



# On the oldest asteroid families in the main belt

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Accepted 2016 March 2. Received 2016 March 2; in original form 2015 December 3

## ABSTRACT

Asteroid families are groups of minor bodies produced by high-velocity collisions. After the initial dispersions of the parent bodies fragments, their orbits evolve because of several gravitational and non-gravitational effects, such as diffusion in mean-motion resonances, Yarkovsky and Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effects, close encounters of collisions, etc. The subsequent dynamical evolution of asteroid family members may cause some of the original fragments to travel beyond the conventional limits of the asteroid family. Eventually, the whole family will dynamically disperse and no longer be recognizable. A natural question that may arise concerns the time-scales for dispersion of large families. In particular, what is the oldest still recognizable family in the main belt? Are there any families that may date from the late stages of the late heavy bombardment and that could provide clues on our understanding of the primitive Solar system? In this work, we investigate the dynamical stability of seven of the allegedly oldest families in the asteroid main belt. Our results show that none of the seven studied families has a nominally mean estimated age older than 2.7 Gyr, assuming standard values for the parameters describing the strength of the Yarkovsky force. Most ‘paleo-families’ that formed between 2.7 and 3.8 Gyr would be characterized by a very shallow size–frequency distribution, and could be recognizable only if located in a dynamically less active region (such as that of the Koronis family). V-type asteroids in the central main belt could be compatible with a formation from a paleo-Eunomia family.

**Key words:** celestial mechanics – minor planets, asteroids: general.

## 1 INTRODUCTION

Asteroid families are born out of collisions. They ‘die’ when the fragments formed in the collisional event disperse so far because of gravitational or non-gravitational forces that the family is no longer recognizable as a group. The time needed to disperse small groups has been the subject of several studies (from our group, see for instance Carruba 2010a; Carruba, Machuca & Gasparino 2011; Carruba et al. 2015). Less attention has been given to the dispersion of larger families. Recent studies (Brasil et al. 2015) suggested that no family could have likely survived the late heavy bombardment (LHB hereafter, at least 3.8 Gyr ago), in the jumping Jupiter scenario, which set an upper limit on the maximum possible age of any asteroid family. In this work, we focus our attention on the families whose estimated age, according to Brož et al. (2013) could possibly date from just after the LHB, and whose existence,

if confirmed, could provide precious clues on the early stages of our Solar system.

Brož et al. (2013) identified 12 families whose age estimate might have been compatible with an origin during the LHB, or just after the Maria, Eunomia, Koronis, Themis, Hygiea, Meliboea, Ursula, Fringilla, Alauda, Sylvia, Camilla, and Hermione families. The Sylvia family and the proposed long-lost groups around Camilla and Hermione in the Cybele region were recently studied in Carruba et al. (2015), and will not be treated in detail here. That work found that, while all asteroids in the Cybele region were most likely lost during the jumping Jupiter phase of the model of planetary migration of Nesvorný, Vokrouhlický & Morbidelli (2013), some of the largest fragments ( $D > 5$  km, with  $D$  the body diameter) of a hypothetical post-LHB Sylvia family may have remained in the Cybele region, but the identification of these groups could be difficult. Due to local dynamics, any Camilla and Hermione families would disperse in time-scales of the order of 1.5 Gyr. The Fringilla family is a rather small group (134 members, according to Nesvorný, Brož & Carruba 2015) in the outer main belt. Since we are already studying the larger Themis, Meliboea, Alauda, and Ursula families in the outer main belt, and since the group is not large enough for the techniques

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used in this work, we will not further investigate this family in this paper. The Hygiea family was studied at length in Carruba et al. (2014a), that found a maximum possible age of 3.6 Gyr, just a bit younger than the minimum currently believed epoch of the last stages of the LHB, i.e. 3.8 Gyr (Bottke et al. 2012; also the age of the Hygiea family should be most likely younger than 3.6 Gyr, because of the long-term effect of close encounters with 10 Hygiea and of the stochastic Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect of Bottke et al. 2015, this last effect not considered in Carruba et al. 2014a). Age estimates for the other families listed in Brož et al. (2013) are however known with substantial uncertainty, and were obtained before Bottke et al. (2015) modelled the so-called stochastic YORP effect, in which the shapes of asteroids changes between YORP cycles, and whose effect is generally to reduce the estimates of the given asteroid family age. Understanding which, if any, of the proposed ‘Brož seven’ oldest groups (Maria, Eunomia, Koronis, Themis, Meliboea, Ursula, Alauda) could have survived since the LHB, and how, is the main goal of this work.

