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Weighted Kaplan-Meier estimators motivating to estimate HIV-1 RNA reduction censored by a limit of detection

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Summary

Measuring the magnitude of reduction in HIV-1 RNA levels accurately is difficult because many patients have a censored reduction due to the limit of detection (LOD) of the virologic assay being employed. The use of censored methods has improved the analysis of such reductions compared with crude methods but implies independent censoring. For HIV-1 RNA reduction data, the value at which a patient's HIV-1 RNA reduction becomes censored is mainly determined by the patient's baseline HIV-1 RNA level. We suggest two possibilities based on modification of the redistribution to the right algorithm to handle the situation of dependence either from a single continuous marker, i.e., the baseline HIV-1 RNA level, or from multiple markers. Two series of simulation, one in the HIV-1 RNA setting and one in the classical censoring setting, compared performance of the previous methods with our suggestions. Our proposed estimators show good performances when the dependent censoring is due to LOD. Overall, in the classical censoring setting, our suggestions perform as well as other methods including the Inverse Probability of Censoring Weighted (IPCW) and the Kaplan-Meier imputation with Bootstrap (KMIB). We applied those estimators to estimate the HIV-1 RNA reduction at week 8 of 502 patients who received a raltegravir-containing regimen and to data from the Mayo Clinic trial in primary biliary cirrhosis.

KEYWORDS:
dependent censoring; weighted Kaplan-Meier estimator; limit of detection, HIV-1 RNA reduction

1 | INTRODUCTION

Efficacy of antiretroviral treatments is currently assessed by measuring plasma HIV-1 RNA levels. In many clinical trials, efficacy is measured in comparing proportions of patients having 'undetectable' viral load, i.e. HIV-1 RNA below the limit of detection (LOD), in the k randomized groups at a prespecified primary follow-up. Nevertheless, the average magnitude of reduction in HIV-1 RNA level from baseline to a prespecified time point remains an important secondary endpoint.[1] Specifically this endpoint is still used in early development of antiretroviral drugs,[2] in comparing early decline of viral load,[3,4] in building clinically relevant genotype interpretation of resistance,[5,6] and as complement to define the virologic response in studies investigating risk factors in treatment-experienced HIV-1 infected patients.[7,8]

Measuring the HIV-1 RNA reduction accurately, however, is difficult because some patients have a censored reduction of HIV-1 RNA due to the LOD of virologic assays being employed.[1] The percentage of patients having a censored reduction has

markedly increased with increasing potency of antiretroviral therapy despite decreasing values for LOD. For example, consider a patient having an HIV-1 RNA measurement of 5,000 copies/ml at baseline, and a measurement below the LOD of 50 copies/ml at the primary follow-up time. For this patient the actual reduction in HIV-1 RNA level is unknown, it is only know to be greater than 2 $\log_{10}$ copies/ml. The crude method that consists to define all HIV-1 RNA levels below the LOD as equal to the LOD leads to serious bias.[1] Classical survival methods, including Kaplan-Meier (KM) method and log-rank test, have been proposed to estimate the magnitude of viral load reduction.[1] Censoring by the LOD, however, provides a loss of information and there is the potential for bias in HIV-1 RNA reduction estimates if the censoring is dependent to the magnitude of reduction. In our context, the value at which a patient's HIV-1 RNA reduction becomes censored is mainly determined by the patient's baseline HIV-1 RNA level, i.e., dependent censoring.[9,10]

In classical survival analysis, several approaches have been proposed to recover some the lost of information due to censoring using prognostic covariates. A few of these methods use information from the prognostic covariates directly without modeling survival or censoring[11,12,13] or use working models to summarize prognostic covariates[13,14] to define homogeneous risk groups to improve estimation of the marginal survival distribution. Another approach was to incorporate the probability of censoring to improve the estimation of the marginal survival distribution.[15,16] The weighted Kaplan-Meier (WKM) approach from Malani,[11] using disease markers, suggested a modification of the redistribution to the right algorithm[17] as a new approach to recover information for censored individuals. A similar approach was also introduced by Murray and Tsiatis.[12] The modification generalizes the Kaplan-Meier method in the presence of auxiliary information available from a disease marker. The method has been introduced when the disease marker is categorical or that continuous markers be categorized.

In this work, we suggest two other possibilities of handling a continuous marker to provide a weighted Kaplan-Meier estimator. All estimators are compared through two examples and a simulation study. Our motivating example concerns a cohort of 502 treatment-experienced HIV-1 infected patients receiving a raltegravir-based regimen. The study was originally designed to investigate factors associated with virological response and mutations selected at failure.[8] In the present work, we want to estimate the HIV-1 RNA reduction from baseline to week 8. A first simulation study investigated the performance of the different estimators in such settings. Although, our suggestions were motivated by the above example, we also applied and investigated performance of our suggestions in the classical censoring settings. Data for the second example comes from the Mayo Clinic trial in primary biliary cirrhosis. A second series of simulation study investigated performance of all estimators in the classical survival setting suggested by Hsu and Taylor.[14] This paper is organized as follows. In Section 2, we describe the different methods including our suggestions in the general setting of classical censoring. The two example and results with the different estimators are displayed in Section 3. Results of a series of two simulation studies are summarized in Section 4. Section 5 provides some elements for discussion.

2 | METHODS

Let T denote time to the outcome of interest in the study, and C the censoring time. The observed data consists of $Y_i = \min(T_i, C_i)$ and $\delta_i = I(T_i \leq C_i)$, where $I(.)$ is the indicator function and for i in 1:N. To simplify the notations, it is supposed that there is no tie for the T_i and C_i. Let J be the number of distinct events and $T_{(j)}$, $j \in \{1, \ldots, J\}$, the ordered failure times. Let L, $L \in \{0, \ldots, N - J\}$ be the number of distinct censored times with $C_{(l)}$, $l \in \{1, \ldots, L\}$, the L ordered censored times. The goal is to estimate $S(t) = P(T > t)$. In the HIV-1 RNA reduction setting, the goal is to estimate the probability of having a reduction greater than x $\log_{10}$ copies/ml.

2.1 | Kaplan-Meier estimator and the redistribution algorithm

Let $w_i(t)$ denote the weight associated with the ith individual at time t. The redistribution algorithm starts by assigning a weight of $1/N$ to all individuals, i.e., $w_i(0) = 1/N$ for all i. Without any censoring the weight remains unchanged until the end of the study and the survival function drops by $1/N$ at each failure time. In the case of censoring, this process is continued by moving through the ordered failure times until the time of the first censored observation. At this time, the weight of the censored individual is re-allocated equally to the remaining individuals in the risk set as follows $w_i(C_{(l)}) = w_i(C_{(l-1)}) + \frac{w_i(C_{(l-1)})}{n(C_{(l)})}$, where $n(C_{(l)}) = \sum_i I(Y_i > C_{(l)})$ and where $w(C_{(0)}) = 1/N$. The weights are modified similarly at each subsequent censoring[17] and the KM estimator is simply $\hat{S}_{KM}(t) = \sum_i I(Y_i > t) w_i(t)$.

2.2 | Malani estimator for a discrete time-independent marker

The basic idea is now to distinguish between censored individuals with different prognoses as measured by a marker. The estimator was originally proposed by Malani for both time-dependent and time-independent marker.[11] For ease of presentation and because our motivating example involves a time-independent marker, we introduce the Malani estimator only for the latter case. So let Z_i denote the value of the marker for the ith individual recorded at the start of the study and assume first that Z_i is discrete and takes G different values denoted by z_g ($g = 1, \ldots, \mathrm{G}$). The estimate of the survival function is

$$\hat{S}_M(t) = \sum_{g=1}^{\mathrm{G}} \theta_g \hat{S}_{KM}(t|z_g) \tag{2.1}$$

where $\hat{S}_{KM}(t|z_g)$ denotes the KM estimate of $\mathrm{pr}(T_i > T|Z_i = z_g)$, $\theta_g = n_g/N$ is the proportion of patients in each group defined by the marker value, with n_g the number of individuals with marker value z_g at the start of the study. Then the redistribution to the right algorithm is modified to take account of the extra information: the weight of each censored observation is divided equally among only those individuals in the risk set who have the same value of the marker that the censored individual at the censoring time.[11] In this simple case, the resulting estimator involves stratifying the data according to different prognosis groups based on the marker, estimating the survival function separately for each group, and then combining the results by taking a weighted average. The asymptotic variance was introduced by Malani[11] and Murray and Tsiatis[12] and can be consistently estimated by

$$\sum_{g=1}^{G} \theta_g^2 \text{ est. var}(\hat{S}_{KM}(t|z_g)) + \frac{1}{N}\sum_{g=1}^{G} \theta_g (\hat{S}_{KM}(t|z_g))^2 - \frac{1}{N}\hat{S}_M(t)^2 \tag{2.2}$$

where est. var $(\hat{S}_{KM}(t|z_g))$ is obtained using Greewood's formula. Different authors show that the estimator is consistent under an assumption that censoring is independent conditional on Z.[11,12]

2.3 | The case of a continuous marker

Now suppose that we have several continuous markers recorded at patient's entry. First we reduce these markers values into a single value in a similar way of Hsu and Taylor.[14] The procedure can be summarized into the following steps. Step 1: fit a working model to the observed failure time and the observed censoring time (observed event times are treated as censored observations), respectively. Step 2: compute the risk score for both working models. Step 3: perform principal component analysis on the two standardized risk scores to generate two orthogonal components. Step 4: the first component is used to calculate the neighbours closest to the component value of the censored individual. Step 6: perform the weighted Kaplan-Meier using the approaches described below. The choice of the model for the two working models depends on the mechanism of censoring. For our motivating example we choose a parametric model using a normal distribution.[1] In the case of a single or very strong prognostic factor, for example the patient's baseline HIV-1 RNA level of the censored individual, the value of that marker itself may replace the first component in the derivation of our estimators. We use the Cox PH model for classical survival data.

Two approaches can be envisaged for this, such as

(i) redistributing the weight among the k neighbours closest to the first component value of the censored individual, or

(ii) redistributing the weight among all the individuals but according to a function giving more weight to individuals having a first component value close to the value of the censored individual.

Note that for the first possibility the redistribution among the k neighbours can be balanced (equal weight is redistributed for the k individuals) or unbalanced (as in (ii) given more weight to individuals having a first component close to the first component value of the censored individual).

Intuitively the neighbours approach seems promising and flexible. For the individual censored at time $C_{(l)}$, the first step consists of determining the genuine number of neighbours available. This number is equal to $k^{(l)} = \min(k, n(C_{(l)}))$, where k is the desired number of neighbours. It is clear that at the end of the follow-up for estimating the tail of $S(t)$ the number of available neighbours can be less than k. The criterion to select the $k^{(l)}$ neighbours of the individual censored at time $C_{(l)}$ according to the first component value ($pca1$) can be summarized by $d_i^{(l)} = \left|pca1_i - pca1_{(l)}\right|$, $i \in \mathcal{R}(C_{(l)})$ where $pca1_{(l)}$ denotes the first component value of the censored individual, and $\mathcal{R}(C_{(l)})$ denotes the set of the individuals at risk at time $C_{(l)}$.

Defining $\mathcal{K}^{(l)} \subset \mathcal{R}(C_{(l)})$ the set of the $k^{(l)}$ individuals with a minimum criterion $d^{(l)}$, the weight $w_{(l)}(C_{(l)})$ of the lth censored individual is redistributed according to

$$\begin{aligned} \forall i \in \mathcal{K}^{(l)}, \quad & w_i(C_{(l)}) = w_i(C_{(l-1)}) + w_{(l)}(C_{(l)}) \cdot f(pca1_i, pca1_{(l)}) \\ \forall i \notin \mathcal{K}^{(l)}, \quad & w_i(C_{(l)}) = w_i(C_{(l-1)}) \end{aligned} \tag{2.3}$$

where $f(pca1_i, pca1_{(l)})$ is a weighted function.
Malani suggests that $f(pca1_i, pca1_{(l)}) = 1/k^{(l)}, \forall i \in \mathcal{K}^{(l)}$ which means that the weight is equally distributed among these $k^{(l)}$ neighbours.[11] Other functions, however, can be employed leading to an unbalanced redistribution of the weight. In particular we suggest

$$f(pca1_i, pca1_{(l)}) = \frac{1/d_i^{(l)}}{\sum_{i' \in \mathcal{K}^{(l)}} 1/d_{i'}^{(l)}}, \text{ and } f(pca1_i, pca1_{(l)}) = \frac{1/r_i^{(l)}}{\sum_{i' \in \mathcal{K}^{(l)}} 1/r_{i'}^{(l)}} \tag{2.4}$$

where $r_i^{(l)}$ corresponds to the rank of individual i among the $k^{(l)}$ neighbours defined according to the criterion $d^{(l)}$. In the first suggestion the weight received by an individual in the risk set, and belonging to the $k^{(l)}$ neighbours, is proportional to the inverse of the distance between its first component value and the first component value of the censored individual while in the second suggestion the weight is proportional to the inverse of the rank of that distance.

Our second suggestion is to consider that the weight of a censored individual be redistributed among all individuals at risk but not equally, individuals having a first component value close to the value of the censored individual receiving more weight than the others. The weight is then redistributed according to

$$\forall i \in \mathcal{R}(C_{(l)}), \quad w_i(C_{(l)}) = w_i(C_{(l-1)}) + w_{(l)}(C_{(l)}) \cdot f(pca1_i, pca1_{(l)}, \sigma) \tag{2.5}$$

where $f(pca1_i, pca1_j, \sigma)$ is based on a normal distribution centered on $pca1_i$

$$f(pca1_i, pca1_{(l)}, \sigma) = \frac{\exp\left\{-\frac{(pca1_i - pca1_{(l)})^2}{2\sigma^2}\right\}}{\sum_{i' \in \mathcal{R}(C_{(l)})} \exp\left\{-\frac{(pca1_{i'} - pca1_{(l)})^2}{2\sigma^2}\right\}} \tag{2.6}$$

for the weight redistribution. All these estimators can be considered as weighted Kaplan-Meier estimates (WKM) and estimating the survival function at time t is still done by summing the corresponding weights of the individuals still at risk at time t. Of note, although the standard Kaplan-Meier estimator can handle tied data, in the setting of our motivating example (HIV-1 RNA reduction) tied data are very rare. Redistribution of the weights among the k neighbours, using a uniform distribution, according to the inverse of the distance, or to the inverse of the rank will be noted $\text{WKM}_{U,x\%}$, $\text{WKM}_{D,x\%}$ and $\text{WKM}_{R,x\%}$, with $x = k/N$ where k is the desired number of neighbours from the total sample size. The estimator based on the redistribution using a normal distribution is called $\text{WKM}_{\mathcal{N}}(\sigma)$.

2.4 | Existing methods

The KMIB estimator used two working proportional hazards models to summarize the association between prognostic covariates and event and censoring times into two risk scores.[13] One is for the association between the covariates and the event times; the other is for the association between the covariates and the censoring times. The two risk scores are weighted to define the distance between subjects, which is then used to select an imputing risk set for each censored observation. We choose a weight of 0.8 and 0.2 for failure and censoring risk scores, respectively, when defining the distance between subjects and the size of the imputing risk set for each censored observation is set at 5.[13,14] The event times are imputed using a multiple imputation strategy including a Bootstrap stage. The authors showed that imputing event times for censored observation through the use of the two risk scores can induce a double robustness property in the estimation of the survival distribution. Specifically, if one of the two working models is correctly specified, the survival estimate derived from the imputed data sets is consistent under defined conditions. More details of the procedure can be found in Hsu *et al.*[13] In our setting the two working models involved only a single prognostic factor, that is the baseline HIV-1 RNA level.

The same idea was further used to propose a second estimator.[14] They proposed using principal component analysis on the two standardized risk scores (centered and scaled) to derive two orthogonal components (linear combinations of two risk scores) and then categorized these two components separately based on their percentiles into $I * J$ groups, where I is the number of categories for the first component and J is the number of categories for the second component.[14] The WKM estimator can then

TABLE 1 Data analysis of the motivating example (N=248): estimation of the parameters used in the KM estimators using two working parametric regression models with a normal distribution (top) or using two working Cox PH models (bottom).

	Regression models with a normal distribution					
	Failure time model			Censoring time model		
Covariates	Estimate	SE	p-value	Estimate	SE	p-value
Baseline viral load	0.605	0.131	<.01	1.023	0.019	<.01
Baseline CD4×10^{-2}	0.092	0.054	0.09	0.014	0.008	0.07
Age	0.017	0.100	0.09	0.002	0.001	0.11
NRTIs	-0.008	0.093	0.93	0.013	0.014	0.34
NNRTIs	0.044	0.196	0.82	0.003	0.028	0.91
Previous ARVs used	-0.077	0.026	<.01	0.004	0.003	0.20

	Cox propotional hazard models					
	Failure time model			Censoring time model		
Covariates	Estimate	SE	p-value	Estimate	SE	p-value
Baseline viral load	-0.783	0.161	<.01	-8.774	0.693	<.01
Baseline CD4×10^{-2}	-0.099	0.060	0.10	-0.141	0.065	0.03
Age	-0.020	0.010	0.04	-0.023	0.012	0.06
NRTIs	0.043	0.090	0.63	-0.196	0.109	0.07
NNRTIs	-0.061	0.181	0.74	-0.014	0.238	0.95
Previous ARVs used	0.073	0.025	<.01	-0.003	0.027	0.91

be derived based on the $I * J$ categorized groups. In the presence of a single prognostic factor, e.g., the baseline HIV-1 RNA level, then J is fixed to 1 and the estimator is noted $\text{WKM}_{I,1}$. More details of the procedure can be found in Hsu and Taylor.[14]

The IPCW estimator is

$$\hat{S}(t) = \prod_{j:T_{(j)}<t} \left\{ 1 - \frac{1/\{\hat{K}^Z_{(j)}(T_{(j)})\}}{\sum_{i\in\mathcal{R}(T_{(j)})} 1/\{\hat{K}^Z_i(T_{(j)})\}} \right\} \tag{2.7}$$

where $\mathcal{R}(T_{(j)})$ is the risk set at time $T_{(j)}$ and $\hat{K}^Z_i(t)$ is a the conditional probability of being uncensored by time t given Z_{ip} derived from a proportional hazards model for censoring time using p prognostic variables as the covariates.

3 | EXAMPLES

3.1 | Motivating example

The data consist of 502 patients who received a raltegravir-containing regimen.[8] Overall, 254 (51%) patients had a censored reduction due to an HIV-1 RNA level at week 8 below the LOD. Both 20 copies/ml and 50 copies/ml were used as LOD in the several departments of virology involved in the study. The crude method, in which all values below the LOD were replaced by the value of 20 or 50 copies/ml, leads to a mean reduction of 2.3 $\log_{10}$ copies/mL (variance of 0.88) and median reduction of 2.4 $\log_{10}$ copies/ml (interquartile range of 1.8 to 2.9). The HIV-1 RNA reduction distributions is given in the supplementary material (supplementary Figure 1). Six variables were analysed with the working models: baseline HIV-1 RNA, baseline CD4 cell count, age (years), the number of Nucleoside Reverse Transcriptase Inhibitors (NRTIs) and of Non Nucleoside Reverse Transcriptase Inhibitors (NNRTIs) used in the raltegravir-containing regimen and the total number of antiretroviral (ARVs) drugs used in the treatment's history. Results of working models are shown in the supplementary Table I and the presence of the same significant variables in both censoring and failure time models indicate the potential for dependent censoring for these data.[14]

TABLE 2 Data analysis of a random sample of PBC data: estimation of two working Cox PH models on a sample (N=250) of data from the Mayo Clinic trial in primary biliary cirrhosis (PBC data)

	Failure time model			Censoring time model		
Covariates	Estimate	SE	p-value	Estimate	SE	p-value
Age	0.017	0.010	0.11	-0.025	0.009	<.01
Sex	0.745	0.264	0.01	-0.821	0.396	0.04
Albumin	-0.663	0.257	0.01	-0.686	0.260	<.01
Protime*	0.586	0.108	<.01	-0.714	0.150	<.01
Stage	0.498	0.146	<.01	0.042	0.104	0.68

* Standardised blood cloting time

A conceptual departure of HIV-1 RNA reduction data from time-to-event data is that the reduction in HIV-1 RNA level can be negative, corresponding to an increase HIV-1 RNA level from baseline. This difficulty is circumvented by simply adding any single number to each HIV-1 RNA reduction such that all observations are positive and then substracting this number after the analysis. In our data, $+2\log_{10}$ copies/ml was added to all viral load reduction. The Kaplan-Meier and other estimators previously introduced are applied to data of the 502 patients of the raltegravir study (supplementary Figure 2). It is difficult, however, to evaluate the performance of the different estimators without a gold standard represented by a fully observed analysis (analysis without censoring). Most of the estimators provide slightly higher or higher estimates of the HIV-1 RNA reduction compared with the partially observed analysis (KM PO) that is the analysis with censored and uncensored observations. It is well know, however, that censored reduction data by a LOD tend to underestimate the true HIV-1 RNA reduction, especially when around half of patients have a censored reduction. [1]

We then selected the 248 observations having an uncensored reduction. From this subset we obtain the Fully Observed (FO) analysis in deriving the KM estimates which is treated as the gold standard. Of note, as the true survival function is unknown in our motivating example, the KM FO obtained from the selected sample does not reflect the true HIV-1 RNA reduction. We then applied two LOD of virologic assays (100 nd 200 copies/ml) that were employed few years ago to provide artificially censored observations. For each observation, the LOD of 100 or 200 copies/ml was generated at random. All HIV-1 RNA measurements at week 8 between 20 and the LOD are considered censored at LOD. That leads to 109 (44%) censored reduction out of 248 observations. Results of the working models needed for the KM estimators are provided in Table I. Parameter estimates of the working Cox models are used by the IPCW, KMIB and $\text{WKM}_{I,J}$ estimators whereas $\text{WKM}_{f,x\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$ used parameter estimates from the regression with a normal distribution. From these results we conclude that baseline HIV-1 RNA is the main prognostic factor of both uncensored and censored viral load reduction.

Figure 1 display the estimated curves for the FO analysis and the other estimators introduced above. In the top of Figure 1 parameter estimates of the six variables are used to derive the corresponding estimators whereas in the bottom only the baseline HIV-1 RNA is used. $\text{WKM}_{I,J}$ methods provided poor estimates of the HIV-1 RNA reduction even poorer than the KM PO estimator. IPCW and KMIB estimators provide almost similar to estimates than KM PO. Our suggestions using $x = 5$ and 10% for $\text{WKM}_{u,x\%}$ and $\sigma = 0.1$ and 0.25 for $\text{WKM}_{\mathcal{N}}(\sigma)$ provide very promising alternatives to estimate the HIV-1 RNA reduction with estimates much closer to the gold standard (KM FO) than all other estimators. Using only the baseline value of HIV-1 RNA provided almost similar to estimates for most methods.

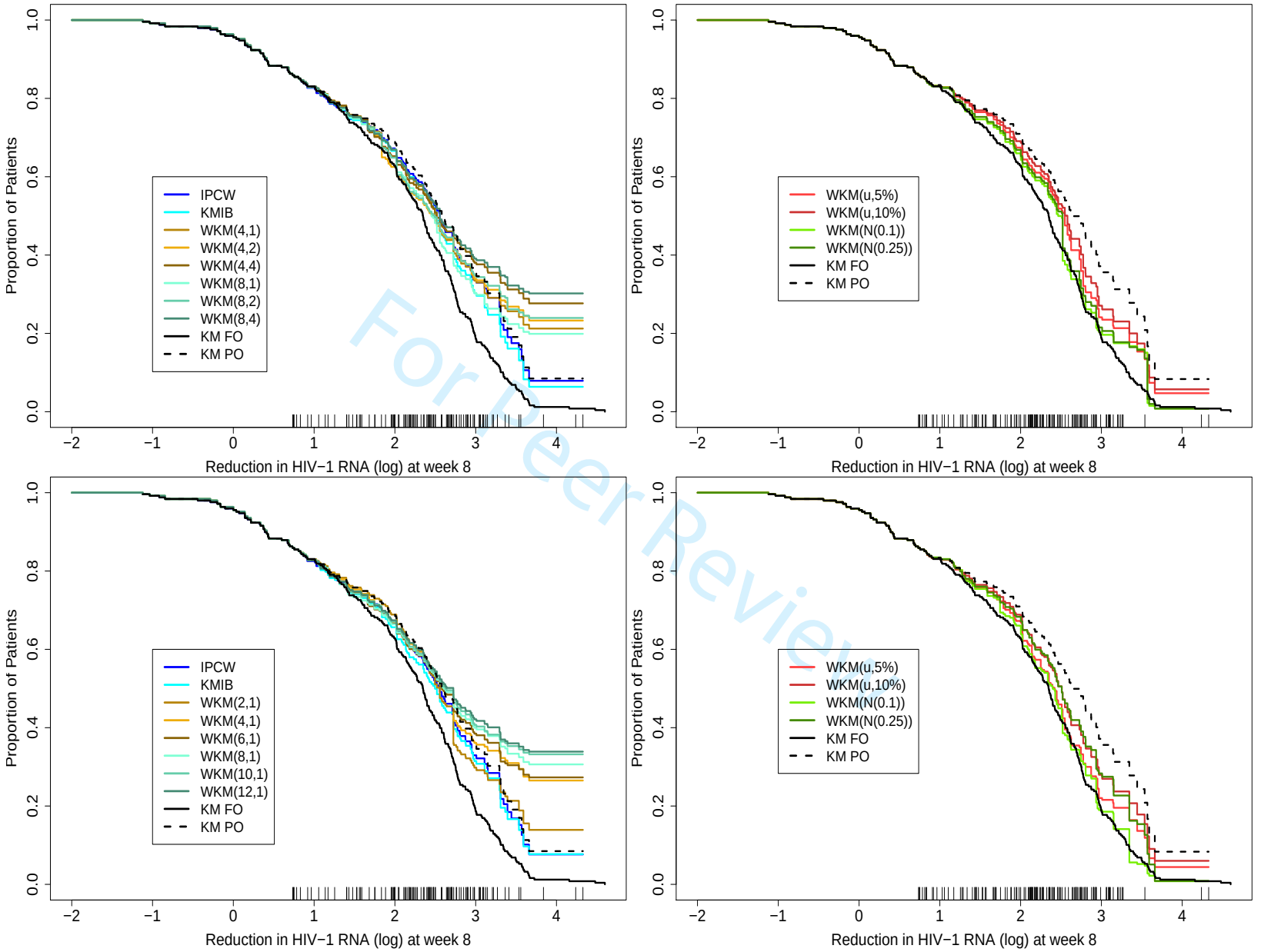

FIGURE 1 Estimates of the reduction in HIV-1 RNA (in $\log_{10}$ copies/ml) at week 8 in the 248 uncensored observations applying a LOD of 100 or 200 copies/ml. Top: using the six covariates in the working models; bottom: using only the baseline HIV-1 RNA in the working models. Marks on the *x*-axis indicated times of censored observations.

3.2 | Example from Primary Biliary Cirrhosis Data

In this example data comes from the Mayo Clinic trial in primary biliary cirrhosis (PBC) of the liver conducted between 1974 and 1984 and including 424 patients. Those data are available in the Survival R package. A subset of 250 patients was randomly selected including 102 deaths (59% censoring). A random subset is used because (i) we used the event death (removing the status transplant in the original dataset), (ii) we removed observations having missing data (variables stage and protime), and (iii) we selected a random sample to have variables strongly associated in both models (failure time model and censoring time model) indicating potential for dependent censoring according to Hsu and Taylor.[14] The following five variables were used in the two working PH models: age, sex, serum albumin, protime (standardised blood clotting time) and histologic stage of disease. Parameter estimates of the two working models suggest a potential for dependent censoring for these data (Table II). Estimates of the survival curve are provided in Figure 2. In this example, as pointed out above, our estimators $\text{WKM}_{f,x\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$ used parameter estimates from the Cox PH models to compute principal component analysis as for the $\text{WKM}_{I,J}$ estimators. All methods, except the KMIB estimator, provided slightly higher estimated survival compared with the PO analysis. KMIB method provided similar estimates than the KM PO analysis.

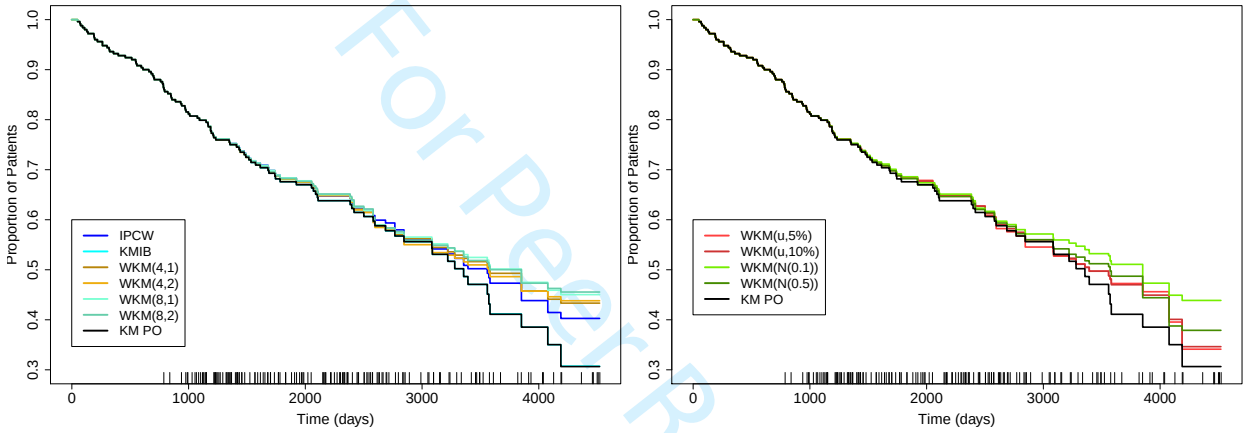

FIGURE 2 Estimates of the reduction in HIV-1 RNA (in $\log_{10}$ copies/ml) at week 8 derived KM Fully Observed, KM partially observed, IPCW, KMIB, $\text{WKM}_{I,J}$ and $\text{WKM}_{U,X\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$ estimators for the 257 patients with an uncensored reduction but applying a LOD of 200 copies/ml (2.3 $\log_{10}$ copies/ml). Marks on the x-axis indicated times of censored observations.

4 | SIMULATION STUDY

4.1 | Simulation methods

We performed two series of simulation study to investigate the performance of our proposed WKM estimators. The first series investigates performance of the methods in the settings of assessing HIV-1 RNA reduction whereas the second series is carried out in the classical survival settings of two competing risks (event time and censored time). For the latter series, we used exactly the same simulation settings and parameter values than Hsu and Taylor.[14] Details of the simulation study may be found in[14] and in the R code is available upon request.

In the first series, five hypothetical prognostic covariates $(Z_1, \ldots, Z_5)$ are independently generated. Three variables (Z_1, Z_2 and Z_3) are generated from a $\mathcal{N}(0,1)$ distribution, Z_4 is generated from a $U(0,5)$ distribution and Z_5 takes value from 0 to 30 based on a normal distribution. Z_1 represents baseline HIV-1 RNA, Z_2 baseline $\sqrt{CD4}$ count and Z_3 age. Z_4 and Z_5 represents the number of NRTIs used and the number of previous ARVs used, respectively. The uncensored viral load reduction at week 8 is generated from a normal $(3 + 0.347Z_1 + 0.437Z_2 + 0.099Z_3 + 0.102Z_4 - 0.041Z_5, 1)$. Parameter values were derived from the analysis of our example after standardization of the baseline HIV-1 RNA from a $\mathcal{N}(5.39, 0.80)$ distribution, baseline $\sqrt{CD4}$ count from a $\mathcal{N}(13.8, 6.7)$ distribution and age from a $\mathcal{N}(46.8, 10)$ distribution. For each hypothetical observation the HIV-1

TABLE 3 Monte Carlo results for estimating a hypothetical HIV-1 RNA reduction. Estimators are derived using the **five covariates** $Z_1 - Z_5$ with N=200.

