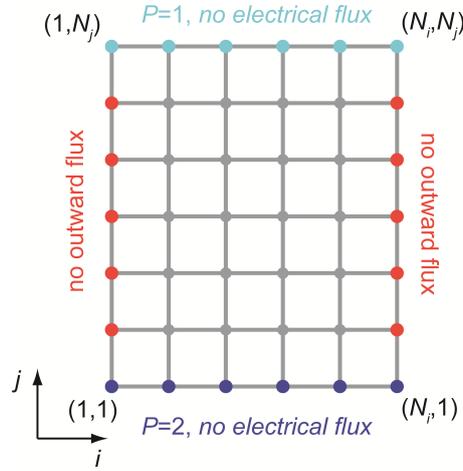
290 with  $a_1 = 5.6$  S L m<sup>-1</sup> mol<sup>-1</sup>,  $a_2 = 0.27$  S L m<sup>-1</sup> mol<sup>-1</sup> °C<sup>-1</sup>,  $a_3 = -1.51 \times 10^{-4}$  S L  
 291 m<sup>-1</sup> mol<sup>-1</sup> °C<sup>-2</sup>,  $a_4 = 2.36$  (S L m<sup>-1</sup> mol<sup>-1</sup>)<sup>3/2</sup>,  $a_5 = 0.099$  (S L m<sup>-1</sup> mol<sup>-1</sup> °C<sup>-1</sup>)<sup>3/2</sup>,  
 292  $a_6 = 0.214$  (mol<sup>-1</sup>)<sup>-1/2</sup>, and in which the ionic concentration and the temperature are  
 293 expressed in mol L<sup>-1</sup> and °C, respectively.

### 294 3.2 2D pore network and related equation system

295 We consider a 2D pore network as shown in Fig. 2. At each node  $(i, j)$  of the grid,  
 296 we applied *Kirchhoff* [1845]'s law for the conservation of the mass and of the electrical  
 297 charge, which yields:

$$\left\{ \begin{array}{l} -\gamma_{i-1,j \rightarrow i,j}^h (P_{i,j} - P_{i-1,j}) + \gamma_{i-1,j \rightarrow i,j}^c (V_{i,j} - V_{i-1,j}) \\ \quad -\gamma_{i+1,j \rightarrow i,j}^h (P_{i,j} - P_{i+1,j}) + \gamma_{i+1,j \rightarrow i,j}^c (V_{i,j} - V_{i+1,j}) \\ \quad -\gamma_{i,j-1 \rightarrow i,j}^h (P_{i,j} - P_{i,j-1}) + \gamma_{i,j-1 \rightarrow i,j}^c (V_{i,j} - V_{i,j-1}) \\ \quad -\gamma_{i,j+1 \rightarrow i,j}^h (P_{i,j} - P_{i,j+1}) + \gamma_{i,j+1 \rightarrow i,j}^c (V_{i,j} - V_{i,j+1}) = 0 \\ \gamma_{i-1,j \rightarrow i,j}^c (P_{i,j} - P_{i-1,j}) - \gamma_{i-1,j \rightarrow i,j}^e (V_{i,j} - V_{i-1,j}) \\ \quad \gamma_{i+1,j \rightarrow i,j}^h (P_{i,j} - P_{i+1,j}) - \gamma_{i+1,j \rightarrow i,j}^e (V_{i,j} - V_{i+1,j}) \\ \quad \gamma_{i,j-1 \rightarrow i,j}^h (P_{i,j} - P_{i,j-1}) - \gamma_{i,j-1 \rightarrow i,j}^e (V_{i,j} - V_{i,j-1}) \\ \quad \gamma_{i,j+1 \rightarrow i,j}^h (P_{i,j} - P_{i,j+1}) - \gamma_{i,j+1 \rightarrow i,j}^e (V_{i,j} - V_{i,j+1}) = 0 \end{array} \right. \quad (30)$$

299 where  $\gamma_{x \rightarrow y}$  is the modified conductance of the tube linking node  $x$  to node  $y$ . With the  
 300 appropriate boundary conditions (i.e., no fluxes over the lateral boundaries, no inflowing  
 301 electrical flux at the upstream boundary and no outflowing electrical flux at the down-  
 302 stream boundary), we obtain a linear system whose unknowns are the  $N_i \times N_j$  hydraulic  
 303 pressure values at the nodes, the  $N_i \times N_j$  electrical potential values at the nodes, the value  
 304 of the electrical potential  $V_u$  in the upstream reservoir, and the value of the electrical po-  
 305 tential  $V_d$  in the downstream reservoir. Note that all the tubes connecting two nodes have  
 306 the same length  $l$ . See Appendix A for the full derivation of the system.



307 **Figure 2.** Scheme of the pore network organization and the boundary conditions used in our simulations.  
 308 Note that all tubes have the same length  $l$ .

### 309 3.3 Pore size distribution

310 In this work, we investigate the effect of four different pore size distributions on  
 311 streaming current generation: fractal, exponential symmetric, lognormal and double log-  
 312 normal (i.e., bimodal). Note that we first built the networks for a pore size range between  
 313 1 and 100  $\mu\text{m}$  (Fig. 3), then we shifted this range towards smaller pores in order to ob-  
 314 tain smaller permeabilities while keeping constant the ratio  $\alpha = R_{max}/R_{min}$ . Hence, we  
 315 obtained five different permeabilities for each pore size distribution.

### 3.3.1 Fractal distribution

We start with a fractal pore size distribution (Fig. 3a) as many geological porous media exhibit frequency distribution skewed towards smaller pore radii [Dullien, 2012]. It is also the pore size distribution used by Guarracino and Jougnot [2018] to develop their analytical model (i.e., Eq. 24).

The cumulative size distribution of pores whose radii are greater than or equal to  $R$  (m) is assumed to obey the following fractal law [Tyler and Wheatcraft, 1990; Yu et al., 2003; Guarracino et al., 2014]:

$$N(R) = \left( \frac{R_{REV}}{R} \right)^D, \quad (31)$$

where  $D$  is the fractal dimension of pore size with  $1 < D < 2$  and  $0 < R_{min} \leq R \leq R_{max} < R_{REV}$ . Differentiating (31) with respect to  $R$  we obtain the number of pores whose radii are in the infinitesimal range  $R$  to  $R + dR$ :

$$dN = -DR_{REV}^D R^{-D-1} dR, \quad (32)$$

where the negative sign implies that the number of pores decreases with the increase of pore radius  $R$ . In fact, the resulting distribution is a decreasing exponential in a semilogarithmic space.

### 3.3.2 Exponential symmetric distribution

To generate the exponential symmetric distribution (Fig. 3b), we contracted the fractal distribution over one decade, we shifted it to the range 10-100  $\mu\text{m}$ , then we added the symmetric part over the range 1-10  $\mu\text{m}$  to obtain the exponentially increasing part, and finally we normalized the distribution to get a cumulative distribution comprised between 0 and 1.

### 3.3.3 Lognormal distribution

The lognormal distribution (Fig. 3c) is so that the decimal logarithm of the radius is normally distributed, as done in Maineult et al. [2017]. The probability  $P$  that  $\log_{10}(R)$  is less than  $X$  is given by:

$$P(\log_{10}(R) \leq X) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{X - \log_{10}(R_{peak})}{s\sqrt{2}} \right), \quad (33)$$

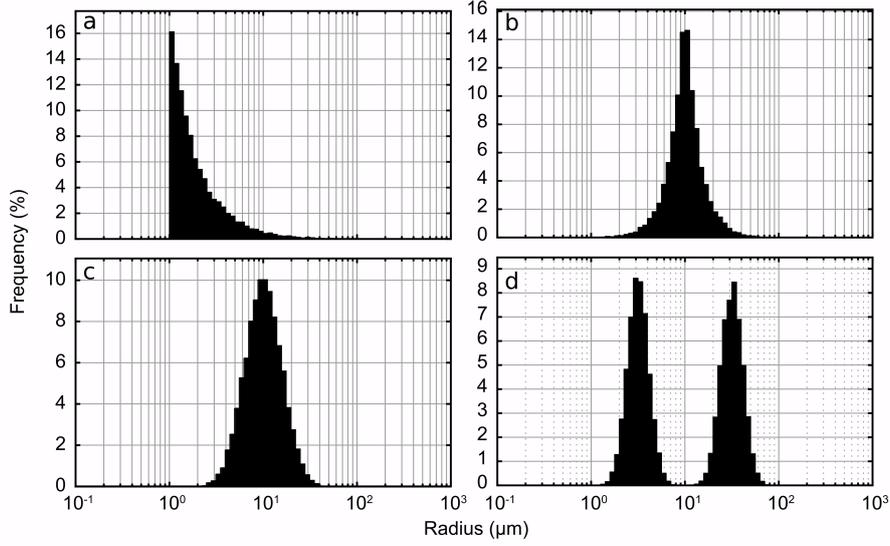
where  $R_{peak}$  is the value of the radius associated to the peak of the distribution, and  $s$  is the standard deviation.

### 3.3.4 Double lognormal distribution

The double lognormal distribution (Fig. 3d) is the sum of two lognormal distributions with the same standard deviation  $s$ , and writes :

$$P(\log_{10}(R) \leq X) = \frac{1}{2} + \frac{1}{4} \operatorname{erf} \left( \frac{X - \log_{10}(R_{peak,1})}{s_1 \sqrt{2}} \right) + \frac{1}{4} \operatorname{erf} \left( \frac{X - \log_{10}(R_{peak,2})}{s_2 \sqrt{2}} \right), \quad (34)$$

where the bimodal distribution is obtained through the choice of the two peaks for the distribution  $R_{peak,1}$  and  $R_{peak,2}$ .



**Figure 3.** Pore size distributions used in this work: (a) fractal ( $D = 1.5$ ), (b) exponential symmetric, (c) lognormal ( $R_{peak} = 10\mu\text{m}$  and  $s = 0.45973$ ), and (d) double lognormal ( $R_{peak,1} = 3.166\mu\text{m}$ ,  $R_{peak,2} = 31.66\mu\text{m}$ , and  $s_1 = s_2 = s/2$ ). Note that the different permeabilities are obtained by shifting the distribution towards smaller pores but keeping constant the ratio  $\alpha = R_{max}/R_{min}$ .