This paper is so divided: in Section 2, we identified the seven ‘Brož’ families in a new space of proper elements and Sloan Digital Sky Survey-Moving Object Catalog data, fourth release (SDSS-MOC4) *gri* slope and  $z' - i'$  colours, and use the method of Yarkovsky isolines to obtain estimates of maximum possible ages of these groups. In Section 3, we use Monte Carlo methods (see Carruba et al. 2015) to obtain refined estimates of the family age and ejection velocity parameters of the same families. Section 4 deals with the dynamical evolution and dispersion times of fictitious simulated ‘Brož’ groups since the latest phases of the LHB, when the stochastic YORP effect (Bottke et al. 2015) and past changes in the solar luminosity (Carruba et al. 2015) are accounted for. Finally, in the last section, we present our conclusions.

## 2 FAMILY IDENTIFICATION

In order to obtain age estimates of the seven Brož families, we first need to obtain good memberships of these groups. Any method used should aim to reduce the number of possible interlopers, objects that may be part of the dynamical family, but have taxonomical properties inconsistent with that of the majority of the members. DeMeo & Carry (2013) recently introduced a new classification method, based on the Bus-DeMeo taxonomic system, that employs SDSS-MOC4 (Ivezić et al. 2002) *gri* slope and  $z' - i'$  colours. In that article, the authors used the photometric data obtained in the five filters  $u'$ ,  $g'$ ,  $r'$ ,  $i'$ , and  $z'$ , from 0.3 to 1.0  $\mu\text{m}$ , to obtain values of  $z' - i'$  colours and spectral slopes over the  $g'$ ,  $r'$ , and  $i'$  reflectance values, that were then used to assign to each asteroid a likely spectral type. Since the authors were interested in dealing with a complete sample, they limited their analysis to asteroids with absolute magnitude  $H$  higher than 15.3, which roughly corresponds to asteroids with diameters larger than 5 km, and for which the SDSS-MOC4 data set is supposed to be complete. Since Carruba et al. (2015) showed that the vast majority of asteroids that could survive 3.8 Gyr of dynamical evolution in the post-LHB scenario should have diameters larger than 5 km, we believe that this choice of limit in  $H$  could also be suitable for this research. For completeness, we also included asteroids with  $H < 12.00$  with known spectral types from the planetary data system (Neese 2010) that are not part of SDSS-MOC4. These objects were assigned values of the *gri* slope and  $z' - i'$  colours at the centre of the range of those of each given class. We refer the reader to DeMeo & Carry (2013) and Carruba et al. (2014a) for more details on the procedure to obtain *gri* slope and  $z' - i'$  colours, and on how to assign asteroids spectral classes based on these data.

Once a complete set of data with proper elements, obtained from the AstDys site (<http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo>, accessed on 2015 April 4th (Knežević & Milani 2003), taxonomical and SDSS-MOC4 data on *gri* slope and  $z' - i'$  colours has been computed for asteroids in each given asteroid family region, family membership is obtained using the hierarchical clustering method (HCM) of Bendjoya & Zappalá (2002) in an extended domain of proper elements and *gri* slope and  $z' - i'$  colours, where distances were computed using the new distance metric:

$$d_{\text{md}} = \sqrt{d^2 + C_{\text{SPV}}[(f_{\text{gri}}\Delta\text{gri})^2 + (\Delta(i - z))^2]}, \quad (1)$$

where  $d$  is the standard distance metrics in proper element domain defined in Zappalá et al. (1995) as