	True value: 0.5				True value: 0.35				True value: 0.2			
Method	Est [1]	R bias[2]	MSE[3]	CR[4]	Est	R bias	MSE	CR	Est	R bias	MSE	CR
LOD is either 100 copies/ml (Pr=70%) or 200 copies/ml (Pr=30%) leading to an overall censoring rate=45%.												
	Censoring rate=24%[5]				Censoring rate=34%				Censoring rate=42%			
Existing methods												
KM FO	0.498	-0.2	0.0013	95.4	0.349	-0.1	0.0012	94.3	0.199	-0.1	0.0009	93.6
KM PO	0.520	2.0	0.0019	92.4	0.379	2.9	0.0027	89.1	0.236	3.6	0.0037	87.9
IPCW	0.515	1.5	0.0019	92.1	0.374	2.4	0.0026	89.8	0.231	3.1	0.0038	86.8
KMIB	0.506	0.6	0.0018	93.9	0.363	1.3	0.0023	93.6	0.220	2.0	0.0031	91.5
$WKM_{4.1}$	0.486	-1.4	0.0026	88.4	0.349	-0.1	0.0033	86.6	0.242	4.2	0.0059	75.8
$WKM_{4.2}$	0.517	1.7	0.0025	87.5	0.406	5.6	0.0058	69.7	0.329	12.9	0.0197	20.0
$WKM_{8.1}$	0.491	-0.9	0.0024	89.2	0.363	1.3	0.0030	87.9	0.266	6.6	0.0078	62.3
$WKM_{8.2}$	0.529	2.9	0.0027	85.1	0.427	7.7	0.0081	50.1	0.360	16.0	0.0278	4.2
Proposed methods												
$WKM_{U,15\%}$	0.493	-0.7	0.0017	94.1	0.346	-0.4	0.0020	92.6	0.208	0.8	0.0023	91.9
$WKM_{U,20\%}$	0.496	-0.4	0.0016	94.7	0.351	0.1	0.0019	93.5	0.213	1.3	0.0024	92.5
$WKM_{\mathcal{N}(1)}$	0.501	0.1	0.0016	94.8	0.354	0.4	0.0019	93.8	0.211	1.1	0.0025	91.6
LOD is either 100 copies/ml (Pr=30%) or 200 copies/ml (Pr=70%) leading to an overall censoring rate=50%.												
	Censoring rate=30%				Censoring rate=40%				Censoring rate=47%			
Existing methods												
KM FO	0.499	-0.1	0.0013	94.1	0.349	-0.1	0.0013	94.1	0.200	0.0	0.0009	93.9
KM PO	0.525	2.5	0.0024	89.9	0.385	3.5	0.0034	89.5	0.240	4.0	0.0053	84.8
IPCW	0.515	1.5	0.0042	82.8	0.374	2.4	0.0054	79.8	0.232	3.2	0.0083	76.7
KMIB	0.509	0.9	0.0022	92.4	0.367	1.7	0.0029	90.4	0.225	2.5	0.0046	88.3
$WKM_{4,1}$	0.487	-1.3	0.0033	85.7	0.354	0.4	0.0043	82.5	0.256	5.6	0.0084	70.6
$WKM_{4,2}$	0.526	2.6	0.0036	82.7	0.424	7.4	0.0090	55.7	0.354	15.4	0.0275	14.0
$WKM_{8,1}$	0.494	-0.6	0.0029	88.0	0.371	2.1	0.0039	82.6	0.284	8.4	0.0111	51.8
$WKM_{8,2}$	0.540	4.0	0.0039	76.2	0.446	9.6	0.0119	36.4	0.386	18.6	0.0373	2.8
Proposed methods												
$WKM_{U,15\%}$	0.493	-0.7	0.0021	92.9	0.347	-0.3	0.0023	92.1	0.210	1.0	0.0033	90.7
$WKM_{U,20\%}$	0.496	-0.4	0.0020	93.2	0.353	0.3	0.0022	93.4	0.216	1.6	0.0035	90.9
$WKM_{\mathcal{N}(1)}$	0.502	0.2	0.0019	93.3	0.357	0.7	0.0023	92.2	0.214	1.4	0.0037	89.2
LOD is either 200 copies/ml (Pr=70%) or 500 copies/ml (Pr=30%) leading to an overall censoring rate=57%.												
	Censoring rate=39%				Censoring rate=49%				Censoring rate=56%			
Existing methods												
KM FO	0.499	-0.1	0.0013	93.8	0.348	-0.2	0.0012	94.1	0.199	-0.1	0.0009	93.3
KM PO	0.535	3.5	0.0032	85.3	0.394	4.4	0.0047	84.6	0.250	5.0	0.0083	83.3
IPCW	0.528	2.8	0.0031	86.6	0.387	3.7	0.0045	85.8	0.245	4.5	0.0085	83.3
KMIB	0.517	1.7	0.0027	91.7	0.374	2.4	0.0038	90.7	0.233	3.3	0.0071	85.5
$WKM_{4.1}$	0.486	-1.4	0.0047	80.3	0.359	0.9	0.0058	80.7	0.277	7.7	0.0129	62.7
$WKM_{4.2}$	0.552	5.2	0.0059	69.3	0.464	11.4	0.0167	33.1	0.409	20.9	0.0476	4.6
$WKM_{8.1}$	0.496	-0.4	0.0039	84.4	0.381	3.1	0.0058	78.0	0.311	11.1	0.0177	39.5
$WKM_{8.2}$	0.569	6.9	0.0073	55.3	0.490	14.0	0.0224	13.8	0.446	24.6	0.0634	0.4
Proposed methods												
$WKM_{U.15\%}$	0.492	-0.8	0.0025	92.1	0.347	-0.3	0.0028	93.1	0.216	1.6	0.0050	90.6
$WKM_{U.20\%}$	0.497	-0.3	0.0023	92.8	0.355	0.5	0.0027	93.3	0.222	2.2	0.0053	89.6
$WKM_{\mathcal{N}(1)}$	0.507	0.7	0.0022	91.7	0.361	1.1	0.0029	92.2	0.220	2.0	0.0057	88.0

[1] Average of 1000 point estimates [2] Relative bias ($100 \times (Bias/S(t))$ [3] Mean square error
[4] Coverage rate of 1000 95 per cent confidence intervals [5] Censoring rate at the time of the true value of $S(t)$

TABLE 4 Monte Carlo results for estimating a hypothetical HIV-1 RNA reduction. Estimators are derived using **only** Z_1 **representing the baseline HIV-1 RNA** with N=200.

	True value: 0.5				True value: 0.35				True value: 0.2			
Method	Est [1]	R bias[2]	MSE[3]	CR[4]	Est	R bias	MSE	CR	Est	R bias	MSE	CR
LOD is either 100 copies/ml (Pr=70%) or 200 copies/ml (Pr=30%) leading to an overall censoring rate=45%.												
	Censoring rate=24%[5]				Censoring rate=34%				Censoring rate=42%			
Existing methods												
KM FO	0.498	-0.2	0.0013	95.4	0.349	-0.1	0.0012	94.3	0.199	-0.1	0.0009	93.6
KM PO	0.520	2.0	0.0019	92.4	0.379	2.9	0.0027	89.1	0.236	3.6	0.0037	87.9
IPCW	0.515	1.5	0.0019	91.5	0.373	2.3	0.0026	90.1	0.230	3.0	0.0037	87.8
KMIB	0.512	1.2	0.0019	92.8	0.371	2.1	0.0025	92.3	0.229	2.9	0.0037	88.9
$WKM_{2.1}$	0.511	1.1	0.0020	93.2	0.386	3.6	0.0052	78.8	0.306	10.6	0.0153	40.8
$WKM_{4.1}$	0.527	2.7	0.0029	85.4	0.425	7.5	0.0083	57.3	0.356	15.6	0.0274	12.6
$WKM_{8.1}$	0.543	4.3	0.0037	78.0	0.450	10.0	0.0122	33.7	0.390	19.0	0.0383	1.4
$WKM_{10.1}$	0.546	4.6	0.0040	73.8	0.455	10.5	0.0131	27.0	0.396	19.6	0.0406	0.6
Proposed methods												
$WKM_{U,15\%}$	0.509	0.9	0.0018	93.5	0.366	1.6	0.0024	92.5	0.226	2.6	0.0032	90.4
$WKM_{U,20\%}$	0.510	1.0	0.0017	93.6	0.368	1.8	0.0023	92.3	0.228	2.8	0.0032	89.7
$WKM_{\mathcal{N}(1)}$	0.511	1.1	0.0017	93.3	0.369	1.9	0.0023	92.3	0.227	2.7	0.0033	88.5
LOD is either 100 copies/ml (Pr=30%) or 200 copies/ml (Pr=70%) leading to an overall censoring rate=50%.												
	Censoring rate=30%				Censoring rate=40%				Censoring rate=47%			
Existing methods												
KM FO	0.499	-0.1	0.0013	94.1	0.349	-0.1	0.0013	94.1	0.200	0.0	0.0009	93.9
KM PO	0.525	2.5	0.0024	89.9	0.385	3.5	0.0034	89.5	0.240	4.0	0.0053	84.8
IPCW	0.514	1.4	0.0039	83.6	0.374	2.4	0.0052	79.3	0.233	3.3	0.0079	77.8
KMIB	0.516	1.6	0.0026	89.9	0.375	2.5	0.0037	87.7	0.232	3.2	0.0060	83.2
$WKM_{2,1}$	0.513	1.3	0.0028	91.1	0.394	4.4	0.0082	65.5	0.314	11.4	0.0199	40.1
$WKM_{4,1}$	0.533	3.3	0.0043	77.6	0.434	8.4	0.0116	51.3	0.369	16.9	0.0339	17.2
$WKM_{8,1}$	0.550	5.0	0.0051	69.8	0.460	11.0	0.0153	32.7	0.404	20.4	0.0450	3.4
$WKM_{10,1}$	0.555	5.5	0.0056	65.7	0.467	11.7	0.0167	27.6	0.412	21.2	0.0485	1.8
Proposed methods												
$WKM_{U.15\%}$	0.512	1.2	0.0022	92.6	0.371	2.1	0.0030	89.6	0.231	3.1	0.0047	87.9
$WKM_{U.20\%}$	0.513	1.3	0.0021	92.7	0.373	2.3	0.0029	89.6	0.232	3.2	0.0047	87.3
$WKM_{\mathcal{N}(1)}$	0.515	1.5	0.0021	92.5	0.374	2.4	0.0030	89.2	0.232	3.2	0.0049	86.5
LOD is either 200 copies/ml (70%) or 500 copies/ml (30%) leading to an overall censoring rate=57%.												
	Censoring rate=39%				Censoring rate=49%				Censoring rate=56%			
Existing methods												
KM FO	0.499	-0.1	0.0013	93.8	0.348	-0.2	0.0012	94.1	0.199	-0.1	0.0009	93.3
KM PO	0.535	3.5	0.0032	85.3	0.394	4.4	0.0047	84.6	0.250	5.0	0.0083	83.3
IPCW	0.527	2.7	0.0031	86.5	0.386	3.6	0.0045	85.7	0.244	4.4	0.0084	83.3
KMIB	0.526	2.6	0.0030	88.8	0.384	3.4	0.0044	87.6	0.243	4.3	0.0082	84.7
$WKM_{2.1}$	0.526	2.6	0.0053	83.2	0.440	9.0	0.0147	47.3	0.353	15.3	0.0327	27.2
$WKM_{4.1}$	0.558	5.8	0.0077	59.5	0.475	12.5	0.0210	30.9	0.424	22.4	0.0567	8.7
$WKM_{8.1}$	0.584	8.4	0.0101	45.0	0.512	16.2	0.0298	11.9	0.472	27.2	0.0781	0.5
$WKM_{10.1}$	0.592	9.2	0.0111	37.3	0.522	17.2	0.0327	6.3	0.485	28.5	0.0846	0.1
Proposed methods												
$WKM_{U.15\%}$	0.519	1.9	0.0029	89.7	0.378	2.8	0.0039	88.3	0.239	3.9	0.0072	86.4
$WKM_{U.20\%}$	0.521	2.1	0.0027	89.8	0.381	3.1	0.0038	88.8	0.241	4.1	0.0073	86.4
$WKM_{\mathcal{N}(1)}$	0.523	2.3	0.0028	89.2	0.382	3.2	0.0040	87.9	0.241	4.1	0.0075	85.2

[1] Average of 1000 point estimates [2] Relative bias ($100 \times (Bias/S(t))$ [3] Mean square error
[4] Coverage rate of 1000 95 per cent confidence intervals [5] Censoring rate at the time of the true value of $S(t)$

RNA at week 8 is calculated using both the baseline HIV-1 RNA (Z_1) and the uncensored reduction. For each subject a LOD of 100, 200 or 500 copies/ml is randomly assigned with different probabilities. We can then distinguish between censored (HIV-1 RNA at week 8 < LOD) and uncensored (HIV-1 RNA at week 8 > LOD) reduction. To select observations having plausible data in our settings, all observations with a baseline HIV-1 RNA < 3 or > 7, or with an uncensored reduction < 1.5 or > 6.5 were deleted. Due to that selection, to analyse approximately N observations we generate $N \times 1.2$ observations. Weighted estimators are derived using the 5 covariates used to generate the data but also using only the main prognostic factor of censoring Z_1 (misspecification of the model) representing the baseline HIV-1 RNA level.

The second series used the same settings and parameter values than the simulation of Hsu and Taylor (see [14] for more details). Briefly, five hypothetical covariates are generated from a $U(0,1)$ distribution. In particular, they investigated the effects of misspecification of the two link functions on survival estimates. In a situation that the link functions are correctly specified for the two working PH models, i.e. the true failure and censoring time models are from a PH family, the event time is generated from a hypothetical PH model conditional on prognostic covariates, where the hazard function is $\lambda(t) = 4t^3 \exp(-2.0Z_1 + 0.5Z_2 - 2.0Z_3 + 2.0Z_4 + 2.0Z_5)$ and the censoring time is generated from a hazard function $\lambda_c(t) = 3t^2 \exp(-3.0Z_1 + 0.5Z_2 - 2.0Z_3 + 1.5Z_4 + 2.0Z_5)$ under a dependent censoring scenario. In a situation where the link functions are incorrectly specified for the two working PH models, i.e. the true failure and censoring time models are not from a PH family, the event time is generated from lognormal $(0.1 - 2Z_1 + 0.5Z_2 - 2Z_3 + 2Z_4 + 2Z_5, 1)$ and the censoring time is generated from lognormal $(0.08 - 2.5Z_1 + 0.5Z_2 - 2Z_3 + 2.5Z_4 + 2Z_5, 1)$. Misspecification of the failure time model is investigated using only Z_1, Z_2 and Z_3 in the fitted model. We do not investigate misspecification of the censoring time model.

For each of 1000 independent such simulated data sets we derived the estimators introduced above. We studied different values of x for $\text{WKM}_{f,x\%}$ and of σ for $\text{WKM}_{\mathcal{N}}(\sigma)$. For the first series, values for the 'true' reduction, equivalent to $S(t)$ in the classical survival analysis, are derived on the basis of a simulated datasets of 100,000 observations analyzed without censoring. For the second series, the true survival curve is derived from the distribution used. For the FO analysis, treated as the gold standard, the KM estimates is derived before any censoring. The KM PO is derived after censoring. The Grenwwod's formula is used to estimate the standard error for both estimators. For the KMIB method, based on previous work, the number of imputation is set at 10 and the size of each imputing risk set is set at 5.[13] The weights on risk scores derived from working failure time and censoring time models are set at 0.8 and 0.2, respectively. Standard errors for the KMIB estimator were computed using the Rubin's rules.[18] Standard errors for $\text{WKM}_{I,J}$ were computed using the formula of Malani[11] given in Section (2.2). Bootstrap resampling (150 samples) was used to compute standard errors for our proposed estimators ($\text{WKM}_{f,x\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$) and for the IPCW method. We also investigated the influence of the principal component analysis in our method. Then, we also derived our proposed estimators using directly the risk score for the failure time model without doing principal component analysis.

Performance of the different estimators are compared through relative bias ($100\times$[Bias/$S(t)$]) with Bias = $(1/1000)(\sum_{i=1}^{1000}(\hat{S}_i(t) - S(t)))$, where $S(t)$ is given by the KM FO and $S_i(t)$ denote the estimated survival function obtained from the ith simulated dataset using a given method. We also determined the mean standard error (MSE=$(1/1000)(\sum_{i=1}^{1000}(\hat{S}_i(t) - S(t))^2)$) of the estimators across the 1000 simulated datasets. Finally, we computed the coverage rates, i.e., the proportion of 95% confidence intervals that covered the KM FO for each method. Performance of all these methods are displayed at the middle and end of survival curve when $S(t) = 0.5$, $S(t) = 0.35$ and $S(t) = 0.2$.

4.2 | Simulation results

Results for the first series of simulation, i.e. estimating an hypothetical HIV-1 RNA reduction, are displayed in Tables 3 and 4 for N=200 (complete results are displayed in supplementary tables 2-9 for N=200 and tables 16-23 for N=100). Using three distinct LOD (100, 200 and 500 copies/ml) leads to overall censoring rate from 45 to 65%. As expected the KM PO analysis provided lower performance compared with the FO analysis (gold standard) with biased estimates and a lower coverage rate than the nominal level. Bias (coverage rate) increases (decreases) with increasing censoring rate from $S(t) = 0.5$ to $S(t) = 0.2$ and for an increasing overall censoring rate from 45 to 65%. $\text{WKM}_{I,J}$ method shows poor performances with a large relative bias and very low coverage rates (below 70%) especially for the tail of the curve. Performances decrease with increasing value of both I and J and, as for all estimators, with increasing censoring rate. Performances markedly decrease when only Z_1 is used in the two working Cox PH models (Table 4). For example, with an overall censoring rate of 50%, the coverage rate moves from 70.6 to 17.2% and the relative bias from 5.6 to 16.9% for $\text{WKM}_{4,1}$. Of note, $\text{WKM}_{I,J}$ method provides poor performances in term of coverage probability even at the median of the curve ($S(t) = 0.5$) with 24% censoring at that time.

In contrast, KMIB method provides much better performance with low coverage rates than the nominal value only for the tail of the curve and for a level of censoring greater than 50%. In general, performances are almost similar in terms of bias and coverage rate when only Z_1 is used to derive the KMIB estimator. Surprisingly, in this setting, IPCW method provided similar or lower performances than the PO analysis. There is no clear trend of the performance when the censoring increase since performances decrease from an overall censoring rate of 45% to 50% but then increase from 50% to 57% and decrease again from 57% to 64% (supplementary Table 5). On the other hand, the use of Z_1 as single variable in the Cox model provides almost similar performances than the use of the 5 variables in the model.

In general, best performances are obtained for $\text{WKM}_{U,x\%}$ ($x = 15$ and 20%) and $\text{WKM}_{\mathcal{N}(1)}$. $\text{WKM}_{D,x\%}$ and $\text{WKM}_{R,x\%}$ estimators may provide reasonable estimates but with poorer or slightly poorer performance than $\text{WKM}_{U,x\%}$ (supplementary tables 2-9). For $\text{WKM}_{f,x\%}$ ($f = D$ and R) the relative bias tends to decrease with increasing number of neighbours for $S(t) = 0.50$ and 0.35. For the tail of the curve, the relative bias is smallest around $x = 10-20\%$. For $\text{WKM}_{U,x\%}$ the relative bias has an U-shape with lowest bias around 10%. For $\text{WKM}_{\mathcal{N}(\sigma)}$, lowest relative bias are obtained with $\sigma = 0.5$ and 1 but better coverage rates are obtained with $\sigma = 1$. Performances of $\text{WKM}_{U,x\%}$ ($x = 15$ and 20%) and $\text{WKM}_{\mathcal{N}(1)}$ are similar or slightly better than KMIB method with an overall censoring of 45%. These estimators, however, outperform the KMIB method when the overall censoring is greater than 50%. Whatever the values of x for $\text{WKM}_{U,x\%}$ and of σ for $\text{WKM}_{\mathcal{N}(\sigma)}$, except $\sigma = 0.1$, these estimators lead to better performances than $\text{WKM}_{I,J}$. In contrast, KMIB method provided better performance than $\text{WKM}_{R,x\%}$ and $\text{WKM}_{D,x\%}$. Performances of our estimators in terms of coverage rate are decreased for $\text{WKM}_{R,x\%}$, $\text{WKM}_{D,x\%}$ and $\text{WKM}_{\mathcal{N}(0.1)}$ when Z_1 alone is used instead of the five variables (supplementary Tables 6-9). $\text{WKM}_{U,x\%}$ with $x \geq 5\%$ and $\text{WKM}_{\mathcal{N}(\sigma)}$ with $\sigma \geq 0.5$, provide almost similar performances with the use of Z_1 alone.

It is difficult to summarize all the results obtained with a smaller sample size (N=100) except that for all methods MSE was higher than with N=200 (supplementary Tables 16-23). In general, for $\text{WKM}_{D,x\%}$ relative bias were similar or slightly lower and coverage probabilities were similar or slightly higher with N=100 compared with N=200. For $\text{WKM}_{R,x\%}$, $\text{WKM}_{U,x\%}$ and $\text{WKM}_{\mathcal{N}(\sigma)}$ performances were roughly similar except in few occasions. Similar conclusions were found when only Z_1, representing the baseline HIV-1 RNA, was used in both failure and censoring models (mis-specified models).

In general, we also obtained good performances when the risk score for the failure time model is used without a principal component analysis (supplementary tables 30-33). For example, the relative bias is low and coverage probabilities not far from the nominal level for $\text{WKM}_{U,10-15\%}$ and $\text{WKM}_{\mathcal{N}(0.5-1)}$. Of note, when most of the weights of censored observations are redistributed among less observations ($\text{WKM}_{U,x\%}$ with $x \leq 5\%$ and $\text{WKM}_{\mathcal{N}(\sigma)}$ with $\sigma \leq 0.5$) performances are better or slightly better compared with the use of a principal component analysis. In contrast, with $x > 5\%$ and $\sigma > 0.5$ performances are similar or slightly lower than with the use of principal component analysis. With a high level of censoring (overall censoring rate of 64%), however, the use of PCA tends to provide slightly better performances for the tail of the curve.

As expected, results of the second series (Table 5) provide similar results for $\text{WKM}_{I,J}$, IPCW and KMIB methods than those found in Hsu and Taylor.[14] Complete results for this second series of simulation are displayed in supplementary Tables 10-15 for N=200 and supplementary Tables 24-29 for N=100. When the link functions are correctly specified $\text{WKM}_{4,1}$, IPCW and KMIB methods show good performances although for $\text{WKM}_{I,J}$ with $I > 4$ or $J > 1$ performances decrease for the tail of the curve ($S(t) = 0.20$). Our estimators $\text{WKM}_{U,x\%}$, with $x = 5$ and 10%, and $\text{WKM}_{\mathcal{N}(0.5)}$ provide similar performances than $\text{WKM}_{4,1}$, IPCW and KMIB estimators. When the link functions are uncorrectly specified, $\text{WKM}_{I,J}$ method has increasing bias and decreasing coverage rates with increasing censoring rate from $S(t) = 0.5$ to $S(t) = 0.2$. IPCW method show also lower coverage rates than the nominal value. $\text{WKM}_{U,x\%}$ ($x = 5$ and 10%) and $\text{WKM}_{\mathcal{N}(0.5)}$ provide similar results than the KMIB method with lower coverage rate than the nominal value for the tail of the survival curve.

As for the other methods, the use of Z_1-Z_3 in the failure time model (supplementary Tables 13-15), performances of our proposed methods are poorer than with a correct specification of the failure time model uing Z_1-Z_5 (supplementary Tables 10-12). In general, however, their performances are still better or slightly better than those provided by the $\text{WKM}_{I,J}$ method mainly for th e tail of the survival curve. As above, MSE increased for all methods with a smaller ample size (N=100, supplementary tables 24-29). For our estimators, performances when N=100 are roughly similar or slightly lower than whith N=200 and still comparable to $\text{WKM}_{I,J}$ for $\text{WKM}_{U,5-10\%}$ and $\text{WKM}_{\mathcal{N}(0.5)}$. The direct use of the risk score of the failure time model without a principal component analysis provides, overall, similar results than using the first component of a principal component analysis analysis (supplementary tables 34-36).

In summary, when censoring is due to a LOD for the reduction of variable approximately normally distributed $\text{WKM}_{U,x\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$ can provide reasonable estimates of such a reduction. In contrast, $\text{WKM}_{D,x\%}$ and $\text{WKM}_{R,x\%}$ do not produce always reasonable estimates mainly for the tail of the curve. In general, our estimators are robust to misspecification of both working

models and the direct use of the risk score of the failure time model provides also roughly similar results than the use of the first component of a principal component analysis. KMIB method can also produce reasonable estimate up to an overall censoring rate lower than 50%. The $\text{WKM}_{I,J}$ method provide large bias and poor coverage rates especially for the tail of the curve. IPCW method does not provide good performance although better than $\text{WKM}_{I,J}$. In the classical survival settings, the $\text{WKM}_{I,J}$ method can provide reasonable survival estimates mainly when the link functions are correctly specified. When the link functions are uncorrectly specified there an increasing bias and decreasing coverage rates for the tail of the curve. IPCW and KMIB methods can produce reasonable survival estimates, the latter method being more robust to misspecification of the link functions. Our estimators $\text{WKM}_{U,x\%}$ and $\text{WKM}_{\mathcal{N}}(\sigma)$ can produce reasonable survival estimates mostly similar to IPCW and KMIB methods. In the case of mis-specification of the failure time model, our estimators provide similar performances than other methods. Again the direct use of the risk score of the failure time model provides reasonable estimates of the survival function in most cases.

5 | DISCUSSION

We investigate in this work a procedure based on the WKM estimator proposed by Malani[11] to estimate survival curve in the presence of dependent censoring. The procedure is based on principal components of standardized risks scores estimated from working models. Our simulations results, however, show good performances using directly the risk score without any principal component analysis. A similar procedure has been used to categorize observations in a certain number of groups.[14] Such groups are defined at time 0 and are time independent. Our procedure is more flexible redistributing weights among observations still at risk when censored observations occurred. When the objective is to estimate an HIV-1 RNA reduction censored by a LOD, a parametric regression model with a normal distribution is chosen for both working models on event and censored times. When the goal is to estimate a classical survival curve, two working Cox PH models are used. Our proposition was motivated to estimate HIV-1 RNA reduction censored by a LOD in HIV-1 infected patients. Our estimators $\text{WKM}_{U,x\%}$ and $\text{WKM}_{\mathcal{N}(\sigma)}$ can provide reasonable estimates of such a reduction. In particular, with appropriate parameters our suggestions have lower relative bias and better coverage rates than existing methods. In the classical survival settings, our method can also provide reasonable estimates showing bias and coverage rates mostly similar to IPCW and KMIB methods.

Most of the existing methods used the Cox proportional hazards model to derive their estimators. A previous work has shown that all these estimators have some bias when event and censoring models are not from a PH family (link functions incorrectly specified). We show that the bias is more pronounced and the coverage rates lower for the tail of the survival curve. In our motivating example, given the level of the baseline HIV-1 RNA and the LOD of the assay used by the department of virology, the maximum uncensored reduction is known as the baseline value minus the LOD. Beyond this value the HIV-1 RNA reduction is censored. In a simulation study, similar censoring would be obtained using a uniform distribution for censoring times. Then, the censoring model is not from a PH family. Thus, our motivating example, as well as the first series of the simulation study, correspond to a situation where the link functions are incorrectly specified. Consequently, the double robustness properties of some estimators does not hold since the true model is not from a PH family. That explains the bias found in our study simulation for most estimators although, for few situations, the relative bias is important only for the tail of the curve. The bias is more pronounced for both $\text{WKM}_{I,J}$ estimators than for KMIB and IPCW estimators. In the classical survival settings, we found the same results as Hsu and Taylor (2010), i.e. $\text{WKM}_{I,J}$, KMIB and IPCW methods provide good performance mainly in the situation where the link functions are correctly specified. Our suggestions provide also reasonable estimates of the survival curve.

For all methods, one has to choose the variables included in models used to derive the estimators. IPCW method has the main advantage of not choosing any other parameters. For the KMIB method we have to choose weights for failure and censoring risks as well as the size of the imputing risk set. The number of groups should be determined for the $\text{WKM}_{I,J}$ method, and, in general, the bias increases with increasing number of groups. The choice of the number of neighbours and the value of σ is a limitation of our method. Our results have shown that the risk score can be used directly without a principal component analysis. As for $\text{WKM}_{I,J}$, one have to choose both failure and censoring time models. Further studies are required to investigate other situations involving other variables having non-normal distributions. For $\text{WKM}_{U,x\%}$, it is clear that especially at the end of the follow-up, the number of available neighbours can be less than the desired number which may explained the large variability for the tail of the curve.

As shown in our motivating example the main prognostic factor of censoring is the baseline HIV-1 RNA level though other variables may be involved. Then, one can use only this variable to derive all estimators. In this case, our motivating example, as well as simulation results, show that, in general, $\text{WKM}_{U,x\%}$, $\text{WKM}_{\mathcal{N}}(\sigma)$ and KMIB estimators have relatively good performance.

Of course, our method can be used in many other situations. Variables following a normal distributions are common and the difficulty of measuring a variable below a LOD is a common problem in medical and biological studies. It occurs for other variables than the viral load in the management of HIV-1 infected patients.[19] For example, assays to quantify most plasma cytokines and chemokines (IL-1β, IL-2, IL6-M, ...) used a LOD. The difficulty of quantifying a variable below a LOD occurs also in environmental studies.[20,21]

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TABLE 5 Monte Carlo results for estimating classical survival curves with five time independent covariates and dependent censoring.

The link functions **correctly** specified. Overall censoring rate=35% and N=200.

