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Alex O. C. Davis, Valerian Thiel, Brian J. Smith. Measuring the quantum state of a photon pair entangled in frequency and time. *Optica*, 2020, 7 (10), pp. 1317-1322. 10.1364/OPTICA.396933 . hal-03018709

**HAL Id: hal-03018709**

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Submitted on 23 Nov 2020

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## Measuring the quantum state of a photon pair entangled in frequency and time

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Received 4 May 2020; revised 3 August 2020; accepted 7 September 2020 (Doc. ID 396933); published 30 September 2020

Quantum entanglement plays a prominent role in both foundational physics and emerging quantum technologies. Light is especially promising as a platform for experimental realizations of high-dimensional entanglement, for which the time-frequency degree of freedom provides a natural encoding. Here, we propose and demonstrate a technique to determine the full quantum state of a pair of photons entangled in the time-frequency domain. Our approach, based on spectral shearing interferometry, is entirely self-referenced. To test our system, we measure a photon-pair source with nonlocal spectral phase that results in entanglement between the photons, in which the time when either photon is detected is correlated with the frequency of the other photon. The results demonstrate an effective new tool for exploring the temporal and spectral characteristics of multipartite quantum systems exhibiting high-dimensional entanglement. © 2020 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

<https://doi.org/10.1364/OPTICA.396933>

### 1. INTRODUCTION

Entanglement is not only a fundamental signature of quantum physics, but an essential resource in emerging quantum technologies [1]. High-dimensional entangled states of light are of interest as a resource for a range of tasks, such as quantum computation [2], quantum key distribution (QKD) [3], quantum communications [4], and dense coding [5]. The ability to fully characterize such states is of widespread interest both for the development and implementation of such technologies and to enable tests of the foundations of quantum theory [6].

Quantum correlations between photon pairs can arise in any of the physical degrees of freedom of light, including polarization [7], spatial mode [8], and the longitudinal or time-frequency (TF) mode. The TF degree of freedom has recently gained significant interest as a basis for encoding quantum information due to its large accessible dimensionality and compatibility with integrated optical platforms [9]. High-dimensional TF entangled photon pairs, which can display tight correlations in time and frequency, have emerged as a useful resource for QKD [3,10], quantum-enhanced sensing [11], nonlinear spectroscopy [12], and quantum range-finding protocols [13]. In addition, high-dimensional entangled systems offer advantages over entangled qubit systems in foundational tests based on violation of generalized Bell inequalities [14].

These protocols require well-characterized sources and detectors of entangled light, the development and evaluation of which demands a measurement scheme that is sensitive to nonclassical correlations [15,16]. Additionally, the intermediate devices that

form the building blocks of quantum networks, such as quantum repeaters and memories, must be verified to conserve the fragile nonlocal coherences of their input states [17]. A complete characterization scheme for TF entangled photons is therefore of considerable interest for the development of quantum technologies that employ optical TF encoding.

Measurements that achieve partial characterization of joint spectral-temporal properties of two-photon states have been recently demonstrated [18,19]. Joint spectral intensity (JSI) measurement, which gives the probability of detecting the photons with given frequencies, is now a standard characterization method [15]. The joint temporal intensity (JTI), which gives the probability of detecting the photons at given arrival times, is more challenging to measure for broadband light, as the timescales involved can be on the order of tens to hundreds of femtoseconds. Both of these measurements are insensitive to phase and hence cannot reveal the full extent of possible nonclassical correlations [15]. Taken together, joint measurements in both frequency and time have enabled partial characterization of entangled ultrafast photon pairs [19], but still cannot provide the full two-photon state. Recently, complete characterization of restricted classes of two-photon states with frequency correlations has been achieved by either time-resolved [20] or frequency-resolved [21] two-photon interference measurements. This approach is limited to nearly degenerate, tightly correlated photon-pair sources with joint temporal amplitudes that depend only on the two-photon time difference and cannot address each photon independently. A recent demonstration employing sum frequency generation with classical

reference fields achieved full characterization, at the experimental price of prohibitive group velocity and phase matching conditions and stable and well-characterized reference fields [22].

Here, we demonstrate a self-referencing method to fully characterize the state of an entangled two-photon system based upon our recent approach to single-photon state characterization by electro-optic shearing interferometry [23,24]. In a previous experiment, we used this method to reconstruct the complex spectral amplitude  $\psi(\omega)$  of a classical field mode occupied by a heralded single photon. Here, we incorporate this photon-level classical field mode reconstruction into a coincident detection scheme to fully characterize the TF state of a spectrally entangled photon pair. This is achieved by performing spectrally resolved measurements of one photon in coincidence with an interferometric measurement of the other that is sensitive to the spectral phase. This method remains fully self-referenced, introduces no constraints on the mode overlap between the two photons themselves, and relies on single-photon interference enabling nonlocal correlations to be directly observed. This approach to determine the quantum state of photon pairs will enable further development of novel ways for generating, manipulating, and measuring high-dimensional quantum systems. We anticipate that this work will unlock new opportunities for both quantum technology and fundamental studies of quantum mechanics with entangled quantum systems.