### 3.4 Petrophysical parameters computation

In our numerical simulations, we impose a hydraulic pressure gradient and obtain the resulting voltage values  $V_u$  and  $V_d$ . It is then trivial to compute the corresponding electrokinetic coupling coefficient using,

$$C_{EK} = \frac{\Delta V}{\Delta P} = \frac{V_d - V_u}{P_{i,N_j} - P_{i,1}} = \frac{V_d - V_u}{2 - 1} = V_d - V_u. \quad (35)$$

Then, the effective excess charge density is obtained by modifying Eq. 3:

$$\hat{Q}_v = -\frac{\eta_w \sigma C_{EK}}{k}. \quad (36)$$

where the permeability is deduced from the pore network simulation. As we neglect the surface electrical conductivity, Eq. 36 can then be expressed by,

$$\hat{Q}_v = -\frac{\eta_w \sigma_w C_{EK}}{kF}. \quad (37)$$

where  $F$  is the formation factor, also deduced from the pore network simulation. Note that, as we neglect the surface conductivity of the medium, the formation factor is the ratio between the pore network and the pore water electrical conductivities:  $F = \sigma_w / \sigma$ . The computation of  $k/\phi$  and  $F\phi$  are described in Appendix B.

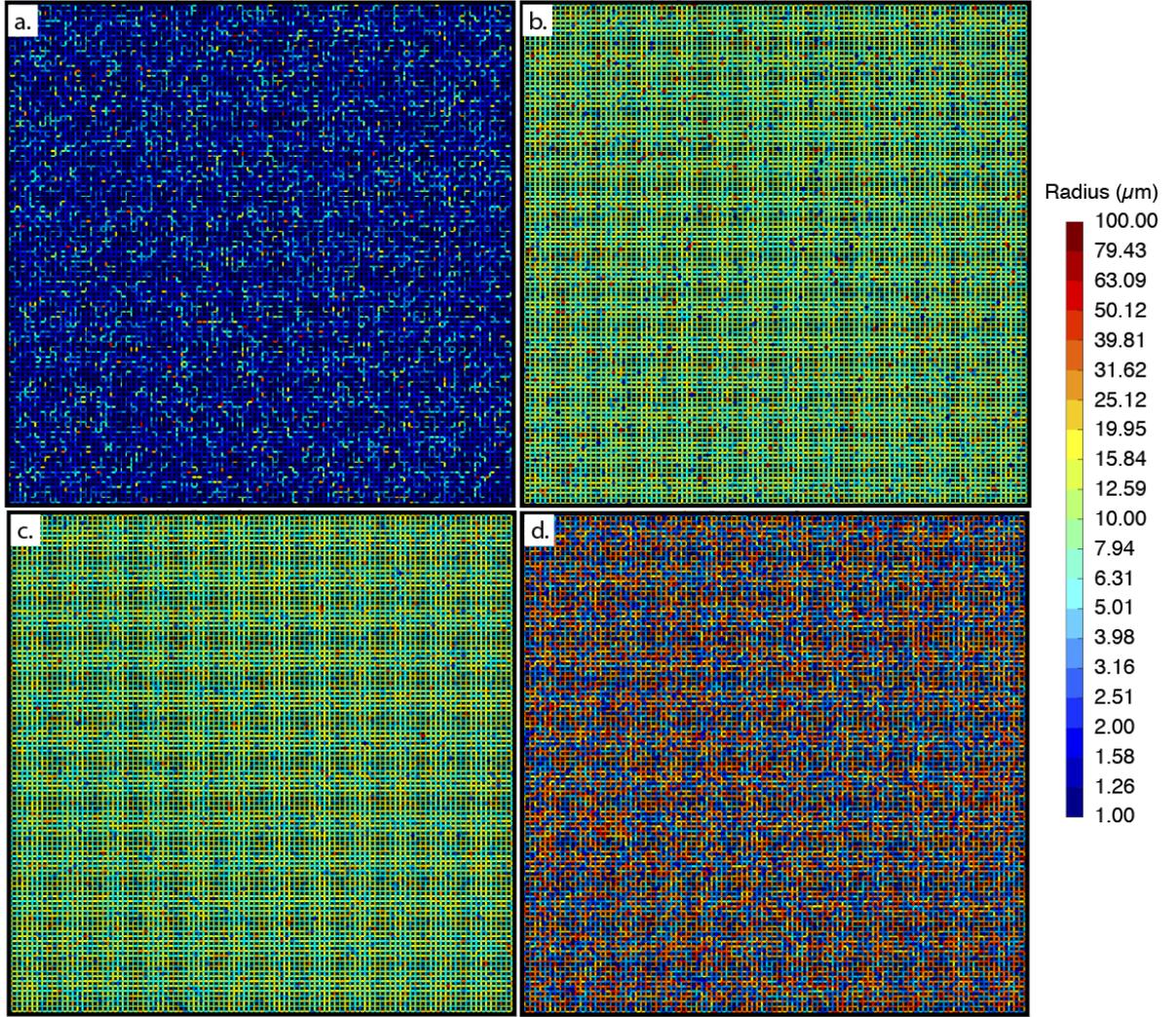
## 4 Numerical results

The simulations were run once for each given distribution (5 pore size distributions for each of the 4 types) and concentration (9 different concentrations) by solving the linear system described in the previous section; that is results for 180 pore networks with a size of  $100 \times 100$ . The results obtained from these simulations can be found in Appendix C. In our simulations, the temperature is fixed to  $20^\circ\text{C}$ . This section presents the simulation results on the effect of the pore size distributions on the two electrokinetic coupling parameters,  $C_{EK}$  and  $\hat{Q}_v$ , for a large range of permeabilities (from  $10^{-16}$  to  $10^{-10}$   $\text{m}^2$ ) and ionic concentrations (from  $10^{-4}$  to  $1$   $\text{mol L}^{-1}$ ).

### 4.1 Influence of the pore size distribution on the permeability

The pore size distribution has a major impact on the pore network effective permeability. As one can see on Figs. 3 and 4, for a given range of capillary radius (i.e., from 1 to 100  $\mu\text{m}$ ), the fractal distribution contains a much higher number of thin capillaries than the exponential symmetric and the lognormal distributions. This yields a smaller effective permeability of the 2D pore network with fractal pore size distribution. By its bimodal nature, double lognormal networks (Figs. 3d) contain both larger and smaller pores than the exponential symmetric and lognormal networks (Figs. 3b and c). However, Fig. 4d shows that their random distribution yields that larger pores are isolated from each other by smaller pores, hence yielding a smaller effective permeability of the double lognormal networks.

Given the important similarity between the exponential symmetric and lognormal pore size distribution (Figs. 3b and c), it is not surprising that both networks have similar permeabilities.



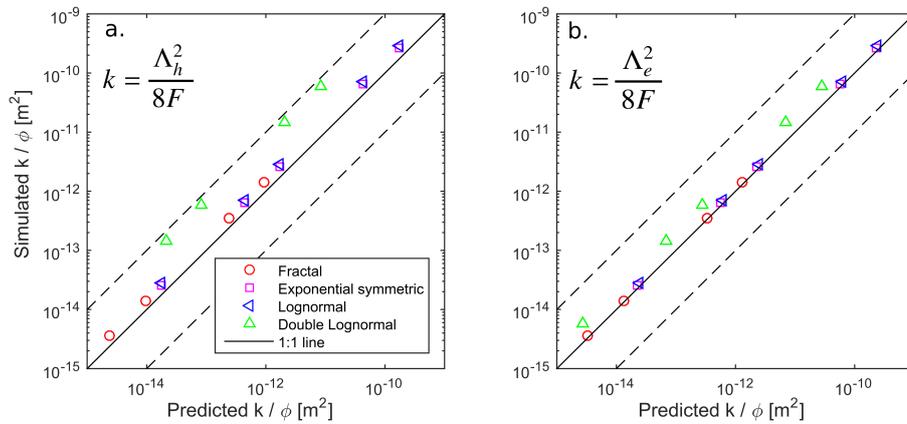
388 **Figure 4.** Examples of the pore networks used in this work: (a) fractal, (b) exponential symmetric, (c)  
 389 lognormal, and (d) double lognormal (in these examples, the capillary sizes range from 1 to 100  $\mu m$ ). Note  
 390 that the size of the networks was  $100 \times 100$  nodes. See the corresponding frequency pore size distributions in  
 391 Fig. 3.

395 The Johnson's length [*Schwartz et al.*, 1989],  $\Lambda$  (m), is a petrophysical parameter  
 396 that has been shown to be representative of a medium permeability. *Revil and Cathles*  
 397 [1999] proposes a simple model to predict the medium permeability:

$$k = \frac{\Lambda^2}{8F}. \quad (38)$$

398 Figures 5a and b compare the permeability resulting from the pore network simulations  
 399 and the ones predicted by the model of *Revil and Cathles* [1999] (Eq. 38) using the hy-  
 400 draulic ( $\Lambda_h$ ) and electrical ( $\Lambda_e$ ) Johnson's lengths deduced from the pore network sim-

401 ulations [see *Bernabé and Revil, 1995*, and Appendix B], respectively. One can see that  
 402 the model from *Revil and Cathles [1999]* tends to overpredict the effective permeabilities  
 403 of the networks, except for the double lognormal network permeabilities predicted by  $\Lambda_h$ .  
 404 Nevertheless, both predictions are rather good (within half an order of magnitude), show-  
 405 ing the interest of Eq. (38) to characterize a porous medium [see also the discussions in  
 406 *Maineult et al., 2018*].

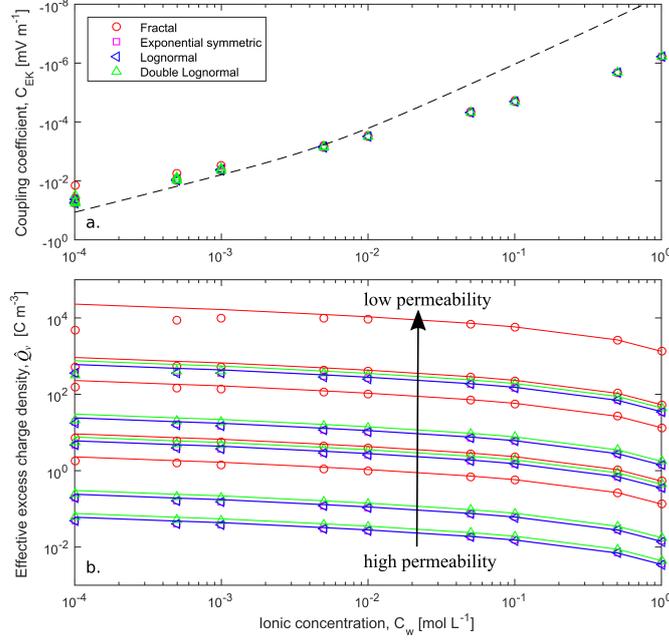


407 **Figure 5.** Comparison between the simulated permeabilities (normalized by the porosities) with the pore  
 408 network model and the ones predicted by the model of *Revil and Cathles [1999]* based on the (a) hydraulic,  
 409  $\Lambda_h$ , and (b) electrical,  $\Lambda_e$ , Johnson's lengths, respectively (see definitions in Appendix B). The solid black  
 410 line corresponds to the 1:1 line, while the dashed lines correspond to the one order of magnitude range.