	True value: 0.5				True value: 0.35				True value: 0.2			
Method	Est [1]	R bias[2]	MSE[3]	CR[4]	Est	R bias	MSE	CR	Est	R bias	MSE	CR
	Censoring rate=26%[5]				Censoring rate=29%				Censoring rate=32%			
Existing methods												
KM FO	0.499	-0.1	0.0012	94.5	0.350	0.0	0.0012	93.9	0.198	-0.2	0.0008	93.4
KM PO	0.565	6.5	0.0059	60.6	0.425	7.5	0.0074	55.3	0.268	6.8	0.0062	60.0
IPCW	0.500	0.0	0.0018	93.0	0.352	0.2	0.0018	94.4	0.201	0.1	0.0013	94.2
KMIB	0.510	1.0	0.0018	94.0	0.364	1.4	0.0018	93.4	0.212	1.2	0.0013	94.3
$WKM_{4,1}$	0.505	0.5	0.0018	94.3	0.357	0.7	0.0017	93.4	0.211	1.1	0.0014	92.9
$WKM_{4,2}$	0.509	0.9	0.0019	93.2	0.365	1.5	0.0020	91.7	0.221	2.1	0.0019	88.6
$WKM_{4,4}$	0.516	1.6	0.0020	92.2	0.376	2.6	0.0025	87.9	0.237	3.7	0.0029	80.2
$WKM_{8,1}$	0.504	0.4	0.0018	94.0	0.360	1.0	0.0019	91.9	0.217	1.7	0.0017	90.6
$WKM_{8,2}$	0.513	1.3	0.0019	92.2	0.374	2.4	0.0024	87.9	0.236	3.6	0.0028	80.6
$WKM_{16,1}$	0.510	1.0	0.0019	92.4	0.371	2.1	0.0023	89.1	0.233	3.3	0.0026	82.7
Proposed methods												
$WKM_{U,5\%}$	0.504	0.4	0.0017	93.7	0.355	0.5	0.0016	92.7	0.203	0.3	0.0011	93.5
$WKM_{U,10\%}$	0.510	1.0	0.0018	93.7	0.361	1.1	0.0017	93.3	0.209	0.9	0.0012	93.5
$WKM_{\mathcal{N}(0.5)}$	0.509	0.9	0.0017	93.9	0.360	1.0	0.0017	93.5	0.206	0.6	0.0012	94.0

The link functions **uncorrectly** specified. Overall censoring rate=51% and N=200

	True value: 0.5				True value: 0.35				True value: 0.2			
Method	Est [1]	R bias[2]	MSE[3]	CR[4]	Est	R bias	MSE	CR	Est	R bias	MSE	CR
	Censoring rate=34%				Censoring rate=40%				Censoring rate=46%			
Existing methods												
KM FO	0.498	-0.2	0.0012	94.8	0.349	-0.1	0.0011	94.9	0.199	-0.1	0.0008	94.3
KM PO	0.603	10.3	0.0121	29.6	0.470	12.0	0.0163	24.3	0.318	11.8	0.0163	29.6
IPCW	0.489	-1.1	0.0066	81.7	0.334	-1.6	0.0073	77.1	0.186	-1.4	0.0053	81.3
KMIB	0.521	2.1	0.0024	91.5	0.377	2.7	0.0029	90.3	0.23	3.0	0.0029	89.0
$WKM_{4,1}$	0.515	1.5	0.0026	90.4	0.374	2.4	0.0034	88.1	0.243	4.3	0.0046	81.2
$WKM_{4,2}$	0.522	2.2	0.0029	88.5	0.389	3.9	0.0043	81.9	0.266	6.6	0.0071	65.7
$WKM_{4,4}$	0.535	3.5	0.0035	82.8	0.409	5.9	0.0061	66.5	0.294	9.4	0.0116	38.3
$WKM_{8,1}$	0.518	1.8	0.0027	89.2	0.382	3.2	0.0038	85.1	0.257	5.7	0.0060	69.2
$WKM_{8,2}$	0.532	3.2	0.0033	83.8	0.406	5.6	0.0058	68.8	0.291	9.1	0.0109	40.5
$WKM_{16,1}$	0.529	2.9	0.0031	84.4	0.402	5.2	0.0053	70.7	0.286	8.6	0.0099	43.9
Proposed methods												
$WKM_{U,5\%}$	0.511	1.1	0.0021	92.1	0.363	1.3	0.0023	90.6	0.216	1.6	0.0021	89.6
$WKM_{U,10\%}$	0.519	1.9	0.0023	91.8	0.373	2.3	0.0026	90.4	0.227	2.7	0.0024	89.2
$WKM_{\mathcal{N}(0.5)}$	0.520	2.0	0.0023	90.4	0.372	2.2	0.0026	90.1	0.220	2.0	0.0024	88.5

[1] Average of 1000 point estimates [2] Relative bias ($100 \times (Bias/S(t))$) [3] Mean square error
[4] Coverage rate of 1000 95 per cent confidence intervals [5] Censoring rate at the time of the true value of $S(t)$

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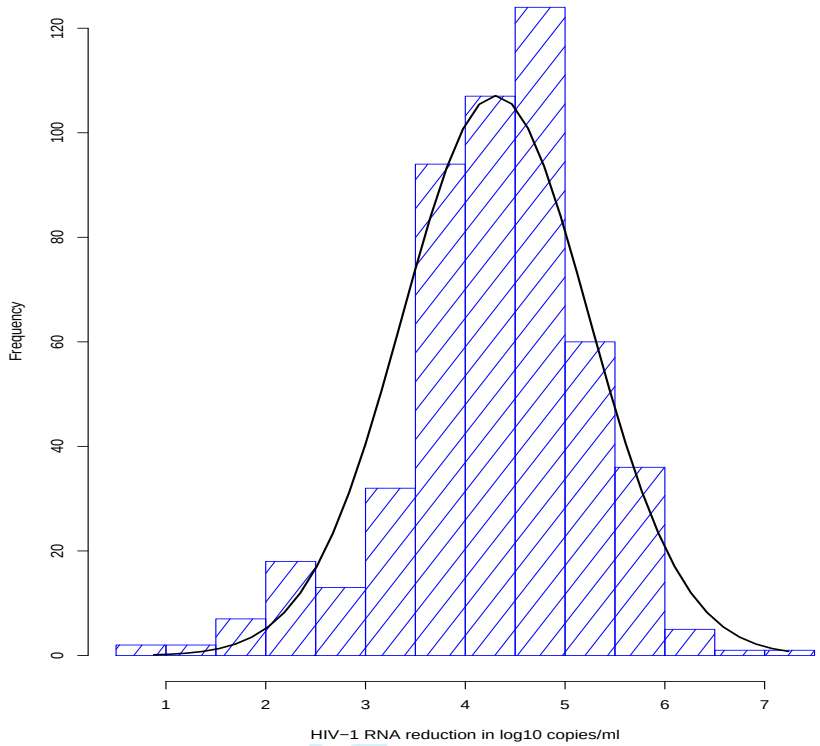

Figure 1. Distribution of the HIV-1 RNA reduction at week 8 in the 502 patients receiving a raltegravir-based regimen using the crude method

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Table 1. Data analysis. top: estimation of two working regression models with a normal distribution on the HIV-1 RNA reduction in the 502 observations; bottom: estimation of two working Cox PH models

Regression models with a normal distribution

	Failure time model			Censoring time model		
Covariates	Estimate	SE	p-value	Estimate	SE	p-value
Baseline viral load	0.324	0.091	<.01	0.980	0.010	<.01
Baseline CD4$\times 10^{-2}$	0.159	0.038	<.01	0.003	0.004	0.41
Age	0.013	0.007	0.05	0.001	0.001	0.17
NRTIs	0.109	0.066	0.10	-0.014	0.007	0.04
NNRTIs	0.219	0.137	0.11	0.001	0.014	0.95
Previous ARVs used	-0.057	0.017	<.01	-0.007	0.002	<.01

Cox proportional hazard models

	Failure time model			Censoring time model		
Covariates	Estimate	SE	p-value	Estimate	SE	p-value
Baseline viral load	-0.599	0.121	<.01	-7.686	0.361	<.01
Baseline CD4$\times 10^{-2}$	-0.197	0.048	<.01	-0.020	0.038	0.59
Age	-0.008	0.006	0.17	-0.011	0.007	0.12
NRTIs	-0.095	0.067	0.15	0.203	0.069	<.01
NNRTIs	-0.254	0.134	0.06	0.067	0.138	0.63
Previous ARVs used	0.056	0.017	<.01	0.078	0.019	<.01

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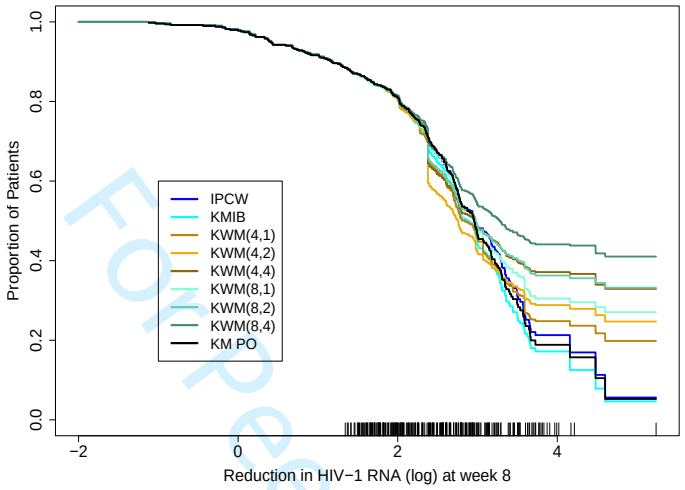
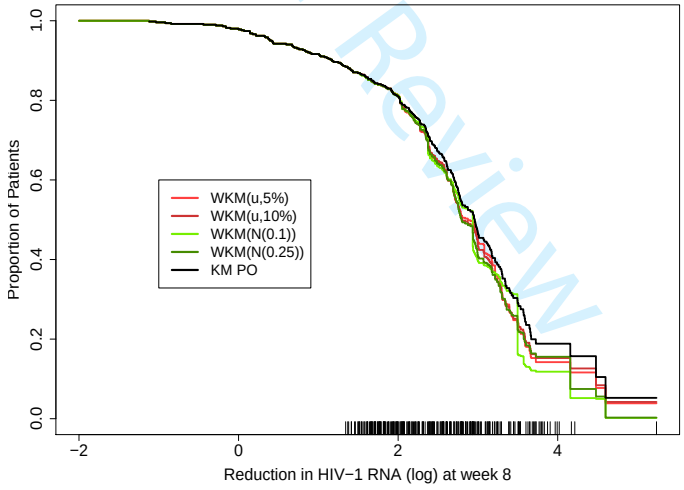


(a)



(b)

Figure 2. Estimates of the reduction in HIV-1 RNA (in $\log_{10}$ copies/ml) at week 8 in the 502 patients of whom 254 (51%) have a censored HIV-1 RNA reduction.

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Table 2. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **70% LOD=100 copies/ml and 30% LOD=200 copies/ml** leading to an overall censoring rate of **45%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=24%¶				True value: 0.35 Censoring rate=34%				True value: 0.2 Censoring rate=42%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.498	-0.2	0.0013	95.4	0.349	-0.1	0.0012	94.3	0.199	-0.1	0.0009	93.6
KM PO	0.520	2.0	0.0019	92.4	0.379	2.9	0.0027	89.1	0.236	3.6	0.0037	87.9
IPCW	0.515	1.5	0.0019	92.1	0.374	2.4	0.0026	89.8	0.231	3.1	0.0038	86.8
KMIB	0.506	0.6	0.0018	93.9	0.363	1.3	0.0023	93.6	0.220	2.0	0.0031	91.5
$WKM_{4,1}$	0.486	-1.4	0.0026	88.4	0.349	-0.1	0.0033	86.6	0.242	4.2	0.0059	75.8
$WKM_{4,2}$	0.517	1.7	0.0025	87.5	0.406	5.6	0.0058	69.7	0.329	12.9	0.0197	20.0
$WKM_{4,4}$	0.536	3.6	0.0031	82.2	0.439	8.9	0.0100	41.1	0.374	17.4	0.0324	2.3
$WKM_{8,1}$	0.491	-0.9	0.0024	89.2	0.363	1.3	0.0030	87.9	0.266	6.6	0.0078	62.3
$WKM_{8,2}$	0.529	2.9	0.0027	85.1	0.427	7.7	0.0081	50.1	0.360	16.0	0.0278	4.2
$WKM_{16,1}$	0.502	0.2	0.0020	90.5	0.383	3.3	0.0034	83.0	0.298	9.8	0.0123	32.2
$WKM_{D,2.5\%}$	0.481	-1.9	0.0026	84.5	0.327	-2.3	0.0037	71.3	0.184	-1.6	0.0041	55.5
$WKM_{D,5\%}$	0.483	-1.7	0.0024	86.8	0.330	-2.0	0.0032	75.1	0.187	-1.3	0.0034	60.9
$WKM_{D,10\%}$	0.486	-1.4	0.0021	88.4	0.334	-1.6	0.0027	78.9	0.192	-0.8	0.0028	65.7
$WKM_{D,15\%}$	0.488	-1.2	0.0020	89.7	0.337	-1.3	0.0025	80.9	0.196	-0.4	0.0027	68.5
$WKM_{D,20\%}$	0.490	-1.0	0.0019	90.8	0.340	-1.0	0.0023	82.7	0.199	-0.1	0.0026	71.0
$WKM_{D,30\%}$	0.493	-0.7	0.0018	92.2	0.345	-0.5	0.0021	84.4	0.203	0.3	0.0026	71.7
$WKM_{D,40\%}$	0.496	-0.4	0.0017	92.7	0.348	-0.2	0.0021	85.9	0.205	0.5	0.0026	71.9
$WKM_{D,50\%}$	0.498	-0.2	0.0017	93.2	0.350	0.0	0.0021	86.6	0.207	0.7	0.0026	71.5
$WKM_{R,2.5\%}$	0.481	-1.9	0.0026	85.8	0.327	-2.3	0.0037	75.0	0.184	-1.6	0.0040	66.3
$WKM_{R,5\%}$	0.483	-1.7	0.0024	87.2	0.330	-2.0	0.0032	78.1	0.187	-1.3	0.0033	73.5
$WKM_{R,10\%}$	0.485	-1.5	0.0021	88.6	0.334	-1.6	0.0028	81.5	0.192	-0.8	0.0028	79.5
$WKM_{R,15\%}$	0.487	-1.3	0.0020	89.9	0.337	-1.3	0.0025	84.8	0.196	-0.4	0.0026	82.9
$WKM_{R,20\%}$	0.489	-1.1	0.0019	90.2	0.339	-1.1	0.0024	86.4	0.198	-0.2	0.0026	84.6
$WKM_{R,30\%}$	0.492	-0.8	0.0018	91.1	0.343	-0.7	0.0022	87.6	0.202	0.2	0.0025	85.6
$WKM_{R,40\%}$	0.494	-0.6	0.0017	91.6	0.346	-0.4	0.0021	88.1	0.204	0.4	0.0025	86.1
$WKM_{R,50\%}$	0.496	-0.4	0.0017	92.4	0.348	-0.2	0.0021	88.9	0.205	0.5	0.0025	86.3
$WKM_{U,2.5\%}$	0.482	-1.8	0.0024	88.8	0.329	-2.1	0.0034	81.3	0.185	-1.5	0.0035	78.6
$WKM_{U,5\%}$	0.485	-1.5	0.0021	90.5	0.334	-1.6	0.0028	86.9	0.191	-0.9	0.0027	86.7
$WKM_{U,10\%}$	0.490	-1.0	0.0019	93.4	0.340	-1.0	0.0023	90.2	0.201	0.1	0.0023	91.5
$WKM_{U,15\%}$	0.493	-0.7	0.0017	94.1	0.346	-0.4	0.0020	92.6	0.208	0.8	0.0023	91.9
$WKM_{U,20\%}$	0.496	-0.4	0.0016	94.7	0.351	0.1	0.0019	93.5	0.213	1.3	0.0024	92.5
$WKM_{U,30\%}$	0.502	0.2	0.0016	94.6	0.359	0.9	0.0019	93.9	0.221	2.1	0.0027	91.0
$WKM_{U,40\%}$	0.507	0.7	0.0016	94.2	0.366	1.6	0.0020	93.7	0.226	2.6	0.0029	90.6
$WKM_{U,50\%}$	0.511	1.1	0.0016	93.8	0.370	2.0	0.0022	92.7	0.229	2.9	0.0032	89.7
$WKM_{\mathcal{N}(0.1)}$	0.478	-2.2	0.0033	82.5	0.323	-2.7	0.0051	70.8	0.175	-2.5	0.0072	51.7
$WKM_{\mathcal{N}(0.5)}$	0.488	-1.2	0.0019	91.1	0.337	-1.3	0.0024	89.	0.191	-0.9	0.0030	84.9
$WKM_{\mathcal{N}(1)}$	0.501	0.1	0.0016	94.8	0.354	0.4	0.0019	93.8	0.211	1.1	0.0025	91.6
$WKM_{\mathcal{N}(2)}$	0.512	1.2	0.0017	93.6	0.369	1.9	0.0022	92.9	0.226	2.6	0.0030	90.4
$WKM_{\mathcal{N}(5)}$	0.519	1.9	0.0018	92.5	0.377	2.7	0.0026	90.5	0.234	3.4	0.0036	88.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 3. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **30% LOD=100 copies/ml and 70% LOD=200 copies/ml** leading to an overall censoring rate of **50%**

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=30%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=47%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	94.1	0.349	-0.1	0.0013	94.1	0.200	0.0	0.0009	93.9
KM PO	0.525	2.5	0.0024	89.9	0.385	3.5	0.0034	89.5	0.240	4.0	0.0053	84.8
IPCW	0.515	1.5	0.0042	82.8	0.374	2.4	0.0054	79.8	0.232	3.2	0.0083	76.7
KMIB	0.509	0.9	0.0022	92.4	0.367	1.7	0.0029	90.4	0.225	2.5	0.0046	88.3
$WKM_{4,1}$	0.487	-1.3	0.0033	85.7	0.354	0.4	0.0043	82.5	0.256	5.6	0.0084	70.6
$WKM_{4,2}$	0.526	2.6	0.0036	82.7	0.424	7.4	0.0090	55.7	0.354	15.4	0.0275	14.0
$WKM_{4,4}$	0.550	5.0	0.0046	70.8	0.462	11.2	0.0151	24.4	0.404	20.4	0.0443	1.0
$WKM_{8,1}$	0.494	-0.6	0.0029	88.0	0.371	2.1	0.0039	82.6	0.284	8.4	0.0111	51.8
$WKM_{8,2}$	0.540	4.0	0.0039	76.2	0.446	9.6	0.0119	36.4	0.386	18.6	0.0373	2.8
$WKM_{16,1}$	0.507	0.7	0.0025	88.7	0.395	4.5	0.0049	73.8	0.320	12.0	0.0176	22.7
$WKM_{D,2.5\%}$	0.477	-2.3	0.0035	80.9	0.325	-2.5	0.0047	66.7	0.184	-1.6	0.0051	52.9
$WKM_{D,5\%}$	0.479	-2.1	0.0031	82.9	0.328	-2.2	0.0040	70.5	0.188	-1.2	0.0043	57.6
$WKM_{D,10\%}$	0.483	-1.7	0.0027	85.3	0.332	-1.8	0.0033	73.9	0.194	-0.6	0.0038	62.3
$WKM_{D,15\%}$	0.486	-1.4	0.0025	87.1	0.336	-1.4	0.0029	78.1	0.198	-0.2	0.0036	63.0
$WKM_{D,20\%}$	0.488	-1.2	0.0023	87.8	0.340	-1.0	0.0027	80.1	0.201	0.1	0.0036	63.8
$WKM_{D,30\%}$	0.492	-0.8	0.0022	89.4	0.345	-0.5	0.0025	82.0	0.205	0.5	0.0036	63.8
$WKM_{D,40\%}$	0.495	-0.5	0.0021	90.2	0.348	-0.2	0.0025	83.3	0.207	0.7	0.0036	64.2
$WKM_{D,50\%}$	0.498	-0.2	0.0020	90.8	0.351	0.1	0.0024	84.0	0.209	0.9	0.0037	64.8
$WKM_{R,2.5\%}$	0.477	-2.3	0.0034	82.5	0.325	-2.5	0.0046	70.9	0.184	-1.6	0.0050	63.8
$WKM_{R,5\%}$	0.479	-2.1	0.0031	84.9	0.328	-2.2	0.0039	76.1	0.188	-1.2	0.0042	69.4
$WKM_{R,10\%}$	0.483	-1.7	0.0027	86.8	0.332	-1.8	0.0033	80.3	0.193	-0.7	0.0037	76.2
$WKM_{R,15\%}$	0.485	-1.5	0.0025	88.0	0.336	-1.4	0.0030	83.5	0.197	-0.3	0.0036	78.6
$WKM_{R,20\%}$	0.487	-1.3	0.0024	88.6	0.339	-1.1	0.0028	85.6	0.200	0.0	0.0035	80.0
$WKM_{R,30\%}$	0.49	-1.0	0.0022	89.4	0.343	-0.7	0.0025	87.1	0.204	0.4	0.0035	82.0
$WKM_{R,40\%}$	0.493	-0.7	0.0021	90.7	0.347	-0.3	0.0024	87.9	0.206	0.6	0.0035	83.1
$WKM_{R,50\%}$	0.496	-0.4	0.0020	91.1	0.349	-0.1	0.0024	87.9	0.208	0.8	0.0036	83.7
$WKM_{U,2.5\%}$	0.479	-2.1	0.0031	86.7	0.327	-2.3	0.0041	79.9	0.186	-1.4	0.0045	75.6
$WKM_{U,5\%}$	0.482	-1.8	0.0027	89.4	0.332	-1.8	0.0033	86.3	0.192	-0.8	0.0037	84.1
$WKM_{U,10\%}$	0.488	-1.2	0.0023	91.7	0.340	-1.0	0.0026	90.7	0.203	0.3	0.0033	89.8
$WKM_{U,15\%}$	0.493	-0.7	0.0021	92.9	0.347	-0.3	0.0023	92.1	0.210	1.0	0.0033	90.7
$WKM_{U,20\%}$	0.496	-0.4	0.0020	93.2	0.353	0.3	0.0022	93.4	0.216	1.6	0.0035	90.9
$WKM_{U,30\%}$	0.503	0.3	0.0019	93.5	0.363	1.3	0.0022	92.4	0.224	2.4	0.0039	90.6
$WKM_{U,40\%}$	0.51	1.0	0.0019	93.2	0.370	2.0	0.0025	91.0	0.229	2.9	0.0043	88.8
$WKM_{U,50\%}$	0.515	1.5	0.0020	92.8	0.375	2.5	0.0027	90.5	0.233	3.3	0.0046	87.1
$WKM_{\mathcal{N}(0.1)}$	0.473	-2.7	0.0045	77.8	0.319	-3.1	0.0069	64.1	0.175	-2.5	0.0093	48.6
$WKM_{\mathcal{N}(0.5)}$	0.487	-1.3	0.0024	90.6	0.336	-1.4	0.0029	88.6	0.193	-0.7	0.0042	81.0
$WKM_{\mathcal{N}(1)}$	0.502	0.2	0.0019	93.3	0.357	0.7	0.0023	92.2	0.214	1.4	0.0037	89.2
$WKM_{\mathcal{N}(2)}$	0.516	1.6	0.0021	92.5	0.374	2.4	0.0027	90.4	0.230	3.0	0.0045	88.0
$WKM_{\mathcal{N}(5)}$	0.523	2.3	0.0023	90.6	0.383	3.3	0.0032	88.6	0.238	3.8	0.0051	85.5

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 4. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **70% LOD=200 copies/ml and 30% LOD=500 copies/ml** leading to an overall censoring rate of **57%**

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=39%¶				True value: 0.35 Censoring rate=49%				True value: 0.2 Censoring rate=56%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	93.8	0.348	-0.2	0.0012	94.1	0.199	-0.1	0.0009	93.3
KM PO	0.535	3.5	0.0032	85.3	0.394	4.4	0.0047	84.6	0.250	5.0	0.0083	83.3
IPCW	0.528	2.8	0.0031	86.6	0.387	3.7	0.0045	85.8	0.245	4.5	0.0085	83.3
KMIB	0.517	1.7	0.0027	91.7	0.374	2.4	0.0038	90.7	0.233	3.3	0.0071	85.5
$WKM_{4,1}$	0.486	-1.4	0.0047	80.3	0.359	0.9	0.0058	80.7	0.277	7.7	0.0129	62.7
$WKM_{4,2}$	0.552	5.2	0.0059	69.3	0.464	11.4	0.0167	33.1	0.409	20.9	0.0476	4.6
$WKM_{4,4}$	0.580	8.0	0.0087	49.2	0.507	15.7	0.0272	7.2	0.463	26.3	0.0720	0.2
$WKM_{8,1}$	0.496	-0.4	0.0039	84.4	0.381	3.1	0.0058	78.0	0.311	11.1	0.0177	39.5
$WKM_{8,2}$	0.569	6.9	0.0073	55.3	0.490	14.0	0.0224	13.8	0.446	24.6	0.0634	0.4
$WKM_{16,1}$	0.514	1.4	0.0033	84.6	0.411	6.1	0.0075	64.7	0.354	15.4	0.0275	11.1
$WKM_{D,2.5\%}$	0.472	-2.8	0.0049	72.6	0.316	-3.4	0.0066	57.2	0.186	-1.4	0.0066	50.0
$WKM_{D,5\%}$	0.474	-2.6	0.0042	77.4	0.320	-3.0	0.0054	61.8	0.190	-1.0	0.0057	56.0
$WKM_{D,10\%}$	0.479	-2.1	0.0036	81.1	0.327	-2.3	0.0043	66.5	0.196	-0.4	0.0051	58.7
$WKM_{D,15\%}$	0.482	-1.8	0.0032	84.2	0.332	-1.8	0.0038	70.2	0.201	0.1	0.0050	59.8
$WKM_{D,20\%}$	0.486	-1.4	0.0029	85.4	0.337	-1.3	0.0035	73.5	0.204	0.4	0.0050	60.9
$WKM_{D,30\%}$	0.491	-0.9	0.0026	87.7	0.343	-0.7	0.0033	75.3	0.209	0.9	0.0052	61.4
$WKM_{D,40\%}$	0.495	-0.5	0.0025	88.5	0.347	-0.3	0.0032	77.5	0.211	1.1	0.0053	62.0
$WKM_{D,50\%}$	0.499	-0.1	0.0024	88.5	0.350	0.0	0.0031	78.4	0.213	1.3	0.0054	62.3
$WKM_{R,2.5\%}$	0.472	-2.8	0.0048	75.5	0.318	-3.2	0.0065	62.8	0.187	-1.3	0.0065	64.1
$WKM_{R,5\%}$	0.475	-2.5	0.0041	79.4	0.322	-2.8	0.0053	69.8	0.191	-0.9	0.0056	72.2
$WKM_{R,10\%}$	0.479	-2.1	0.0036	83.2	0.328	-2.2	0.0043	74.8	0.197	-0.3	0.0051	78.5
$WKM_{R,15\%}$	0.482	-1.8	0.0032	85.1	0.332	-1.8	0.0038	79.3	0.201	0.1	0.0050	81.5
$WKM_{R,20\%}$	0.485	-1.5	0.0030	86.1	0.336	-1.4	0.0035	82.4	0.204	0.4	0.0050	82.9
$WKM_{R,30\%}$	0.489	-1.1	0.0027	88.0	0.342	-0.8	0.0032	84.7	0.209	0.9	0.0051	83.3
$WKM_{R,40\%}$	0.493	-0.7	0.0025	89.1	0.346	-0.4	0.0031	86.0	0.211	1.1	0.0052	84.0
$WKM_{R,50\%}$	0.496	-0.4	0.0024	88.9	0.349	-0.1	0.0031	86.5	0.213	1.3	0.0053	84.2
$WKM_{U,2.5\%}$	0.474	-2.6	0.0043	83.8	0.320	-3.0	0.0057	75.5	0.190	-1.0	0.0058	78.7
$WKM_{U,5\%}$	0.478	-2.2	0.0036	87.3	0.327	-2.3	0.0043	84.2	0.197	-0.3	0.0049	87.6
$WKM_{U,10\%}$	0.486	-1.4	0.0029	91.3	0.338	-1.2	0.0032	90.6	0.208	0.8	0.0047	91.3
$WKM_{U,15\%}$	0.492	-0.8	0.0025	92.1	0.347	-0.3	0.0028	93.1	0.216	1.6	0.0050	90.6
$WKM_{U,20\%}$	0.497	-0.3	0.0023	92.8	0.355	0.5	0.0027	93.3	0.222	2.2	0.0053	89.6
$WKM_{U,30\%}$	0.507	0.7	0.0021	91.9	0.367	1.7	0.0029	92.3	0.231	3.1	0.0060	88.5
$WKM_{U,40\%}$	0.515	1.5	0.0022	91.1	0.376	2.6	0.0032	90.2	0.237	3.7	0.0067	87.4
$WKM_{U,50\%}$	0.522	2.2	0.0025	90.5	0.382	3.2	0.0036	88.6	0.241	4.1	0.0071	86.5
$WKM_{\mathcal{N}(0.1)}$	0.468	-3.2	0.0064	72.2	0.310	-4.0	0.0103	56.2	0.172	-2.8	0.0133	45.7
$WKM_{\mathcal{N}(0.5)}$	0.485	-1.5	0.0029	90.0	0.334	-1.6	0.0038	86.5	0.194	-0.6	0.0060	83.6
$WKM_{\mathcal{N}(1)}$	0.507	0.7	0.0022	91.7	0.361	1.1	0.0029	92.2	0.220	2.0	0.0057	88.0
$WKM_{\mathcal{N}(2)}$	0.524	2.4	0.0026	89.3	0.382	3.2	0.0037	88.4	0.239	3.9	0.0070	86.9
$WKM_{\mathcal{N}(5)}$	0.533	3.3	0.0031	86.1	0.392	4.2	0.0045	85.5	0.248	4.8	0.0080	84.6