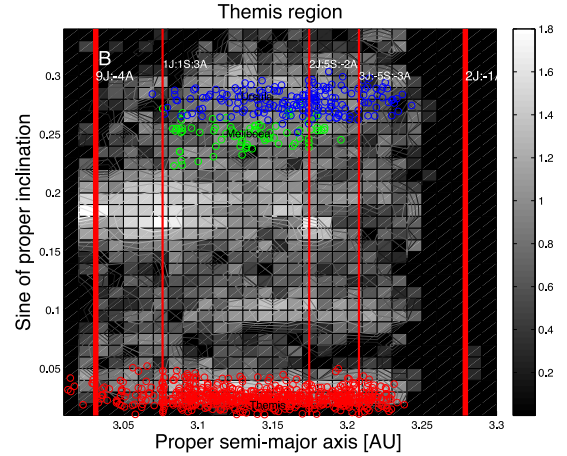
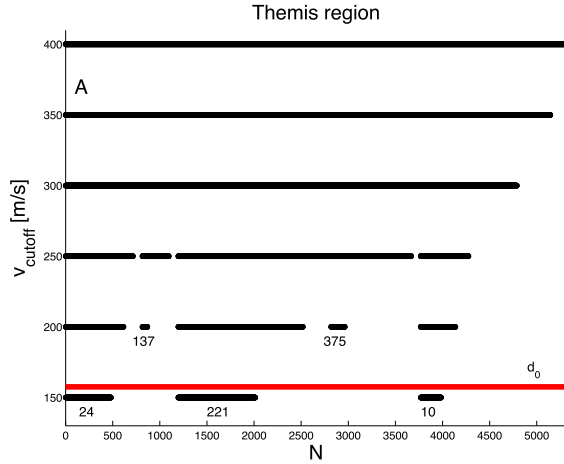
$$d = na\sqrt{k_1\left(\frac{\Delta a}{a}\right)^2 + k_2(\Delta e)^2 + k_3(\Delta \sin(i))^2}, \quad (2)$$

where  $n$  is the asteroid mean motion;  $\Delta x$  the difference in proper  $a$ ,  $e$ , and  $\sin(i)$ ; and  $k_1$ ,  $k_2$ ,  $k_3$  are weighting factors, defined as  $k_1 = 5/4$ ,  $k_2 = 2$ ,  $k_3 = 2$  in Zappalá et al. (1990, 1995).  $\Delta\text{gri}$  and  $\Delta(i - z)$  are difference between two neighbouring asteroids *gri* slopes and  $z' - i'$  colours,  $C_{\text{SPV}}$  is a constant equal to  $10^6$  (see also Carruba et al. 2013, for a discussion on the choice of this constant) and  $f_{\text{gri}} = 0.027$  is a normalization factor to account for differences among the mean values of differences in *gri* slope and  $z' - i'$  colours (different values of  $f_{\text{gri}}$  in the range 0.05–0.001 have been used without substantially affecting the output of the method). We assigned families to their regions, defined as in Brož et al. (2013) as the central main belt, the pristine region, the outer main belt and the outer highly inclined region, and compute proper elements and *gri* slope and  $z' - i'$  colours for all asteroids in each given region. We then computed nominal distance cutoff and stalactite diagrams with the standard techniques described in Carruba (2010b), for all the seven Brož families, and then assigned values of diameters and geometric albedos from the *WISE* catalogue (Wright et al. 2010) to family members for which this information is available [other objects were assigned the values of geometric albedo of the largest object in the family, and diameters computed using equation (1) in Carruba et al. 2003]. Finally the method of the Yarkovsky isolines (see also Carruba et al. 2013) is applied to obtain an estimate of the maximum possible age of each given family. In the next subsection, we will discuss in detail the method for families in the outer main belt, results are similar for other regions.

### 2.1 The outer main belt: the themis, meliboea, and ursula families

The outer main belt is defined in Nesvorný et al. (2015) as the region between the 7J:-3A and 2J:-1A mean-motion resonances and  $\sin(i) < 0.3$ . Three possible old C-complex families have been proposed in this region: the Themis, Meliboea, and Ursula groups. We identified 5472 asteroids with either taxonomical or SDSS-MOC4 information in the area, 3524 of which with data in the *WISE* data set, and 2896 (82.2 per cent) with  $p_V < 0.15$ . The value of the minimal distance cutoff  $d_0$  was of  $157.5 \text{ m s}^{-1}$  and the minimal number of objects to have a group statistically significant was 25.

Fig. 1, panel A, displays a stalactite diagram for this region. The families of Themis, Meliboea, and Ursula were all visible at the nominal distance cutoff  $d_0$ , but not for lower values of  $d$ . 137 Meliboea merges with the local background at  $d = 300 \text{ m s}^{-1}$ . The family of 375 Ursula merges with Eos at cutoff of  $250 \text{ m s}^{-1}$ , and with other minor groups at  $d = 225 \text{ m s}^{-1}$ , so we choose to work



**Figure 1.** Panel A: stalactite diagram for the 5472 asteroids in the Themis region. Asteroids belonging to a family are identified by black dots. Panel B: contour plot of number density of asteroids in the  $(a, \sin(i))$  plane. Red, green and blue circles show the location of members of the Themis, Meliboea, and Ursula families, respectively.

with a cutoff of  $220 \text{ m s}^{-1}$ . Other important families in the region were those of 221 Eos and 10 Hygiea. Panel B of Fig. 1 displays a contour plot of number density of asteroids in the  $(a, \sin(i))$  plane. Whither tones are associated with higher values of local number density. For our grid in this domain, we used 30 steps of 0.01 au in  $a$ , starting from  $a = 3.0$  au, and 34 steps of 0.01 in  $\sin(i)$ , starting from  $\sin(i) = 0.0$ . The members of the identified Themis, Meliboea, and Ursula families, after taxonomical interlopers were removed (there were none in these cases), are shown as red, green, and blue circles, respectively.