#### 411 **4.2 Evolution of the coupling parameters with the ionic concentration and perme-** 412 **ability**

413 Figure 6a presents the evolution of the coupling coefficient as a function of the pore  
 414 water ionic concentration. The simulation results clearly indicate that the NaCl ionic con-  
 415 centration drives the amplitude of the coupling coefficient, while the influence of pore  
 416 size distribution is rather small (from less than 1% for  $1 \text{ mol L}^{-1}$  up to 66% for  $10^{-4} \text{ mol}$   
 417  $\text{L}^{-1}$ ). This is consistent with the Helmholtz-Smoluchowski equation (Eq. 2) that contains

418 two parameters which are concentration dependent, the  $\zeta$ -potential (Eq. 28) and the pore  
 419 water electrical conductivity (Eq. 29), but none related to the medium geometrical proper-  
 420 ties.



421 **Figure 6.** Simulation results presented as (a) electrokinetic coupling coefficient and (b) effective excess  
 422 charge density as a function of the ionic concentrations for the different pore size distributions. In the (a) sub-  
 423 subplot, the dashed black line corresponds to the empirical relationship proposed by *Linde et al.* [2007] (Eq. 39).  
 424 In the (b) subplot, the solid lines in colors correspond to the model predictions of *Guarracino and Jougnot*  
 425 [2018] (Eq. 24).

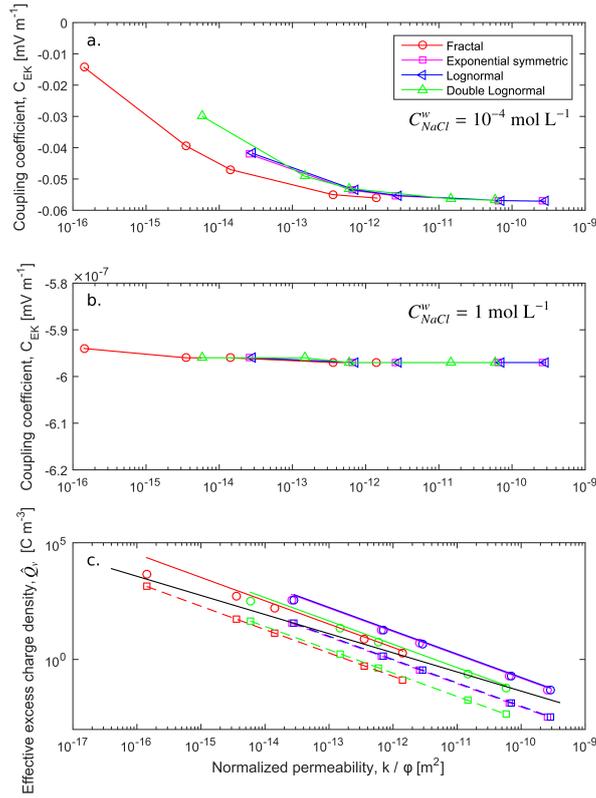
426 *Linde et al.* [2007] proposed an empirical model depending only on the pore water  
 427 ionic concentration (through its electrical conductivity) based on a large data set of cou-  
 428 pling coefficients:

$$\log |C_{EK}| = b_1 + b_2 \log(\sigma_w) + b_3 \log(\sigma_w)^2, \quad (39)$$

429 where  $b_1 = -0.895$ ,  $b_2 = -1.319$ , and  $b_3 = -0.1227$ . Fig. 6a shows that this empirical model  
 430 matches rather well for ionic concentrations between  $10^{-4}$  to  $10^{-2}$  mol L<sup>-1</sup>, clearly con-  
 431 firming that ionic concentration is the main driver.

432 Figures 7a and b show that the variation of  $C_{EK}$  as a function of the network per-  
 433 meability (hence of the network pore size distribution, see previous subsection) strongly

434 depends on the ionic concentration. Indeed,  $C_{EK}$  diminishes importantly when permeabil-  
 435 ity increases at low salinity ( $C_{NaCl}^w = 10^{-4} \text{ mol L}^{-1}$  in Fig. 7a), but it barely varies for  
 436 higher salinity ( $C_{NaCl}^w = 1 \text{ mol L}^{-1}$  in Fig. 7b). As for the permeabilities,  $C_{EK}$  for the  
 437 exponential symmetric and lognormal networks are very similar, while the fractal distribu-  
 438 tion has a very different behaviour, probably related to the larger number of smaller pores.



439 **Figure 7.** Electrokinetic coupling coefficient as a function of the permeability normalized by the porosity  
 440 for (a)  $C_w = 10^{-4} \text{ mol L}^{-1}$  and (b)  $C_w = 1 \text{ mol L}^{-1}$  from our numerical simulation. (c) Effective excess  
 441 charge density as a function of the permeability normalized by the porosity for the different pore size distri-  
 442 butions for  $C_w = 10^{-4}$  and  $1 \text{ mol L}^{-1}$ . Note that each point corresponds to the simulation result for a given  
 443 network. On the (c) subplot, the solid and dashed colored lines correspond to the model predictions of Guar-  
 444 racino and Jougnot [2018] (Eq. 24) for  $C_w = 10^{-4} \text{ mol L}^{-1}$  and  $C_w = 1 \text{ mol L}^{-1}$ , respectively; while the  
 445 single black solid line is the prediction from Jardani et al. [2007] with a fixed porosity  $\phi = 0.4$ .

446 Contrarily to the electrokinetic coupling coefficient, the effective excess charge den-  
 447 sity computed from Eq. (37) strongly depends both on ionic concentration and network

448 permeability. Figures 6b and 7c show that the permeability is the most important param-  
 449 eter controlling the magnitude of  $\hat{Q}_v$ : a decrease of 4 orders of magnitude in permeabil-  
 450 ity yields an increase of 4 orders of magnitude for  $\hat{Q}_v$ . This behaviour is consistent with  
 451 experimental data and models from the literature [e.g., *Titov et al.*, 2002; *Jardani et al.*,  
 452 2007; *Jougnot et al.*, 2012]. The influence of the ionic concentration on the effective ex-  
 453 cess charge density is also consistent with experimental data from the literature: an in-  
 454 crease of 4 orders of magnitude in the ionic concentration yields a decrease of around 1  
 455 order of magnitude for  $\hat{Q}_v$  [e.g., *Pengra et al.*, 1999; *Jougnot et al.*, 2015; *Cherubini et al.*,  
 456 2018].

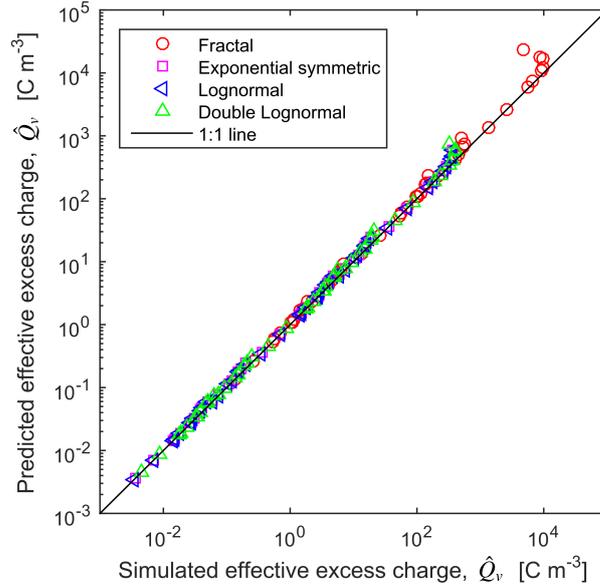
### 457 **4.3 Testing the model of *Guarracino and Jougnot* [2018]**

458 The dependence of the effective excess charge on both the permeability and the pore  
 459 water ionic concentration is discussed in details in *Guarracino and Jougnot* [2018] and  
 460 taken into account in their model (Eq. 24). Figures 6b and 7c show the very good agree-  
 461 ment between the  $\hat{Q}_v$  obtained from the network simulations and the one predicted by the  
 462 *Guarracino and Jougnot* [2018]’s model as a function of the ionic concentration and per-  
 463 meability, respectively. All the parameters needed for the model (Eq. 24) are either input  
 464 parameters ( $C^w$ , thus  $\zeta$  and  $l_D$ , from Eqs. 28 and 18, respectively) or calculated outputs  
 465 from the simulations ( $k/\phi$ , from Eq. 63). Following the proposition of *Guarracino and*  
 466 *Jougnot* [2018], we use the *Winsauer et al.* [1952] model to determine the hydraulic tortu-  
 467 osity from:

$$\tau = \sqrt{F\phi}. \quad (40)$$

468 Therefore, none of the parameters were fitted in order to obtain these predictions in very  
 469 good agreement with the computations from our numerical simulations. Note that the *Jar-*  
 470 *dani et al.* [2007]’s model corresponds fairly well to an average trend, regardless the net-  
 471 work and the ionic concentration.

472 Figure 8 represents the same data (i.e., for all networks and ionic concentrations)  
 473 along a 1:1 line. One can notice that the model slightly overpredicts the numerical effec-  
 474 tive excess charge for very high  $\hat{Q}_v$ , that is for low permeability and low ionic concentra-  
 475 tion. This can be explained by the model limitation: the capillary radius has to be signifi-  
 476 cantly larger than the Debye length  $R \gg 5l_D$ .



477 **Figure 8.** Comparison between the simulated effective excess charge density with the pore network model  
 478 and the one predicted by the analytical model of *Guarracino and Jougnot* [2018]. The solid black line corre-  
 479 sponds to the 1:1 line.

#### 480 **4.4 Limitation of the model *Guarracino and Jougnot* [2018] in small pores at low** 481 **ionic concentration**

482 In this subsection, we investigate why the largest misfits are obtained for the highest  
 483 values of effective excess charge, that is, for the lowest ionic concentrations (i.e., thickest  
 484 diffuse layers) and for the lowest permeabilities (i.e., smallest pore sizes). In Fig. 8, one  
 485 can see that it is especially the case for the fractal distribution, where the amount of small  
 486 pores is larger than in the other distributions (see Fig. 3).