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 5. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **30% LOD=200 copies/ml and 70% LOD=500 copies/ml** leading to an overall censoring rate of **64%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=49%¶				True value: 0.35 Censoring rate=58%				True value: 0.2 Censoring rate=63%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	93.6	0.350	0.0	0.0011	93.9	0.200	0.0	0.0008	94.4
KM PO	0.545	4.5	0.0040	84.7	0.406	5.6	0.0065	82.0	0.263	6.3	0.0161	68.2
IPCW	0.524	2.4	0.0065	79.2	0.385	3.5	0.0097	76.8	0.248	4.8	0.0190	74.7
KMIB	0.522	2.2	0.0031	91.6	0.383	3.3	0.0052	88.9	0.246	4.6	0.0138	78.9
$WKM_{4,1}$	0.486	-1.4	0.0056	80.1	0.371	2.1	0.0081	77.3	0.302	10.2	0.0196	55.2
$WKM_{4,2}$	0.562	6.2	0.0078	65.6	0.484	13.4	0.0227	28.6	0.437	23.7	0.0616	4.5
$WKM_{4,4}$	0.599	9.9	0.0126	36.5	0.536	18.6	0.0376	3.8	0.501	30.1	0.0941	0.1
$WKM_{8,1}$	0.501	0.1	0.0046	82.3	0.398	4.8	0.0082	75.4	0.342	14.2	0.0269	29.9
$WKM_{8,2}$	0.584	8.4	0.0099	46.2	0.514	16.4	0.0304	8.4	0.479	27.9	0.0816	0.1
$WKM_{16,1}$	0.524	2.4	0.0040	82.6	0.436	8.6	0.0115	55.0	0.393	19.3	0.0418	4.5
$WKM_{D,2.5\%}$	0.467	-3.3	0.0058	66.9	0.318	-3.2	0.0079	52.6	0.194	-0.6	0.0106	47.0
$WKM_{D,5\%}$	0.470	-3.0	0.0049	70.5	0.322	-2.8	0.0065	57.5	0.199	-0.1	0.0099	49.3
$WKM_{D,10\%}$	0.476	-2.4	0.0040	77.8	0.330	-2.0	0.0052	63.5	0.206	0.6	0.0097	50.0
$WKM_{D,15\%}$	0.481	-1.9	0.0035	81.9	0.336	-1.4	0.0046	67.0	0.211	1.1	0.0098	50.6
$WKM_{D,20\%}$	0.485	-1.5	0.0031	83.9	0.341	-0.9	0.0043	70.1	0.214	1.4	0.0100	51.4
$WKM_{D,30\%}$	0.491	-0.9	0.0028	86.7	0.348	-0.2	0.0041	73.1	0.219	1.9	0.0103	51.0
$WKM_{D,40\%}$	0.496	-0.4	0.0026	88.7	0.353	0.3	0.0040	74.5	0.222	2.2	0.0106	50.7
$WKM_{D,50\%}$	0.500	0.0	0.0025	89.5	0.356	0.6	0.0040	74.5	0.225	2.5	0.0109	50.0
$WKM_{R,2.5\%}$	0.467	-3.3	0.0057	71.3	0.319	-3.1	0.0077	61.9	0.195	-0.5	0.0106	60.0
$WKM_{R,5\%}$	0.470	-3.0	0.0048	76.3	0.323	-2.7	0.0064	68.6	0.199	-0.1	0.0100	67.5
$WKM_{R,10\%}$	0.476	-2.4	0.0040	82.0	0.330	-2.0	0.0052	75.7	0.206	0.6	0.0097	71.8
$WKM_{R,15\%}$	0.480	-2.0	0.0035	85.0	0.336	-1.4	0.0047	79.2	0.210	1.0	0.0098	73.4
$WKM_{R,20\%}$	0.483	-1.7	0.0032	86.8	0.340	-1.0	0.0044	81.6	0.214	1.4	0.0099	73.9
$WKM_{R,30\%}$	0.489	-1.1	0.0028	88.9	0.347	-0.3	0.0041	83.6	0.218	1.8	0.0102	74.0
$WKM_{R,40\%}$	0.494	-0.6	0.0026	89.7	0.351	0.1	0.0040	84.6	0.221	2.1	0.0105	74.2
$WKM_{R,50\%}$	0.497	-0.3	0.0025	90.7	0.354	0.4	0.0040	85.2	0.224	2.4	0.0107	74.6
$WKM_{U,2.5\%}$	0.469	-3.1	0.0051	80.1	0.321	-2.9	0.0067	75.6	0.198	-0.2	0.0100	75.6
$WKM_{U,5\%}$	0.475	-2.5	0.0040	87.8	0.329	-2.1	0.0053	85.6	0.206	0.6	0.0096	82.1
$WKM_{U,10\%}$	0.484	-1.6	0.0031	91.7	0.342	-0.8	0.0040	90.6	0.217	1.7	0.0098	82.9
$WKM_{U,15\%}$	0.492	-0.8	0.0026	92.9	0.353	0.3	0.0036	92.5	0.226	2.6	0.0104	82.2
$WKM_{U,20\%}$	0.499	-0.1	0.0023	93.9	0.362	1.2	0.0036	92.4	0.233	3.3	0.0111	80.9
$WKM_{U,30\%}$	0.511	1.1	0.0022	94.9	0.376	2.6	0.0040	91.7	0.242	4.2	0.0123	79.1
$WKM_{U,40\%}$	0.521	2.1	0.0025	93.5	0.385	3.5	0.0045	89.5	0.249	4.9	0.0134	77.2
$WKM_{U,50\%}$	0.529	2.9	0.0028	91.4	0.392	4.2	0.0051	87.7	0.253	5.3	0.0142	76.6
$WKM_{\mathcal{N}(0.1)}$	0.460	-4.0	0.0084	67.8	0.307	-4.3	0.0139	52.9	0.179	-2.1	0.0180	44.3
$WKM_{\mathcal{N}(0.5)}$	0.483	-1.7	0.0032	90.4	0.335	-1.5	0.0050	86.6	0.203	0.3	0.0111	79.6
$WKM_{\mathcal{N}(1)}$	0.511	1.1	0.0023	94.2	0.368	1.8	0.0040	91.8	0.232	3.2	0.0118	79.9
$WKM_{\mathcal{N}(2)}$	0.532	3.2	0.0030	89.9	0.392	4.2	0.0052	87.3	0.252	5.2	0.0141	76.6
$WKM_{\mathcal{N}(5)}$	0.542	4.2	0.0037	85.0	0.403	5.3	0.0062	82.5	0.261	6.1	0.0157	75.1

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 6. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200). 70% LOD=100 copies/ml and 30% LOD=200 copies/ml** leading to an overall censoring rate of **45%**

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=24%¶				True value: 0.35 Censoring rate=34%				True value: 0.2 Censoring rate=42%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.498	-0.2	0.0013	95.4	0.349	-0.1	0.0012	94.3	0.199	-0.1	0.0009	93.6
KM PO	0.520	2.0	0.0019	92.4	0.379	2.9	0.0027	89.1	0.236	3.6	0.0037	87.9
IPCW	0.515	1.5	0.0019	91.5	0.373	2.3	0.0026	90.1	0.230	3.0	0.0037	87.8
KMIB	0.512	1.2	0.0019	92.8	0.371	2.1	0.0025	92.3	0.229	2.9	0.0037	88.9
$WKM_{2,1}$	0.511	1.1	0.0020	93.2	0.386	3.6	0.0052	78.8	0.306	10.6	0.0153	40.8
$WKM_{4,1}$	0.527	2.7	0.0029	85.4	0.425	7.5	0.0083	57.3	0.356	15.6	0.0274	12.6
$WKM_{6,1}$	0.535	3.5	0.0034	81.2	0.440	9.0	0.0105	43.7	0.375	17.5	0.0334	5.5
$WKM_{8,1}$	0.543	4.3	0.0037	78.0	0.450	10.0	0.0122	33.7	0.390	19.0	0.0383	1.4
$WKM_{10,1}$	0.546	4.6	0.0040	73.8	0.455	10.5	0.0131	27.0	0.396	19.6	0.0406	0.6
$WKM_{12,1}$	0.550	5.0	0.0043	71.8	0.460	11.0	0.0140	22.0	0.403	20.3	0.0431	0.3
$WKM_{D,2.5\%}$	0.503	0.3	0.0028	82.4	0.358	0.8	0.0045	62.5	0.220	2.0	0.0060	42.3
$WKM_{D,5\%}$	0.505	0.5	0.0023	86.4	0.361	1.1	0.0035	69.2	0.221	2.1	0.0044	51.1
$WKM_{D,10\%}$	0.506	0.6	0.0020	90.0	0.363	1.3	0.0028	76.3	0.222	2.2	0.0036	57.0
$WKM_{D,15\%}$	0.507	0.7	0.0019	91.5	0.364	1.4	0.0026	79.8	0.224	2.4	0.0034	60.5
$WKM_{D,20\%}$	0.508	0.8	0.0018	91.9	0.366	1.6	0.0025	81.3	0.225	2.5	0.0034	61.4
$WKM_{D,30\%}$	0.510	1.0	0.0017	92.6	0.368	1.8	0.0024	82.8	0.226	2.6	0.0034	62.8
$WKM_{D,40\%}$	0.511	1.1	0.0017	93.4	0.369	1.9	0.0024	84.0	0.227	2.7	0.0034	62.9
$WKM_{D,50\%}$	0.512	1.2	0.0017	93.8	0.370	2.0	0.0024	84.4	0.228	2.8	0.0034	62.6
$WKM_{R,2.5\%}$	0.503	0.3	0.0029	84.9	0.358	0.8	0.0048	64.7	0.220	2.0	0.0062	49.6
$WKM_{R,5\%}$	0.505	0.5	0.0023	87.8	0.360	1.0	0.0035	74.4	0.220	2.0	0.0045	61.9
$WKM_{R,10\%}$	0.506	0.6	0.0020	89.3	0.362	1.2	0.0029	80.4	0.222	2.2	0.0037	73.2
$WKM_{R,15\%}$	0.507	0.7	0.0019	90.4	0.364	1.4	0.0026	83.6	0.223	2.3	0.0034	76.7
$WKM_{R,20\%}$	0.508	0.8	0.0018	90.4	0.365	1.5	0.0025	84.7	0.225	2.5	0.0034	78.2
$WKM_{R,30\%}$	0.509	0.9	0.0018	91.0	0.367	1.7	0.0024	87.0	0.226	2.6	0.0034	80.4
$WKM_{R,40\%}$	0.510	1.0	0.0017	91.2	0.368	1.8	0.0024	88.0	0.227	2.7	0.0034	81.1
$WKM_{R,50\%}$	0.511	1.1	0.0017	91.3	0.369	1.9	0.0024	88.0	0.228	2.8	0.0034	81.3
$WKM_{U,2.5\%}$	0.504	0.4	0.0026	88.8	0.359	0.9	0.0041	79.1	0.220	2.0	0.0053	68.2
$WKM_{U,5\%}$	0.506	0.6	0.0021	90.9	0.362	1.2	0.0031	87.1	0.221	2.1	0.0039	83.5
$WKM_{U,10\%}$	0.507	0.7	0.0019	92.1	0.364	1.4	0.0026	91.4	0.224	2.4	0.0032	89.7
$WKM_{U,15\%}$	0.509	0.9	0.0018	93.5	0.366	1.6	0.0024	92.5	0.226	2.6	0.0032	90.4
$WKM_{U,20\%}$	0.510	1.0	0.0017	93.6	0.368	1.8	0.0023	92.3	0.228	2.8	0.0032	89.7
$WKM_{U,30\%}$	0.512	1.2	0.0017	93.8	0.371	2.1	0.0023	92.6	0.230	3.0	0.0033	89.2
$WKM_{U,40\%}$	0.514	1.4	0.0017	93.3	0.374	2.4	0.0024	91.6	0.232	3.2	0.0034	88.5
$WKM_{U,50\%}$	0.516	1.6	0.0018	93.3	0.376	2.6	0.0025	90.8	0.233	3.3	0.0035	88.0
$WKM_{\mathcal{N}(0.1)}$	0.501	0.1	0.0042	77.7	0.355	0.5	0.0075	54.5	0.216	1.6	0.0130	30.9
$WKM_{\mathcal{N}(0.5)}$	0.507	0.7	0.0019	91.1	0.363	1.3	0.0026	90.4	0.221	2.1	0.0039	82.2
$WKM_{\mathcal{N}(1)}$	0.511	1.1	0.0017	93.3	0.369	1.9	0.0023	92.3	0.227	2.7	0.0033	88.5
$WKM_{\mathcal{N}(2)}$	0.516	1.6	0.0018	92.8	0.374	2.4	0.0024	91.3	0.232	3.2	0.0035	88.5
$WKM_{\mathcal{N}(5)}$	0.519	1.9	0.0019	92.7	0.378	2.8	0.0026	89.7	0.235	3.5	0.0036	87.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 7. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction N(200)**. **30% LOD=100 copies/ml and 70% LOD=200 copies/ml** leading to an overall censoring rate of **50%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=30%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=47%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	94.1	0.349	-0.1	0.0013	94.1	0.200	0.0	0.0009	93.9
KM PO	0.525	2.5	0.0024	89.9	0.385	3.5	0.0034	89.5	0.240	4.0	0.0053	84.8
IPCW	0.514	1.4	0.0039	83.6	0.374	2.4	0.0052	79.3	0.233	3.3	0.0079	77.8
KMIB	0.516	1.6	0.0026	89.9	0.375	2.5	0.0037	87.7	0.232	3.2	0.0060	83.2
$WKM_{2,1}$	0.513	1.3	0.0028	91.1	0.394	4.4	0.0082	65.5	0.314	11.4	0.0199	40.1
$WKM_{4,1}$	0.533	3.3	0.0043	77.6	0.434	8.4	0.0116	51.3	0.369	16.9	0.0339	17.2
$WKM_{6,1}$	0.543	4.3	0.0049	72.6	0.450	10.0	0.0136	42.7	0.390	19.0	0.0402	7.5
$WKM_{8,1}$	0.550	5.0	0.0051	69.8	0.460	11.0	0.0153	32.7	0.404	20.4	0.0450	3.4
$WKM_{10,1}$	0.555	5.5	0.0056	65.7	0.467	11.7	0.0167	27.6	0.412	21.2	0.0485	1.8
$WKM_{12,1}$	0.559	5.9	0.0058	64.1	0.472	12.2	0.0178	21.4	0.419	21.9	0.0512	1.1
$WKM_{D,2.5\%}$	0.509	0.9	0.0042	74.7	0.366	1.6	0.0070	52.9	0.224	2.4	0.0089	35.7
$WKM_{D,5\%}$	0.509	0.9	0.0035	78.9	0.367	1.7	0.0054	59.1	0.224	2.4	0.0070	40.5
$WKM_{D,10\%}$	0.510	1.0	0.0029	83.7	0.368	1.8	0.0042	66.4	0.226	2.6	0.0058	44.7
$WKM_{D,15\%}$	0.511	1.1	0.0026	86.0	0.369	1.9	0.0037	70.7	0.227	2.7	0.0054	48.7
$WKM_{D,20\%}$	0.512	1.2	0.0024	88.2	0.370	2.0	0.0035	73.2	0.228	2.8	0.0053	50.5
$WKM_{D,30\%}$	0.513	1.3	0.0023	90.2	0.372	2.2	0.0033	75.1	0.230	3.0	0.0052	51.8
$WKM_{D,40\%}$	0.515	1.5	0.0022	90.8	0.374	2.4	0.0032	76.7	0.231	3.1	0.0052	52.7
$WKM_{D,50\%}$	0.516	1.6	0.0022	90.8	0.375	2.5	0.0032	76.7	0.232	3.2	0.0052	52.9
$WKM_{R,2.5\%}$	0.509	0.9	0.0043	77.0	0.367	1.7	0.0072	59.7	0.225	2.5	0.0092	47.3
$WKM_{R,5\%}$	0.509	0.9	0.0035	81.7	0.366	1.6	0.0055	67.6	0.225	2.5	0.0072	56.6
$WKM_{R,10\%}$	0.510	1.0	0.0029	83.8	0.367	1.7	0.0043	73.4	0.227	2.7	0.0059	65.7
$WKM_{R,15\%}$	0.510	1.0	0.0026	85.3	0.369	1.9	0.0038	77.8	0.228	2.8	0.0055	70.0
$WKM_{R,20\%}$	0.511	1.1	0.0025	86.9	0.370	2.0	0.0035	78.8	0.229	2.9	0.0053	72.3
$WKM_{R,30\%}$	0.513	1.3	0.0023	87.9	0.372	2.2	0.0033	81.8	0.230	3.0	0.0052	74.3
$WKM_{R,40\%}$	0.514	1.4	0.0022	88.9	0.373	2.3	0.0032	83.2	0.231	3.1	0.0052	75.0
$WKM_{R,50\%}$	0.515	1.5	0.0022	89.3	0.374	2.4	0.0032	83.1	0.232	3.2	0.0052	75.3
$WKM_{U,2.5\%}$	0.509	0.9	0.0038	85.3	0.366	1.6	0.0061	75.2	0.225	2.5	0.0078	66.6
$WKM_{U,5\%}$	0.509	0.9	0.0029	87.8	0.367	1.7	0.0044	83.8	0.226	2.6	0.0059	79.9
$WKM_{U,10\%}$	0.510	1.0	0.0024	91.4	0.369	1.9	0.0034	88.4	0.229	2.9	0.0049	87.2
$WKM_{U,15\%}$	0.512	1.2	0.0022	92.6	0.371	2.1	0.0030	89.6	0.231	3.1	0.0047	87.9
$WKM_{U,20\%}$	0.513	1.3	0.0021	92.7	0.373	2.3	0.0029	89.6	0.232	3.2	0.0047	87.3
$WKM_{U,30\%}$	0.516	1.6	0.0021	92.5	0.377	2.7	0.0029	90	0.235	3.5	0.0048	86.5
$WKM_{U,40\%}$	0.519	1.9	0.0021	92.1	0.379	2.9	0.0030	89	0.236	3.6	0.0049	85.6
$WKM_{U,50\%}$	0.521	2.1	0.0022	91.4	0.381	3.1	0.0031	88.9	0.238	3.8	0.005	85.5
$WKM_{\mathcal{N}(0.1)}$	0.508	0.8	0.0068	67.6	0.363	1.3	0.0135	40.6	0.223	2.3	0.0208	27.2
$WKM_{\mathcal{N}(0.5)}$	0.510	1.0	0.0024	89.4	0.367	1.7	0.0035	85.9	0.227	2.7	0.0058	78.0
$WKM_{\mathcal{N}(1)}$	0.515	1.5	0.0021	92.5	0.374	2.4	0.0030	89.2	0.232	3.2	0.0049	86.5
$WKM_{\mathcal{N}(2)}$	0.520	2.0	0.0022	91.8	0.380	3.0	0.0031	89.0	0.237	3.7	0.0050	86.3
$WKM_{\mathcal{N}(5)}$	0.524	2.4	0.0023	90.2	0.384	3.4	0.0033	88.5	0.240	4.0	0.0052	85.1

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 8. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **70% LOD=200 copies/ml and 30% LOD=500 copies/ml** leading to an overall censoring rate of **57%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=39%¶				True value: 0.35 Censoring rate=49%				True value: 0.2 Censoring rate=56%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	93.8	0.348	-0.2	0.0012	94.1	0.199	-0.1	0.0009	93.3
KM PO	0.535	3.5	0.0032	85.3	0.394	4.4	0.0047	84.6	0.250	5.0	0.0083	83.3
IPCW	0.527	2.7	0.0031	86.5	0.386	3.6	0.0045	85.7	0.244	4.4	0.0084	83.3
KMIB	0.526	2.6	0.0030	88.8	0.384	3.4	0.0044	87.6	0.243	4.3	0.0082	84.7
$WKM_{2,1}$	0.526	2.6	0.0053	83.2	0.440	9.0	0.0147	47.3	0.353	15.3	0.0327	27.2
$WKM_{4,1}$	0.558	5.8	0.0077	59.5	0.475	12.5	0.0210	30.9	0.424	22.4	0.0567	8.7
$WKM_{6,1}$	0.576	7.6	0.0092	50.5	0.501	15.1	0.0269	17.7	0.457	25.7	0.0710	2.0
$WKM_{8,1}$	0.584	8.4	0.0101	45.0	0.512	16.2	0.0298	11.9	0.472	27.2	0.0781	0.5
$WKM_{10,1}$	0.592	9.2	0.0111	37.3	0.522	17.2	0.0327	6.3	0.485	28.5	0.0846	0.1
$WKM_{12,1}$	0.596	9.6	0.0116	33.2	0.529	17.9	0.0345	3.6	0.493	29.3	0.0887	0.0
$WKM_{D,2.5\%}$	0.512	1.2	0.0056	63.8	0.370	2.0	0.0083	47.5	0.230	3.0	0.0104	37.3
$WKM_{D,5\%}$	0.513	1.3	0.0042	72.5	0.372	2.2	0.0059	56.0	0.232	3.2	0.0085	43.5
$WKM_{D,10\%}$	0.515	1.5	0.0034	79.8	0.374	2.4	0.0047	63.3	0.234	3.4	0.0076	45.3
$WKM_{D,15\%}$	0.517	1.7	0.0031	83.3	0.376	2.6	0.0043	66.2	0.236	3.6	0.0074	46.2
$WKM_{D,20\%}$	0.518	1.8	0.0030	84.5	0.378	2.8	0.0041	69.3	0.237	3.7	0.0074	47.5
$WKM_{D,30\%}$	0.520	2.0	0.0028	86.2	0.380	3.0	0.0040	70.2	0.238	3.8	0.0074	48.1
$WKM_{D,40\%}$	0.522	2.2	0.0028	86.6	0.382	3.2	0.0041	70.9	0.240	4.0	0.0075	49.1
$WKM_{D,50\%}$	0.524	2.4	0.0028	86.7	0.383	3.3	0.0041	70.8	0.240	4.0	0.0076	49.4
$WKM_{R,2.5\%}$	0.512	1.2	0.0058	67.3	0.369	1.9	0.0087	50.1	0.23	3.0	0.0107	45.9
$WKM_{R,5\%}$	0.513	1.3	0.0043	77.0	0.371	2.1	0.0061	64.0	0.232	3.2	0.0085	61.2
$WKM_{R,10\%}$	0.515	1.5	0.0034	82.7	0.373	2.3	0.0048	74.2	0.234	3.4	0.0076	69.7
$WKM_{R,15\%}$	0.516	1.6	0.0032	83.3	0.375	2.5	0.0043	78.1	0.236	3.6	0.0074	72.7
$WKM_{R,20\%}$	0.517	1.7	0.0030	84.1	0.377	2.7	0.0041	78.6	0.237	3.7	0.0073	74.9
$WKM_{R,30\%}$	0.520	2.0	0.0028	85.2	0.379	2.9	0.0040	80.3	0.238	3.8	0.0074	77.0
$WKM_{R,40\%}$	0.521	2.1	0.0028	85.1	0.381	3.1	0.0040	81.2	0.239	3.9	0.0074	77.4
$WKM_{R,50\%}$	0.523	2.3	0.0028	85.7	0.382	3.2	0.0041	81.2	0.240	4.0	0.0075	77.8
$WKM_{U,2.5\%}$	0.512	1.2	0.0051	79.8	0.370	2.0	0.0073	69.8	0.231	3.1	0.0096	67.6
$WKM_{U,5\%}$	0.514	1.4	0.0038	85.9	0.372	2.2	0.0052	83.8	0.233	3.3	0.0078	82.6
$WKM_{U,10\%}$	0.517	1.7	0.0031	89.0	0.375	2.5	0.0042	88.3	0.236	3.6	0.0072	86.4
$WKM_{U,15\%}$	0.519	1.9	0.0029	89.7	0.378	2.8	0.0039	88.3	0.239	3.9	0.0072	86.4
$WKM_{U,20\%}$	0.521	2.1	0.0027	89.8	0.381	3.1	0.0038	88.8	0.241	4.1	0.0073	86.4
$WKM_{U,30\%}$	0.524	2.4	0.0027	89.1	0.385	3.5	0.0039	87.6	0.244	4.4	0.0075	86.0
$WKM_{U,40\%}$	0.528	2.8	0.0028	88.0	0.388	3.8	0.0041	85.6	0.246	4.6	0.0077	85.2
$WKM_{U,50\%}$	0.530	3.0	0.0029	87.3	0.390	4.0	0.0043	85.8	0.247	4.7	0.0079	84.9
$WKM_{\mathcal{N}(0.1)}$	0.510	1.0	0.0078	62.7	0.365	1.5	0.0143	41.3	0.230	3.0	0.0224	26.7
$WKM_{\mathcal{N}(0.5)}$	0.517	1.7	0.0032	88.1	0.375	2.5	0.0047	84.8	0.236	3.6	0.0083	79.8
$WKM_{\mathcal{N}(1)}$	0.523	2.3	0.0028	89.2	0.382	3.2	0.0040	87.9	0.241	4.1	0.0075	85.2
$WKM_{\mathcal{N}(2)}$	0.530	3.0	0.0029	87.0	0.389	3.9	0.0043	85.8	0.247	4.7	0.0079	84.8
$WKM_{\mathcal{N}(5)}$	0.534	3.4	0.0032	85.4	0.393	4.3	0.0046	84.9	0.249	4.9	0.0082	84.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 9. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **30% LOD=200 copies/ml and 70% LOD=500 copies/ml** leading to an overall censoring rate of **64%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=49%¶				True value: 0.35 Censoring rate=58%				True value: 0.2 Censoring rate=63%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	93.6	0.350	0.0	0.0011	93.9	0.200	0.0	0.0008	94.4
KM PO	0.545	4.5	0.0040	84.7	0.406	5.6	0.0065	82.0	0.263	6.3	0.0161	68.2
IPCW	0.524	2.4	0.0061	79.1	0.385	3.5	0.0091	76.9	0.249	4.9	0.0188	74.6
KMIB	0.529	2.9	0.0041	87.9	0.388	3.8	0.0065	85.9	0.252	5.2	0.0157	78.3
$WKM_{2,1}$	0.533	3.3	0.0082	73.4	0.447	9.7	0.0193	45.0	0.360	16.0	0.0400	34.5
$WKM_{4,1}$	0.563	6.3	0.0101	60.6	0.482	13.2	0.0254	39.3	0.431	23.1	0.0636	14.5
$WKM_{6,1}$	0.582	8.2	0.0114	51.9	0.510	16.0	0.0316	22.4	0.468	26.8	0.0791	3.8
$WKM_{8,1}$	0.589	8.9	0.0122	48.9	0.520	17.0	0.0340	16.7	0.483	28.3	0.0860	1.2
$WKM_{10,1}$	0.597	9.7	0.0131	42.5	0.530	18.0	0.0372	11.6	0.496	29.6	0.0930	0.2
$WKM_{12,1}$	0.603	10.3	0.0137	36.8	0.538	18.8	0.0392	7.8	0.506	30.6	0.0978	0.0
$WKM_{D,2.5\%}$	0.516	1.6	0.0073	59.2	0.375	2.5	0.0110	42.6	0.240	4.0	0.0167	34.9
$WKM_{D,5\%}$	0.518	1.8	0.0057	66.4	0.377	2.7	0.0084	50.8	0.241	4.1	0.0149	37.9
$WKM_{D,10\%}$	0.520	2.0	0.0045	73.6	0.380	3.0	0.0068	55.2	0.244	4.4	0.0141	37.5
$WKM_{D,15\%}$	0.522	2.2	0.0041	76.8	0.383	3.3	0.0062	57.9	0.246	4.6	0.0140	38.0
$WKM_{D,20\%}$	0.523	2.3	0.0037	78.8	0.385	3.5	0.0059	59.6	0.247	4.7	0.0141	37.8
$WKM_{D,30\%}$	0.526	2.6	0.0035	82.2	0.387	3.7	0.0057	62.2	0.249	4.9	0.0142	38.5
$WKM_{D,40\%}$	0.528	2.8	0.0034	83.5	0.389	3.9	0.0057	63.5	0.251	5.1	0.0144	38.7
$WKM_{D,50\%}$	0.530	3.0	0.0034	84.3	0.391	4.1	0.0058	63.5	0.252	5.2	0.0145	38.7
$WKM_{R,2.5\%}$	0.516	1.6	0.0075	64.9	0.376	2.6	0.0114	53.2	0.240	4.0	0.0170	47.9
$WKM_{R,5\%}$	0.518	1.8	0.0057	72.7	0.378	2.8	0.0085	64.0	0.242	4.2	0.0151	58.0
$WKM_{R,10\%}$	0.520	2.0	0.0046	77.7	0.381	3.1	0.0070	70.6	0.244	4.4	0.0143	61.7
$WKM_{R,15\%}$	0.521	2.1	0.0042	79.4	0.383	3.3	0.0063	73.6	0.246	4.6	0.0141	64.2
$WKM_{R,20\%}$	0.523	2.3	0.0038	80.8	0.384	3.4	0.0060	75.1	0.247	4.7	0.0141	65.7
$WKM_{R,30\%}$	0.525	2.5	0.0035	83.8	0.387	3.7	0.0058	76.8	0.249	4.9	0.0142	66.2
$WKM_{R,40\%}$	0.527	2.7	0.0034	84.2	0.389	3.9	0.0058	77.8	0.250	5.0	0.0144	66.6
$WKM_{R,50\%}$	0.529	2.9	0.0034	84.7	0.390	4.0	0.0058	77.8	0.251	5.1	0.0145	66.5
$WKM_{U,2.5\%}$	0.518	1.8	0.0066	78.7	0.376	2.6	0.0097	73.1	0.241	4.1	0.0157	65.9
$WKM_{U,5\%}$	0.520	2.0	0.0049	84.7	0.380	3.0	0.0072	82.3	0.243	4.3	0.0143	76.3
$WKM_{U,10\%}$	0.522	2.2	0.0038	89.1	0.384	3.4	0.0058	86.5	0.247	4.7	0.0138	78.4
$WKM_{U,15\%}$	0.525	2.5	0.0034	89.7	0.388	3.8	0.0054	87.3	0.250	5.0	0.0140	77.5
$WKM_{U,20\%}$	0.527	2.7	0.0032	90.5	0.391	4.1	0.0053	86.9	0.253	5.3	0.0142	76.1
$WKM_{U,30\%}$	0.532	3.2	0.0031	89.7	0.395	4.5	0.0055	85.9	0.256	5.6	0.0147	75.7
$WKM_{U,40\%}$	0.536	3.6	0.0033	88.4	0.399	4.9	0.0058	84.5	0.258	5.8	0.0151	75.5
$WKM_{U,50\%}$	0.539	3.9	0.0035	87.2	0.401	5.1	0.0060	83.4	0.260	6.0	0.0154	75.2
$WKM_{\mathcal{N}(0.1)}$	0.509	0.9	0.0139	51.9	0.370	2.0	0.0247	34.2	0.241	4.1	0.0363	25.6
$WKM_{\mathcal{N}(0.5)}$	0.521	2.1	0.0039	89.1	0.381	3.1	0.0064	84.0	0.246	4.6	0.0149	72.3
$WKM_{\mathcal{N}(1)}$	0.531	3.1	0.0032	89.8	0.393	4.3	0.0055	86.1	0.254	5.4	0.0146	75.1
$WKM_{\mathcal{N}(2)}$	0.539	3.9	0.0035	86.8	0.401	5.1	0.0060	84.0	0.259	5.9	0.0154	74.5
$WKM_{\mathcal{N}(5)}$	0.544	4.4	0.0039	84.4	0.405	5.5	0.0064	81.6	0.262	6.2	0.0159	74.6

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 10. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=200)**.