Finally, we also apply the method of Yarkovsky isolines (Carruba et al. 2013) to the Themis, Meliboea, and Ursula families, using standard values of the parameters of this force for C-type groups from Brož et al. (2013), table 2. Results are shown in Fig. 2 for the

Themis family. The red and blue lines are the expected displacement of family members over 3.8 and 4.6 Gyr because of the Yarkovsky effect and close encounters with massive asteroids, assumed equal to 0.01 au for simplicity (Carruba et al. 2014a). All asteroids were assumed to be initially at the family barycentre. This method yields maximum ages of 4.6 Gyr for the three Themis, Meliboea, and Ursula families. Information on number of confirmed members and interlopers is given in Table 1, among other quantities.

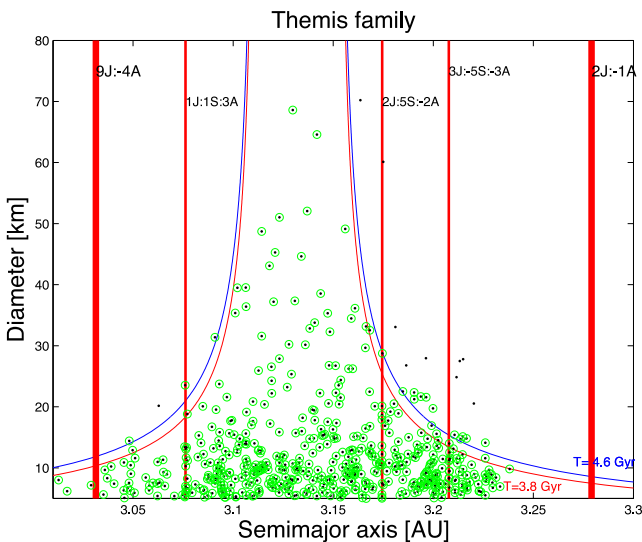
### 3 CHRONOLOGY

Monte Carlo methods to obtain estimates of the family age and ejection velocity parameters were pioneered by Vokrouhlický et al. (2006a,b,c) for the Eos and other asteroid groups. They were recently modified to account for the ‘stochastic’ version of the YORP effect (Bottke et al. 2015), and for changes in the past values of solar luminosity (Vokrouhlický et al. 2006a) for a study of dynamical groups in the Cybele region (Carruba et al. 2015). Age estimates obtained including the stochastic YORP effect in Carruba et al. (2015) were (i) of better quality with respect those obtained with the static version of YORP for old families, in terms of confidence level, and (ii) tend to produce younger age estimates. We refer the reader to Bottke et al. (2015) and Carruba et al. (2015) for a more in depth description of the method. Essentially, the semimajor axis distribution of various fictitious families is evolved under the influence of the Yarkovsky, both diurnal and seasonal versions, and YORP effect and occasionally other effects such as close encounters with massive asteroids (Carruba et al. 2014a) or changes in past solar luminosity values (Carruba et al. 2015). This method, however, ignores the effect of planetary perturbations. The newly obtained distributions of a C-target function computed with the relationship:

$$0.2H = \log_{10}(\Delta a/C), \quad (3)$$

are then compared to the current C distribution of real family members using a  $\chi^2$ -like variable  $\psi_{\Delta C}$  (Vokrouhlický et al. 2006a,b,c), whose minimum value is associated with the best-fitted solution.