487 Therefore, we consider the smallest investigated capillaries ( $R = 0.1\mu\text{m}$ ) filled by  
 488 a pore water containing the lowest ionic concentration of NaCl,  $C_{NaCl}^w = 10^{-4} \text{ mol L}^{-1}$   
 489 (i.e.,  $l_D = 3.04 \times 10^{-8} \text{ m}$ , hence  $R = 3.29l_D < 4l_D$ ), i.e., the most extreme case for  
 490 the present study. Then, we use the numerical code of *Leroy and Mainault* [2018] to solve  
 491 for the Poisson-Boltzmann equation in an infinite charged cylinder and the  $\zeta$ -potential is  
 492  $\zeta = -89.8 \text{ mV}$  following *Jaafar et al.* [2009] (Eq. 28). Figures 9 and 10 illustrate the  
 493 limitation of the Debye-Hückel approximation used by *Guarracino and Jougnot* [2018]

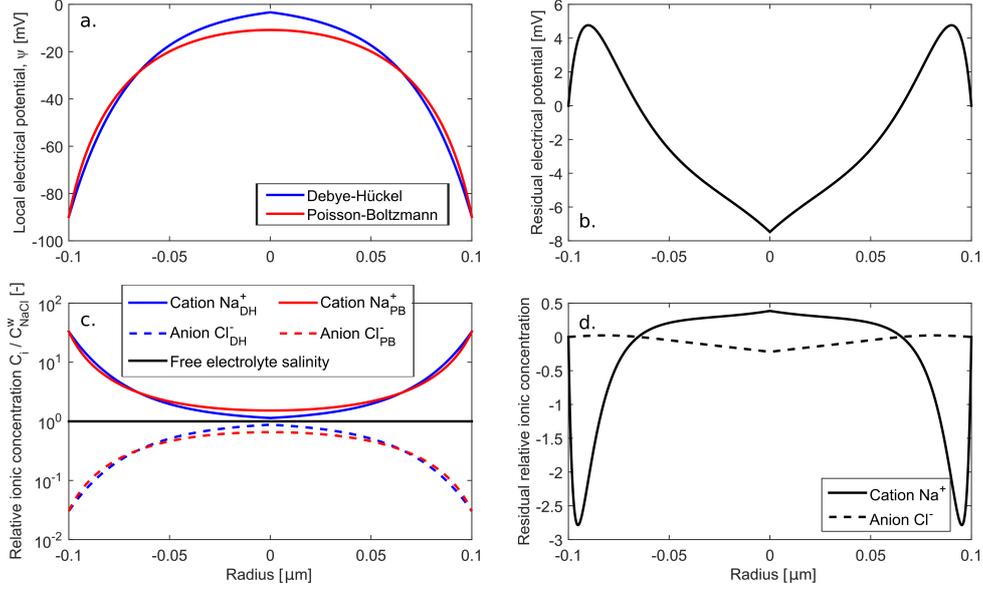
494 by comparing its results to the Poisson-Boltzmann numerical resolution using *Leroy and*  
495 *Maineult* [2018].

496 Figure 9a compares the local electrical potential calculated with the Debye-Hückel  
497 approximation (Eq. 17) and the general Poisson-Boltzmann (Eq. 12), while Figure 9b dis-  
498 plays the corresponding residual potential. Given that  $R < 4l_D$ , one can see that  $\psi \neq 0$   
499 mV in the middle of the pore, this implies that the EDL overlap [e.g., *Gonçalvès et al.,*  
500 2007]. The effect on the local electrical potential is substantial: the residual is close to  
501 50% at the center of the pore. This has a significant effect on the distribution of the ions  
502 as shown in Figs. 9c and d. For  $R = 0.1\mu\text{m}$  and  $C_{NaCl}^w = 10^{-4} \text{ mol L}^{-1}$ , one can see that  
503 there is no free electrolyte, therefore the local ionic concentrations are different from the  
504 bulk water concentrations  $C_{Na} \gg C_{Na}^w$  and  $C_{Cl} \ll C_{Cl}^w$  in the entire capillary. Conse-  
505 quently, the distribution of the excess charge density  $\bar{Q}_v$  calculated from Eq. 19 in a small  
506 capillary for low concentrations is strongly affected by the Debye-Hückel approximation  
507 (Fig. 10b and c). This example on the most extreme case used in the previous simulation  
508 clearly demonstrates why the model of *Guarracino and Jougnot* [2018] cannot correctly  
509 predict the effective excess charge density in pores such as  $R < 5l_D$ , that is when the thin  
510 double layer assumption is not respected.

## 523 5 Discussion and conclusion

524 In the present paper, we present numerical simulations of streaming current gen-  
525 eration in water saturated 2D pore networks with different pore size distributions, hence  
526 different permeabilities (from  $10^{-16}$  to  $10^{-10} \text{ m}^2$ ). We performed the simulations to ob-  
527 tain the electrokinetic coupling coefficients for pore water having a NaCl concentrations  
528 ranging from  $10^{-4}$  to  $1 \text{ mol L}^{-1}$ . From these simulations we deduced the effective excess  
529 charge density from the corresponding coupling coefficient and performed a detailed anal-  
530 ysis of the behaviour of these two electrokinetic coupling parameters.

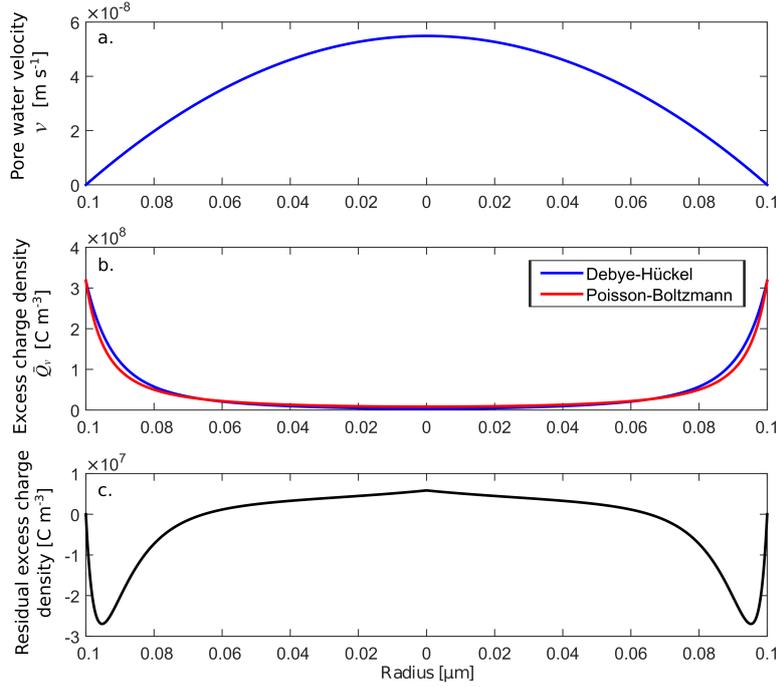
531 Our first finding is that the pore size distribution has a primary influence on the  
532 medium's permeability (Fig. 5) as expected from the literature, but almost no influence  
533 on the electrokinetic coupling coefficient (Figs. 6a and 7b). This is consistent with the  
534 widely used model of Helmholtz-Smoluchowski (Eq. 2) which does not include any in-  
535 formation nor parameters about the medium's texture and has been proven to be useful in  
536 a large range of natural geological media (as long as the surface conductivity can be ne-



511 **Figure 9.** Comparison between the Debye-Hückel approximation and the Poisson-Boltzmann equation  
 512 to compute (a) the electrical potential distribution and (c) the ionic species relative concentration  
 513 distribution in a small capillary ( $R = 10^{-7}$  m) containing a NaCl electrolyte with  $C_{NaCl}^w = 10^{-4}$  mol L<sup>-1</sup> (i.e.,  
 514  $l_D = 3.04 \times 10^{-8}$  m). (b) and (d) show the corresponding residual electrical potential and relative ionic  
 515 concentration, respectively. Note that the  $x$ -axis is a modified coordinate  $r' = R - r$  such as  $r' = 0$  m in the  
 516 middle of the capillary.

537 neglected). It is therefore clear that the pore water chemistry is the main driver for the  $C_{EK}$   
 538 as proposed by the empirical model of *Linde et al.* [2007].

539 On the contrary, the pore size distribution has a strong influence on the effective ex-  
 540 cess charge density through the permeability, as it was expected from both empirical [e.g.,  
 541 *Titov et al.*, 2002; *Jardani et al.*, 2007; *Cherubini et al.*, 2018] and theoretical evidence  
 542 [e.g., *Jougnot et al.*, 2012; *Guarracino and Jougnot*, 2018]. When considering Eq. 3 and  
 543 Eq. 24 [*Guarracino and Jougnot*, 2018], it is clear that the permeability simplifies out in  
 544 the electrokinetic coupling coefficient  $C_{EK}$ . One can also note that the analytical model  
 545 of *Guarracino and Jougnot* [2018], originally defined for fractal media, performs well for  
 546 any kind of pore size distribution (even double porosity ones) given that this information  
 547 is included in the model through the medium's permeability and porosity that appear ex-  
 548 plicitely.



517 **Figure 10.** (a) Distribution of the pore water velocity in a small capillary ( $R = 10^{-7}$  m) following  
518 Poiseuille's law. (b) Comparison of the excess charge density distribution obtained from the Debye-Hückel  
519 approximation and the numerical Poisson-Boltzmann resolution in the same capillary ( $R = 10^{-7}$  m) contain-  
520 ing a NaCl electrolyte with  $C_{NaCl}^w = 10^{-4}$  mol L $^{-1}$  (i.e.,  $l_D = 3.04 \times 10^{-8}$ m), and (c) the corresponding  
521 residual. Note that the  $x$ -axis is a modified coordinate  $r' = R - r$  such as  $r' = 0$  m in the middle of the  
522 capillary.

549 Nevertheless, the observations from the previous paragraphs are not valid for very  
550 small pores filled by pore water with a low ionic concentration, that is  $C^w < 10^{-3}$  mol L $^{-1}$   
551 (Figs. 6a and 7a). Indeed, when the salinity decreases and if the medium has small pores  
552 (Fig. 7a),  $C_{EK}$  becomes highly dependent on the permeability. This behaviour is consis-  
553 tent with the previous work of *Bernabé* [1998] on pore networks, but also with the exper-  
554 imental results of *Jouniaux and Pozzi* [1995b] (using a very resistive water). This effect is  
555 directly related to the EDL in the pore space: when  $l_D$  becomes important in comparison  
556 to the pore radius ( $R < 4l_D$ ), the diffuse layers from both sides of the capillary start to  
557 overlap, yielding a strong effect on the amount of excess charge that can be dragged by the  
558 water flow (e.g. Figs. 9 and 10). Such effect also impacts the performance of the model

559 of *Guarracino and Jougnot* [2018] to reproduce the simulated effective excess charge den-  
560 sities (Fig. 8).

561 In geological media and under most environmental conditions (i.e. groundwater for  
562 human consumption or subsurface reservoirs),  $10^{-4}$  mol L<sup>-1</sup> represents an extreme case  
563 scenario [e.g., *McCleskey*, 2011]. Indeed, ionic strengths (i.e., a proxy for ionic concen-  
564 tration) in potable water typically vary between  $10^{-3}$  and  $10^{-2}$  mol L<sup>-1</sup>, while reservoirs  
565 can be saturated with brines having much higher ionic concentrations depending on the  
566 formation. Therefore, the assumption of  $R \gg 4l_D$  can be considered valid in most natural  
567 systems, which allows the use of the model recently proposed by *Guarracino and Jougnot*  
568 [2018] (valid for  $R > 5l_D$ ).