Overall censoring rate=35% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=26%¶				True value: 0.35 Censoring rate=29%				True value: 0.2 Censoring rate=32%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0012	94.5	0.350	0.0	0.0012	93.9	0.198	-0.2	0.0008	93.4
KM PO	0.565	6.5	0.0059	60.6	0.425	7.5	0.0074	55.3	0.268	6.8	0.0062	60.0
IPCW	0.500	0.0	0.0018	93.0	0.352	0.2	0.0018	94.4	0.201	0.1	0.0013	94.2
KMIB	0.510	1.0	0.0018	94.0	0.364	1.4	0.0018	93.4	0.212	1.2	0.0013	94.3
$WKM_{4,1}$	0.505	0.5	0.0018	94.3	0.357	0.7	0.0017	93.4	0.211	1.1	0.0014	92.9
$WKM_{4,2}$	0.509	0.9	0.0019	93.2	0.365	1.5	0.0020	91.7	0.221	2.1	0.0019	88.6
$WKM_{4,4}$	0.516	1.6	0.0020	92.2	0.376	2.6	0.0025	87.9	0.237	3.7	0.0029	80.2
$WKM_{8,1}$	0.504	0.4	0.0018	94.0	0.360	1.0	0.0019	91.9	0.217	1.7	0.0017	90.6
$WKM_{8,2}$	0.513	1.3	0.0019	92.2	0.374	2.4	0.0024	87.9	0.236	3.6	0.0028	80.6
$WKM_{16,1}$	0.510	1.0	0.0019	92.4	0.371	2.1	0.0023	89.1	0.233	3.3	0.0026	82.7
$WKM_{D,2.5\%}$	0.499	-0.1	0.0018	92.0	0.350	0.0	0.0017	90.2	0.200	0.0	0.0011	88.1
$WKM_{D,5\%}$	0.501	0.1	0.0017	92.6	0.352	0.2	0.0016	90.1	0.201	0.1	0.0011	88.7
$WKM_{D,10\%}$	0.505	0.5	0.0017	93.1	0.355	0.5	0.0017	90.4	0.204	0.4	0.0011	88.6
$WKM_{D,15\%}$	0.507	0.7	0.0018	93.0	0.358	0.8	0.0017	90.1	0.206	0.6	0.0012	88.5
$WKM_{D,20\%}$	0.510	1.0	0.0018	93.2	0.361	1.1	0.0018	90.2	0.209	0.9	0.0012	88.1
$WKM_{D,30\%}$	0.514	1.4	0.0019	92.5	0.366	1.6	0.0019	89.3	0.212	1.2	0.0014	86.8
$WKM_{D,40\%}$	0.518	1.8	0.0020	91.7	0.370	2.0	0.0021	88.3	0.216	1.6	0.0015	86.1
$WKM_{D,50\%}$	0.522	2.2	0.0022	90.6	0.373	2.3	0.0022	87.8	0.218	1.8	0.0016	84.9
$WKM_{R,2.5\%}$	0.500	0.0	0.0018	92.6	0.351	0.1	0.0016	91.3	0.200	0.0	0.0011	89.2
$WKM_{R,5\%}$	0.502	0.2	0.0017	92.7	0.352	0.2	0.0016	91.3	0.201	0.1	0.0011	90.6
$WKM_{R,10\%}$	0.505	0.5	0.0017	93.1	0.355	0.5	0.0016	91.3	0.203	0.3	0.0011	91.7
$WKM_{R,15\%}$	0.507	0.7	0.0017	93.5	0.358	0.8	0.0017	91.2	0.206	0.6	0.0012	91.9
$WKM_{R,20\%}$	0.509	0.9	0.0018	92.4	0.360	1.0	0.0017	91.7	0.208	0.8	0.0012	91.4
$WKM_{R,30\%}$	0.513	1.3	0.0018	92.3	0.364	1.4	0.0018	91.5	0.212	1.2	0.0013	91.2
$WKM_{R,40\%}$	0.517	1.7	0.0019	91.5	0.368	1.8	0.0020	91.2	0.215	1.5	0.0014	90.2
$WKM_{R,50\%}$	0.520	2.0	0.0020	90.8	0.372	2.2	0.0021	90.3	0.218	1.8	0.0015	88.9
$WKM_{U,2.5\%}$	0.501	0.1	0.0017	92.8	0.352	0.2	0.0016	92.3	0.201	0.1	0.0011	92.2
$WKM_{U,5\%}$	0.504	0.4	0.0017	93.7	0.355	0.5	0.0016	92.7	0.203	0.3	0.0011	93.5
$WKM_{U,10\%}$	0.510	1.0	0.0018	93.7	0.361	1.1	0.0017	93.3	0.209	0.9	0.0012	93.5
$WKM_{U,15\%}$	0.516	1.6	0.0019	92.9	0.367	1.7	0.0019	92.8	0.214	1.4	0.0014	92.9
$WKM_{U,20\%}$	0.521	2.1	0.0021	91.6	0.372	2.2	0.0021	92.0	0.219	1.9	0.0016	91.2
$WKM_{U,30\%}$	0.531	3.1	0.0026	87.6	0.383	3.3	0.0027	87.9	0.230	3.0	0.0022	87.2
$WKM_{U,40\%}$	0.539	3.9	0.0032	83.5	0.393	4.3	0.0036	80.6	0.239	3.9	0.0029	82.9
$WKM_{U,50\%}$	0.547	4.7	0.0038	77.7	0.402	5.2	0.0044	74.1	0.247	4.7	0.0036	76.7
$WKM_{\mathcal{N}(0.1)}$	0.497	-0.3	0.0018	93.2	0.347	-0.3	0.0017	91.6	0.197	-0.3	0.0012	90.5
$WKM_{\mathcal{N}(0.5)}$	0.509	0.9	0.0017	93.9	0.360	1.0	0.0017	93.5	0.206	0.6	0.0012	94.0
$WKM_{\mathcal{N}(1)}$	0.529	2.9	0.0025	88.1	0.381	3.1	0.0026	88.3	0.224	2.4	0.0019	88.9
$WKM_{\mathcal{N}(2)}$	0.550	5.0	0.0041	74.5	0.405	5.5	0.0048	71.5	0.248	4.8	0.0037	76.1
$WKM_{\mathcal{N}(5)}$	0.562	6.2	0.0055	64.2	0.421	7.1	0.0068	59.0	0.264	6.4	0.0056	61.5

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 11. Monte Carlo results for estimating **classical survival curve** with dependent censoring **(N=200)**.

Censoring rate=51% and the link functions **uncorrectly** specified.

	True value: 0.5 Censoring rate=34%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=46%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.498	-0.2	0.0012	94.8	0.349	-0.1	0.0011	94.9	0.199	-0.1	0.0008	94.3
KM PO	0.603	10.3	0.0121	29.6	0.470	12.0	0.0163	24.3	0.318	11.8	0.0163	29.6
IPCW	0.489	-1.1	0.0066	81.7	0.334	-1.6	0.0073	77.1	0.186	-1.4	0.0053	81.3
KMIB	0.521	2.1	0.0024	91.5	0.377	2.7	0.0029	90.3	0.23	3.0	0.0029	89.0
$WKM_{4,1}$	0.515	1.5	0.0026	90.4	0.374	2.4	0.0034	88.1	0.243	4.3	0.0046	81.2
$WKM_{4,2}$	0.522	2.2	0.0029	88.5	0.389	3.9	0.0043	81.9	0.266	6.6	0.0071	65.7
$WKM_{4,4}$	0.535	3.5	0.0035	82.8	0.409	5.9	0.0061	66.5	0.294	9.4	0.0116	38.3
$WKM_{8,1}$	0.518	1.8	0.0027	89.2	0.382	3.2	0.0038	85.1	0.257	5.7	0.0060	69.2
$WKM_{8,2}$	0.532	3.2	0.0033	83.8	0.406	5.6	0.0058	68.8	0.291	9.1	0.0109	40.5
$WKM_{16,1}$	0.529	2.9	0.0031	84.4	0.402	5.2	0.0053	70.7	0.286	8.6	0.0099	43.9
$WKM_{D,2.5\%}$	0.505	0.5	0.0023	89.5	0.356	0.6	0.0025	82.0	0.208	0.8	0.0022	75.4
$WKM_{D,5\%}$	0.507	0.7	0.0022	90.0	0.358	0.8	0.0024	83.5	0.211	1.1	0.0021	77.1
$WKM_{D,10\%}$	0.512	1.2	0.0022	90.2	0.363	1.3	0.0024	83.6	0.216	1.6	0.0022	76.0
$WKM_{D,15\%}$	0.516	1.6	0.0023	89.9	0.368	1.8	0.0025	82.8	0.222	2.2	0.0023	74.5
$WKM_{D,20\%}$	0.519	1.9	0.0024	89.2	0.373	2.3	0.0027	81.4	0.226	2.6	0.0026	72.8
$WKM_{D,30\%}$	0.526	2.6	0.0026	86.7	0.381	3.1	0.0031	78.8	0.233	3.3	0.0030	69.7
$WKM_{D,40\%}$	0.532	3.2	0.0030	84.8	0.387	3.7	0.0035	75.5	0.238	3.8	0.0034	66.5
$WKM_{D,50\%}$	0.538	3.8	0.0033	82.7	0.392	4.2	0.0040	72.0	0.242	4.2	0.0038	62.7
$WKM_{R,2.5\%}$	0.505	0.5	0.0023	89.3	0.356	0.6	0.0025	84.0	0.208	0.8	0.0022	80.5
$WKM_{R,5\%}$	0.507	0.7	0.0022	89.9	0.359	0.9	0.0024	85.0	0.211	1.1	0.0021	82.1
$WKM_{R,10\%}$	0.511	1.1	0.0022	89.7	0.363	1.3	0.0024	86.9	0.216	1.6	0.0021	83.5
$WKM_{R,15\%}$	0.515	1.5	0.0022	89.8	0.368	1.8	0.0025	86.7	0.221	2.1	0.0023	83.7
$WKM_{R,20\%}$	0.518	1.8	0.0023	89.6	0.372	2.2	0.0026	85.6	0.225	2.5	0.0025	83.3
$WKM_{R,30\%}$	0.524	2.4	0.0025	87.3	0.379	2.9	0.0029	84.4	0.232	3.2	0.0029	80.7
$WKM_{R,40\%}$	0.530	3.0	0.0028	85.1	0.385	3.5	0.0033	80.9	0.238	3.8	0.0033	77.2
$WKM_{R,50\%}$	0.535	3.5	0.0031	82.8	0.390	4.0	0.0037	77.9	0.242	4.2	0.0036	75.4
$WKM_{U,2.5\%}$	0.506	0.6	0.0022	90.7	0.358	0.8	0.0024	87.9	0.210	1.0	0.0021	86.6
$WKM_{U,5\%}$	0.511	1.1	0.0021	92.1	0.363	1.3	0.0023	90.6	0.216	1.6	0.0021	89.6
$WKM_{U,10\%}$	0.519	1.9	0.0023	91.8	0.373	2.3	0.0026	90.4	0.227	2.7	0.0024	89.2
$WKM_{U,15\%}$	0.527	2.7	0.0027	88.1	0.382	3.2	0.0031	87.8	0.238	3.8	0.0031	85.2
$WKM_{U,20\%}$	0.535	3.5	0.0031	85.9	0.391	4.1	0.0037	83.5	0.248	4.8	0.0041	80.0
$WKM_{U,30\%}$	0.549	4.9	0.0042	79.1	0.409	5.9	0.0055	71.5	0.266	6.6	0.0062	65.7
$WKM_{U,40\%}$	0.562	6.2	0.0056	68.8	0.425	7.5	0.0076	58.9	0.280	8.0	0.0084	54.3
$WKM_{U,50\%}$	0.574	7.4	0.0072	58.0	0.438	8.8	0.0098	48.3	0.291	9.1	0.0105	46.5
$WKM_{\mathcal{N}(0.1)}$	0.500	0.0	0.0025	89.3	0.350	0.0	0.0027	85.2	0.203	0.3	0.0027	79.2
$WKM_{\mathcal{N}(0.5)}$	0.520	2.0	0.0023	90.4	0.372	2.2	0.0026	90.1	0.220	2.0	0.0024	88.5
$WKM_{\mathcal{N}(1)}$	0.550	5.0	0.0043	77.5	0.406	5.6	0.0053	73.7	0.253	5.3	0.0049	77.1
$WKM_{\mathcal{N}(2)}$	0.581	8.1	0.0082	51.6	0.442	9.2	0.0106	45.5	0.289	8.9	0.0102	49.1
$WKM_{\mathcal{N}(5)}$	0.598	9.8	0.0113	33.1	0.464	11.4	0.0151	27.7	0.312	11.2	0.0149	32.5

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 12. Monte Carlo results for estimating **classical survival curve** with independent censoring **(N=200)**.

Censoring rate=40% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=35%¶				True value: 0.35 Censoring rate=37%				True value: 0.2 Censoring rate=39%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	94.2	0.349	-0.1	0.0012	93.5	0.199	-0.1	0.0008	93.2
KM PO	0.500	0.0	0.0019	95.6	0.348	-0.2	0.0018	94.1	0.199	-0.1	0.0014	94.2
IPCW	0.500	0.0	0.0017	94.5	0.348	-0.2	0.0017	93.4	0.198	-0.2	0.0013	94.9
KMIB	0.501	0.1	0.0018	94.5	0.349	-0.1	0.0018	92.8	0.199	-0.1	0.0013	93.2
$WKM_{4.1}$	0.499	-0.1	0.0018	94.6	0.348	-0.2	0.0018	93.9	0.199	-0.1	0.0013	93.9
$WKM_{4.2}$	0.499	-0.1	0.0018	95.1	0.348	-0.2	0.0018	92.8	0.199	-0.1	0.0013	93.3
$WKM_{4.4}$	0.500	0.0	0.0018	94.3	0.350	0.0	0.0019	92.1	0.203	0.3	0.0014	92.4
$WKM_{8.1}$	0.499	-0.1	0.0018	94.6	0.349	-0.1	0.0018	93.0	0.201	0.1	0.0013	92.9
$WKM_{8.2}$	0.500	0.0	0.0018	94.3	0.350	0.0	0.0018	91.9	0.203	0.3	0.0014	91.4
$WKM_{16.1}$	0.500	0.0	0.0018	94.1	0.351	0.1	0.0019	92.0	0.205	0.5	0.0014	92.4
$WKM_{D.2.5\%}$	0.500	0.0	0.0019	90.9	0.349	-0.1	0.002	87.7	0.199	-0.1	0.0014	84.6
$WKM_{D.5\%}$	0.500	0.0	0.0019	91.8	0.349	-0.1	0.0019	88.0	0.199	-0.1	0.0014	85.7
$WKM_{D.10\%}$	0.500	0.0	0.0019	92.1	0.349	-0.1	0.0019	88.0	0.198	-0.2	0.0014	86.5
$WKM_{D.15\%}$	0.500	0.0	0.0019	92.2	0.349	-0.1	0.0019	88.3	0.198	-0.2	0.0013	86.3
$WKM_{D.20\%}$	0.501	0.1	0.0019	92.3	0.348	-0.2	0.0019	88.5	0.197	-0.3	0.0013	86.3
$WKM_{D.30\%}$	0.501	0.1	0.0019	92.7	0.348	-0.2	0.0019	88.9	0.197	-0.3	0.0013	86.4
$WKM_{D.40\%}$	0.501	0.1	0.0019	92.4	0.348	-0.2	0.0019	89.2	0.196	-0.4	0.0013	86.3
$WKM_{D.50\%}$	0.501	0.1	0.0018	92.3	0.348	-0.2	0.0018	89.1	0.196	-0.4	0.0013	86.4
$WKM_{R.2.5\%}$	0.500	0.0	0.0019	90.6	0.349	-0.1	0.0019	88.8	0.199	-0.1	0.0014	87.3
$WKM_{R.5\%}$	0.500	0.0	0.0019	91.2	0.349	-0.1	0.0019	89.5	0.199	-0.1	0.0014	88.9
$WKM_{R.10\%}$	0.500	0.0	0.0018	91.9	0.349	-0.1	0.0019	90.0	0.198	-0.2	0.0013	90.1
$WKM_{R.15\%}$	0.500	0.0	0.0018	92.2	0.348	-0.2	0.0018	90.3	0.198	-0.2	0.0013	90.2
$WKM_{R.20\%}$	0.500	0.0	0.0018	92.5	0.348	-0.2	0.0018	90.7	0.197	-0.3	0.0013	90.5
$WKM_{R.30\%}$	0.501	0.1	0.0018	93.0	0.348	-0.2	0.0018	91.4	0.197	-0.3	0.0013	91.0
$WKM_{R.40\%}$	0.501	0.1	0.0018	93.4	0.348	-0.2	0.0018	91.6	0.196	-0.4	0.0013	91.3
$WKM_{R.50\%}$	0.501	0.1	0.0018	93.5	0.348	-0.2	0.0018	92.0	0.196	-0.4	0.0013	91.5
$WKM_{U.2.5\%}$	0.500	0.0	0.0019	93.2	0.349	-0.1	0.0019	90.7	0.199	-0.1	0.0014	90.1
$WKM_{U.5\%}$	0.499	-0.1	0.0018	94.5	0.348	-0.2	0.0019	91.9	0.198	-0.2	0.0013	92.0
$WKM_{U.10\%}$	0.500	0.0	0.0018	94.5	0.348	-0.2	0.0018	92.8	0.197	-0.3	0.0013	92.9
$WKM_{U.15\%}$	0.501	0.1	0.0018	94.8	0.348	-0.2	0.0018	93.4	0.196	-0.4	0.0013	93.2
$WKM_{U.20\%}$	0.501	0.1	0.0018	94.7	0.348	-0.2	0.0018	93.6	0.195	-0.5	0.0013	93.5
$WKM_{U.30\%}$	0.502	0.2	0.0018	94.6	0.347	-0.3	0.0018	93.8	0.194	-0.6	0.0014	93.5
$WKM_{U.40\%}$	0.502	0.2	0.0019	94.8	0.347	-0.3	0.0018	93.8	0.194	-0.6	0.0014	93.3
$WKM_{U.50\%}$	0.502	0.2	0.0019	95.4	0.347	-0.3	0.0018	93.8	0.194	-0.6	0.0014	93.4
$WKM_{\mathcal{N}(0.1)}$	0.499	-0.1	0.0018	93.6	0.349	-0.1	0.0019	90.8	0.199	-0.1	0.0014	90.3
$WKM_{\mathcal{N}(0.5)}$	0.500	0.0	0.0018	94.7	0.348	-0.2	0.0018	93.3	0.197	-0.3	0.0013	92.8
$WKM_{\mathcal{N}(1)}$	0.501	0.1	0.0018	94.7	0.348	-0.2	0.0018	94.1	0.196	-0.4	0.0013	94.0
$WKM_{\mathcal{N}(2)}$	0.501	0.1	0.0018	95.9	0.348	-0.2	0.0018	93.5	0.197	-0.3	0.0013	94.1
$WKM_{\mathcal{N}(5)}$	0.500	0.0	0.0019	95.4	0.348	-0.2	0.0018	93.8	0.198	-0.2	0.0014	93.3

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 13. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=200)**.

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Overall censoring rate=35% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=26%¶				True value: 0.35 Censoring rate=29%				True value: 0.2 Censoring rate=32%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0012	94.5	0.350	0.0	0.0012	93.9	0.198	-0.2	0.0008	93.4
KM PO	0.565	6.5	0.0059	60.6	0.425	7.5	0.0074	55.3	0.268	6.8	0.0062	60.0
IPCW	0.535	3.5	0.0030	83.4	0.387	3.7	0.0033	83.0	0.229	2.9	0.0026	86.7
$WKM_{4.1}$	0.521	2.1	0.0022	91.4	0.372	2.2	0.0023	91.7	0.221	2.1	0.0018	92.8
$WKM_{4.2}$	0.515	1.5	0.0019	93.1	0.370	2.0	0.0021	91.3	0.225	2.5	0.0020	89.6
$WKM_{4.4}$	0.518	1.8	0.0020	92.4	0.378	2.8	0.0025	87.8	0.239	3.9	0.0030	79.1
$WKM_{8.1}$	0.519	1.9	0.0022	91.2	0.374	2.4	0.0024	91.3	0.226	2.6	0.0022	88.6
$WKM_{8.2}$	0.518	1.8	0.0021	90.8	0.379	2.9	0.0027	86.2	0.240	4.0	0.0031	78.9
$WKM_{16.1}$	0.523	2.3	0.0024	88.8	0.382	3.2	0.0029	84.9	0.241	4.1	0.0033	76.9
$WKM_{D.2.5\%}$	0.515	1.5	0.0021	90.4	0.365	1.5	0.0020	87.7	0.211	1.1	0.0014	86.2
$WKM_{D.5\%}$	0.516	1.6	0.0021	90.3	0.367	1.7	0.0020	88.5	0.213	1.3	0.0015	85.5
$WKM_{D.10\%}$	0.518	1.8	0.0021	90.2	0.369	1.9	0.0021	88.4	0.215	1.5	0.0015	84.6
$WKM_{D.15\%}$	0.520	2.0	0.0021	89.8	0.371	2.1	0.0022	87.3	0.217	1.7	0.0016	84.9
$WKM_{D.20\%}$	0.522	2.2	0.0022	89.3	0.373	2.3	0.0023	86.6	0.219	1.9	0.0017	83.9
$WKM_{D.30\%}$	0.525	2.5	0.0023	88.4	0.377	2.7	0.0024	84.6	0.223	2.3	0.0018	82.3
$WKM_{D.40\%}$	0.528	2.8	0.0025	87.2	0.380	3.0	0.0026	83.0	0.225	2.5	0.0020	80.6
$WKM_{D.50\%}$	0.531	3.1	0.0026	86.1	0.383	3.3	0.0028	81.1	0.228	2.8	0.0021	80.2
$WKM_{R.2.5\%}$	0.515	1.5	0.0021	90.0	0.365	1.5	0.0020	89.0	0.211	1.1	0.0014	88.7
$WKM_{R.5\%}$	0.516	1.6	0.0020	89.6	0.366	1.6	0.0020	89.8	0.213	1.3	0.0014	88.9
$WKM_{R.10\%}$	0.518	1.8	0.0021	89.7	0.369	1.9	0.0020	90.0	0.215	1.5	0.0015	88.9
$WKM_{R.15\%}$	0.520	2.0	0.0021	89.3	0.371	2.1	0.0021	90.2	0.217	1.7	0.0015	88.8
$WKM_{R.20\%}$	0.521	2.1	0.0022	88.8	0.372	2.2	0.0022	89.9	0.219	1.9	0.0016	88.1
$WKM_{R.30\%}$	0.524	2.4	0.0023	87.7	0.376	2.6	0.0023	88.5	0.222	2.2	0.0018	87.4
$WKM_{R.40\%}$	0.527	2.7	0.0024	86.7	0.379	2.9	0.0025	86.8	0.224	2.4	0.0019	86.5
$WKM_{R.50\%}$	0.529	2.9	0.0025	86.0	0.382	3.2	0.0027	85.1	0.227	2.7	0.0020	85.3
$WKM_{U.2.5\%}$	0.515	1.5	0.0020	91.0	0.366	1.6	0.0020	91.4	0.212	1.2	0.0014	91.4
$WKM_{U.5\%}$	0.517	1.7	0.0020	91.2	0.368	1.8	0.0020	91.9	0.214	1.4	0.0015	91.9
$WKM_{U.10\%}$	0.522	2.2	0.0022	90.4	0.373	2.3	0.0022	91.8	0.219	1.9	0.0016	91.5
$WKM_{U.15\%}$	0.526	2.6	0.0023	89.5	0.378	2.8	0.0024	91.4	0.224	2.4	0.0018	90.2
$WKM_{U.20\%}$	0.529	2.9	0.0025	87.9	0.382	3.2	0.0027	88.7	0.228	2.8	0.0021	88.3
$WKM_{U.30\%}$	0.536	3.6	0.0030	84.5	0.391	4.1	0.0033	82.7	0.237	3.7	0.0027	83.8
$WKM_{U.40\%}$	0.543	4.3	0.0035	80.5	0.399	4.9	0.0041	76.9	0.244	4.4	0.0034	78.7
$WKM_{U.50\%}$	0.549	4.9	0.0040	75.3	0.406	5.6	0.0048	70.8	0.251	5.1	0.0040	74.0
$WKM_{\mathcal{N}(0.1)}$	0.514	1.4	0.0021	90.7	0.364	1.4	0.0020	90.6	0.210	1.0	0.0014	90.8
$WKM_{\mathcal{N}(0.5)}$	0.522	2.2	0.0022	89.2	0.373	2.3	0.0022	91.4	0.218	1.8	0.0016	91.0
$WKM_{\mathcal{N}(1)}$	0.537	3.7	0.0030	84.5	0.390	4.0	0.0033	82.7	0.233	3.3	0.0025	85.0
$WKM_{\mathcal{N}(2)}$	0.553	5.3	0.0045	71.7	0.410	6.0	0.0053	68.0	0.252	5.2	0.0042	72.6
$WKM_{\mathcal{N}(5)}$	0.563	6.3	0.0056	62.9	0.422	7.2	0.0069	58.0	0.265	6.5	0.0058	60.7

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 14. Monte Carlo results for estimating **classical survival curve** with dependent censoring **(N=200)**.

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Censoring rate=51% and the link functions **uncorrectly** specified.

	True value: 0.5 Censoring rate=34%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=46%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.498	-0.2	0.0012	94.8	0.349	-0.1	0.0011	94.9	0.199	-0.1	0.0008	94.3
KM PO	0.603	10.3	0.0121	29.6	0.470	12.0	0.0163	24.3	0.318	11.8	0.0163	29.6
IPCW	0.566	6.6	0.0065	62.5	0.418	6.8	0.0077	63.0	0.257	5.7	0.0069	72.5
$WKM_{4.1}$	0.535	3.5	0.0033	86.6	0.392	4.2	0.0044	82.0	0.251	5.1	0.0051	77.0
$WKM_{4.2}$	0.528	2.8	0.0030	86.8	0.394	4.4	0.0046	79.1	0.266	6.6	0.0070	64.4
$WKM_{4.4}$	0.537	3.7	0.0035	80.9	0.411	6.1	0.0063	66.2	0.294	9.4	0.0114	38.4
$WKM_{8.1}$	0.535	3.5	0.0033	85.6	0.397	4.7	0.0047	77.4	0.265	6.5	0.0067	64.2
$WKM_{8.2}$	0.537	3.7	0.0035	82.0	0.411	6.1	0.0062	67.0	0.293	9.3	0.0111	39.4
$WKM_{16.1}$	0.542	4.2	0.0039	79.4	0.412	6.2	0.0063	65.1	0.290	9.0	0.0106	41.0
$WKM_{D.2.5\%}$	0.525	2.5	0.0028	86.3	0.376	2.6	0.0033	77.4	0.224	2.4	0.0029	70.5
$WKM_{D.5\%}$	0.526	2.6	0.0028	86.6	0.379	2.9	0.0033	78.0	0.227	2.7	0.0029	70.3
$WKM_{D.10\%}$	0.530	3.0	0.0029	85.7	0.383	3.3	0.0034	77.1	0.233	3.3	0.0031	68.4
$WKM_{D.15\%}$	0.533	3.3	0.0031	84.8	0.387	3.7	0.0036	75.5	0.237	3.7	0.0034	66.3
$WKM_{D.20\%}$	0.536	3.6	0.0032	83.5	0.390	4.0	0.0038	73.5	0.241	4.1	0.0037	62.7
$WKM_{D.30\%}$	0.541	4.1	0.0036	81.5	0.396	4.6	0.0043	70.8	0.247	4.7	0.0042	57.0
$WKM_{D.40\%}$	0.545	4.5	0.0039	78.4	0.402	5.2	0.0049	66.8	0.251	5.1	0.0047	54.5
$WKM_{D.50\%}$	0.549	4.9	0.0043	75.6	0.406	5.6	0.0053	63.9	0.254	5.4	0.0051	51.4
$WKM_{R.2.5\%}$	0.525	2.5	0.0028	85.7	0.376	2.6	0.0033	80.6	0.224	2.4	0.0029	77.7
$WKM_{R.5\%}$	0.527	2.7	0.0028	85.5	0.379	2.9	0.0033	80.9	0.227	2.7	0.0029	78.7
$WKM_{R.10\%}$	0.530	3.0	0.0029	85.0	0.383	3.3	0.0034	80.7	0.232	3.2	0.0031	77.8
$WKM_{R.15\%}$	0.532	3.2	0.0030	83.5	0.386	3.6	0.0035	79.2	0.236	3.6	0.0033	77.2
$WKM_{R.20\%}$	0.535	3.5	0.0032	82.8	0.389	3.9	0.0037	78.6	0.240	4.0	0.0036	76.6
$WKM_{R.30\%}$	0.540	4.0	0.0035	81.0	0.395	4.5	0.0042	74.6	0.246	4.6	0.0041	72.8
$WKM_{R.40\%}$	0.544	4.4	0.0038	77.6	0.400	5.0	0.0047	72.6	0.250	5.0	0.0045	69.9
$WKM_{R.50\%}$	0.547	4.7	0.0041	75.7	0.404	5.4	0.0051	69.7	0.253	5.3	0.0049	67.2
$WKM_{U.2.5\%}$	0.526	2.6	0.0028	87.3	0.378	2.8	0.0033	84.5	0.226	2.6	0.0029	83.5
$WKM_{U.5\%}$	0.529	2.9	0.0028	87.8	0.383	3.3	0.0033	85.4	0.232	3.2	0.0030	86.2
$WKM_{U.10\%}$	0.536	3.6	0.0032	85.7	0.391	4.1	0.0038	82.6	0.242	4.2	0.0036	83.9
$WKM_{U.15\%}$	0.542	4.2	0.0036	82.1	0.398	4.8	0.0044	79.3	0.251	5.1	0.0045	78.2
$WKM_{U.20\%}$	0.548	4.8	0.0041	78.8	0.406	5.6	0.0051	74.5	0.260	6.0	0.0055	71.2
$WKM_{U.30\%}$	0.559	5.9	0.0052	71.8	0.420	7.0	0.0069	64.4	0.274	7.4	0.0075	58.3
$WKM_{U.40\%}$	0.569	6.9	0.0065	62.2	0.433	8.3	0.0088	52.9	0.286	8.6	0.0095	51.1
$WKM_{U.50\%}$	0.579	7.9	0.0079	54.0	0.443	9.3	0.0107	43.9	0.295	9.5	0.0112	43.5
$WKM_{\mathcal{N}(0.1)}$	0.524	2.4	0.0029	87.0	0.375	2.5	0.0035	82.7	0.223	2.3	0.0032	80.2
$WKM_{\mathcal{N}(0.5)}$	0.539	3.9	0.0034	83.8	0.393	4.3	0.0040	81.5	0.239	3.9	0.0036	83.0
$WKM_{\mathcal{N}(1)}$	0.563	6.3	0.0057	67.1	0.420	7.0	0.0071	62.8	0.267	6.7	0.0066	66.2
$WKM_{\mathcal{N}(2)}$	0.586	8.6	0.0092	44.9	0.449	9.9	0.0119	38.7	0.296	9.6	0.0116	42.8
$WKM_{\mathcal{N}(5)}$	0.600	10.0	0.0115	32.1	0.466	11.6	0.0154	26.6	0.314	11.4	0.0153	31.5

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 15. Monte Carlo results for estimating **classical survival curve** with independent censoring **(N=200)**.

