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Physics of Weibel-mediated relativistic collisionless shocks

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We develop a comprehensive theoretical model of relativistic collisionless pair shocks mediated by the current filamentation instability. We notably characterize the noninertial frame in which this instability is of a mostly magnetic nature, and describe at a microscopic level the deceleration and heating of the incoming background plasma through its collisionless interaction with the electromagnetic turbulence. Our model compares well to large-scale 2D3V PIC simulations, and provides an important touchstone for the phenomenology of such plasma systems.

**Introduction**—Though of mundane occurrence in space plasmas, collisionless shock waves represent outstanding phenomena owing to their inherent complexity and many astrophysical repercussions [1]. Mediated by collective electromagnetic interactions, whose nature and origin form the focus of active investigations [2], they seemingly accelerate charged particles to high energies in a broad variety of sources [3], giving rise to a rich phenomenology at the core of high-energy and multi-messenger astrophysics. The electromagnetic counterpart of the gravitational wave event GW170817 is thus interpreted as the synchrotron radiation of electrons energized at the unmagnetized, relativistic shock wave triggered by the neutron star coalescence [4]. In parallel, collisionless shocks have become central topics in high-power laser-plasma experiments, which might well generate and study such structures in the near future [5].

In the absence of a significant background magnetic field, the physics of the shock is governed by an electromagnetic microturbulence driven by a current filamentation instability (CFI), as predicted [6], and as observed in *ab initio* simulations [7]. This microturbulence dissipates the ordered kinetic energy of the unshocked plasma, just as it governs the acceleration of particles to suprathermal energies. The latter, in turn, induce electromagnetic instabilities in the upstream region [8], thereby ensuring the self-sustained and (quasi-) stationary nature of the shock. Recent theoretical models have discussed the formation of the shock [9], the structure of the microturbulence [10], or the early-time shock transition in the sub-relativistic regime [11], but a detailed microphysical picture of well-formed shocks remains missing.

In this Letter, we present a comprehensive theoretical model for unmagnetized, relativistic collisionless shock waves, such as those expected at the boundary of relativistic astrophysical jets. Specifically, we provide a microphysical description of the deceleration and nonadiabatic heating of the background plasma in the shock precursor, and of the dynamics of the microturbulence and suprathermal particles. Our model relies on the observation that there exists a noninertial frame (hereafter “Weibel frame”) in which the microturbulence is essentially magnetostatic. Introducing such a frame allows one to derive proper transport equations for the background and suprathermal particles. Our arguments are shown to agree with dedicated high-resolution, large-scale particle-in-cell (PIC) simulations conducted using the code CALDER [12] in a 2D3V geometry (2D in configuration space, 3D in momentum space) [13]. We restrict ourselves to the case of a shock propagating in an electron-positron plasma, but discuss how the results can be generalized to electron-ion plasmas.

We describe the 1D profile of a formed shock, assumed stationary in the shock front rest frame \( R_x \). The precursor is defined as the region where the background plasma coexists with a population of suprathermal particles, characterized by their pressure \( \xi_p \), normalized to the incoming momentum flux density \( F_{\perp \infty} \equiv \gamma_{\perp \infty}^2 \beta_{\perp \infty}^2 n_{\infty} m_e c^2 \) (with \( \gamma_{\perp \infty}, \beta_{\perp \infty} \), and \( n_{\infty} \) denoting, respectively, the Lorentz factor of the unshocked background plasma, its normal velocity and its proper density), and a self-generated electromagnetic microturbulence of energy density \( \epsilon_B \) (also in units of \( F_{\infty} \)) [14]. Both \( \xi_p \) and \( \epsilon_B \) vary with the distance \( x \) to the shock. Figure 1 plots their (transversely averaged) profiles extracted from a PIC simulation, in which the background plasma is injected with \( \gamma_{\perp \infty} = 173 \) (i.e., \( \gamma_{\perp \infty}|_{\text{d}} = 100 \) in the simulation frame, which coincides with the downstream rest frame) and proper temperature \( T_p = 10^{-2} m_e c^2/k_B \). Distances are in units of \( c/\omega_p = c/\left(4\pi n_{\infty} e^2/m_e\right)^{1/2} \).