569 In addition to the intrinsic limitation of the model proposed by *Guarracino and*  
570 *Jougnot* [2018], the fact that we neglect the surface conductivity in Eq. 37 even for the  
571 lowest ionic concentration and smaller pores can also contribute to the misfit. Further  
572 developments of the present 2D pore network code should also include an explicit cal-  
573 culation of the surface conductivity for the determination of the effective excess charge  
574 density. This would open the possibility of studying the behaviour of micro-porous me-  
575 dia such as clay rocks. Additional improvements on our pore network modeling approach  
576 could also allow further studies, among which: relating pore lengths to pore sizes to mimic  
577 more natural observations (e.g., small pore sizes are usually related to small pore length),  
578 considering connectivities higher than 4 for each nodes. Nevertheless, despite all these  
579 limitations, the two approaches that we consider here converge towards similar predictions,  
580 and this is remarkable, since they are totally independent. Further works will require the  
581 overcoming of these limitations, and also to implement 3D network, in order to produce  
582 synthetic media closer to real ones. A more advance approach would be extracting pore  
583 networks that replicates the pore space obtain from rock sample imagery [e.g., *Bryant and*  
584 *Blunt*, 1992] to solve for the electrokinetic coupling.

585 We believe that the present study will help to better understand the theoretical links  
586 between the electrokinetic coupling coefficient and the effective excess charge approaches,  
587 providing a mechanistic study of the streaming potential generation under water saturated  
588 conditions. In the future, we will try to extend this approach and the corresponding study  
589 for partially saturated conditions [see *Jougnot et al.*, 2012; *Soldi et al.*, 2019].

## Appendix A: Pressure and electrical potential equations in the pore network

Inside the network, that is for the indexes  $(i, j) \in [2, N_i - 1] \times [2, N_j - 1]$ , Eq. 30 is rewritten as,

$$\begin{cases} \gamma_{i-1,j \rightarrow i,j}^h P_{i-1,j} + \gamma_{i+1,j \rightarrow i,j}^h P_{i+1,j} - \kappa_{i,j}^h P_{i,j} + \gamma_{i,j-1 \rightarrow i,j}^h P_{i,j-1} + \gamma_{i,j+1 \rightarrow i,j}^h P_{i,j+1} \\ \quad - \gamma_{i-1,j \rightarrow i,j}^c V_{i-1,j} - \gamma_{i+1,j \rightarrow i,j}^c V_{i+1,j} + \kappa_{i,j}^c V_{i,j} - \gamma_{i,j-1 \rightarrow i,j}^c V_{i,j-1} - \gamma_{i,j+1 \rightarrow i,j}^c V_{i,j+1} = 0 \\ -\gamma_{i-1,j \rightarrow i,j}^c P_{i-1,j} - \gamma_{i+1,j \rightarrow i,j}^c P_{i+1,j} + \kappa_{i,j}^c P_{i,j} - \gamma_{i,j-1 \rightarrow i,j}^c P_{i,j-1} - \gamma_{i,j+1 \rightarrow i,j}^c P_{i,j+1} \\ \quad + \gamma_{i-1,j \rightarrow i,j}^e V_{i-1,j} + \gamma_{i+1,j \rightarrow i,j}^e V_{i+1,j} - \kappa_{i,j}^e V_{i,j} + \gamma_{i,j-1 \rightarrow i,j}^e V_{i,j-1} + \gamma_{i,j+1 \rightarrow i,j}^e V_{i,j+1} = 0 \end{cases} \quad (41)$$

with,

$$\begin{cases} \kappa_{i,j}^h = (\gamma_{i-1,j \rightarrow i,j}^h + \gamma_{i+1,j \rightarrow i,j}^h + \gamma_{i,j-1 \rightarrow i,j}^h + \gamma_{i,j+1 \rightarrow i,j}^h) \\ \kappa_{i,j}^c = (\gamma_{i-1,j \rightarrow i,j}^c + \gamma_{i+1,j \rightarrow i,j}^c + \gamma_{i,j-1 \rightarrow i,j}^c + \gamma_{i,j+1 \rightarrow i,j}^c) \\ \kappa_{i,j}^e = (\gamma_{i-1,j \rightarrow i,j}^e + \gamma_{i+1,j \rightarrow i,j}^e + \gamma_{i,j-1 \rightarrow i,j}^e + \gamma_{i,j+1 \rightarrow i,j}^e) \end{cases} \quad (42)$$

in  $i = 1$  (no outward current) and  $j \in [2, N_j - 1]$ , we have

$$\begin{cases} \gamma_{2,j \rightarrow 1,j}^h P_{2,j} - \kappa_{1,j}^h P_{1,j} + \gamma_{1,j-1 \rightarrow 1,j}^h P_{1,j-1} + \gamma_{1,j+1 \rightarrow 1,j}^h P_{1,j+1} \\ \quad - \gamma_{2,j \rightarrow 1,j}^c V_{2,j} + \kappa_{1,j}^c V_{1,j} - \gamma_{1,j-1 \rightarrow 1,j}^c V_{1,j-1} + \gamma_{1,j+1 \rightarrow 1,j}^c V_{1,j+1} = 0 \\ -\gamma_{2,j \rightarrow 1,j}^c P_{2,j} + \kappa_{1,j}^c P_{1,j} - \gamma_{1,j-1 \rightarrow 1,j}^c P_{1,j-1} - \gamma_{1,j+1 \rightarrow 1,j}^c P_{1,j+1} \\ \quad + \gamma_{2,j \rightarrow 1,j}^e V_{2,j} - \kappa_{1,j}^e V_{1,j} - \gamma_{1,j-1 \rightarrow 1,j}^e V_{1,j-1} + \gamma_{1,j+1 \rightarrow 1,j}^e V_{1,j+1} = 0 \end{cases} \quad (43)$$

with

$$\begin{cases} \kappa_{1,j}^h = (\gamma_{2,j \rightarrow 1,j}^h + \gamma_{1,j-1 \rightarrow 1,j}^h + \gamma_{1,j+1 \rightarrow 1,j}^h) \\ \kappa_{1,j}^c = (\gamma_{2,j \rightarrow 1,j}^c + \gamma_{1,j-1 \rightarrow 1,j}^c + \gamma_{1,j+1 \rightarrow 1,j}^c) \\ \kappa_{1,j}^e = (\gamma_{2,j \rightarrow 1,j}^e + \gamma_{1,j-1 \rightarrow 1,j}^e + \gamma_{1,j+1 \rightarrow 1,j}^e) \end{cases} \quad (44)$$

in  $i = N_i$  (no outward current) and  $j \in [2, N_j - 1]$ , we have

$$\begin{cases} \gamma_{N_i-1,j \rightarrow N_i,j}^h P_{N_i-1,j} - \kappa_{N_i,j}^h P_{N_i,j} + \gamma_{N_i,j-1 \rightarrow N_i,j}^h P_{N_i,j-1} + \gamma_{N_i,j+1 \rightarrow N_i,j}^h P_{N_i,j+1} \\ \quad - \gamma_{N_i-1,j \rightarrow N_i,j}^c V_{N_i-1,j} + \kappa_{N_i,j}^c V_{N_i,j} - \gamma_{N_i,j-1 \rightarrow N_i,j}^c V_{N_i,j-1} + \gamma_{N_i,j+1 \rightarrow N_i,j}^c V_{N_i,j+1} = 0 \\ -\gamma_{N_i-1,j \rightarrow N_i,j}^c P_{N_i-1,j} + \kappa_{N_i,j}^c P_{N_i,j} - \gamma_{N_i,j-1 \rightarrow N_i,j}^c P_{N_i,j-1} - \gamma_{N_i,j+1 \rightarrow N_i,j}^c P_{N_i,j+1} \\ \quad + \gamma_{N_i-1,j \rightarrow N_i,j}^e V_{N_i-1,j} - \kappa_{N_i,j}^e V_{N_i,j} - \gamma_{N_i,j-1 \rightarrow N_i,j}^e V_{N_i,j-1} + \gamma_{N_i,j+1 \rightarrow N_i,j}^e V_{N_i,j+1} = 0 \end{cases} \quad (45)$$

602 with

$$603 \quad \begin{cases} \kappa_{N_i,j}^h = (\gamma_{N_i-1,j \rightarrow N_i,j}^h + \gamma_{N_i,j-1 \rightarrow N_i,j}^h + \gamma_{N_i,j+1 \rightarrow N_i,j}^h) \\ \kappa_{N_i,j}^c = (\gamma_{N_i-1,j \rightarrow N_i,j}^c + \gamma_{N_i,j-1 \rightarrow N_i,j}^c + \gamma_{N_i,j+1 \rightarrow N_i,j}^c) \\ \kappa_{N_i,j}^e = (\gamma_{N_i-1,j \rightarrow N_i,j}^e + \gamma_{N_i-1,j-1 \rightarrow N_i-1,j}^e + \gamma_{N_i-1,j+1 \rightarrow N_i-1,j}^e) \end{cases} \quad (46)$$

604 In  $j = 1$ , the following conditions are imposed for the hydraulic pressure and electrical potential:  
605

$$\begin{cases} P_{i,1} = 2 \\ V_{i,1} = V_u \end{cases}, \quad (47)$$

606 There is no inflowing electrical current, that is:

$$\sum_{i=1}^{N_i} J_{i,1 \rightarrow i,2} l = \sum_{i=1}^{N_i} (\gamma_{i,1 \rightarrow i,2}^c (P_{i,2} - P_{i,1}) - \gamma_{i,1 \rightarrow i,2}^e (V_{i,2} - V_{i,1})) = 0, \quad (48)$$

607 which yields:

$$-\sum_{i=1}^{N_i} \gamma_{i,1 \rightarrow i,2}^c P_{i,1} + \left( \sum_{i=1}^{N_i} \gamma_{i,1 \rightarrow i,2}^e \right) V_u + \sum_{i=1}^{N_i} \gamma_{i,1 \rightarrow i,2}^c P_{i,2} - \sum_{i=1}^{N_i} \gamma_{i,1 \rightarrow i,2}^e V_{i,2} = 0. \quad (49)$$

608 Finally, in  $j = N_j$ , the conditions are:

$$\begin{cases} P_{i,N_j} = 1 \\ V_{i,N_j} = V_d \end{cases}, \quad (50)$$

609 There is no outflowing electrical current, that is:

$$\sum_{i=1}^{N_i} J_{i,N_j-1 \rightarrow i,N_j} l = \sum_{i=1}^{N_i} (\gamma_{i,N_j-1 \rightarrow i,N_j}^c (P_{i,N_j} - P_{i,N_j-1}) - \gamma_{i,N_j-1 \rightarrow i,N_j}^e (V_{i,N_j} - V_{i,N_j-1})) = 0, \quad (51)$$

610 which yields:

$$-\sum_{i=1}^{N_i} \gamma_{i,N_j-1 \rightarrow i,N_j}^c P_{i,N_j-1} + \left( \sum_{i=1}^{N_i} \gamma_{i,N_j-1 \rightarrow i,N_j}^e \right) V_{i,N_j-1} + \sum_{i=1}^{N_i} \gamma_{i,N_j-1 \rightarrow i,N_j}^c P_{i,N_j} - \sum_{i=1}^{N_i} \gamma_{i,N_j-1 \rightarrow i,N_j}^e V_d = 0. \quad (52)$$

611 The set of equations described above (Eqs. 41-47, 49-50, 52) forms a linear system.  
612 The unknowns are the hydraulic pressure,  $P_{i,j}$ , and the electrical potential,  $V_{i,j}$ , at all  
613 nodes and the two boundary electrical potentials  $V_u$  and  $V_d$ .