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Censoring rate=40% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=35%¶				True value: 0.35 Censoring rate=37%				True value: 0.2 Censoring rate=39%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.499	-0.1	0.0013	94.2	0.349	-0.1	0.0012	93.5	0.199	-0.1	0.0008	93.2
KM PO	0.500	0.0	0.0019	95.6	0.348	-0.2	0.0018	94.1	0.199	-0.1	0.0014	94.2
IPCW	0.499	-0.1	0.0018	94.5	0.348	-0.2	0.0018	94.1	0.198	-0.2	0.0013	94.3
$WKM_{4.1}$	0.499	-0.1	0.0018	95.6	0.348	-0.2	0.0018	93.6	0.198	-0.2	0.0013	93.8
$WKM_{4.2}$	0.499	-0.1	0.0017	94.8	0.348	-0.2	0.0018	93.3	0.200	0.0	0.0013	93.7
$WKM_{4.4}$	0.499	-0.1	0.0018	94.6	0.350	0.0	0.0018	92.8	0.204	0.4	0.0014	92.5
$WKM_{8.1}$	0.499	-0.1	0.0018	95.0	0.348	-0.2	0.0018	93.4	0.199	-0.1	0.0013	94.1
$WKM_{8.2}$	0.499	-0.1	0.0018	94.3	0.349	-0.1	0.0018	92.8	0.203	0.3	0.0014	93.2
$WKM_{16.1}$	0.499	-0.1	0.0018	94.2	0.349	-0.1	0.0019	93.1	0.202	0.2	0.0014	93.4
$WKM_{D.2.5\%}$	0.499	-0.1	0.0019	91.8	0.348	-0.2	0.0020	87.2	0.198	-0.2	0.0015	82.9
$WKM_{D.5\%}$	0.499	-0.1	0.0018	92.3	0.348	-0.2	0.0019	87.8	0.198	-0.2	0.0014	84.4
$WKM_{D.10\%}$	0.499	-0.1	0.0018	92.8	0.348	-0.2	0.0019	87.6	0.198	-0.2	0.0014	84.8
$WKM_{D.15\%}$	0.499	-0.1	0.0018	92.9	0.348	-0.2	0.0018	88.0	0.198	-0.2	0.0014	85.0
$WKM_{D.20\%}$	0.499	-0.1	0.0018	92.8	0.348	-0.2	0.0018	87.9	0.198	-0.2	0.0014	84.9
$WKM_{D.30\%}$	0.500	0.0	0.0018	93.0	0.348	-0.2	0.0018	88.4	0.198	-0.2	0.0014	84.9
$WKM_{D.40\%}$	0.500	0.0	0.0018	92.9	0.348	-0.2	0.0018	88.6	0.198	-0.2	0.0014	84.8
$WKM_{D.50\%}$	0.500	0.0	0.0018	92.9	0.348	-0.2	0.0018	88.5	0.198	-0.2	0.0014	85.0
$WKM_{R.2.5\%}$	0.499	-0.1	0.0019	92.5	0.348	-0.2	0.0019	88	0.198	-0.2	0.0014	86.5
$WKM_{R.5\%}$	0.499	-0.1	0.0018	93.1	0.348	-0.2	0.0019	89.2	0.198	-0.2	0.0014	87.7
$WKM_{R.10\%}$	0.499	-0.1	0.0018	92.9	0.348	-0.2	0.0018	90.4	0.198	-0.2	0.0013	89.2
$WKM_{R.15\%}$	0.499	-0.1	0.0018	93.2	0.348	-0.2	0.0018	90.6	0.198	-0.2	0.0013	89.8
$WKM_{R.20\%}$	0.500	0.0	0.0018	93.3	0.348	-0.2	0.0018	90.7	0.198	-0.2	0.0013	90.1
$WKM_{R.30\%}$	0.500	0.0	0.0018	93.1	0.348	-0.2	0.0018	90.9	0.197	-0.3	0.0013	90.9
$WKM_{R.40\%}$	0.500	0.0	0.0018	93.3	0.348	-0.2	0.0018	91.0	0.197	-0.3	0.0013	90.8
$WKM_{R.50\%}$	0.500	0.0	0.0018	93.5	0.348	-0.2	0.0018	91.2	0.197	-0.3	0.0013	90.7
$WKM_{U.2.5\%}$	0.499	-0.1	0.0019	93.9	0.348	-0.2	0.0019	91.8	0.198	-0.2	0.0014	91.0
$WKM_{U.5\%}$	0.499	-0.1	0.0018	94.2	0.348	-0.2	0.0018	92.7	0.198	-0.2	0.0014	93.1
$WKM_{U.10\%}$	0.500	0.0	0.0018	94.7	0.348	-0.2	0.0018	93.2	0.198	-0.2	0.0013	93.9
$WKM_{U.15\%}$	0.500	0.0	0.0018	95.1	0.348	-0.2	0.0018	93.7	0.197	-0.3	0.0013	94.2
$WKM_{U.20\%}$	0.500	0.0	0.0018	94.7	0.348	-0.2	0.0018	94.5	0.197	-0.3	0.0013	94.4
$WKM_{U.30\%}$	0.501	0.1	0.0018	95.0	0.348	-0.2	0.0018	94.3	0.197	-0.3	0.0013	94.0
$WKM_{U.40\%}$	0.501	0.1	0.0018	94.9	0.348	-0.2	0.0018	94.1	0.197	-0.3	0.0014	93.8
$WKM_{U.50\%}$	0.501	0.1	0.0019	95.3	0.348	-0.2	0.0018	94.1	0.197	-0.3	0.0014	93.9
$WKM_{\mathcal{N}(0.1)}$	0.499	-0.1	0.0018	94.0	0.348	-0.2	0.0019	91.7	0.198	-0.2	0.0014	90.1
$WKM_{\mathcal{N}(0.5)}$	0.499	-0.1	0.0018	94.7	0.348	-0.2	0.0018	93.7	0.197	-0.3	0.0013	93.7
$WKM_{\mathcal{N}(1)}$	0.500	0.0	0.0018	95.0	0.348	-0.2	0.0018	94.0	0.197	-0.3	0.0013	94.3
$WKM_{\mathcal{N}(2)}$	0.500	0.0	0.0018	95.6	0.348	-0.2	0.0018	94.1	0.198	-0.2	0.0014	94.0
$WKM_{\mathcal{N}(5)}$	0.500	0.0	0.0019	95.5	0.348	-0.2	0.0018	93.9	0.199	-0.1	0.0014	93.6

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 16. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100)**. **70% LOD=100 copies/ml and 30% LOD=200 copies/ml** leading to an overall censoring rate of **45%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=24%¶				True value: 0.35 Censoring rate=34%				True value: 0.2 Censoring rate=42%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.521	2.1	0.0037	93.3	0.382	3.2	0.0046	92.0	0.235	3.5	0.0062	90.6
$WKM_{4.1}$	0.491	-0.9	0.0047	88.5	0.359	0.9	0.0056	86.6	0.258	5.8	0.0100	75.2
$WKM_{4.2}$	0.526	2.6	0.0049	86.2	0.424	7.4	0.0100	69.7	0.353	15.3	0.0280	25.7
$WKM_{8.1}$	0.500	0.0	0.0043	90.8	0.380	3.0	0.0056	85.4	0.292	9.2	0.0136	60.3
$WKM_{D.3\%}$	0.487	-1.3	0.0048	88.2	0.334	-1.6	0.0062	76.4	0.188	-1.2	0.0063	66.1
$WKM_{D.5\%}$	0.488	-1.2	0.0045	87.9	0.337	-1.3	0.0056	79.5	0.191	-0.9	0.0058	69.7
$WKM_{D.10\%}$	0.490	-1.0	0.0042	89.6	0.341	-0.9	0.0049	82.1	0.196	-0.4	0.0052	73.9
$WKM_{D.20\%}$	0.493	-0.7	0.0038	91.4	0.346	-0.4	0.0044	84.3	0.203	0.3	0.0049	75.3
$WKM_{D.30\%}$	0.496	-0.4	0.0037	92.3	0.351	0.1	0.0042	85.7	0.206	0.6	0.0049	75.7
$WKM_{D.40\%}$	0.499	-0.1	0.0036	92.0	0.354	0.4	0.0041	87.4	0.209	0.9	0.0050	76.4
$WKM_{D.50\%}$	0.501	0.1	0.0035	92.4	0.356	0.6	0.0041	87.4	0.210	1.0	0.0050	76.2
$WKM_{R.3\%}$	0.487	-1.3	0.0048	87.5	0.335	-1.5	0.0061	79.2	0.188	-1.2	0.0063	73.2
$WKM_{R.5\%}$	0.488	-1.2	0.0045	88.0	0.337	-1.3	0.0055	82.2	0.191	-0.9	0.0056	78.0
$WKM_{R.10\%}$	0.490	-1.0	0.0041	89.5	0.341	-0.9	0.0048	85.6	0.196	-0.4	0.0051	82.1
$WKM_{R.20\%}$	0.493	-0.7	0.0038	90.1	0.346	-0.4	0.0043	87.8	0.202	0.2	0.0048	85.6
$WKM_{R.30\%}$	0.495	-0.5	0.0037	91.1	0.350	0.0	0.0041	89.3	0.205	0.5	0.0048	86.5
$WKM_{R.40\%}$	0.498	-0.2	0.0036	91.0	0.353	0.3	0.0040	90.0	0.208	0.8	0.0048	87.2
$WKM_{R.50\%}$	0.500	0.0	0.0035	91.1	0.355	0.5	0.0040	90.7	0.209	0.9	0.0048	87.7
$WKM_{U.3\%}$	0.488	-1.2	0.0046	89.0	0.336	-1.4	0.0057	83.1	0.190	-1.0	0.0058	80.1
$WKM_{U.5\%}$	0.489	-1.1	0.0042	89.7	0.339	-1.1	0.0050	87.6	0.194	-0.6	0.0051	85.5
$WKM_{U.10\%}$	0.493	-0.7	0.0038	91.7	0.346	-0.4	0.0042	91.0	0.202	0.2	0.0047	89.6
$WKM_{U.20\%}$	0.498	-0.2	0.0035	92.3	0.355	0.5	0.0038	93.1	0.213	1.3	0.0047	92.4
$WKM_{U.30\%}$	0.504	0.4	0.0034	93.1	0.363	1.3	0.0038	94.1	0.220	2.0	0.0050	92.2
$WKM_{U.40\%}$	0.508	0.8	0.0034	93.2	0.369	1.9	0.0039	94.0	0.225	2.5	0.0053	91.9
$WKM_{U.50\%}$	0.512	1.2	0.0034	93.2	0.373	2.3	0.0041	93.3	0.229	2.9	0.0056	90.9
$WKM_{\mathcal{N}(0.1)}$	0.483	-1.7	0.0058	85.6	0.327	-2.3	0.0083	74.3	0.177	-2.3	0.0109	57.8
$WKM_{\mathcal{N}(0.5)}$	0.491	-0.9	0.0039	91.4	0.341	-0.9	0.0047	89.3	0.192	-0.8	0.0059	82.8
$WKM_{\mathcal{N}(1)}$	0.502	0.2	0.0034	92.8	0.358	0.8	0.0039	93.1	0.211	1.1	0.0049	90.6
$WKM_{\mathcal{N}(2)}$	0.513	1.3	0.0035	92.8	0.372	2.2	0.0042	93.0	0.225	2.5	0.0054	91.7
$WKM_{\mathcal{N}(5)}$	0.519	1.9	0.0036	92.5	0.380	3.0	0.0045	91.4	0.233	3.3	0.0060	90.5

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Statistics in Medicine

Table 17. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100)**. **30% LOD=100 copies/ml and 70% LOD=200 copies/ml** leading to an overall censoring rate of **50%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=30%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=47%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.525	2.5	0.0040	92.0	0.386	3.6	0.0053	90.7	0.239	3.9	0.0081	87.9
$WKM_{4.1}$	0.492	-0.8	0.0055	87.7	0.367	1.7	0.0072	82.3	0.276	7.6	0.0139	69.0
$WKM_{4.2}$	0.536	3.6	0.0058	83.9	0.442	9.2	0.0132	60.4	0.380	18	0.0372	16.5
$WKM_{8.1}$	0.504	0.4	0.0048	88.5	0.391	4.1	0.0074	80.8	0.313	11.3	0.0192	51.0
$WKM_{D.3\%}$	0.484	-1.6	0.0057	84.9	0.332	-1.8	0.0073	72.9	0.193	-0.7	0.0084	62.2
$WKM_{D.5\%}$	0.485	-1.5	0.0052	86.1	0.334	-1.6	0.0066	74.9	0.195	-0.5	0.0077	63.8
$WKM_{D.10\%}$	0.487	-1.3	0.0047	88.1	0.339	-1.1	0.0057	79.6	0.199	-0.1	0.0070	66.9
$WKM_{D.20\%}$	0.492	-0.8	0.0043	89.8	0.345	-0.5	0.0049	82.9	0.205	0.5	0.0066	70.1
$WKM_{D.30\%}$	0.495	-0.5	0.0041	90.8	0.350	0.0	0.0046	84.7	0.209	0.9	0.0066	71.8
$WKM_{D.40\%}$	0.498	-0.2	0.0039	91.5	0.354	0.4	0.0046	85.5	0.211	1.1	0.0066	72.2
$WKM_{D.50\%}$	0.501	0.1	0.0039	91.6	0.356	0.6	0.0045	85.8	0.213	1.3	0.0067	71.8
$WKM_{R.3\%}$	0.484	-1.6	0.0056	85.4	0.332	-1.8	0.0073	75.2	0.193	-0.7	0.0083	68.5
$WKM_{R.5\%}$	0.485	-1.5	0.0051	86.4	0.334	-1.6	0.0065	78.1	0.195	-0.5	0.0076	73.7
$WKM_{R.10\%}$	0.487	-1.3	0.0047	88.0	0.338	-1.2	0.0056	83.4	0.199	-0.1	0.0068	79.1
$WKM_{R.20\%}$	0.491	-0.9	0.0043	89.3	0.345	-0.5	0.0049	86.4	0.205	0.5	0.0065	84.2
$WKM_{R.30\%}$	0.495	-0.5	0.0040	90.1	0.349	-0.1	0.0046	88.3	0.209	0.9	0.0064	85.3
$WKM_{R.40\%}$	0.497	-0.3	0.0039	90.5	0.353	0.3	0.0045	89.2	0.211	1.1	0.0065	86.3
$WKM_{R.50\%}$	0.500	0.0	0.0038	90.6	0.355	0.5	0.0045	89.6	0.213	1.3	0.0065	86.7
$WKM_{U.3\%}$	0.484	-1.6	0.0053	87.6	0.334	-1.6	0.0068	81.1	0.194	-0.6	0.0079	76.0
$WKM_{U.5\%}$	0.486	-1.4	0.0048	89.3	0.337	-1.3	0.0059	85.9	0.197	-0.3	0.0071	82.7
$WKM_{U.10\%}$	0.491	-0.9	0.0043	91.4	0.344	-0.6	0.0049	89.9	0.205	0.5	0.0063	89.6
$WKM_{U.20\%}$	0.498	-0.2	0.0039	92.4	0.355	0.5	0.0042	93.4	0.216	1.6	0.0062	91.9
$WKM_{U.30\%}$	0.505	0.5	0.0037	92.2	0.365	1.5	0.0042	93.6	0.223	2.3	0.0066	92.2
$WKM_{U.40\%}$	0.511	1.1	0.0036	93.0	0.371	2.1	0.0044	93.0	0.228	2.8	0.0070	91.2
$WKM_{U.50\%}$	0.515	1.5	0.0037	92.9	0.376	2.6	0.0046	92.2	0.232	3.2	0.0073	90.7
$WKM_{\mathcal{N}(0.1)}$	0.479	-2.1	0.0072	82.1	0.323	-2.7	0.0104	69.6	0.180	-2.0	0.0139	55.4
$WKM_{\mathcal{N}(0.5)}$	0.489	-1.1	0.0044	90.6	0.339	-1.1	0.0055	89.4	0.194	-0.6	0.0079	82.4
$WKM_{\mathcal{N}(1)}$	0.503	0.3	0.0037	92.4	0.359	0.9	0.0044	93.4	0.214	1.4	0.0067	90.6
$WKM_{\mathcal{N}(2)}$	0.516	1.6	0.0038	92.6	0.375	2.5	0.0047	92.5	0.229	2.9	0.0073	91.1
$WKM_{\mathcal{N}(5)}$	0.523	2.3	0.0040	91.6	0.384	3.4	0.0052	91.0	0.237	3.7	0.0079	89.4

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 18. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100)**. **70% LOD=200 copies/ml and 30% LOD=500 copies/ml** leading to an overall censoring rate of **57%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=39%¶				True value: 0.35 Censoring rate=49%				True value: 0.2 Censoring rate=56%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.534	3.4	0.0051	90.5	0.396	4.6	0.0072	88.9	0.252	5.2	0.0140	79.4
$WKM_{4.1}$	0.498	-0.2	0.0074	82.6	0.384	3.4	0.0101	78.5	0.312	11.2	0.0234	59.5
$WKM_{4.2}$	0.561	6.1	0.0088	73.9	0.481	13.1	0.0228	43.0	0.432	23.2	0.0600	8.1
$WKM_{8.1}$	0.515	1.5	0.0063	85.0	0.413	6.3	0.0110	74.4	0.354	15.4	0.0317	34.5
$WKM_{D.3\%}$	0.481	-1.9	0.0077	78.2	0.332	-1.8	0.0098	65.7	0.200	0.0	0.0126	55.4
$WKM_{D.5\%}$	0.482	-1.8	0.0069	80.5	0.334	-1.6	0.0086	68.6	0.203	0.3	0.0118	58.1
$WKM_{D.10\%}$	0.485	-1.5	0.0061	84.1	0.339	-1.1	0.0073	73.9	0.208	0.8	0.0110	60.5
$WKM_{D.20\%}$	0.491	-0.9	0.0054	86.4	0.347	-0.3	0.0063	78.3	0.215	1.5	0.0108	62.2
$WKM_{D.30\%}$	0.496	-0.4	0.0050	88.2	0.353	0.3	0.0060	81.1	0.219	1.9	0.0109	61.8
$WKM_{D.40\%}$	0.500	0.0	0.0048	89.8	0.357	0.7	0.0059	82.0	0.222	2.2	0.0111	62.2
$WKM_{D.50\%}$	0.503	0.3	0.0047	90.3	0.360	1.0	0.0059	82.8	0.223	2.3	0.0112	62.4
$WKM_{R.3\%}$	0.481	-1.9	0.0076	79.5	0.333	-1.7	0.0096	70.3	0.201	0.1	0.0125	63.3
$WKM_{R.5\%}$	0.482	-1.8	0.0069	82.1	0.335	-1.5	0.0084	75.3	0.204	0.4	0.0117	68.2
$WKM_{R.10\%}$	0.486	-1.4	0.0061	84.2	0.340	-1.0	0.0072	80.7	0.209	0.9	0.0109	73.7
$WKM_{R.20\%}$	0.491	-0.9	0.0054	87.1	0.347	-0.3	0.0062	84.9	0.215	1.5	0.0106	77.8
$WKM_{R.30\%}$	0.495	-0.5	0.0050	88.0	0.353	0.3	0.0059	87.3	0.219	1.9	0.0108	78.2
$WKM_{R.40\%}$	0.499	-0.1	0.0048	89.2	0.357	0.7	0.0058	88.2	0.221	2.1	0.0109	78.4
$WKM_{R.50\%}$	0.502	0.2	0.0047	89.8	0.359	0.9	0.0058	88.4	0.223	2.3	0.0110	78.6
$WKM_{U.3\%}$	0.482	-1.8	0.0071	83.5	0.334	-1.6	0.0089	77.8	0.202	0.2	0.0119	71.3
$WKM_{U.5\%}$	0.485	-1.5	0.0062	87.0	0.338	-1.2	0.0075	84.4	0.207	0.7	0.0110	79.1
$WKM_{U.10\%}$	0.490	-1.0	0.0054	89.1	0.346	-0.4	0.0062	88.6	0.215	1.5	0.0103	84.2
$WKM_{U.20\%}$	0.500	0.0	0.0047	91.0	0.360	1.0	0.0054	92.9	0.226	2.6	0.0107	86.4
$WKM_{U.30\%}$	0.509	0.9	0.0044	92.2	0.371	2.1	0.0055	93.1	0.234	3.4	0.0115	85.5
$WKM_{U.40\%}$	0.516	1.6	0.0044	91.6	0.378	2.8	0.0058	92.5	0.240	4.0	0.0121	85.0
$WKM_{U.50\%}$	0.522	2.2	0.0045	90.9	0.384	3.4	0.0061	91.5	0.244	4.4	0.0127	84.5
$WKM_{\mathcal{N}(0.1)}$	0.475	-2.5	0.0105	75.4	0.320	-3.0	0.0149	63.2	0.187	-1.3	0.0203	49.7
$WKM_{\mathcal{N}(0.5)}$	0.488	-1.2	0.0058	88.4	0.339	-1.1	0.0073	86.7	0.203	0.3	0.0123	78.3
$WKM_{\mathcal{N}(1)}$	0.507	0.7	0.0045	91.8	0.364	1.4	0.0056	92.7	0.225	2.5	0.0113	84.5
$WKM_{\mathcal{N}(2)}$	0.524	2.4	0.0046	90.8	0.384	3.4	0.0062	91.6	0.242	4.2	0.0126	84.7
$WKM_{\mathcal{N}(5)}$	0.532	3.2	0.0050	89.6	0.393	4.3	0.0070	89.1	0.250	5.0	0.0137	83.1

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Statistics in Medicine

Table 19. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100)**. **30% LOD=200 copies/ml and 70% LOD=500 copies/ml** leading to an overall censoring rate of **64%**.

Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=49%¶				True value: 0.35 Censoring rate=58%				True value: 0.2 Censoring rate=63%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.544	4.4	0.0065	86.8	0.403	5.3	0.0098	87.8	0.268	6.8	0.0232	67.5
$WKM_{4.1}$	0.506	0.6	0.0105	75.4	0.400	5.0	0.0156	71.5	0.344	14.4	0.0361	49.7
$WKM_{4.2}$	0.582	8.2	0.0128	62.4	0.509	15.9	0.0325	32.6	0.471	27.1	0.0817	5.5
$WKM_{8.1}$	0.528	2.8	0.0090	77.4	0.438	8.8	0.0174	64.9	0.393	19.3	0.0483	25.8
$WKM_{D.3\%}$	0.480	-2.0	0.0098	73.1	0.331	-1.9	0.0129	61.0	0.216	1.6	0.0193	52.0
$WKM_{D.5\%}$	0.482	-1.8	0.0089	75.8	0.334	-1.6	0.0116	63.4	0.219	1.9	0.0184	53.7
$WKM_{D.10\%}$	0.487	-1.3	0.0078	80.7	0.341	-0.9	0.0100	68.4	0.224	2.4	0.0177	55.2
$WKM_{D.20\%}$	0.495	-0.5	0.0067	84.4	0.350	0.0	0.0088	72.8	0.231	3.1	0.0177	54.9
$WKM_{D.30\%}$	0.500	0.0	0.0062	85.9	0.357	0.7	0.0083	74.9	0.235	3.5	0.0180	54.3
$WKM_{D.40\%}$	0.505	0.5	0.0060	86.7	0.361	1.1	0.0082	75.8	0.238	3.8	0.0183	53.6
$WKM_{D.50\%}$	0.508	0.8	0.0059	87.0	0.364	1.4	0.0082	76.4	0.240	4.0	0.0186	53.5
$WKM_{R.3\%}$	0.480	-2.0	0.0098	75.1	0.331	-1.9	0.0127	67.7	0.216	1.6	0.0192	60.8
$WKM_{R.5\%}$	0.482	-1.8	0.0088	77.9	0.334	-1.6	0.0113	71.6	0.219	1.9	0.0182	65.8
$WKM_{R.10\%}$	0.487	-1.3	0.0077	82.0	0.341	-0.9	0.0098	77.1	0.224	2.4	0.0175	69.9
$WKM_{R.20\%}$	0.494	-0.6	0.0068	84.5	0.350	0.0	0.0086	82.3	0.230	3.0	0.0175	73.1
$WKM_{R.30\%}$	0.499	-0.1	0.0063	86.1	0.355	0.5	0.0082	84.9	0.234	3.4	0.0178	73.2
$WKM_{R.40\%}$	0.503	0.3	0.0060	87.0	0.360	1.0	0.0081	86.0	0.237	3.7	0.0181	73.9
$WKM_{R.50\%}$	0.506	0.6	0.0058	87.1	0.362	1.2	0.0080	86.4	0.239	3.9	0.0183	74.2
$WKM_{U.3\%}$	0.482	-1.8	0.0093	80.5	0.333	-1.7	0.0117	74.4	0.218	1.8	0.0186	69.1
$WKM_{U.5\%}$	0.485	-1.5	0.0083	84.1	0.338	-1.2	0.0103	81.7	0.222	2.2	0.0176	76.1
$WKM_{U.10\%}$	0.493	-0.7	0.0070	87.5	0.349	-0.1	0.0087	87.3	0.230	3.0	0.0173	80.4
$WKM_{U.20\%}$	0.505	0.5	0.0059	89.5	0.365	1.5	0.0077	91.0	0.242	4.2	0.0181	81.6
$WKM_{U.30\%}$	0.515	1.5	0.0055	90.1	0.377	2.7	0.0078	91.2	0.250	5.0	0.0193	80.2
$WKM_{U.40\%}$	0.523	2.3	0.0055	89.9	0.385	3.5	0.0081	90.6	0.256	5.6	0.0204	79.1
$WKM_{U.50\%}$	0.530	3.0	0.0057	89.1	0.390	4.0	0.0086	90.4	0.26	6.0	0.0212	78.4
$WKM_{\mathcal{N}(0.1)}$	0.471	-2.9	0.0138	71.3	0.315	-3.5	0.0212	57.1	0.202	0.2	0.0277	53.1
$WKM_{\mathcal{N}(0.5)}$	0.490	-1.0	0.0074	86.9	0.341	-0.9	0.0105	82.8	0.221	2.1	0.0192	75.3
$WKM_{\mathcal{N}(1)}$	0.513	1.3	0.0057	89.9	0.369	1.9	0.0080	89.4	0.242	4.2	0.0190	79.4
$WKM_{\mathcal{N}(2)}$	0.532	3.2	0.0059	89.1	0.390	4.0	0.0087	89.5	0.259	5.9	0.0212	78.6
$WKM_{\mathcal{N}(5)}$	0.542	4.2	0.0064	87.1	0.400	5.0	0.0096	88.3	0.267	6.7	0.0228	77.8

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Prepared using simauth.cls

Statistics
in Medicine

Table 20. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100). 70% LOD=100 copies/ml and 30% LOD=200 copies/ml** leading to an overall censoring rate of **45%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=24%¶				True value: 0.35 Censoring rate=34%				True value: 0.2 Censoring rate=42%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.521	2.1	0.0037	93.3	0.382	3.2	0.0046	92.0	0.235	3.5	0.0062	90.6
$WKM_{4.1}$	0.513	1.3	0.0039	92.6	0.391	4.1	0.0086	78.3	0.299	9.9	0.0183	53.3
$WKM_{4.2}$	0.528	2.8	0.0052	86.7	0.423	7.3	0.0114	67.0	0.349	14.9	0.0287	34.9
$WKM_{8.1}$	0.545	4.5	0.0059	81.6	0.450	10	0.0146	53.3	0.386	18.6	0.0394	11.9
$WKM_{D.3\%}$	0.508	0.8	0.0055	85.6	0.366	1.6	0.0079	67.1	0.219	1.9	0.0091	55.9
$WKM_{D.5\%}$	0.508	0.8	0.0049	87.0	0.367	1.7	0.0066	73.8	0.219	1.9	0.0075	60.8
$WKM_{D.10\%}$	0.509	0.9	0.0043	90.4	0.368	1.8	0.0054	79.9	0.221	2.1	0.0062	66.5
$WKM_{D.20\%}$	0.510	1.0	0.0038	92.4	0.370	2.0	0.0047	85.3	0.224	2.4	0.0058	69.7
$WKM_{D.30\%}$	0.511	1.1	0.0037	93.7	0.372	2.2	0.0045	86.5	0.226	2.6	0.0058	70.7
$WKM_{D.40\%}$	0.512	1.2	0.0036	93.9	0.373	2.3	0.0044	86.9	0.227	2.7	0.0058	70.7
$WKM_{D.50\%}$	0.513	1.3	0.0036	93.6	0.374	2.4	0.0045	87.2	0.227	2.7	0.0058	71.0
$WKM_{R.3\%}$	0.508	0.8	0.0056	86.9	0.366	1.6	0.0082	71.6	0.219	1.9	0.0094	60.5
$WKM_{R.5\%}$	0.508	0.8	0.0049	87.8	0.367	1.7	0.0067	77.0	0.220	2.0	0.0077	70.0
$WKM_{R.10\%}$	0.509	0.9	0.0043	89.5	0.368	1.8	0.0055	83.5	0.222	2.2	0.0064	78.1
$WKM_{R.20\%}$	0.510	1.0	0.0038	90.1	0.370	2.0	0.0047	86.7	0.224	2.4	0.0058	83.2
$WKM_{R.30\%}$	0.511	1.1	0.0037	90.6	0.371	2.1	0.0045	88.0	0.226	2.6	0.0058	84.8
$WKM_{R.40\%}$	0.512	1.2	0.0036	91.5	0.372	2.2	0.0045	88.4	0.227	2.7	0.0058	85.4
$WKM_{R.50\%}$	0.513	1.3	0.0036	92.0	0.373	2.3	0.0044	89.1	0.227	2.7	0.0058	85.9
$WKM_{U.3\%}$	0.508	0.8	0.0052	89.5	0.366	1.6	0.0074	80.0	0.219	1.9	0.0085	71.1
$WKM_{U.5\%}$	0.508	0.8	0.0046	89.8	0.367	1.7	0.0061	85.7	0.221	2.1	0.0068	81.8
$WKM_{U.10\%}$	0.509	0.9	0.0040	90.6	0.369	1.9	0.0049	90.0	0.223	2.3	0.0058	88.8
$WKM_{U.20\%}$	0.511	1.1	0.0036	92.7	0.372	2.2	0.0043	92.0	0.227	2.7	0.0056	91.2
$WKM_{U.30\%}$	0.513	1.3	0.0035	93.4	0.374	2.4	0.0042	93.0	0.230	3.0	0.0057	91.6
$WKM_{U.40\%}$	0.515	1.5	0.0035	93.1	0.377	2.7	0.0043	92.2	0.231	3.1	0.0058	91.2
$WKM_{U.50\%}$	0.517	1.7	0.0035	93.0	0.378	2.8	0.0044	91.7	0.233	3.3	0.0059	90.8
$WKM_{\mathcal{N}(0.1)}$	0.506	0.6	0.0081	79.6	0.363	1.3	0.0133	54.6	0.216	1.6	0.0192	36.9
$WKM_{\mathcal{N}(0.5)}$	0.508	0.8	0.0041	90.9	0.367	1.7	0.0054	88.1	0.220	2.0	0.0073	80.6
$WKM_{\mathcal{N}(1)}$	0.511	1.1	0.0035	93.1	0.372	2.2	0.0044	91.8	0.226	2.6	0.0059	89.4
$WKM_{\mathcal{N}(2)}$	0.516	1.6	0.0035	92.8	0.377	2.7	0.0044	91.9	0.231	3.1	0.0059	91.2
$WKM_{\mathcal{N}(5)}$	0.520	2.0	0.0036	92.6	0.381	3.1	0.0046	91.2	0.234	3.4	0.0061	90.4