The “Weibel frame”—The leading micro-instability in driving the shock transition is the Weibel-type transverse CFI [6, 8], which fragments the background plasma into current filaments along the shock normal, surrounded by transverse electromagnetic fields \( \delta E_{\perp} \) (radial) and \( \delta B_{\perp} \) (toroidal). This instability is essentially magnetic, i.e.,...
\[\delta B_\perp^2 - \delta E_\perp^2 > 0.\] Along with \(\delta E_\perp \cdot \delta B_\perp = 0,\) this implies that, at a given point \(x,\) one can define a local reference frame, denoted \(R_w,\) where \(\delta E_\perp = 0.\) Far from the shock front, however, the transverse CFI might be superseded by electrostatic two-stream or oblique modes \([8],\) thus compromising the very existence of \(R_w.\) We indeed observe a finite \(\delta E_\perp,\) yet its energy density is well subordinate relative to that of \(\delta B_\perp\) in the near precursor. We therefore omit \(\delta E_\perp\) for now, but we will comment on its possible role further on. Our model thus describes the turbulence as a collection of magnetostatic modes transverse to the flow in \(R_w.\)

Figure 2 displays the downstream-frame 4-velocity \(u_{w|d} = \gamma_{w|d} \beta_{w|d} of R_w,\) as extracted from the PIC simulation through the ratio \(\delta E_y/\delta B_y^1/2 = \beta_{w|d}\) (where averaging is done over the transverse dimension). That \(|\beta_{w|d}| < 1\) confirms that \(R_w\) is well defined, at least in the near precursor \(x \lesssim 10^3c/\omega_p,\) where it can be measured unambiguously, and where the shock transition mainly takes place. The spatial dependence of \(u_{w|d}\) indicates that \(R_w\) is not globally inertial, which bears critical consequences for plasma heating, as explained below.

Obtaining a theoretical estimate of \(\beta_w\) turns out to be a nontrivial task. We determine this velocity through two approaches \([15]:\) (i) we search for a frame, at each point along the precursor, where the fastest-growing CFI mode computed from the kinetic linear dispersion relation has a vanishing electrostatic component; (ii) we search for a frame in which we can describe the nonlinear stage of the CFI as a locally stationary pressure equilibrium between the plasma, the beam and purely magnetic structures. Both approaches yield rather comparable estimates, \(\beta_{w|d} \approx \xi_b,\) with one important implication: \(R_w\) moves at subrelativistic velocities relative to the background plasma, hence at relativistic velocities towards the shock front, with \(\gamma_w \lesssim \gamma_p.\) The magnitude of \(\beta_{w|d}\) proves to be a central element of our model.

Figure 2 clearly illustrates these features: in PIC simulations, the background plasma moves slightly faster than \(R_w,\) and at \(x \gtrsim 300c/\omega_p,\) both 4-velocities remain close to each other; the relative velocity \(\beta_{w|d}\) between the \(R_w\) frame and the background plasma is always subrelativistic where it can be measured accurately; finally, our theoretical estimates of \(\beta_{w|d}\) agree well with the simulation data. Comparison of Figs. 1 and 2 also confirms that \(\xi_b\) provides a reasonable guide for the scaling of \(\beta_{w|d}\). That \(|\beta_{w|d}| < 1\) results from the large asymmetry between the background and suprathermal plasmas: in \(R_w,\) the latter forms a tenuous beam of large-inertia particles, which undergo small-angle scattering off the microturbulence: the former is comparatively dense and cold over most of the precursor, and its particles are mostly trapped in the magnetic filaments.

The deceleration of the background plasma—A nonvanishing \(\xi_b\) implies a nonvanishing \(\beta_{w|d},\) so that the frame \(R_w\) never exactly coincides with the rest frame of the background plasma, which nevertheless keeps relaxing in \(R_w\) through scattering. Hence, the finite pressure of the beam leads to the progressive deceleration of \(R_w,\) and, in turn, of the background plasma. This offers a view of how, at the kinetic level in \(R_w,\) momentum is transferred from the suprathermal beam to the background plasma. This explanation departs from the standard picture in which the CFI builds up a magnetized barrier in the shock rest frame, which hails and isotropizes the incoming plasma particles \([6].\) As a matter of fact, if the scattering center frame were exactly static in the shock
frame, Fermi acceleration would not occur.