614 **Appendix B: Numerical determination of the pore network permeability, formation**  
615 **factor, and Johnson's lengths**

616 For a laminar flow, i.e. following Poiseuille's law, the hydraulic flux  $F_{x \rightarrow y}$  through a  
617 capillary linking two nodes  $x$  and  $y$  writes:

$$F_{x \rightarrow y} = \frac{\pi R_{x \rightarrow y}^4}{8\eta_w} \frac{P_x - P_y}{l} = g_{x \rightarrow y}^h (P_x - P_y). \quad (53)$$

618 The length of the capillary,  $l$ , is eliminated by introducing a modified hydraulic flux de-  
619 fined as:

$$\Phi_{x \rightarrow y}^h = F_{x \rightarrow y} l = \frac{\pi R_{x \rightarrow y}^4}{8\eta_w} (P_x - P_y) = \gamma_{x \rightarrow y}^h (P_x - P_y). \quad (54)$$

620 Neglecting the surface electrical conductivity, the electrical flux  $J_{x \rightarrow y}$  corresponds to:

$$J_{x \rightarrow y} = \sigma_w \pi R_{x \rightarrow y}^2 \frac{V_x - V_y}{l} = g_{x \rightarrow y}^e (V_x - V_y). \quad (55)$$

621 The length of the capillary,  $l$ , is eliminated by introducing a modified electrical flux de-  
622 fined as:

$$\Phi_{x \rightarrow y}^e = \frac{J_{x \rightarrow y} l}{\sigma_w} = \pi R_{x \rightarrow y}^2 (V_x - V_y) = \gamma_{x \rightarrow y}^e (V_x - V_y). \quad (56)$$

623 At any node in the square network, *Kirchhoff* [1845]'s law yields

$$Z_{i,j-1 \rightarrow i,j} + Z_{i-1,j \rightarrow i,j} + Z_{i+1,j \rightarrow i,j} + Z_{i,j+1 \rightarrow i,j} = 0. \quad (57)$$

624 with  $Z$  standing for  $F$  or  $J$ , respectively. Eq. 53 or 55, leads to

$$a_{i,j-1 \rightarrow i,j} X_{i,j-1} + a_{i-1,j \rightarrow i,j} X_{i-1,j} - (a_{i,j-1 \rightarrow i,j} + a_{i-1,j \rightarrow i,j} + a_{i+1,i \rightarrow i,j} + a_{i,j+1 \rightarrow i,j}) \quad (58)$$

$$+ a_{i+1,j \rightarrow i,j} X_{i+1,j} + a_{i,j+1 \rightarrow i,j} X_{i,j+1} = 0.$$

625 with  $a = R^4$  and  $X = P$  or  $a = R^2$  and  $X = V$  for the hydraulic or the electrical case,  
626 respectively.

627 For the nodes at the border of the network, Eq. 58 is easily modified to take into  
628 account the boundary conditions (i.e., no outward flow for  $i = 1$  and  $i = N_i$ ,  $P = 1$  or  
629  $V = 1$  for  $j = 1$ , and  $P = 0$  or  $V = 0$  for  $j = N_j$ ).

630 A linear system is obtained; the  $N_i N_j$  unknowns are the hydraulic pressure or elec-  
631 trical potential at the nodes of the network. Once the system is solved, the modified fluxes  
632 can be computed using Eqs. 54 or 56.

633 The effective permeability of the pore network  $k$  ( $\text{m}^2$ ) is then computed using Darcy's  
634 law:

$$k = \frac{\eta_w Q L}{S |\Delta P|} = \frac{\eta_w}{l^2} \frac{N_j - 1}{N_i - 1} \frac{\Phi_{\Sigma \text{out/in}}^h}{|\Delta P|}, \quad (59)$$

635 where  $Q$  is the hydraulic flux,  $L$  the length of the network along the flux direction (i.e.,  
636 the  $j$ -direction),  $S$  the transversal section, and the total out-flowing and in-flowing fluxes  
637 are given by:

$$\begin{cases} \Phi_{\Sigma out}^h = \sum_{i=1}^{N_i-1} \Phi_{i, N_j-1 \rightarrow i, N_j}^h \\ \Phi_{\Sigma in}^h = \sum_{i=1}^{N_i-1} \Phi_{i, 1 \rightarrow i, 2}^h \end{cases} \quad (60)$$

638 In order to estimate the section and porosity of the network, we extend the 2D net-  
639 work into a virtual 3D one by adding two vertical capillaries of length  $l/2$  at each node,  
640 but not contributing to the transport. This yields:

$$S = (N_i - 1) l^2 \quad (61)$$

$$\phi = \frac{\left( (N_i - 1) N_j + (N_j - 1) N_i + N_i N_j \right) \pi \langle R^2 \rangle l}{(N_i - 1) (N_j - 1) l^3} \quad (62)$$

642 Extracting  $l^2$  from Eq. 62 and given that  $|\Delta P| = 1$ , the effective permeability can be  
643 determined by:

$$\frac{k}{\phi} = \frac{\eta_w}{\pi \langle R^2 \rangle} \frac{(N_j - 1)^2}{(N_i - 1) N_j + (N_j - 1) N_i + N_i N_j} \Phi_{\Sigma out/in}^h \quad (63)$$

644 Given that the surface conductivity can be neglected, the formation factor  $F$  of the  
645 network can be computed by:

$$\frac{1}{F} = \frac{\sigma}{\sigma_w} = \frac{1}{\sigma_w} \frac{JL}{S |\Delta V|} = \frac{1}{l^2} \frac{N_j - 1}{N_i - 1} \frac{\Phi_{\Sigma out/in}^e}{|\Delta V|}. \quad (64)$$

646 Then, considering that  $|\Delta V| = 1$ , the formation factor is then defined by:

$$\frac{1}{F\phi} = \frac{1}{\pi \langle R^2 \rangle} \frac{(N_j - 1)^2}{(N_i - 1) N_j + (N_j - 1) N_i + N_i N_j} \Phi_{\Sigma out/in}^e \quad (65)$$

647 The Johnson's length,  $\Lambda$  (m), is a petrophysical parameter proposed by *Schwartz*  
648 *et al.* [1989] that quantifies a representative length of a porous medium. Following *Bern-*  
649 *abé and Revil* [1995], we computed two Johnson's lengths for each of our networks:

$$\Lambda_h = \frac{\sum_{i=1}^{N_t} R_i^2 |\Delta P_i|^2}{\sum_{i=1}^{N_t} R_i |\Delta P_i|} \quad (66)$$

650 and

$$\Lambda_e = \frac{\sum_{i=1}^{N_t} R_i^2 |\Delta V_i|^2}{\sum_{i=1}^{N_t} R_i |\Delta V_i|} \quad (67)$$

651 where  $N_t$  is the total number of nodes and  $\Delta P_i$  (resp.  $\Delta V_i$ ) is the gradient of hydraulic  
652 pressure (resp. electrical potential) between the two ends of capillary I (of radius  $R_i$ ). By  
653 definition, the hydraulic and electrical Johnson's lengths are based on the hydraulic (Eq.  
654 66) and the electrical potentials (Eq. 67), respectively. These two lengths are expected to  
655 have close values.

656

**Appendix C: Simulation results**

657

This table regroups all the numerical results from the simulation of the present study

658

for the different types of pore size distributions: fractal (Fract.), exponential symmetric

659

(Exp. Sym.), lognormal (Log.), and double lognormal (Dbl. Log.).

Type	$R$ range ( $\mu\text{m}$ )	$C_{NaCl}^w$ (mol/L)	$C_{EK}$ (mV/m)	$k/\phi$ (mD)	$F \times \phi$ (-)	$\sigma_w$ (S/m)	$\hat{Q}_v$ (C/m <sup>3</sup> )	$\Lambda_h$ ( $\mu\text{m}$ )	$\Lambda_e$ ( $\mu\text{m}$ )
Fract.	0.1-10	0.0001	-140.6379	1.44E-01	23.51	1.09E-03	4.674E+03	0.1337	0.1567
Fract.	0.1-10	0.0005	-52.6636	1.44E-01	23.51	5.42E-03	8.708E+03	0.1337	0.1567
Fract.	0.1-10	0.001	-29.6833	1.44E-01	23.51	1.08E-02	9.781E+03	0.1337	0.1567
Fract.	0.1-10	0.005	-6.1136	1.44E-01	23.51	5.32E-02	9.914E+03	0.1337	0.1567
Fract.	0.1-10	0.01	-2.8766	1.44E-01	23.51	1.05E-01	9.219E+03	0.1337	0.1567
Fract.	0.1-10	0.05	-0.4461	1.44E-01	23.51	4.99E-01	6.789E+03	0.1337	0.1567
Fract.	0.1-10	0.1	-0.1902	1.44E-01	23.51	9.61E-01	5.575E+03	0.1337	0.1567
Fract.	0.1-10	0.5	-0.0209	1.44E-01	23.51	4.12E+00	2.626E+03	0.1337	0.1567
Fract.	0.1-10	1	-0.0058	1.44E-01	23.51	7.49E+00	1.331E+03	0.1337	0.1567
Fract.	0.5-50	0.0001	-387.0505	3.60E+00	23.51	1.09E-03	5.146E+02	0.6687	0.7836
Fract.	0.5-50	0.0005	-82.9874	3.60E+00	23.51	5.42E-03	5.489E+02	0.6687	0.7836
Fract.	0.5-50	0.001	-40.1124	3.60E+00	23.51	1.08E-02	5.287E+02	0.6687	0.7836
Fract.	0.5-50	0.005	-6.7928	3.60E+00	23.51	5.32E-02	4.406E+02	0.6687	0.7836
Fract.	0.5-50	0.01	-3.0721	3.60E+00	23.51	1.05E-01	3.938E+02	0.6687	0.7836
Fract.	0.5-50	0.05	-0.4560	3.60E+00	23.51	4.99E-01	2.776E+02	0.6687	0.7836
Fract.	0.5-50	0.1	-0.1929	3.60E+00	23.51	9.61E-01	2.261E+02	0.6687	0.7836
Fract.	0.5-50	0.5	-0.0210	3.60E+00	23.51	4.12E+00	1.056E+02	0.6687	0.7836
Fract.	0.5-50	1	-0.0058	3.60E+00	23.51	7.49E+00	5.344E+01	0.6687	0.7836
Fract.	1-100	0.0001	-461.1766	1.44E+01	23.51	1.09E-03	1.532E+02	1.3374	1.5672
Fract.	1-100	0.0005	-88.5209	1.44E+01	23.51	5.42E-03	1.463E+02	1.3374	1.5672
Fract.	1-100	0.001	-41.7734	1.44E+01	23.51	1.08E-02	1.376E+02	1.3374	1.5672
Fract.	1-100	0.005	-6.8843	1.44E+01	23.51	5.32E-02	1.116E+02	1.3374	1.5672
Fract.	1-100	0.01	-3.0976	1.44E+01	23.51	1.05E-01	9.925E+01	1.3374	1.5672
Fract.	1-100	0.05	-0.4573	1.44E+01	23.51	4.99E-01	6.958E+01	1.3374	1.5672
Fract.	1-100	0.1	-0.1932	1.44E+01	23.51	9.61E-01	5.662E+01	1.3374	1.5672
Fract.	1-100	0.5	-0.0210	1.44E+01	23.51	4.12E+00	2.640E+01	1.3374	1.5672