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Statistics in Medicine

Table 21. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100)**. **30% LOD=100 copies/ml and 70% LOD=200 copies/ml** leading to an overall censoring rate pf **50%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=30%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=47%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.525	2.5	0.0040	92.0	0.386	3.6	0.0053	90.7	0.239	3.9	0.0081	87.9
$WKM_{4.1}$	0.516	1.6	0.0050	90.9	0.402	5.2	0.0113	68.5	0.310	11.0	0.0234	50.0
$WKM_{4.2}$	0.537	3.7	0.0066	81.1	0.436	8.6	0.0145	61.3	0.366	16.6	0.0363	32.1
$WKM_{8.1}$	0.555	5.5	0.0073	75.6	0.465	11.5	0.0183	46.6	0.407	20.7	0.0485	9.5
$WKM_{D.3\%}$	0.508	0.8	0.0072	78.9	0.365	1.5	0.0107	59.9	0.223	2.3	0.0127	47.0
$WKM_{D.5\%}$	0.509	0.9	0.0062	84.0	0.367	1.7	0.0087	67.6	0.223	2.3	0.0107	51.7
$WKM_{D.10\%}$	0.509	0.9	0.0051	88.5	0.369	1.9	0.0067	74.1	0.225	2.5	0.0087	59.5
$WKM_{D.20\%}$	0.511	1.1	0.0043	90.7	0.371	2.1	0.0055	80.2	0.227	2.7	0.0080	63.8
$WKM_{D.30\%}$	0.513	1.3	0.0041	91.8	0.374	2.4	0.0052	83.2	0.229	2.9	0.0079	65.0
$WKM_{D.40\%}$	0.515	1.5	0.0040	92.8	0.375	2.5	0.0051	84.2	0.230	3.0	0.0079	65.6
$WKM_{D.50\%}$	0.516	1.6	0.0040	92.8	0.376	2.6	0.0051	84.5	0.231	3.1	0.0079	65.7
$WKM_{R.3\%}$	0.509	0.9	0.0073	82.5	0.365	1.5	0.0107	64.8	0.223	2.3	0.0127	56.0
$WKM_{R.5\%}$	0.509	0.9	0.0062	85.0	0.366	1.6	0.0088	71.6	0.223	2.3	0.0107	64.7
$WKM_{R.10\%}$	0.510	1.0	0.0051	87.3	0.368	1.8	0.0068	78.0	0.225	2.5	0.0088	74.7
$WKM_{R.20\%}$	0.511	1.1	0.0044	89.1	0.371	2.1	0.0056	83.8	0.227	2.7	0.0080	81.3
$WKM_{R.30\%}$	0.513	1.3	0.0041	89.7	0.373	2.3	0.0052	86.6	0.229	2.9	0.0078	83.2
$WKM_{R.40\%}$	0.514	1.4	0.0040	90.2	0.375	2.5	0.0051	87.8	0.230	3.0	0.0078	83.6
$WKM_{R.50\%}$	0.516	1.6	0.0040	90.2	0.376	2.6	0.0051	88.4	0.231	3.1	0.0078	83.7
$WKM_{U.3\%}$	0.509	0.9	0.0066	87.9	0.366	1.6	0.0096	76.5	0.222	2.2	0.0114	68.0
$WKM_{U.5\%}$	0.510	1.0	0.0055	89.0	0.368	1.8	0.0075	81.8	0.224	2.4	0.0094	79.0
$WKM_{U.10\%}$	0.511	1.1	0.0045	90.7	0.370	2.0	0.0057	88.1	0.226	2.6	0.0078	87.9
$WKM_{U.20\%}$	0.514	1.4	0.0039	92.2	0.374	2.4	0.0048	91.4	0.231	3.1	0.0074	89.7
$WKM_{U.30\%}$	0.516	1.6	0.0038	92.4	0.378	2.8	0.0048	92.0	0.233	3.3	0.0076	89.7
$WKM_{U.40\%}$	0.519	1.9	0.0038	92.1	0.380	3.0	0.0049	91.6	0.235	3.5	0.0077	90.1
$WKM_{U.50\%}$	0.521	2.1	0.0038	92.1	0.382	3.2	0.0050	91.4	0.236	3.6	0.0078	90.0
$WKM_{\mathcal{N}(0.1)}$	0.506	0.6	0.0104	73.2	0.361	1.1	0.0176	48.2	0.216	1.6	0.0247	35.9
$WKM_{\mathcal{N}(0.5)}$	0.510	1.0	0.0045	90.6	0.368	1.8	0.0063	86.9	0.224	2.4	0.0096	79.8
$WKM_{\mathcal{N}(1)}$	0.514	1.4	0.0038	92.4	0.375	2.5	0.0050	91.5	0.230	3.0	0.0078	88.4
$WKM_{\mathcal{N}(2)}$	0.520	2.0	0.0039	92.1	0.381	3.1	0.0050	91.3	0.235	3.5	0.0078	89.8
$WKM_{\mathcal{N}(5)}$	0.524	2.4	0.0040	91.5	0.385	3.5	0.0052	90.8	0.238	3.8	0.0080	89.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Statistics
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Table 22. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100). 70% LOD=200 copies/ml and 30% LOD=500 copies/ml** leading to an overall censoring rate of **57%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=39%¶				True value: 0.35 Censoring rate=49%				True value: 0.2 Censoring rate=56%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.350	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.534	3.4	0.0051	90.5	0.396	4.6	0.0072	88.9	0.252	5.2	0.0140	79.4
$WKM_{4.1}$	0.532	3.2	0.0073	85.9	0.442	9.2	0.0178	58.0	0.352	15.2	0.0365	40.6
$WKM_{4.2}$	0.563	6.3	0.0099	71.3	0.482	13.2	0.0248	42.4	0.428	22.8	0.0609	15.5
$WKM_{8.1}$	0.589	8.9	0.0120	60.4	0.517	16.7	0.0328	22.7	0.476	27.6	0.0816	1.4
$WKM_{D.3\%}$	0.513	1.3	0.0090	72.8	0.369	1.9	0.0127	55.6	0.231	3.1	0.0168	45.9
$WKM_{D.5\%}$	0.514	1.4	0.0072	78.9	0.373	2.3	0.0100	62.9	0.233	3.3	0.0148	47.9
$WKM_{D.10\%}$	0.515	1.5	0.0058	85.1	0.376	2.6	0.0078	69.6	0.236	3.6	0.0135	50.1
$WKM_{D.20\%}$	0.517	1.7	0.0051	88.3	0.379	2.9	0.0068	75.5	0.239	3.9	0.0130	51.7
$WKM_{D.30\%}$	0.520	2.0	0.0049	89.8	0.382	3.2	0.0067	77.0	0.241	4.1	0.0131	52.3
$WKM_{D.40\%}$	0.522	2.2	0.0048	91.0	0.384	3.4	0.0066	77.9	0.242	4.2	0.0132	52.5
$WKM_{D.50\%}$	0.523	2.3	0.0048	91.1	0.385	3.5	0.0067	78.6	0.243	4.3	0.0133	52.9
$WKM_{R.3\%}$	0.514	1.4	0.0092	75.6	0.370	2.0	0.0134	58.9	0.233	3.3	0.0176	51.4
$WKM_{R.5\%}$	0.515	1.5	0.0074	81.7	0.373	2.3	0.0105	68.8	0.234	3.4	0.0152	59.1
$WKM_{R.10\%}$	0.515	1.5	0.0059	86.0	0.375	2.5	0.0081	76.8	0.236	3.6	0.0136	67.8
$WKM_{R.20\%}$	0.517	1.7	0.0051	87.5	0.379	2.9	0.0069	83.0	0.239	3.9	0.0130	73.7
$WKM_{R.30\%}$	0.519	1.9	0.0049	88.8	0.381	3.1	0.0067	85.0	0.241	4.1	0.0130	75.9
$WKM_{R.40\%}$	0.521	2.1	0.0048	89.1	0.383	3.3	0.0066	86.3	0.242	4.2	0.0131	76.4
$WKM_{R.50\%}$	0.522	2.2	0.0048	89.2	0.384	3.4	0.0067	86.7	0.243	4.3	0.0132	76.7
$WKM_{U.3\%}$	0.513	1.3	0.0083	83.9	0.370	2.0	0.0118	70.8	0.232	3.2	0.0161	62.3
$WKM_{U.5\%}$	0.515	1.5	0.0066	86.7	0.374	2.4	0.0092	80.3	0.234	3.4	0.0141	75.2
$WKM_{U.10\%}$	0.516	1.6	0.0053	90.0	0.377	2.7	0.0071	88.0	0.238	3.8	0.0129	82.4
$WKM_{U.20\%}$	0.520	2.0	0.0047	91.1	0.382	3.2	0.0063	91.4	0.242	4.2	0.0127	84.1
$WKM_{U.30\%}$	0.523	2.3	0.0046	90.8	0.386	3.6	0.0064	91.3	0.245	4.5	0.0130	84.0
$WKM_{U.40\%}$	0.527	2.7	0.0047	90.8	0.389	3.9	0.0066	90.2	0.247	4.7	0.0133	83.9
$WKM_{U.50\%}$	0.529	2.9	0.0048	91.1	0.391	4.1	0.0068	90.0	0.249	4.9	0.0135	83.7
$WKM_{\mathcal{N}(0.1)}$	0.512	1.2	0.0144	65.0	0.371	2.1	0.0238	42.8	0.238	3.8	0.0348	28.7
$WKM_{\mathcal{N}(0.5)}$	0.515	1.5	0.0056	89.5	0.376	2.6	0.0082	84.6	0.237	3.7	0.0152	75.7
$WKM_{\mathcal{N}(1)}$	0.522	2.2	0.0047	91.6	0.384	3.4	0.0066	90.5	0.243	4.3	0.0134	83.2
$WKM_{\mathcal{N}(2)}$	0.529	2.9	0.0048	90.8	0.391	4.1	0.0068	89.7	0.248	4.8	0.0136	83.2
$WKM_{\mathcal{N}(5)}$	0.533	3.3	0.0051	89.6	0.395	4.5	0.0071	89.2	0.251	5.1	0.0139	83.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

Statistics in Medicine

Table 23. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=100). 30% LOD=200 copies/ml and 70% LOD=500 copies/ml** leading to an overall censoring rate of **64%**.

Estimators are derived using only Z_1 representing the baseline HIV-1 RNA

	True value: 0.5 Censoring rate=49%¶				True value: 0.35 Censoring rate=58%				True value: 0.2 Censoring rate=63%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.497	-0.3	0.0026	93.9	0.35	0.0	0.0023	94.7	0.201	0.1	0.0017	93.7
KM PO	0.544	4.4	0.0065	86.8	0.403	5.3	0.0098	87.8	0.268	6.8	0.0232	67.5
$WKM_{4.1}$	0.544	4.4	0.0121	72.3	0.457	10.7	0.0252	51.7	0.378	17.8	0.0501	42.7
$WKM_{4.2}$	0.578	7.8	0.0136	65.3	0.499	14.9	0.0325	42.5	0.454	25.4	0.0764	15.1
$WKM_{8.1}$	0.603	10.3	0.0161	54.0	0.537	18.7	0.0415	20.9	0.504	30.4	0.0994	2.2
$WKM_{D.3\%}$	0.519	1.9	0.0122	66.0	0.376	2.6	0.0180	48.0	0.248	4.8	0.0270	39.7
$WKM_{D.5\%}$	0.520	2.0	0.0103	69.7	0.378	2.8	0.0151	54.0	0.250	5.0	0.0248	42.1
$WKM_{D.10\%}$	0.521	2.1	0.0083	77.0	0.380	3.0	0.0121	60.7	0.252	5.2	0.0230	42.9
$WKM_{D.20\%}$	0.524	2.4	0.0070	81.2	0.384	3.4	0.0104	67.2	0.255	5.5	0.0222	43.1
$WKM_{D.30\%}$	0.527	2.7	0.0065	84.4	0.387	3.7	0.0099	69.5	0.257	5.7	0.0222	42.7
$WKM_{D.40\%}$	0.529	2.9	0.0063	86.4	0.389	3.9	0.0098	70.5	0.258	5.8	0.0223	43.4
$WKM_{D.50\%}$	0.530	3.0	0.0062	86.7	0.390	4.0	0.0097	71.5	0.259	5.9	0.0224	43.4
$WKM_{R.3\%}$	0.519	1.9	0.0124	70.7	0.375	2.5	0.0182	55.6	0.247	4.7	0.0271	48.6
$WKM_{R.5\%}$	0.519	1.9	0.0104	75.6	0.377	2.7	0.0153	65.2	0.249	4.9	0.0249	54.5
$WKM_{R.10\%}$	0.521	2.1	0.0084	78.9	0.380	3.0	0.0124	73.1	0.252	5.2	0.0230	60.5
$WKM_{R.20\%}$	0.524	2.4	0.0070	81.9	0.383	3.3	0.0105	79.9	0.255	5.5	0.0222	64.9
$WKM_{R.30\%}$	0.526	2.6	0.0065	84.0	0.386	3.6	0.0100	81.6	0.257	5.7	0.0221	67.0
$WKM_{R.40\%}$	0.528	2.8	0.0063	85.2	0.388	3.8	0.0098	82.3	0.258	5.8	0.0222	67.5
$WKM_{R.50\%}$	0.530	3.0	0.0062	85.6	0.389	3.9	0.0098	82.7	0.259	5.9	0.0222	67.9
$WKM_{U.3\%}$	0.519	1.9	0.0111	79.7	0.377	2.7	0.0166	69.1	0.249	4.9	0.0260	58.8
$WKM_{U.5\%}$	0.520	2.0	0.0091	83.5	0.378	2.8	0.0135	79.3	0.251	5.1	0.0237	69.4
$WKM_{U.10\%}$	0.523	2.3	0.0071	86.7	0.382	3.2	0.0105	85.4	0.255	5.5	0.0219	75.8
$WKM_{U.20\%}$	0.528	2.8	0.0061	87.7	0.389	3.9	0.0091	88.9	0.259	5.9	0.0216	77.6
$WKM_{U.30\%}$	0.532	3.2	0.0059	88.2	0.393	4.3	0.0091	89.3	0.262	6.2	0.0220	77.7
$WKM_{U.40\%}$	0.536	3.6	0.0059	88.1	0.396	4.6	0.0092	89.2	0.264	6.4	0.0222	77.9
$WKM_{U.50\%}$	0.538	3.8	0.0061	87.8	0.398	4.8	0.0093	88.7	0.265	6.5	0.0225	77.4
$WKM_{\mathcal{N}(0.1)}$	0.518	1.8	0.0202	59.1	0.373	2.3	0.0350	34.9	0.243	4.3	0.0455	33.9
$WKM_{\mathcal{N}(0.5)}$	0.523	2.3	0.0074	87.6	0.381	3.1	0.0128	81.3	0.252	5.2	0.0245	69.1
$WKM_{\mathcal{N}(1)}$	0.531	3.1	0.0060	88.3	0.390	4.0	0.0095	87.5	0.260	6.0	0.0222	77.2
$WKM_{\mathcal{N}(2)}$	0.538	3.8	0.0061	87.7	0.398	4.8	0.0094	88.7	0.265	6.5	0.0226	77.6
$WKM_{\mathcal{N}(5)}$	0.543	4.3	0.0064	86.9	0.402	5.2	0.0097	88.1	0.268	6.8	0.0231	77.3

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 24. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=100)**.

Overall censoring rate=35% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=26%¶				True value: 0.35 Censoring rate=29%				True value: 0.2 Censoring rate=32%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.502	0.2	0.0024	94.0	0.352	0.2	0.0023	94.1	0.201	0.1	0.0016	93.5
KM PO	0.569	6.9	0.0080	76.6	0.429	7.9	0.0098	73.6	0.272	7.2	0.0083	77.2
IPCW	0.505	0.5	0.0041	90.8	0.356	0.6	0.0038	92.2	0.204	0.4	0.0026	93.7
KMIB	0.516	1.6	0.0037	92.7	0.368	1.8	0.0036	90.2	0.215	1.5	0.0026	93.8
$WKM_{4.1}$	0.511	1.1	0.0037	92.6	0.365	1.5	0.0035	90.4	0.220	2.0	0.0029	92.8
$WKM_{4.2}$	0.519	1.9	0.0039	91.0	0.378	2.8	0.0042	93.1	0.238	3.8	0.0044	87.2
$WKM_{8.1}$	0.514	1.4	0.0039	91.0	0.374	2.4	0.0041	90.3	0.234	3.4	0.0041	87.7
$WKM_{D.3\%}$	0.507	0.7	0.0038	90.4	0.355	0.5	0.0034	90.0	0.202	0.2	0.0023	88.5
$WKM_{D.5\%}$	0.508	0.8	0.0037	91.1	0.356	0.6	0.0034	89.6	0.203	0.3	0.0022	88.6
$WKM_{D.10\%}$	0.511	1.1	0.0037	90.6	0.360	1.0	0.0034	89.4	0.206	0.6	0.0023	89.4
$WKM_{D.20\%}$	0.517	1.7	0.0038	89.9	0.366	1.6	0.0036	88.1	0.212	1.2	0.0024	88.3
$WKM_{D.30\%}$	0.521	2.1	0.0040	89.3	0.371	2.1	0.0038	87.7	0.216	1.6	0.0026	86.8
$WKM_{D.40\%}$	0.525	2.5	0.0042	89.2	0.375	2.5	0.0040	86.7	0.220	2.0	0.0028	86.6
$WKM_{D.50\%}$	0.529	2.9	0.0043	88.8	0.379	2.9	0.0043	90.3	0.223	2.3	0.0030	86.1
$WKM_{R.3\%}$	0.507	0.7	0.0037	90.1	0.355	0.5	0.0034	90.6	0.202	0.2	0.0023	89.5
$WKM_{R.5\%}$	0.508	0.8	0.0037	90.3	0.357	0.7	0.0033	91.0	0.203	0.3	0.0023	90.4
$WKM_{R.10\%}$	0.511	1.1	0.0037	90.0	0.360	1.0	0.0033	90.7	0.206	0.6	0.0023	91.0
$WKM_{R.20\%}$	0.516	1.6	0.0038	89.9	0.366	1.6	0.0035	89.9	0.212	1.2	0.0024	90.8
$WKM_{R.30\%}$	0.521	2.1	0.0039	89.5	0.370	2.0	0.0037	89.0	0.216	1.6	0.0026	90.2
$WKM_{R.40\%}$	0.524	2.4	0.0040	89.1	0.375	2.5	0.0039	89.0	0.220	2.0	0.0028	90.1
$WKM_{R.50\%}$	0.527	2.7	0.0042	88.6	0.378	2.8	0.0041	91.6	0.223	2.3	0.0030	90.2
$WKM_{U.3\%}$	0.507	0.7	0.0037	90.9	0.356	0.6	0.0034	91.6	0.203	0.3	0.0023	91.0
$WKM_{U.5\%}$	0.510	1.0	0.0037	91.3	0.359	0.9	0.0033	92.1	0.205	0.5	0.0022	92.3
$WKM_{U.10\%}$	0.516	1.6	0.0038	91.1	0.365	1.5	0.0034	91.1	0.211	1.1	0.0024	93.0
$WKM_{U.20\%}$	0.526	2.6	0.0042	90.1	0.376	2.6	0.0040	89.8	0.222	2.2	0.0029	91.4
$WKM_{U.30\%}$	0.535	3.5	0.0047	88.4	0.387	3.7	0.0047	87.2	0.233	3.3	0.0037	89.5
$WKM_{U.40\%}$	0.544	4.4	0.0053	85.9	0.398	4.8	0.0057	83.6	0.242	4.2	0.0045	87.3
$WKM_{U.50\%}$	0.551	5.1	0.0059	83.6	0.406	5.6	0.0066	89.9	0.250	5.0	0.0054	84.7
$WKM_{\mathcal{N}(0.1)}$	0.502	0.2	0.0037	90.5	0.350	0.0	0.0035	92.0	0.198	-0.2	0.0024	90.7
$WKM_{\mathcal{N}(0.5)}$	0.513	1.3	0.0036	91.9	0.362	1.2	0.0034	90.6	0.207	0.7	0.0023	93.1
$WKM_{\mathcal{N}(1)}$	0.533	3.3	0.0044	88.9	0.384	3.4	0.0045	82.7	0.226	2.6	0.0033	90.4
$WKM_{\mathcal{N}(2)}$	0.554	5.4	0.0062	82.5	0.409	5.9	0.0070	74.2	0.251	5.1	0.0055	84.7
$WKM_{\mathcal{N}(5)}$	0.566	6.6	0.0076	77.0	0.425	7.5	0.0092	92.6	0.267	6.7	0.0077	77.4

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 25. Monte Carlo results for estimating **classical survival curve** with dependent censoring **(N=100)**.

Censoring rate=51% and the link functions **uncorrectly** specified.

	True value: 0.5 Censoring rate=34%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=46%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.502	0.2	0.0026	93.8	0.353	0.3	0.0025	92.0	0.202	0.2	0.0017	93.0
KM PO	0.607	10.7	0.0148	51.8	0.474	12.4	0.0195	49.5	0.317	11.7	0.0186	38.8
IPCW	0.502	0.2	0.0090	83.0	0.354	0.4	0.0089	83.9	0.202	0.2	0.0069	86.8
KMIB	0.527	2.7	0.0047	92.3	0.383	3.3	0.0054	90.0	0.235	3.5	0.0050	89.9
$WKM_{4.1}$	0.524	2.4	0.0053	88.9	0.390	4.0	0.0067	86.0	0.261	6.1	0.0089	78.0
$WKM_{4.2}$	0.539	3.9	0.0061	84.0	0.415	6.5	0.0094	75.2	0.299	9.9	0.0150	59.2
$WKM_{8.1}$	0.533	3.3	0.0055	86.8	0.407	5.7	0.0083	79.2	0.286	8.6	0.0124	64.2
$WKM_{D.3\%}$	0.511	1.1	0.0047	90.2	0.362	1.2	0.0049	83.3	0.213	1.3	0.0043	75.9
$WKM_{D.5\%}$	0.514	1.4	0.0045	90.5	0.365	1.5	0.0048	83.8	0.216	1.6	0.0042	77.7
$WKM_{D.10\%}$	0.519	1.9	0.0046	90.5	0.371	2.1	0.0048	84.9	0.222	2.2	0.0043	77.5
$WKM_{D.20\%}$	0.527	2.7	0.0048	90.5	0.380	3.0	0.0052	84.2	0.231	3.1	0.0048	75.9
$WKM_{D.30\%}$	0.534	3.4	0.0051	89.3	0.388	3.8	0.0057	81.6	0.239	3.9	0.0055	73.3
$WKM_{D.40\%}$	0.540	4.0	0.0055	88.2	0.395	4.5	0.0063	79.8	0.244	4.4	0.0060	70.6
$WKM_{D.50\%}$	0.545	4.5	0.0059	86.8	0.401	5.1	0.0069	77.3	0.249	4.9	0.0065	68.9
$WKM_{R.3\%}$	0.512	1.2	0.0046	90.1	0.363	1.3	0.0049	84.6	0.213	1.3	0.0042	80.6
$WKM_{R.5\%}$	0.514	1.4	0.0045	90.3	0.365	1.5	0.0048	85.5	0.216	1.6	0.0042	81.8
$WKM_{R.10\%}$	0.519	1.9	0.0045	89.8	0.371	2.1	0.0048	86.4	0.222	2.2	0.0042	83.5
$WKM_{R.20\%}$	0.526	2.6	0.0047	90.0	0.380	3.0	0.0051	86.4	0.231	3.1	0.0047	83.3
$WKM_{R.30\%}$	0.533	3.3	0.0050	89.0	0.388	3.8	0.0056	85.2	0.239	3.9	0.0053	82.2
$WKM_{R.40\%}$	0.539	3.9	0.0053	86.2	0.395	4.5	0.0061	83.3	0.244	4.4	0.0059	81.1
$WKM_{R.50\%}$	0.544	4.4	0.0057	84.4	0.400	5.0	0.0067	81.9	0.249	4.9	0.0064	79.4
$WKM_{U.3\%}$	0.513	1.3	0.0046	91.6	0.364	1.4	0.0048	87.1	0.215	1.5	0.0041	85.2
$WKM_{U.5\%}$	0.517	1.7	0.0045	91.7	0.368	1.8	0.0047	88.9	0.220	2.0	0.0042	86.7
$WKM_{U.10\%}$	0.526	2.6	0.0048	91.4	0.379	2.9	0.0051	89.8	0.231	3.1	0.0046	87.9
$WKM_{U.20\%}$	0.541	4.1	0.0055	88.6	0.397	4.7	0.0063	86.8	0.250	5.0	0.0063	85.2
$WKM_{U.30\%}$	0.555	5.5	0.0067	83.0	0.414	6.4	0.0082	81.1	0.267	6.7	0.0084	80.0
$WKM_{U.40\%}$	0.568	6.8	0.0082	77.0	0.430	8.0	0.0105	73.8	0.281	8.1	0.0107	74.5
$WKM_{U.50\%}$	0.579	7.9	0.0099	71.4	0.443	9.3	0.0128	66.2	0.292	9.2	0.0128	69.9
$WKM_{\mathcal{N}(0.1)}$	0.505	0.5	0.0050	89.4	0.355	0.5	0.0056	84.4	0.204	0.4	0.0052	78.8
$WKM_{\mathcal{N}(0.5)}$	0.524	2.4	0.0045	90.7	0.375	2.5	0.0051	89.0	0.221	2.1	0.0045	86.9
$WKM_{\mathcal{N}(1)}$	0.555	5.5	0.0067	82.0	0.410	6.0	0.0079	82.1	0.254	5.4	0.0071	83.6
$WKM_{\mathcal{N}(2)}$	0.585	8.5	0.0109	66.7	0.447	9.7	0.0136	64.5	0.290	9.0	0.0126	70.8
$WKM_{\mathcal{N}(5)}$	0.603	10.3	0.0140	54.0	0.468	11.8	0.0182	52.5	0.312	11.2	0.0173	59.8

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 26. Monte Carlo results for estimating **classical survival curve** with independent censoring **(N=100)**.

Censoring rate=40% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=35%¶				True value: 0.35 Censoring rate=37%				True value: 0.2 Censoring rate=39%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.501	0.1	0.0024	94.8	0.349	-0.1	0.0021	94.6	0.198	-0.2	0.0015	93.5
KM PO	0.502	0.2	0.0037	94.5	0.350	0.0	0.0036	94.4	0.198	-0.2	0.0028	93.2
IPCW	0.502	0.2	0.0034	94.7	0.350	0.0	0.0033	93.2	0.198	-0.2	0.0027	92.3
KMIB	0.504	0.4	0.0034	95.2	0.350	0.0	0.0034	94.7	0.197	-0.3	0.0026	91.7
$WKM_{4.1}$	0.501	0.1	0.0035	94.5	0.350	0.0	0.0034	94.4	0.200	0.0	0.0027	93.1
$WKM_{4.2}$	0.502	0.2	0.0035	94.8	0.352	0.2	0.0034	93.8	0.203	0.3	0.0028	91.3
$WKM_{8.1}$	0.502	0.2	0.0035	94.3	0.353	0.3	0.0035	93.8	0.204	0.4	0.0029	91.3
$WKM_{D.3\%}$	0.502	0.2	0.0038	90.6	0.351	0.1	0.0037	89.1	0.198	-0.2	0.0030	80.2
$WKM_{D.5\%}$	0.502	0.2	0.0037	91.9	0.351	0.1	0.0036	88.8	0.198	-0.2	0.0029	80.6
$WKM_{D.10\%}$	0.503	0.3	0.0037	92.0	0.351	0.1	0.0035	88.8	0.198	-0.2	0.0029	81.9
$WKM_{D.20\%}$	0.503	0.3	0.0036	92.5	0.351	0.1	0.0035	88.7	0.197	-0.3	0.0028	82.9
$WKM_{D.30\%}$	0.503	0.3	0.0036	92.5	0.350	0.0	0.0035	88.7	0.196	-0.4	0.0028	82.8
$WKM_{D.40\%}$	0.504	0.4	0.0036	92.6	0.350	0.0	0.0035	88.9	0.196	-0.4	0.0028	83.0
$WKM_{D.50\%}$	0.504	0.4	0.0036	92.5	0.350	0.0	0.0035	88.9	0.196	-0.4	0.0028	83.0
$WKM_{R.3\%}$	0.502	0.2	0.0038	90.4	0.351	0.1	0.0037	89.4	0.198	-0.2	0.0029	83.6
$WKM_{R.5\%}$	0.502	0.2	0.0037	91.2	0.351	0.1	0.0035	90.2	0.198	-0.2	0.0028	85.3
$WKM_{R.10\%}$	0.502	0.2	0.0036	92.3	0.350	0.0	0.0035	91.1	0.198	-0.2	0.0028	87.4
$WKM_{R.20\%}$	0.503	0.3	0.0035	93.1	0.350	0.0	0.0034	91.6	0.197	-0.3	0.0027	88.7
$WKM_{R.30\%}$	0.503	0.3	0.0035	93.6	0.350	0.0	0.0034	91.8	0.196	-0.4	0.0027	89.2
$WKM_{R.40\%}$	0.503	0.3	0.0035	93.6	0.350	0.0	0.0034	92.0	0.196	-0.4	0.0027	89.4
$WKM_{R.50\%}$	0.503	0.3	0.0035	93.6	0.350	0.0	0.0034	92.3	0.196	-0.4	0.0027	89.5
$WKM_{U.3\%}$	0.502	0.2	0.0037	91.9	0.351	0.1	0.0036	90.7	0.199	-0.1	0.0029	86.3
$WKM_{U.5\%}$	0.503	0.3	0.0036	92.6	0.351	0.1	0.0035	92.4	0.198	-0.2	0.0028	89.3
$WKM_{U.10\%}$	0.503	0.3	0.0035	94.1	0.350	0.0	0.0034	93.4	0.197	-0.3	0.0027	91.5
$WKM_{U.20\%}$	0.504	0.4	0.0036	94.8	0.350	0.0	0.0035	94.1	0.195	-0.5	0.0027	92.0
$WKM_{U.30\%}$	0.505	0.5	0.0036	94.9	0.349	-0.1	0.0035	93.8	0.194	-0.6	0.0027	91.6
$WKM_{U.40\%}$	0.505	0.5	0.0037	95.0	0.349	-0.1	0.0036	93.9	0.194	-0.6	0.0027	92.2
$WKM_{U.50\%}$	0.504	0.4	0.0037	94.9	0.349	-0.1	0.0036	94.1	0.194	-0.6	0.0027	92.0
$WKM_{\mathcal{N}(0.1)}$	0.502	0.2	0.0036	92.8	0.351	0.1	0.0037	91.4	0.200	0.0	0.0030	87.0
$WKM_{\mathcal{N}(0.5)}$	0.502	0.2	0.0035	94.7	0.350	0.0	0.0034	93.8	0.197	-0.3	0.0027	91.7
$WKM_{\mathcal{N}(1)}$	0.503	0.3	0.0035	95.2	0.349	-0.1	0.0034	93.9	0.195	-0.5	0.0026	92.1
$WKM_{\mathcal{N}(2)}$	0.503	0.3	0.0036	95.0	0.350	0.0	0.0035	94.0	0.196	-0.4	0.0027	91.9
$WKM_{\mathcal{N}(5)}$	0.502	0.2	0.0037	94.4	0.350	0.0	0.0036	93.8	0.198	-0.2	0.0028	92.0

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 27. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=100)**.

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Overall censoring rate=35% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=26%¶				True value: 0.35 Censoring rate=29%				True value: 0.2 Censoring rate=32%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.502	0.2	0.0024	94.0	0.352	0.2	0.0023	94.1	0.201	0.1	0.0016	93.5
KM PO	0.569	6.9	0.0080	76.6	0.429	7.9	0.0098	73.6	0.272	7.2	0.0083	77.2
IPCW	0.539	3.9	0.0053	86.0	0.389	3.9	0.0057	86.9	0.230	3.0	0.0042	90.8
$WKM_{4.1}$	0.526	2.6	0.0043	91.0	0.378	2.8	0.0045	91.2	0.229	2.9	0.0038	90.3
$WKM_{4.2}$	0.523	2.3	0.0042	90.1	0.383	3.3	0.0048	88.4	0.242	4.2	0.0049	85.3
$WKM_{8.1}$	0.527	2.7	0.0045	89.3	0.385	3.5	0.0051	88.1	0.243	4.3	0.0052	84.2
$WKM_{D.3\%}$	0.518	1.8	0.0042	89.8	0.367	1.7	0.0041	87.5	0.212	1.2	0.0029	85.0
$WKM_{D.5\%}$	0.519	1.9	0.0042	89.4	0.368	1.8	0.0040	88.2	0.213	1.3	0.0029	85.5
$WKM_{D.10\%}$	0.521	2.1	0.0041	89.6	0.371	2.1	0.0039	88.1	0.216	1.6	0.0029	85.9
$WKM_{D.20\%}$	0.525	2.5	0.0043	89.2	0.375	2.5	0.0041	88.2	0.220	2.0	0.0030	85.3
$WKM_{D.30\%}$	0.529	2.9	0.0044	88.7	0.380	3.0	0.0043	87.5	0.224	2.4	0.0033	84.7
$WKM_{D.40\%}$	0.532	3.2	0.0046	88.4	0.383	3.3	0.0046	86.9	0.227	2.7	0.0035	83.4
$WKM_{D.50\%}$	0.535	3.5	0.0047	87.7	0.386	3.6	0.0048	86.2	0.230	3.0	0.0037	82.5
$WKM_{R.3\%}$	0.518	1.8	0.0042	89.5	0.368	1.8	0.0041	88.3	0.212	1.2	0.0029	86.8
$WKM_{R.5\%}$	0.519	1.9	0.0041	89.1	0.369	1.9	0.0040	88.8	0.213	1.3	0.0028	88.6
$WKM_{R.10\%}$	0.522	2.2	0.0041	89.5	0.371	2.1	0.0039	89.1	0.216	1.6	0.0028	90.3
$WKM_{R.20\%}$	0.526	2.6	0.0042	89.0	0.376	2.6	0.0041	89.4	0.221	2.1	0.0030	89.3
$WKM_{R.30\%}$	0.529	2.9	0.0043	88.1	0.380	3.0	0.0043	89.4	0.224	2.4	0.0032	89.0
$WKM_{R.40\%}$	0.532	3.2	0.0045	87.3	0.383	3.3	0.0045	88.7	0.228	2.8	0.0034	88.3
$WKM_{R.50\%}$	0.534	3.4	0.0046	86.6	0.386	3.6	0.0047	87.8	0.230	3.0	0.0036	87.7
$WKM_{U.3\%}$	0.518	1.8	0.0041	90.9	0.368	1.8	0.0040	89.5	0.213	1.3	0.0028	89.1
$WKM_{U.5\%}$	0.521	2.1	0.0041	90.7	0.371	2.1	0.0040	90.8	0.215	1.5	0.0028	91.9
$WKM_{U.10\%}$	0.525	2.5	0.0042	90.1	0.375	2.5	0.0040	91.5	0.220	2.0	0.0029	91.6
$WKM_{U.20\%}$	0.533	3.3	0.0046	88.7	0.385	3.5	0.0046	91.0	0.230	3.0	0.0035	90.1
$WKM_{U.30\%}$	0.540	4.0	0.0051	87.1	0.394	4.4	0.0053	88.7	0.239	3.9	0.0042	88.7
$WKM_{U.40\%}$	0.547	4.7	0.0056	84.9	0.402	5.2	0.0062	85.6	0.247	4.7	0.0050	85.8
$WKM_{U.50\%}$	0.553	5.3	0.0062	83.0	0.410	6.0	0.0070	82.6	0.254	5.4	0.0058	83.5
$WKM_{\mathcal{N}(0.1)}$	0.518	1.8	0.0043	89.2	0.366	1.6	0.0042	89.4	0.209	0.9	0.0029	89.1
$WKM_{\mathcal{N}(0.5)}$	0.526	2.6	0.0042	90.0	0.376	2.6	0.0041	90.9	0.219	1.9	0.0029	91.6
$WKM_{\mathcal{N}(1)}$	0.541	4.1	0.0051	87.0	0.393	4.3	0.0053	88.5	0.235	3.5	0.0040	88.9
$WKM_{\mathcal{N}(2)}$	0.557	5.7	0.0066	81.0	0.414	6.4	0.0075	80.4	0.255	5.5	0.0061	82.9
$WKM_{\mathcal{N}(5)}$	0.567	6.7	0.0077	76.6	0.426	7.6	0.0093	74.1	0.268	6.8	0.0078	76.7

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 28. Monte Carlo results for estimating **classical survival curve** with dependent censoring **(N=100)**.