At the fluid level, this momentum transfer can be described via the scattering of suprathermal particles off an effectively magnetized background plasma. The ensuing deceleration of the background plasma can be quantified using the steady-state conservation laws for current and energy-momentum in $\mathcal{R}_s$ [13]. Through its interaction with the suprathermal beam, the background plasma loses an energy flux density $T_b^{\gamma z}$ and a momentum flux density $T_b^{\gamma z}$ between $+\infty$ and a point $x$ in the precursor. By definition of $\xi_b$, one has $T_b^{\gamma z} = T_b^{\gamma z} \sim \xi_b F_\infty$, up to different prefactors of the order of unity. In analogy with the fact that, unless a particle moves at the same velocity $\beta_\infty$ as the plasma, its energy increases by $\gamma^2_{\infty}$ when it is picked up by the latter, one can show that deceleration occurs once $|\beta_\infty T_b^{\gamma z} - T_b^{\gamma z}| \gtrsim F_\infty/\gamma_{\infty}^2$ [16]. Hence, where $\xi_b \gtrsim 1/\gamma_{\infty}^2$, the Lorentz factor $\gamma_p$ drops according to $\gamma_p^2 \xi_b \simeq \text{const}$. Figure 3 shows that this fluid deceleration law is well verified in PIC simulations.

Consequently, the background plasma slows down to subrelativistic velocities once $\xi_b \sim 0.1 - 0.3$. This nicely accounts for the universal – i.e., independent of $\gamma_{\infty}$ – fraction of shock energy injected into the suprathermal population, of typical value $\xi_b \sim 0.1$ in PIC simulations, e.g. [17] and Fig. 1. Such cosmic-ray mediated shocks have been predicted in the subrelativistic limit [18] and observed in nonlinear Monte Carlo simulations [19]. One clear prediction is the existence of a sub-shock: in Fig. 1, the Lorentz factor of the background plasma indeed decreases over thousands of $c/\omega_p$ from $\gamma_{\infty}^0$ down to $\gamma_{\text{sub}} \sim 5$, at which point the shock transition suddenly occurs over $\lesssim 100c/\omega_p$, as discussed further below.

The heating of the background plasma – The noninertial nature of the turbulence frame controls the heating of the background plasma as follows. In $\mathcal{R}_w$, particles are subject to an effective gravity $\times du_w/dx$ directed toward the shock front, and to pitch-angle (elastic) scattering off the turbulence. This gives rise to Joule-like heating wherein gravity plays the role of the driving electric field, while turbulence-induced scattering provides collisional friction. The corresponding physics can be described by a general relativistic Vlasov-Fokker-Planck equation written in a mixed coordinate frame, with spatial variables in $\mathcal{R}_s$ and momenta in $\mathcal{R}_w$ [13, 16]. Here, we simulate this interplay between gravity and friction through a numerical Monte Carlo integration of the analog stochastic dynamical system: $d\mu_w = \sqrt{2\nu_w}|d\mu_w|$ and $dp^{\gamma z}_w = p^{\gamma z}_w d\mu_w - \left(\beta_w p^{\gamma z}_w + p^{\gamma z}_w\right)(du_x/dx) d\mu_w$, with $\mu_w = p^{\gamma z}_w/|p^{\gamma z}_w|$ the pitch-angle cosine, $\nu_w$ the effective pitch-angle scattering frequency (treated as a constant parameter), $\gamma \sim \mathcal{N}(0, 1)$ describing white noise and $dt_w$ a time interval in $\mathcal{R}_w$.