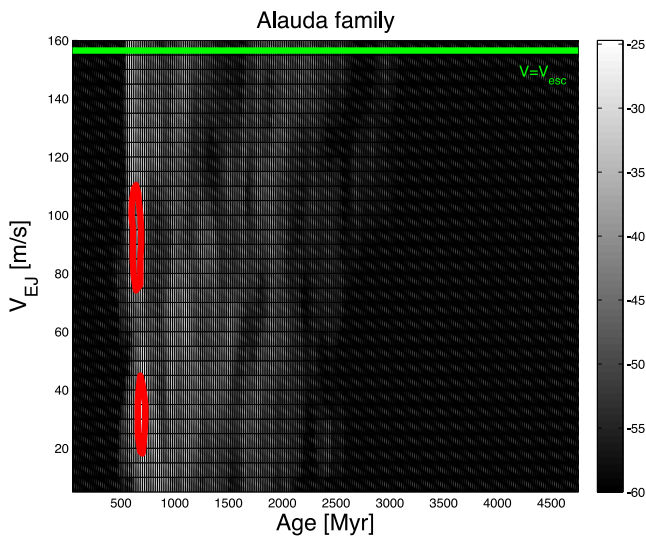
We applied this method to six out of the seven ‘Brož’ families. For the case of the Maria family, since this group was significantly depleted at lower semimajor axis by interaction with the 3J:-1A mean-motion resonance, the number of remaining intervals in the C-target function was too small to allow for a precise determination



**Figure 2.** Yarkovsky isolines age estimates for the Themis family. Black dots identify members of the dynamical group without the taxonomical interlopers, and green circles show the position of members that are not dynamical interlopers. Vertical red lines display the location of mean-motion resonances, the red and blue lines displays isolines of displacement from the family barycentre due to Yarkovsky effect and close encounters with massive asteroids over 3.8 and 4.6 Gyr, respectively.

**Table 1.** Orbital region, spectral complexes, number of asteroids in the orbital region, distance cutoff  $d_0$ , number of taxonomical interlopers, of dynamical interlopers, confirmed members, and maximum age estimates, inferred by Yarkovsky isolines, of the ‘Brož’ seven families.

Family name	Orbital region	Number of asteroids	$d_0$ (m s <sup>-1</sup> )	Spectral complex	Number of tax. interlopers	Number of dyn. interlopers	Number of confirmed members	Maximum age estimate (Gyr)
Eunomia	Central mb	1416	155	S	33	1	1101	3.8
Maria	Central mb	1416	155	S	16	1	386	3.8
Koronis	Pristine zone	1015	135	S	18	1	502	4.6
Themis	Outer mb	5472	220	CX	0	10	642	4.6
Meliboea	Outer mb	5472	220	CX	0	4	78	4.6
Ursula	Outer mb	5472	220	CX	0	11	172	4.6
Alauda	H1 Outer mb	286	250	CX	1	23	122	4.6



**Figure 3.** Target function  $\psi_{\Delta C}$  values in the (Age,  $V_{EJ}$ ) plane for Alauda family. The horizontal green line display the value of the estimated escape velocity from the parent body. The red lines display the contour level of  $\psi_{\Delta C}$  associated with a  $1\sigma$  probability that the simulated and real distribution were compatible.

of the family age. Fig. 3 displays  $\psi_{\Delta C}$  values in the (Age,  $V_{\text{EJ}}$ ) plane for Alauda family. Values of the  $V_{\text{EJ}}$  parameter, that describes the spread in the terminal ejection velocities (Vokrouhlický et al. 2006a,b,c), tend to be lower than the estimated escape velocity from the parent body (Bottke et al. 2015),  $156.5 \text{ m s}^{-1}$  for the case of the Alauda family. To estimate nominal values of the uncertainties associated with our estimate of the age and the  $V_{\text{EJ}}$  parameter, here we used the approach first described in Vokrouhlický et al. (2006a,b,c). First, we computed the number of degrees of freedom of the  $\chi^2$ -like variable, given by the number of intervals in the C distribution with more than 10 asteroids (we require a minimum number of 10 asteroid per C interval so as to avoid the problems associated with dividing by small number when computing  $\psi_{\Delta C}$ ), minus 2, the number of parameters estimated from the distribution. Then, assuming that the  $\psi_{\Delta C}$  probability distribution is given by an incomplete gamma function of arguments  $\psi_{\Delta C}$  and the number of degrees of freedom, we computed the value of  $\psi_{\Delta C}$  associated with a  $1\sigma$  probability (or 68.3 per cent) that the simulated and real distribution were compatible (Press et al. 2001).]