Fract.	1-100	1	-0.0059	1.44E+01	23.51	7.49E+00	1.336E+01	1.3374	1.5672
Fract.	5-500	0.0001	-539.0909	3.60E+02	23.51	1.09E-03	7.167E+00	6.6872	7.8362
Fract.	5-500	0.0005	-93.3707	3.60E+02	23.51	5.42E-03	6.176E+00	6.6872	7.8362
Fract.	5-500	0.001	-43.1767	3.60E+02	23.51	1.08E-02	5.691E+00	6.6872	7.8362
Fract.	5-500	0.005	-6.9587	3.60E+02	23.51	5.32E-02	4.514E+00	6.6872	7.8362
Fract.	5-500	0.01	-3.1182	3.60E+02	23.51	1.05E-01	3.997E+00	6.6872	7.8362
Fract.	5-500	0.05	-0.4583	3.60E+02	23.51	4.99E-01	2.790E+00	6.6872	7.8362
Fract.	5-500	0.1	-0.1935	3.60E+02	23.51	9.61E-01	2.269E+00	6.6872	7.8362
Fract.	5-500	0.5	-0.0210	3.60E+02	23.51	4.12E+00	1.057E+00	6.6872	7.8362
Fract.	5-500	1	-0.0059	3.60E+02	23.51	7.49E+00	5.348E-01	6.6872	7.8362
Fract.	10-1000	0.0001	-550.3921	1.44E+03	23.51	1.09E-03	1.829E+00	13.3744	15.6723
Fract.	10-1000	0.0005	-94.0061	1.44E+03	23.51	5.42E-03	1.554E+00	13.3744	15.6723
Fract.	10-1000	0.001	-43.3571	1.44E+03	23.51	1.08E-02	1.429E+00	13.3744	15.6723
Fract.	10-1000	0.005	-6.9680	1.44E+03	23.51	5.32E-02	1.130E+00	13.3744	15.6723
Fract.	10-1000	0.01	-3.1208	1.44E+03	23.51	1.05E-01	1.000E+00	13.3744	15.6723
Fract.	10-1000	0.05	-0.4584	1.44E+03	23.51	4.99E-01	6.977E-01	13.3744	15.6723
Fract.	10-1000	0.1	-0.1935	1.44E+03	23.51	9.61E-01	5.672E-01	13.3744	15.6723
Fract.	10-1000	0.5	-0.0210	1.44E+03	23.51	4.12E+00	2.642E-01	13.3744	15.6723
Fract.	10-1000	1	-0.0059	1.44E+03	23.51	7.49E+00	1.337E-01	13.3744	15.6723
Exp. Sym.	0.1-10	0.0001	-413.1205	2.62E+01	4.88	1.09E-03	3.636E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.0005	-85.0421	2.62E+01	4.88	5.42E-03	3.724E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.001	-40.7362	2.62E+01	4.88	1.08E-02	3.555E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.005	-6.8276	2.62E+01	4.88	5.32E-02	2.932E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.01	-3.0818	2.62E+01	4.88	1.05E-01	2.615E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.05	-0.4565	2.62E+01	4.88	4.99E-01	1.840E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.1	-0.1930	2.62E+01	4.88	9.61E-01	1.498E+02	0.8264	0.9395
Exp. Sym.	0.1-10	0.5	-0.0210	2.62E+01	4.88	4.12E+00	6.991E+01	0.8264	0.9395
Exp. Sym.	0.1-10	1	-0.0058	2.62E+01	4.88	7.49E+00	3.539E+01	0.8264	0.9395
Exp. Sym.	0.5-50	0.0001	-525.6708	6.55E+02	4.88	1.09E-03	1.851E+01	4.1317	4.6975
Exp. Sym.	0.5-50	0.0005	-92.5948	6.55E+02	4.88	5.42E-03	1.622E+01	4.1317	4.6975
Exp. Sym.	0.5-50	0.001	-42.9554	6.55E+02	4.88	1.08E-02	1.499E+01	4.1317	4.6975
Exp. Sym.	0.5-50	0.005	-6.9471	6.55E+02	4.88	5.32E-02	1.193E+01	4.1317	4.6975
Exp. Sym.	0.5-50	0.01	-3.1150	6.55E+02	4.88	1.05E-01	1.057E+01	4.1317	4.6975

Exp. Sym.	0.5-50	0.05	-0.4581	6.55E+02	4.88	4.99E-01	7.386E+00	4.1317	4.6975
Exp. Sym.	0.5-50	0.1	-0.1934	6.55E+02	4.88	9.61E-01	6.006E+00	4.1317	4.6975
Exp. Sym.	0.5-50	0.5	-0.0210	6.55E+02	4.88	4.12E+00	2.799E+00	4.1317	4.6975
Exp. Sym.	0.5-50	1	-0.0059	6.55E+02	4.88	7.49E+00	1.416E+00	4.1317	4.6975
Exp. Sym.	1-100	0.0001	-543.3634	2.62E+03	4.88	1.09E-03	4.783E+00	8.2635	9.3950
Exp. Sym.	1-100	0.0005	-93.6126	2.62E+03	4.88	5.42E-03	4.099E+00	8.2635	9.3950
Exp. Sym.	1-100	0.001	-43.2455	2.62E+03	4.88	1.08E-02	3.774E+00	8.2635	9.3950
Exp. Sym.	1-100	0.005	-6.9622	2.62E+03	4.88	5.32E-02	2.990E+00	8.2635	9.3950
Exp. Sym.	1-100	0.01	-3.1192	2.62E+03	4.88	1.05E-01	2.647E+00	8.2635	9.3950
Exp. Sym.	1-100	0.05	-0.4583	2.62E+03	4.88	4.99E-01	1.847E+00	8.2635	9.3950
Exp. Sym.	1-100	0.1	-0.1935	2.62E+03	4.88	9.61E-01	1.502E+00	8.2635	9.3950
Exp. Sym.	1-100	0.5	-0.0210	2.62E+03	4.88	4.12E+00	6.997E-01	8.2635	9.3950
Exp. Sym.	1-100	1	-0.0059	2.62E+03	4.88	7.49E+00	3.541E-01	8.2635	9.3950
Exp. Sym.	5-500	0.0001	-558.2680	6.55E+04	4.88	1.09E-03	1.966E-01	41.3174	46.9751
Exp. Sym.	5-500	0.0005	-94.4397	6.55E+04	4.88	5.42E-03	1.654E-01	41.3174	46.9751
Exp. Sym.	5-500	0.001	-43.4797	6.55E+04	4.88	1.08E-02	1.518E-01	41.3174	46.9751
Exp. Sym.	5-500	0.005	-6.9744	6.55E+04	4.88	5.32E-02	1.198E-01	41.3174	46.9751
Exp. Sym.	5-500	0.01	-3.1226	6.55E+04	4.88	1.05E-01	1.060E-01	41.3174	46.9751
Exp. Sym.	5-500	0.05	-0.4585	6.55E+04	4.88	4.99E-01	7.392E-02	41.3174	46.9751
Exp. Sym.	5-500	0.1	-0.1935	6.55E+04	4.88	9.61E-01	6.010E-02	41.3174	46.9751
Exp. Sym.	5-500	0.5	-0.0210	6.55E+04	4.88	4.12E+00	2.799E-02	41.3174	46.9751
Exp. Sym.	5-500	1	-0.0059	6.55E+04	4.88	7.49E+00	1.417E-02	41.3174	46.9751
Exp. Sym.	10-1000	0.0001	-560.1807	2.62E+05	4.88	1.09E-03	4.931E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.0005	-94.5439	2.62E+05	4.88	5.42E-03	4.140E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.001	-43.5092	2.62E+05	4.88	1.08E-02	3.797E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.005	-6.9759	2.62E+05	4.88	5.32E-02	2.996E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.01	-3.1230	2.62E+05	4.88	1.05E-01	2.650E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.05	-0.4585	2.62E+05	4.88	4.99E-01	1.848E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.1	-0.1935	2.62E+05	4.88	9.61E-01	1.502E-02	82.6347	93.9501
Exp. Sym.	10-1000	0.5	-0.0210	2.62E+05	4.88	4.12E+00	6.998E-03	82.6347	93.9501
Exp. Sym.	10-1000	1	-0.0059	2.62E+05	4.88	7.49E+00	3.541E-03	82.6347	93.9501
Log.	0.1-10	0.0001	-410.0958	2.88E+01	4.51	1.09E-03	3.554E+02	0.7898	0.9386
Log.	0.1-10	0.0005	-84.7976	2.88E+01	4.51	5.42E-03	3.656E+02	0.7898	0.9386