One can anticipate the trajectories $u_p$ vs $T_p$ along the plasma world line: for $\nu_w \to +\infty$, the background plasma behaves as a perfect fluid, hence $T_p \propto |u_p|^{-2/3}$, as befits adiabatic 1D compression of a subrelativistic fluid, while if $\nu_w \to 0$, the background plasma remains in its initial state because it cannot experience the effective gravity. Using our model (ii) for $u_p(x)$ (Fig. 2), we obtain numerical predictions for $T_p$ and $u_p$ and compare them to the PIC simulation results in Fig. 4. For an effective $\nu_w = 0.01\omega_p$, the model trajectories satisfactorily reproduce those observed in the PIC simulations as well as the shock jump conditions.

Our PIC simulations reveal that about half of background plasma particles have experienced at least one turnaround while crossing the precursor. Given a pre-
cursor length scale of $\ell_{\mathrm{id}} \approx 2 \times 10^3 \epsilon_i / \omega_p$ for $\gamma_{\infty | d} = 100$ (resp. $\ell_{\mathrm{id}} \approx 10^3 \epsilon_i / \omega_p$ for $\gamma_{\infty | d} = 10$), boosting back to $\mathcal{R}_w$ with typical Lorentz factor $\langle \gamma_{\infty | d} \rangle \sim 30$ over the precursor (resp. $\langle \gamma_{\infty | d} \rangle \sim 10$), where this average is obtained as the value of $\gamma_{\mathrm{id}}$ halfway through the precursor, we estimate $\nu_{\infty} \sim \gamma_{\infty | d} \epsilon_i / \ell_{\mathrm{id}} \sim 1.5 \times 10^{-2} \omega_p$ (resp. $\sim 10^{-2} \omega_p$), which agrees with the value inferred above. Alternatively, assuming marginally untrapped particles, one expects $\nu_{\infty} \sim \epsilon_{\infty} \gamma_{\infty | d} \epsilon_i / \ell_{\mathrm{id}}$, again consistent with the above value for mildly relativistic background plasma particles in $\mathcal{R}_w$. In the case of bound particles oscillating transversely in the filament at the betatron frequency $\omega_{\parallel | w} \sim c (r_{\perp} r_{\parallel | w})^{-1/2}$, and experiencing decoherence of the force on a length scale $r_{\parallel | w}$, we derive $\nu_{\infty} \sim \epsilon_{\infty} \gamma_{\infty | d} \epsilon_i / \ell_{\mathrm{id}} \sim 1/2 (r_{\perp} / r_{\parallel | w}) \omega_p / \gamma_{\infty | d}$, in fair agreement with the above results for a typical aspect ratio $r_{\perp} / r_{\parallel | w} \sim 0.1$.

In the noninertial local plasma rest frame, heating thus occurs from $T_x \ll m_i c^2 / k_B$ up to $\sim \gamma_{\infty} m_i c^2 / k_B$ at the shock. In the shock frame, however, this dissipative dynamics is better seen as the effect of a collisionless viscosity, which transfers momentum from the forward to the transverse directions while preserving the energy per particle, consistent with the shock jump conditions [20]. In this respect, electron heating in pair shocks starkly differs from that in electron-ion shocks: there, the electron population behaves as an open system that draws energy from the ion reservoir, so that electron energization truly occurs in $\mathcal{R}_s$ through their interaction with transverse or longitudinal electromagnetic fields [21, 22].

Although an electrostatic (longitudinal) electric field component indeed appears in our pair simulations, its contribution to the energization of particles in the near precursor ($x \lesssim 10^3 \epsilon_i / \omega_p$) is at best comparable to that of the transverse component, as we have checked using test particles. In the far precursor, the electrostatic contribution is significant, presumably due to the excitation of non-transverse CFI modes, but the amount of heating there is small, see Fig. 1. By contrast, the longitudinal electric field is expected to play an important role in electron-ion shocks [22] because the dependence of $\nu_{\infty}$ on inertia breaks the equivalence of the electron and ion trajectories in the effective gravity field. This might well preheat the electrons up to near equipartition with the ions, as observed numerically [17]. We further speculate that the above physics of slowdown and heating in pair shocks could describe reasonably well the dynamics of the inertia-carrier ions in electron-ion collisionless shocks. Such study is left for further work.