<sup>1</sup> We should caution the reader that other methods to estimate the uncertainties for the estimated parameters are also used in the literature. For instance, one can compute the  $1\sigma$ ,  $2\sigma$ , or  $3\sigma$   $\chi^2$  values and sum these to the minimum observed value of  $\chi^2$  to obtain estimates of the errors at  $1\sigma$ ,  $2\sigma$ , or  $3\sigma$  levels

**Table 2.** Estimated age and ejection velocity parameter from Monte Carlo chronology for six ‘Brož’ families.

Name	Age (Myr)	$V_{\text{EJ}}$ ( $\text{m s}^{-1}$ )	$\psi_{\Delta C}$
Eunomia	$2020^{+650}_{-400}$	$90^{50}_{-90}$	18.24
Koronis	$2360^{+2240}_{-2090}$	$70^{+60}_{-70}$	12.59
Themis	$1500^{+2650}_{-1010}$	$70^{+75}_{-70}$	15.41
Meliboea	$640 \pm 10$	$15^{+10}_{-5}$	7.02
Ursula	$1060^{+3540}_{1060}$	$45 \pm 45$	4.30
Alauda	$640 \pm 50$	$95^{+15}_{-20}$	27.77

**Table 3.** Estimated maximum ages from this work, Brož et al. (2013), and Spoto et al. (2015).

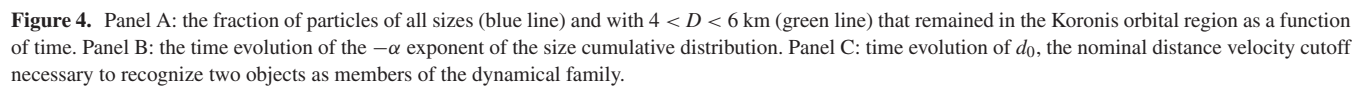
Name	Current estimates (Gyr)	Brož et al. (2013) (Gyr)	Spoto et al. (2015) (Gyr)
Eunomia	2.67	3.00	2.35
Maria		4.00	2.35
Koronis	4.60	3.50	2.24
Themis	4.15	3.50	<4.60
Meliboea	0.65	<3.00	
Ursula	4.60	<3.50	4.52
Alauda	0.69	<3.50	

For the case of the Alauda family, we had 32 C intervals with more than 10 asteroids, and 30 degrees of freedom. The Alauda family should be  $640 \pm 50$  Myr old, with  $V_{\text{EJ}} = 95^{+15}_{-20}$  m s<sup>-1</sup>, and a secondary minimum at lower values of  $V_{\text{EJ}}$ . Our results for the six families are summarized in Table 2, where we display the name of the family, the estimated age, the values of  $V_{\text{EJ}}$ , and the limit used for  $\psi_{\Delta C}$ .

Overall, we found that no family is nominally older than 2.7 Gyr, but uncertainties are too large for the Koronis, Themis, and Ursula families for a positive conclusion to be reached. We compared our results with the maximum age estimates from Brož et al. (2013) and from the more recent work of Spoto, Milani & Knežević (2015), that found estimates for the family ages with a V-shape method in the  $(a, 1/D)$  domain. Table 3 shows the maximum estimated ages

(Press et al. 2001). Since age estimates for very old families, such as those studied in this work, tend to produce shallow minima, here we prefer to use the Vokrouhlický et al. (2006a,b,c) approach, so as to provide a more limited range of estimated values. But larger values of the error estimates are possible.





Name	Fraction of surviving asteroids (all sizes) (per cent)	Fraction of surviving asteroids ( $4 < D < 6$ km) (per cent)	$-\alpha$	$d_0$ (m s $^{-1}$ )
Eunomia	36.8	53.3	1.7	77.3
Maria	72.7	92.0	1.7	63.5
Koronis	76.2	88.3	1.8	42.2
Themis	69.5	89.4	2.0	92.3
Meliboea	9.0	16.6	2.4	375.2
Ursula	65.1	92.1	2.0	137.4
Alauda	22.1	8.5	1.9	171.4



## ACKNOWLEDGEMENTS

We thank the reviewer of this paper, Miroslav Brož, for comments and suggestions that improved the quality of this work. We would like to thank the São Paulo State Science Foundation (FAPESP) that supported this work via the grants 14/06762-2 and 14/24071-7), and the Brazilian National Research Council (CNPq, grant 305453/2011-4). DN acknowledges support from the NASA Solar System Working (SSW) programme. The first author was a visiting scientist at the Southwest Research Institute in Boulder, CO, USA, when this article was written. This publication makes use of data products from the *Wide-field Infrared Survey Explorer WISE* and NEOWISE, which are a joint project of the University of California, Los Angeles, and the Jet Propulsion Laboratory/California Institute of Technology, funded by the National Aeronautics and Space Administration.

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