Log.	0.1-10	0.001	-40.6616	2.88E+01	4.51	1.08E-02	3.493E+02	0.7898	0.9386
Log.	0.1-10	0.005	-6.8234	2.88E+01	4.51	5.32E-02	2.885E+02	0.7898	0.9386
Log.	0.1-10	0.01	-3.0807	2.88E+01	4.51	1.05E-01	2.574E+02	0.7898	0.9386
Log.	0.1-10	0.05	-0.4564	2.88E+01	4.51	4.99E-01	1.811E+02	0.7898	0.9386
Log.	0.1-10	0.1	-0.1930	2.88E+01	4.51	9.61E-01	1.475E+02	0.7898	0.9386
Log.	0.1-10	0.5	-0.0210	2.88E+01	4.51	4.12E+00	6.883E+01	0.7898	0.9386
Log.	0.1-10	1	-0.0058	2.88E+01	4.51	7.49E+00	3.484E+01	0.7898	0.9386
Log.	0.5-50	0.0001	-524.7114	7.20E+02	4.51	1.09E-03	1.819E+01	3.9488	4.6930
Log.	0.5-50	0.0005	-92.5376	7.20E+02	4.51	5.42E-03	1.596E+01	3.9488	4.6930
Log.	0.5-50	0.001	-42.9390	7.20E+02	4.51	1.08E-02	1.476E+01	3.9488	4.6930
Log.	0.5-50	0.005	-6.9462	7.20E+02	4.51	5.32E-02	1.175E+01	3.9488	4.6930
Log.	0.5-50	0.01	-3.1148	7.20E+02	4.51	1.05E-01	1.041E+01	3.9488	4.6930
Log.	0.5-50	0.05	-0.4581	7.20E+02	4.51	4.99E-01	7.272E+00	3.9488	4.6930
Log.	0.5-50	0.1	-0.1934	7.20E+02	4.51	9.61E-01	5.914E+00	3.9488	4.6930
Log.	0.5-50	0.5	-0.0210	7.20E+02	4.51	4.12E+00	2.755E+00	3.9488	4.6930
Log.	0.5-50	1	-0.0059	7.20E+02	4.51	7.49E+00	1.394E+00	3.9488	4.6930
Log.	1-100	0.0001	-542.8505	2.88E+03	4.51	1.09E-03	4.704E+00	7.8977	9.3860
Log.	1-100	0.0005	-93.5834	2.88E+03	4.51	5.42E-03	4.035E+00	7.8977	9.3860
Log.	1-100	0.001	-43.2372	2.88E+03	4.51	1.08E-02	3.715E+00	7.8977	9.3860
Log.	1-100	0.005	-6.9618	2.88E+03	4.51	5.32E-02	2.944E+00	7.8977	9.3860
Log.	1-100	0.01	-3.1191	2.88E+03	4.51	1.05E-01	2.606E+00	7.8977	9.3860
Log.	1-100	0.05	-0.4583	2.88E+03	4.51	4.99E-01	1.819E+00	7.8977	9.3860
Log.	1-100	0.1	-0.1935	2.88E+03	4.51	9.61E-01	1.479E+00	7.8977	9.3860
Log.	1-100	0.5	-0.0210	2.88E+03	4.51	4.12E+00	6.889E-01	7.8977	9.3860
Log.	1-100	1	-0.0059	2.88E+03	4.51	7.49E+00	3.486E-01	7.8977	9.3860
Log.	5-500	0.0001	-558.1596	7.20E+04	4.51	1.09E-03	1.935E-01	39.4883	46.9301
Log.	5-500	0.0005	-94.4338	7.20E+04	4.51	5.42E-03	1.629E-01	39.4883	46.9301
Log.	5-500	0.001	-43.4781	7.20E+04	4.51	1.08E-02	1.494E-01	39.4883	46.9301
Log.	5-500	0.005	-6.9743	7.20E+04	4.51	5.32E-02	1.180E-01	39.4883	46.9301
Log.	5-500	0.01	-3.1225	7.20E+04	4.51	1.05E-01	1.044E-01	39.4883	46.9301
Log.	5-500	0.05	-0.4585	7.20E+04	4.51	4.99E-01	7.278E-02	39.4883	46.9301
Log.	5-500	0.1	-0.1935	7.20E+04	4.51	9.61E-01	5.917E-02	39.4883	46.9301
Log.	5-500	0.5	-0.0210	7.20E+04	4.51	4.12E+00	2.756E-02	39.4883	46.9301

Log.	5-500	1	-0.0059	7.20E+04	4.51	7.49E+00	1.395E-02	39.4883	46.9301
Log.	10-1000	0.0001	-560.1261	2.88E+05	4.51	1.09E-03	4.854E-02	78.9766	93.8603
Log.	10-1000	0.0005	-94.5410	2.88E+05	4.51	5.42E-03	4.076E-02	78.9766	93.8603
Log.	10-1000	0.001	-43.5083	2.88E+05	4.51	1.08E-02	3.738E-02	78.9766	93.8603
Log.	10-1000	0.005	-6.9758	2.88E+05	4.51	5.32E-02	2.949E-02	78.9766	93.8603
Log.	10-1000	0.01	-3.1230	2.88E+05	4.51	1.05E-01	2.609E-02	78.9766	93.8603
Log.	10-1000	0.05	-0.4585	2.88E+05	4.51	4.99E-01	1.819E-02	78.9766	93.8603
Log.	10-1000	0.1	-0.1935	2.88E+05	4.51	9.61E-01	1.479E-02	78.9766	93.8603
Log.	10-1000	0.5	-0.0210	2.88E+05	4.51	4.12E+00	6.890E-03	78.9766	93.8603
Log.	10-1000	1	-0.0059	2.88E+05	4.51	7.49E+00	3.487E-03	78.9766	93.8603
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Dbl. Log.	0.1-10	0.0001	-294.0487	5.96E+00	17.22	1.09E-03	3.223E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.0005	-74.1977	5.96E+00	17.22	5.42E-03	4.046E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.001	-37.3259	5.96E+00	17.22	1.08E-02	4.056E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.005	-6.6296	5.96E+00	17.22	5.32E-02	3.545E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.01	-3.0261	5.96E+00	17.22	1.05E-01	3.198E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.05	-0.4537	5.96E+00	17.22	4.99E-01	2.277E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.1	-0.1923	5.96E+00	17.22	9.61E-01	1.859E+02	0.3398	0.6179
Dbl. Log.	0.1-10	0.5	-0.0210	5.96E+00	17.22	4.12E+00	8.693E+01	0.3398	0.6179
Dbl. Log.	0.1-10	1	-0.0058	5.96E+00	17.22	7.49E+00	4.403E+01	0.3398	0.6179
Dbl. Log.	0.5-50	0.0001	-482.0907	1.49E+02	17.22	1.09E-03	2.114E+01	1.6989	3.0893
Dbl. Log.	0.5-50	0.0005	-89.9010	1.49E+02	17.22	5.42E-03	1.961E+01	1.6989	3.0893
Dbl. Log.	0.5-50	0.001	-42.1772	1.49E+02	17.22	1.08E-02	1.833E+01	1.6989	3.0893
Dbl. Log.	0.5-50	0.005	-6.9060	1.49E+02	17.22	5.32E-02	1.477E+01	1.6989	3.0893
Dbl. Log.	0.5-50	0.01	-3.1036	1.49E+02	17.22	1.05E-01	1.312E+01	1.6989	3.0893
Dbl. Log.	0.5-50	0.05	-0.4576	1.49E+02	17.22	4.99E-01	9.187E+00	1.6989	3.0893
Dbl. Log.	0.5-50	0.1	-0.1933	1.49E+02	17.22	9.61E-01	7.474E+00	1.6989	3.0893
Dbl. Log.	0.5-50	0.5	-0.0210	1.49E+02	17.22	4.12E+00	3.484E+00	1.6989	3.0893
Dbl. Log.	0.5-50	1	-0.0059	1.49E+02	17.22	7.49E+00	1.763E+00	1.6989	3.0893
Dbl. Log.	1-100	0.0001	-519.4521	5.96E+02	17.22	1.09E-03	5.694E+00	3.3978	6.1785
Dbl. Log.	1-100	0.0005	-92.2258	5.96E+02	17.22	5.42E-03	5.029E+00	3.3978	6.1785
Dbl. Log.	1-100	0.001	-42.8496	5.96E+02	17.22	1.08E-02	4.656E+00	3.3978	6.1785
Dbl. Log.	1-100	0.005	-6.9416	5.96E+02	17.22	5.32E-02	3.712E+00	3.3978	6.1785
Dbl. Log.	1-100	0.01	-3.1135	5.96E+02	17.22	1.05E-01	3.291E+00	3.3978	6.1785

Dbl. Log.	1-100	0.05	-0.4581	5.96E+02	17.22	4.99E-01	2.299E+00	3.3978	6.1785
Dbl. Log.	1-100	0.1	-0.1934	5.96E+02	17.22	9.61E-01	1.870E+00	3.3978	6.1785
Dbl. Log.	1-100	0.5	-0.0210	5.96E+02	17.22	4.12E+00	8.713E-01	3.3978	6.1785
Dbl. Log.	1-100	1	-0.0059	5.96E+02	17.22	7.49E+00	4.409E-01	3.3978	6.1785
Dbl. Log.	5-500	0.0001	-553.0960	1.49E+04	17.22	1.09E-03	2.425E-01	16.9892	30.8926
Dbl. Log.	5-500	0.0005	-94.1557	1.49E+04	17.22	5.42E-03	2.054E-01	16.9892	30.8926
Dbl. Log.	5-500	0.001	-43.3994	1.49E+04	17.22	1.08E-02	1.886E-01	16.9892	30.8926
Dbl. Log.	5-500	0.005	-6.9702	1.49E+04	17.22	5.32E-02	1.491E-01	16.9892	30.8926
Dbl. Log.	5-500	0.01	-3.1214	1.49E+04	17.22	1.05E-01	1.320E-01	16.9892	30.8926
Dbl. Log.	5-500	0.05	-0.4584	1.49E+04	17.22	4.99E-01	9.204E-02	16.9892	30.8926
Dbl. Log.	5-500	0.1	-0.1935	1.49E+04	17.22	9.61E-01	7.483E-02	16.9892	30.8926
Dbl. Log.	5-500	0.5	-0.0210	1.49E+04	17.22	4.12E+00	3.486E-02	16.9892	30.8926
Dbl. Log.	5-500	1	-0.0059	1.49E+04	17.22	7.49E+00	1.764E-02	16.9892	30.8926
Dbl. Log.	10-1000	0.0001	-557.5685	5.96E+04	17.22	1.09E-03	6.111E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.0005	-94.4015	5.96E+04	17.22	5.42E-03	5.148E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.001	-43.4689	5.96E+04	17.22	1.08E-02	4.724E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.005	-6.9738	5.96E+04	17.22	5.32E-02	3.730E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.01	-3.1224	5.96E+04	17.22	1.05E-01	3.300E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.05	-0.4585	5.96E+04	17.22	4.99E-01	2.301E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.1	-0.1935	5.96E+04	17.22	9.61E-01	1.871E-02	33.9784	61.7852
Dbl. Log.	10-1000	0.5	-0.0210	5.96E+04	17.22	4.12E+00	8.715E-03	33.9784	61.7852
Dbl. Log.	10-1000	1	-0.0059	5.96E+04	17.22	7.49E+00	4.410E-03	33.9784	61.7852

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