**The shock transition**—The relativistic motion of $\mathcal{R}_w$ relative to $\mathcal{R}_s$ also affects the growth rate of the CFI, which determines the profile of $\epsilon_B$: a background plasma element, subject to the CFI with a growth rate $3 \omega_{\parallel | w}$ in $\mathcal{R}_w$ indeed experiences growth over a timescale $\tau = \gamma_{\infty} / 3 \omega_{\parallel | w}$ in $\mathcal{R}_s$. While the profile of $\epsilon_B(x)$ shows slow growth at large distances, its relatively flat shape at $x \lesssim 10^3 \epsilon_i / \omega_p$ (Fig. 1) is ascribed to saturation by advection of the CFI, i.e., the $e$-folding scale is then larger than $x$. This is confirmed by detailed kinetic calculations of the growth rates [23].

By contrast, the sudden rise in $\epsilon_B$ at the shock transition is commonly interpreted as the buildup of the electromagnetic barrier through microinstabilities. A nagging question is then: why does this occur precisely at the location predicted by the shock trajectory in the lab frame? Our model readily explains this sudden rise as the compression law $\epsilon_B \propto \beta_w^{-2}$ (dashed green), where $\beta_w$ is extracted from the PIC simulation through $(\delta E_s)^{1/2}/(\delta B^2)^{1/2}$, then smoothed. The proportionality factors are ad hoc linear fits to the scaling of $\epsilon_B$ at large $x \gtrsim 100c / \omega_p$.

![FIG. 5. Zoom on the peak of $\epsilon_B$ at the shock, compared to the compression law $\epsilon_B \propto \beta_w^{-2}$ (dashed green), where $\beta_w$ is extracted from the PIC simulation through $(\delta E_s)^{1/2}/(\delta B^2)^{1/2}$, then smoothed. The proportionality factors are ad hoc linear fits to the scaling of $\epsilon_B$ at large $x \gtrsim 100c / \omega_p$.](image)
relativistic motion, implies a large acceleration timescale in the far precursor. This may explain why PIC simulations of limited duration observe Fermi acceleration mainly through grazing orbits on the shock front, where $\gamma_w \sim 1$.

Finally, at the shock transition, $\gamma_w \sim 1$, $\epsilon_B \sim 0.1$ and $p \sim p_{\text{ms}}$, and hence $l_{\text{scatt}} \sim 10 - 100/c\omega_p$. This value, lower than that expected hundreds of $c/\omega_p$ away, where $\epsilon_B \lesssim 0.01$ and $\gamma_w \gg 1$, introduces a scattering barrier that selects the most energetic particles from the shocked thermal pool to form the population of injected suprathermal particles. The quadratic energy dependence of $l_{\text{scatt}}$ implies that $\xi_p(x)$ takes on a powerlaw form, at least in the near precursor (Fig. 1), which accounts for the powerlaw profile for $\gamma_{\text{pid}}$ (Fig. 3). In this context, the core of the sub-shock can be interpreted as resulting from the pressure of suprathermal particles located within one scattering length from the shock front. The background plasma decouples from the microturbulence once $v_w^{-1}$ exceeds the deceleration length scale of $R_w$, before it eventually relaxes in the asymptotic stationary $R_w$ frame, again on a $\sim v_w^{-1}$ length scale. Overall, we infer from Figs. 2 and 3 a sub-shock width of $\sim 100/c\omega_p$, in reasonable agreement with the relaxation length $\nu_w^{-1}$.

In summary, we have presented an analytical microphysical model of the precursor of unmagnetized, relativistic collisionless pair shocks, which sheds new light into the phenomenology of such systems in high-energy and laboratory astrophysics. Our theory, based on the properties of the noninertial “Weibel frame” and benchmarked against large-scale PIC simulations, lends itself to extrapolation to other shock regimes and to the large spatiotemporal scales of astrophysical interest.

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[14] Quantities related to the background plasma (resp. suprathermal beam) are indexed with $p$ (resp. $b$). Also, subscripts $p$ (resp. $b$) refer to quantities measured in the simulation (downstream) rest frame (resp. the background plasma rest frame).