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Censoring rate=51% and the link functions **uncorrectly** specified.

	True value: 0.5 Censoring rate=34%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=46%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.502	0.2	0.0026	93.8	0.353	0.3	0.0025	92.0	0.202	0.2	0.0017	93.0
KM PO	0.607	10.7	0.0148	51.8	0.474	12.4	0.0195	49.5	0.317	11.7	0.0186	58.8
IPCW	0.571	7.1	0.0096	71.3	0.425	7.5	0.0117	72.6	0.261	6.1	0.0106	78.3
$WKM_{4.1}$	0.543	4.3	0.0063	86.4	0.405	5.5	0.0084	80.5	0.268	6.8	0.0097	77.5
$WKM_{4.2}$	0.545	4.5	0.0065	83.9	0.419	6.9	0.0098	74.1	0.298	9.8	0.0148	56.6
$WKM_{8.1}$	0.549	4.9	0.0069	81.7	0.419	6.9	0.0101	73.3	0.293	9.3	0.0135	61.3
$WKM_{D.3\%}$	0.531	3.1	0.0057	86.6	0.382	3.2	0.0067	79.2	0.227	2.7	0.0058	72.5
$WKM_{D.5\%}$	0.532	3.2	0.0056	86.5	0.385	3.5	0.0065	79.4	0.229	2.9	0.0056	72.8
$WKM_{D.10\%}$	0.536	3.6	0.0056	86.4	0.389	3.9	0.0065	79.2	0.235	3.5	0.0057	73.7
$WKM_{D.20\%}$	0.542	4.2	0.0059	85.3	0.396	4.6	0.0070	77.8	0.244	4.4	0.0063	69.6
$WKM_{D.30\%}$	0.547	4.7	0.0063	84.0	0.403	5.3	0.0076	76.3	0.250	5.0	0.0069	67.2
$WKM_{D.40\%}$	0.552	5.2	0.0067	82.5	0.409	5.9	0.0082	73.7	0.254	5.4	0.0075	66.1
$WKM_{D.50\%}$	0.556	5.6	0.0071	81.5	0.413	6.3	0.0087	71.8	0.258	5.8	0.0079	65.2
$WKM_{R.3\%}$	0.531	3.1	0.0057	86.2	0.383	3.3	0.0066	80.5	0.227	2.7	0.0058	77.1
$WKM_{R.5\%}$	0.532	3.2	0.0055	85.7	0.385	3.5	0.0064	80.5	0.230	3.0	0.0055	79.6
$WKM_{R.10\%}$	0.536	3.6	0.0055	85.6	0.389	3.9	0.0064	81.6	0.235	3.5	0.0055	80.8
$WKM_{R.20\%}$	0.541	4.1	0.0058	84.9	0.396	4.6	0.0068	80.0	0.244	4.4	0.0061	80.1
$WKM_{R.30\%}$	0.546	4.6	0.0061	82.8	0.402	5.2	0.0073	79.0	0.250	5.0	0.0067	78.9
$WKM_{R.40\%}$	0.551	5.1	0.0065	81.5	0.408	5.8	0.0079	76.8	0.254	5.4	0.0073	77.6
$WKM_{R.50\%}$	0.555	5.5	0.0069	79.6	0.412	6.2	0.0084	75.1	0.258	5.8	0.0077	76.3
$WKM_{U.3\%}$	0.532	3.2	0.0056	87.3	0.384	3.4	0.0065	83.0	0.229	2.9	0.0056	81.5
$WKM_{U.5\%}$	0.534	3.4	0.0055	87.9	0.387	3.7	0.0063	85.4	0.233	3.3	0.0053	86.1
$WKM_{U.10\%}$	0.540	4.0	0.0057	87.8	0.395	4.5	0.0066	85.5	0.243	4.3	0.0059	85.9
$WKM_{U.20\%}$	0.552	5.2	0.0066	83.1	0.410	6.0	0.0080	81.8	0.260	6.0	0.0077	82.2
$WKM_{U.30\%}$	0.563	6.3	0.0077	79.3	0.424	7.4	0.0098	76.0	0.275	7.5	0.0098	77.3
$WKM_{U.40\%}$	0.574	7.4	0.0091	74.2	0.437	8.7	0.0118	70.5	0.286	8.6	0.0117	72.0
$WKM_{U.50\%}$	0.583	8.3	0.0105	68.6	0.447	9.7	0.0137	63.0	0.295	9.5	0.0135	68.6
$WKM_{\mathcal{N}(0.1)}$	0.530	3.0	0.0059	87.3	0.380	3.0	0.0069	82.4	0.225	2.5	0.0064	78.5
$WKM_{\mathcal{N}(0.5)}$	0.544	4.4	0.0059	86.6	0.396	4.6	0.0069	83.7	0.239	3.9	0.0060	84.8
$WKM_{\mathcal{N}(1)}$	0.567	6.7	0.0083	76.5	0.425	7.5	0.0100	74.6	0.267	6.7	0.0090	79.6
$WKM_{\mathcal{N}(2)}$	0.591	9.1	0.0118	62.6	0.454	10.4	0.0151	59.8	0.297	9.7	0.0140	67.9
$WKM_{\mathcal{N}(5)}$	0.604	10.4	0.0142	52.7	0.470	12.0	0.0186	51.2	0.313	11.3	0.0177	59.1

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 29. Monte Carlo results for estimating **classical survival curve** with independent censoring (**N=100**).

Working failure model mis-specified (using only Z_1, Z_2 and Z_3)
Censoring rate=40% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=35%¶				True value: 0.35 Censoring rate=37%				True value: 0.2 Censoring rate=39%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.501	0.1	0.0024	94.8	0.349	-0.1	0.0021	94.6	0.198	-0.2	0.0015	93.5
KM PO	0.502	0.2	0.0037	94.5	0.350	0.0	0.0036	94.4	0.198	-0.2	0.0028	93.2
IPCW	0.502	0.2	0.0036	94.1	0.350	0.0	0.0035	93.3	0.198	-0.2	0.0028	92.4
$WKM_{4.1}$	0.502	0.2	0.0035	95	0.35	0	0.0035	94.3	0.2	0	0.0028	92.2
$WKM_{4.2}$	0.502	0.2	0.0036	94	0.352	0.2	0.0035	93.8	0.204	0.4	0.0028	92.3
$WKM_{8.1}$	0.503	0.3	0.0036	94.4	0.352	0.2	0.0036	93.5	0.203	0.3	0.003	91.3
$WKM_{D.3\%}$	0.503	0.3	0.0038	92.5	0.351	0.1	0.0040	87.4	0.199	-0.1	0.0033	79.2
$WKM_{D.5\%}$	0.503	0.3	0.0038	92.8	0.351	0.1	0.0038	88.4	0.199	-0.1	0.0031	80.5
$WKM_{D.10\%}$	0.503	0.3	0.0037	92.9	0.351	0.1	0.0037	88.6	0.199	-0.1	0.0030	81.4
$WKM_{D.20\%}$	0.504	0.4	0.0037	93.1	0.351	0.1	0.0037	88.6	0.198	-0.2	0.0030	82.1
$WKM_{D.30\%}$	0.504	0.4	0.0037	93.1	0.351	0.1	0.0036	88.6	0.198	-0.2	0.0029	82.3
$WKM_{D.40\%}$	0.504	0.4	0.0037	92.9	0.351	0.1	0.0036	88.4	0.198	-0.2	0.0029	82.6
$WKM_{D.50\%}$	0.504	0.4	0.0037	92.8	0.351	0.1	0.0036	88.4	0.198	-0.2	0.0029	82.5
$WKM_{R.3\%}$	0.503	0.3	0.0038	92.2	0.351	0.1	0.0039	88.2	0.199	-0.1	0.0032	82.2
$WKM_{R.5\%}$	0.503	0.3	0.0037	92.9	0.351	0.1	0.0037	89.2	0.199	-0.1	0.0030	84.5
$WKM_{R.10\%}$	0.503	0.3	0.0036	92.9	0.351	0.1	0.0036	90.2	0.198	-0.2	0.0029	87.2
$WKM_{R.20\%}$	0.503	0.3	0.0036	92.9	0.351	0.1	0.0035	90.7	0.198	-0.2	0.0029	88.5
$WKM_{R.30\%}$	0.503	0.3	0.0036	93.1	0.351	0.1	0.0035	91.2	0.198	-0.2	0.0028	88.9
$WKM_{R.40\%}$	0.503	0.3	0.0036	93.2	0.350	0.0	0.0035	91.7	0.197	-0.3	0.0028	89.0
$WKM_{R.50\%}$	0.503	0.3	0.0036	93.2	0.350	0.0	0.0035	91.8	0.197	-0.3	0.0028	89.0
$WKM_{U.3\%}$	0.503	0.3	0.0037	92.8	0.351	0.1	0.0038	90.1	0.199	-0.1	0.0031	86.9
$WKM_{U.5\%}$	0.503	0.3	0.0037	93.7	0.350	0.0	0.0036	91.7	0.198	-0.2	0.0029	89.5
$WKM_{U.10\%}$	0.503	0.3	0.0036	94.1	0.350	0.0	0.0035	93.8	0.198	-0.2	0.0028	91.3
$WKM_{U.20\%}$	0.503	0.3	0.0037	94.4	0.350	0.0	0.0035	93.7	0.197	-0.3	0.0028	92.4
$WKM_{U.30\%}$	0.504	0.4	0.0037	94.7	0.350	0.0	0.0035	93.7	0.196	-0.4	0.0027	92.5
$WKM_{U.40\%}$	0.504	0.4	0.0037	94.5	0.350	0.0	0.0036	93.8	0.196	-0.4	0.0028	91.9
$WKM_{U.50\%}$	0.503	0.3	0.0038	94.4	0.35	0.0	0.0036	93.8	0.196	-0.4	0.0028	92.2
$WKM_{\mathcal{N}(0.1)}$	0.503	0.3	0.0036	94.0	0.351	0.1	0.0037	91.5	0.199	-0.1	0.0032	87.0
$WKM_{\mathcal{N}(0.5)}$	0.503	0.3	0.0035	94.9	0.350	0.0	0.0034	93.7	0.198	-0.2	0.0028	91.3
$WKM_{\mathcal{N}(1)}$	0.503	0.3	0.0036	94.5	0.350	0.0	0.0034	93.9	0.197	-0.3	0.0027	92.2
$WKM_{\mathcal{N}(2)}$	0.503	0.3	0.0037	94.9	0.350	0.0	0.0035	93.9	0.198	-0.2	0.0028	92.3
$WKM_{\mathcal{N}(5)}$	0.502	0.2	0.0037	94.2	0.350	0.0	0.0036	93.8	0.198	-0.2	0.0028	92.0

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 30. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200**). **70% LOD=100 copies/ml and 30% LOD=200 copies/ml** leading to an overall censoring rate of **45%**

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Using Risk Score instead of principal component analysis (PCA).
Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=24%¶				True value: 0.35 Censoring rate=34%				True value: 0.2 Censoring rate=42%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.500	0.0	0.0014	93.6	0.351	0.1	0.0012	95.5	0.202	0.2	0.0009	93.7
KM PO	0.523	2.3	0.0022	90.0	0.381	3.1	0.0028	88.9	0.237	3.7	0.0038	87.8
$WKM_{U.2.5\%}$	0.502	0.2	0.0019	91.2	0.352	0.2	0.0021	89.6	0.208	0.8	0.0026	85.4
$WKM_{U.5\%}$	0.502	0.2	0.0019	92.4	0.354	0.4	0.0021	91.3	0.210	1.0	0.0025	89.7
$WKM_{U.10\%}$	0.504	0.4	0.0018	92.9	0.356	0.6	0.0020	92.4	0.215	1.5	0.0025	90.7
$WKM_{U.15\%}$	0.506	0.6	0.0018	93.0	0.359	0.9	0.0020	92.8	0.218	1.8	0.0026	91.0
$WKM_{U.20\%}$	0.507	0.7	0.0018	93.2	0.362	1.2	0.0020	92.3	0.221	2.1	0.0028	90.7
$WKM_{U.30\%}$	0.511	1.1	0.0019	92.9	0.367	1.7	0.0021	92.2	0.225	2.5	0.0030	89.9
$WKM_{U.40\%}$	0.514	1.4	0.0019	92.6	0.371	2.1	0.0023	91.4	0.229	2.9	0.0032	89.4
$WKM_{U.50\%}$	0.516	1.6	0.0020	92.2	0.374	2.4	0.0024	90.8	0.231	3.1	0.0034	88.6
$WKM_{\mathcal{N}(0.1)}$	0.500	0.0	0.0019	91.8	0.350	0.0	0.0023	90.5	0.204	0.4	0.0033	81.7
$WKM_{\mathcal{N}(0.5)}$	0.505	0.5	0.0018	93.0	0.357	0.7	0.0020	92.7	0.211	1.1	0.0026	90.3
$WKM_{\mathcal{N}(1)}$	0.513	1.3	0.0019	92.4	0.367	1.7	0.0022	91.9	0.222	2.2	0.0029	90.1
$WKM_{\mathcal{N}(2)}$	0.519	1.9	0.0021	90.9	0.376	2.6	0.0026	90.1	0.231	3.1	0.0034	88.8
$WKM_{\mathcal{N}(5)}$	0.522	2.2	0.0022	90.1	0.380	3.0	0.0028	88.4	0.236	3.6	0.0037	87.8

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 31. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **30% LOD=100 copies/ml and 70% LOD=200 copies/ml** leading to an overall censoring rate of **50%**

Using Risk Score instead of principal component analysis (PCA).
Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=30%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=47%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.500	0.0	0.0014	93.6	0.351	0.1	0.0012	95.5	0.202	0.2	0.0009	93.7
KM PO	0.528	2.8	0.0026	88.1	0.387	3.7	0.0035	87.5	0.242	4.2	0.0051	85.8
$WKM_{U.2.5\%}$	0.503	0.3	0.0021	91.4	0.355	0.5	0.0026	88.9	0.211	1.1	0.0036	84.5
$WKM_{U.5\%}$	0.504	0.4	0.0021	91.8	0.356	0.6	0.0024	91.4	0.213	1.3	0.0034	88.0
$WKM_{U.10\%}$	0.506	0.6	0.0021	91.7	0.360	1.0	0.0023	92.9	0.218	1.8	0.0034	90.0
$WKM_{U.15\%}$	0.508	0.8	0.0021	92.0	0.363	1.3	0.0024	93.1	0.222	2.2	0.0036	89.6
$WKM_{U.20\%}$	0.510	1.0	0.0021	91.9	0.366	1.6	0.0024	92.8	0.225	2.5	0.0038	89.6
$WKM_{U.30\%}$	0.514	1.4	0.0021	92.2	0.372	2.2	0.0026	91.3	0.230	3.0	0.0040	88.3
$WKM_{U.40\%}$	0.517	1.7	0.0022	91.6	0.376	2.6	0.0027	90.7	0.233	3.3	0.0043	87.6
$WKM_{U.50\%}$	0.520	2.0	0.0023	90.9	0.379	2.9	0.0029	89.9	0.236	3.6	0.0045	87.3
$WKM_{\mathcal{N}(0.1)}$	0.501	0.1	0.0022	91.1	0.351	0.1	0.0028	87.8	0.204	0.4	0.0047	78.6
$WKM_{\mathcal{N}(0.5)}$	0.507	0.7	0.0020	92.1	0.360	1.0	0.0024	92.9	0.213	1.3	0.0036	88.5
$WKM_{\mathcal{N}(1)}$	0.516	1.6	0.0022	91.5	0.372	2.2	0.0026	91.6	0.226	2.6	0.0040	88.9
$WKM_{\mathcal{N}(2)}$	0.524	2.4	0.0024	89.0	0.381	3.1	0.0031	89.1	0.236	3.6	0.0046	86.7
$WKM_{\mathcal{N}(5)}$	0.527	2.7	0.0025	88.4	0.386	3.6	0.0034	86.9	0.241	4.1	0.0050	85.3

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 32. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **70% LOD=200 copies/ml and 30% LOD=500 copies/ml** leading to an overall censoring rate of **57%**

Using Risk Score instead of principal component analysis (PCA).
Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=39%¶				True value: 0.35 Censoring rate=49%				True value: 0.2 Censoring rate=56%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.500	0.0	0.0014	93.6	0.351	0.1	0.0012	95.5	0.202	0.2	0.0009	93.7
KM PO	0.536	3.6	0.0033	84.6	0.396	4.6	0.0048	84.7	0.252	5.2	0.0087	81.3
$WKM_{U.2.5\%}$	0.502	0.2	0.0026	89.1	0.356	0.6	0.0034	87.3	0.217	1.7	0.0059	80.6
$WKM_{U.5\%}$	0.504	0.4	0.0024	91.0	0.359	0.9	0.0032	89.8	0.221	2.1	0.0059	86.0
$WKM_{U.10\%}$	0.507	0.7	0.0024	92.0	0.364	1.4	0.0030	91.9	0.226	2.6	0.0060	87.5
$WKM_{U.15\%}$	0.510	1.0	0.0023	91.7	0.368	1.8	0.0031	92.4	0.230	3.0	0.0063	87.7
$WKM_{U.20\%}$	0.513	1.3	0.0024	91.9	0.372	2.2	0.0032	92.1	0.234	3.4	0.0066	87.0
$WKM_{U.30\%}$	0.518	1.8	0.0024	91.4	0.378	2.8	0.0035	91.1	0.239	3.9	0.0071	85.5
$WKM_{U.40\%}$	0.522	2.2	0.0026	90.2	0.383	3.3	0.0037	89.9	0.242	4.2	0.0075	84.6
$WKM_{U.50\%}$	0.526	2.6	0.0027	89.1	0.387	3.7	0.0040	88.5	0.245	4.5	0.0078	84.3
$WKM_{\mathcal{N}(0.1)}$	0.500	0.0	0.0026	90.5	0.351	0.1	0.0040	86.0	0.206	0.6	0.0079	74.4
$WKM_{\mathcal{N}(0.5)}$	0.509	0.9	0.0023	92.3	0.363	1.3	0.0031	91.9	0.220	2.0	0.0065	85.8
$WKM_{\mathcal{N}(1)}$	0.521	2.1	0.0025	90.7	0.378	2.8	0.0036	90.3	0.235	3.5	0.0071	85.3
$WKM_{\mathcal{N}(2)}$	0.530	3.0	0.0030	86.7	0.390	4.0	0.0043	87.5	0.246	4.6	0.0080	84.3
$WKM_{\mathcal{N}(5)}$	0.535	3.5	0.0032	85.1	0.395	4.5	0.0047	84.7	0.251	5.1	0.0086	83.1

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 33. Monte Carlo results for estimating an hypothetical **HIV-1 RNA reduction (N=200)**. **30% LOD=200 copies/ml and 70% LOD=500 copies/ml** leading to an overall censoring rate of **64%**

Using Risk Score instead of principal component analysis (PCA).
Estimators are derived using the 5 covariates.

	True value: 0.5 Censoring rate=49%¶				True value: 0.35 Censoring rate=58%				True value: 0.2 Censoring rate=63%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.500	0.0	0.0014	93.6	0.351	0.1	0.0012	95.5	0.202	0.2	0.0009	93.7
KM PO	0.544	4.4	0.0044	81.8	0.406	5.6	0.0067	82.2	0.261	6.1	0.0155	69.4
$WKM_{U.2.5\%}$	0.503	0.3	0.0033	87.3	0.359	0.9	0.0048	83.1	0.223	2.3	0.0110	75.2
$WKM_{U.5\%}$	0.505	0.5	0.0031	88.3	0.362	1.2	0.0044	88.2	0.227	2.7	0.0108	80.4
$WKM_{U.10\%}$	0.509	0.9	0.0030	89.8	0.369	1.9	0.0043	89.9	0.234	3.4	0.0113	81.8
$WKM_{U.15\%}$	0.513	1.3	0.0030	89.8	0.375	2.5	0.0044	90.2	0.239	3.9	0.0118	81.7
$WKM_{U.20\%}$	0.517	1.7	0.0030	89.5	0.379	2.9	0.0045	89.9	0.242	4.2	0.0123	80.4
$WKM_{U.30\%}$	0.524	2.4	0.0031	89.0	0.386	3.6	0.0049	88.1	0.247	4.7	0.0131	79.3
$WKM_{U.40\%}$	0.529	2.9	0.0033	87.2	0.392	4.2	0.0053	85.9	0.251	5.1	0.0137	78.6
$WKM_{U.50\%}$	0.534	3.4	0.0036	85.3	0.396	4.6	0.0057	84.4	0.254	5.4	0.0142	77.7
$WKM_{\mathcal{N}(0.1)}$	0.502	0.2	0.0035	87.4	0.357	0.7	0.0056	83.2	0.217	1.7	0.0134	67.3
$WKM_{\mathcal{N}(0.5)}$	0.513	1.3	0.0030	89.7	0.369	1.9	0.0045	89.6	0.230	3.0	0.0120	79.6
$WKM_{\mathcal{N}(1)}$	0.527	2.7	0.0033	87.5	0.387	3.7	0.0051	87.1	0.246	4.6	0.0132	79.1
$WKM_{\mathcal{N}(2)}$	0.538	3.8	0.0039	83.5	0.399	4.9	0.0061	83.4	0.256	5.6	0.0146	76.8
$WKM_{\mathcal{N}(5)}$	0.543	4.3	0.0043	82.2	0.405	5.5	0.0066	81.8	0.260	6.0	0.0154	76.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 34. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=200)**.

Using Risk Score instead of principal component analysis (PCA).
Overall censoring rate=35% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=26%¶				True value: 0.35 Censoring rate=29%				True value: 0.2 Censoring rate=32%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.501	0.1	0.0013	93.3	0.351	0.1	0.0012	94.4	0.199	-0.1	0.0008	94.4
KM PO	0.569	6.9	0.0063	59.6	0.428	7.8	0.0079	53.1	0.269	6.9	0.0063	58.7
$\text{WKM}_{U.2.5\%}$	0.507	0.7	0.0018	92.6	0.356	0.6	0.0018	91.8	0.201	0.1	0.0011	91.3
$\text{WKM}_{U.5\%}$	0.510	1.0	0.0018	92.5	0.358	0.8	0.0018	92.6	0.204	0.4	0.0011	93.0
$\text{WKM}_{U.10\%}$	0.515	1.5	0.0019	92.4	0.364	1.4	0.0019	92.5	0.209	0.9	0.0012	93.6
$\text{WKM}_{U.15\%}$	0.521	2.1	0.0021	91.4	0.370	2.0	0.0021	91.2	0.214	1.4	0.0014	92.8
$\text{WKM}_{U.20\%}$	0.526	2.6	0.0023	89.7	0.375	2.5	0.0024	88.9	0.219	1.9	0.0016	91.0
$\text{WKM}_{U.30\%}$	0.535	3.5	0.0029	84.8	0.386	3.6	0.0031	84.9	0.230	3.0	0.0022	88.2
$\text{WKM}_{U.40\%}$	0.543	4.3	0.0035	79.9	0.396	4.6	0.0040	78.5	0.239	3.9	0.0029	82.3
$\text{WKM}_{U.50\%}$	0.550	5.0	0.0042	75.9	0.405	5.5	0.0049	71.9	0.247	4.7	0.0037	76.8
$\text{WKM}_{\mathcal{N}(0.1)}$	0.506	0.6	0.0018	92.5	0.354	0.4	0.0018	92.1	0.200	0.0	0.0011	91.6
$\text{WKM}_{\mathcal{N}(0.5)}$	0.523	2.3	0.0022	89.8	0.373	2.3	0.0023	89.6	0.215	1.5	0.0014	92.0
$\text{WKM}_{\mathcal{N}(1)}$	0.545	4.5	0.0036	78.6	0.397	4.7	0.0040	76.6	0.237	3.7	0.0028	84.1
$\text{WKM}_{\mathcal{N}(2)}$	0.560	6.0	0.0053	67.9	0.417	6.7	0.0063	61.5	0.257	5.7	0.0048	68.5
$\text{WKM}_{\mathcal{N}(5)}$	0.567	6.7	0.0061	61.2	0.426	7.6	0.0076	53.4	0.267	6.7	0.0060	60.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 35. Monte Carlo results for estimating **classical survival curve** with with dependent censoring **(N=200)**.

Using Risk Score instead of principal component analysis (PCA).
Overall censoring rate=51% and the link functions **uncorrectly** specified.

	True value: 0.5 Censoring rate=34%¶				True value: 0.35 Censoring rate=40%				True value: 0.2 Censoring rate=46%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.502	0.2	0.0013	94.2	0.352	0.2	0.0011	94.6	0.201	0.1	0.0008	95.1
KM PO	0.605	10.5	0.0128	27.3	0.472	12.2	0.0171	22.6	0.321	12.1	0.0169	28.1
$WKM_{U.2.5\%}$	0.508	0.8	0.0023	89.8	0.358	0.8	0.0023	87.6	0.210	1.0	0.0021	86.7
$WKM_{U.5\%}$	0.512	1.2	0.0023	90.4	0.363	1.3	0.0023	91.3	0.216	1.6	0.0021	89.7
$WKM_{U.10\%}$	0.521	2.1	0.0025	89.6	0.373	2.3	0.0026	91.0	0.228	2.8	0.0025	87.6
$WKM_{U.15\%}$	0.529	2.9	0.0028	87.6	0.383	3.3	0.0031	88.0	0.239	3.9	0.0033	83.6
$WKM_{U.20\%}$	0.536	3.6	0.0033	85.0	0.393	4.3	0.0038	84.0	0.250	5.0	0.0042	77.8
$WKM_{U.30\%}$	0.550	5.0	0.0044	77.6	0.411	6.1	0.0056	70.6	0.268	6.8	0.0065	64.3
$WKM_{U.40\%}$	0.564	6.4	0.0059	66.8	0.427	7.7	0.0079	57.3	0.282	8.2	0.0088	53.6
$WKM_{U.50\%}$	0.576	7.6	0.0076	54.0	0.441	9.1	0.0102	45.1	0.294	9.4	0.0109	45.0
$WKM_{\mathcal{N}(0.1)}$	0.505	0.5	0.0025	89.7	0.356	0.6	0.0025	86.5	0.206	0.6	0.0025	82.8
$WKM_{\mathcal{N}(0.5)}$	0.537	3.7	0.0033	83.7	0.390	4.0	0.0037	84.6	0.238	3.8	0.0034	82.9
$WKM_{\mathcal{N}(1)}$	0.570	7.0	0.0068	61.2	0.429	7.9	0.0083	55.1	0.275	7.5	0.0078	59.1
$WKM_{\mathcal{N}(2)}$	0.593	9.3	0.0105	37.1	0.457	10.7	0.0135	32.9	0.305	10.5	0.0132	38.3
$WKM_{\mathcal{N}(5)}$	0.603	10.3	0.0124	29.2	0.470	12.0	0.0164	24.0	0.318	11.8	0.0162	28.8

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$)
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

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Table 36. Monte Carlo results for estimating **classical survival curve** with independent censoring **(N=200)**.

Using Risk Score instead of principal component analysis (PCA).
Censoring rate=40% and the link functions **correctly** specified.

	True value: 0.5 Censoring rate=35%¶				True value: 0.35 Censoring rate=37%				True value: 0.2 Censoring rate=39%			
Method	Est *	R bias†	MSE‡	CR§	Est	R bias	MSE	CR	Est	R bias	MSE	CR
KM FO	0.498	-0.2	0.0012	94.9	0.348	-0.2	0.0011	95.0	0.198	-0.2	0.0008	94.4
KM PO	0.500	0.0	0.0018	95.0	0.348	-0.2	0.0017	95.4	0.198	-0.2	0.0014	93.7
$WKM_{U.2.5\%}$	0.500	0.0	0.0017	93.4	0.348	-0.2	0.0015	92.1	0.198	-0.2	0.0013	90.4
$WKM_{U.5\%}$	0.500	0.0	0.0017	93.4	0.348	-0.2	0.0015	94.1	0.197	-0.3	0.0013	91.8
$WKM_{U.10\%}$	0.501	0.1	0.0017	93.9	0.348	-0.2	0.0015	94.3	0.195	-0.5	0.0013	92.3
$WKM_{U.15\%}$	0.502	0.2	0.0017	94.4	0.347	-0.3	0.0016	94.4	0.193	-0.7	0.0014	92.5
$WKM_{U.20\%}$	0.503	0.3	0.0018	94.4	0.347	-0.3	0.0016	94.9	0.191	-0.9	0.0014	92.2
$WKM_{U.30\%}$	0.504	0.4	0.0018	94.4	0.346	-0.4	0.0017	94.7	0.189	-1.1	0.0014	91.0
$WKM_{U.40\%}$	0.504	0.4	0.0019	94.7	0.345	-0.5	0.0017	95.2	0.189	-1.1	0.0015	91.2
$WKM_{U.50\%}$	0.504	0.4	0.0019	94.5	0.345	-0.5	0.0017	94.9	0.19	-1.0	0.0014	91.6
$WKM_{\mathcal{N}(0.1)}$	0.500	0.0	0.0017	93.6	0.348	-0.2	0.0015	93.8	0.197	-0.3	0.0013	91.2
$WKM_{\mathcal{N}(0.5)}$	0.502	0.2	0.0017	94.5	0.347	-0.3	0.0015	94.7	0.194	-0.6	0.0013	92.5
$WKM_{\mathcal{N}(1)}$	0.502	0.2	0.0018	94.3	0.347	-0.3	0.0016	95.1	0.193	-0.7	0.0014	92.0
$WKM_{\mathcal{N}(2)}$	0.501	0.1	0.0018	95.1	0.347	-0.3	0.0017	95.0	0.195	-0.5	0.0014	93.0
$WKM_{\mathcal{N}(5)}$	0.500	0.0	0.0018	95.1	0.348	-0.2	0.0017	94.3	0.197	-0.3	0.0014	93.2

¶ Censoring rate at the time of the true value of $S(t)$
* Average of 1000 point estimates
† Relative bias ($100 \times (Bias/S(t))$
‡ Mean square error
§ Coverage rate of 1000 95 per cent confidence intervals