A general two-photon pure state  $|\Psi\rangle$  can be expressed in terms of the complex-valued joint spectral amplitude (JSA)  $f(\omega_1, \omega_2)$ , which is also called the two-photon spectral-temporal wave function,

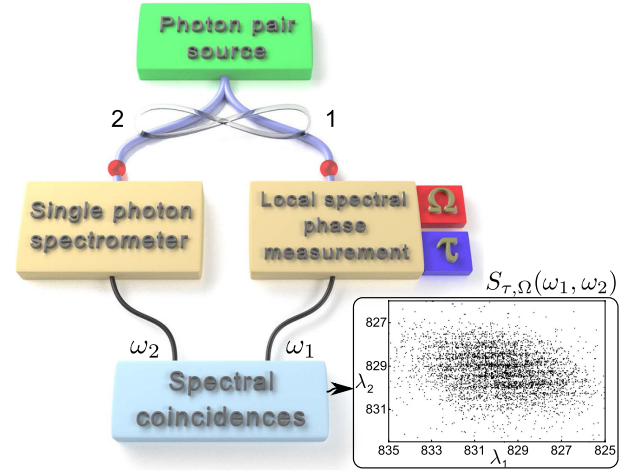
$$|\Psi\rangle = \iint f(\omega_1, \omega_2) \hat{a}_s^\dagger(\omega_2) \hat{a}_i^\dagger(\omega_1) d\omega_1 d\omega_2 |\text{vac}\rangle, \quad (1)$$

where  $\hat{a}_s^\dagger(\omega_2)$  and  $\hat{a}_i^\dagger(\omega_1)$  are bosonic operators that create photons in two orthogonal modes (termed “signal” and “idler”) with frequencies  $\omega_2$  and  $\omega_1$ , respectively, and  $|\text{vac}\rangle$  is the vacuum state of the electromagnetic field. The JSI is given by the square modulus of the JSA,  $S(\omega_1, \omega_2) \equiv |f(\omega_1, \omega_2)|^2$ . Additionally, the JSA is characterized by the joint spectral phase,  $\phi(\omega_1, \omega_2) \equiv \text{Arg}[f(\omega_1, \omega_2)]$ . The joint spectral phase  $\phi(\omega_1, \omega_2)$  may be expanded as a power series about the central frequencies of the two photons  $\omega_{c1}$  and  $\omega_{c2}$ ,

$$\begin{aligned} \phi(\omega_1, \omega_2) = & \phi_{00} + \phi_{10}\tilde{\omega}_1 + \phi_{01}\tilde{\omega}_2 \\ & + \phi_{11}\tilde{\omega}_1\tilde{\omega}_2 + \phi_{20}\tilde{\omega}_1^2 + \phi_{02}\tilde{\omega}_2^2 + \dots, \end{aligned}$$

where  $\omega_1 = \omega_{c1} + \tilde{\omega}_1$ ,  $\omega_2 = \omega_{c2} + \tilde{\omega}_2$ , and  $\phi_{00} \equiv \phi(\omega_{c1}, \omega_{c2})$  is the uncharacterized global phase. This is a complete description of the joint spectral phase in terms of the signal and idler central frequencies and expansion coefficients  $\{\phi_{ij}\}$ . The terms in this expansion with coefficients  $\phi_{0j}$  or  $\phi_{i0}$  are independent of the frequency of one photon and give the classical spectral phase of separable photons. The remaining coefficients  $\phi_{ij}$ , for which  $\{i, j\} \neq 0$ , are collectively referred to here as the *nonlocal spectral phase*. These coefficients encode TF entanglement and are invariant under local phase operations on either photon.

Our approach to measuring  $f(\omega_1, \omega_2)$  is based on the electro-optic spectral shearing interferometer (EOSI). Widely used in classical ultrafast pulse characterization, spectral shearing interferometry is appealing, as the lack of an external optical reference field allows for more rapid characterization with a simpler experimental set-up [25,26]. Our previous demonstration of EOSI



**Fig. 1. Experimental concept.** Spatially separated beams containing the signal and idler photons are measured using frequency-resolved measurements at the output of an electro-optic shearing interferometer registering  $\omega_1$  providing local spectral phase information and a single-photon-counting spectrometer registering  $\omega_2$ . The measured joint spectral interference distribution (inset) follows the expression in Eq. (2).

used electro-optic spectral shearing to extend these benefits to the characterization of single-photon-level pulses [24]. The experimental concept for the full two-photon state characterization incorporates EOSI and is illustrated in Fig. 1. Initially, the idler photon passes through an EOSI comprised of a Mach–Zehnder interferometer, where one path undergoes a spectral shear,  $\Omega$ , and the other path imparts a relative delay,  $\tau$ . The spectral shear is applied using an electro-optic modulator (EOM), the deterministic nature of which ensures unit internal efficiency of the frequency translation. The output of the interferometer is monitored by a single-photon-counting spectrometer registering frequency  $\omega_1$ . The signal photon is simultaneously monitored by a second single-photon-counting spectrometer registering frequency  $\omega_2$ . The JSI of the idler photon upon emerging from one port of the EOSI and the signal photon is then

$$\begin{aligned} S_{\tau, \Omega}(\omega_1, \omega_2) = & \frac{1}{4} \{ S(\omega_1, \omega_2) + S(\omega_1 + \Omega, \omega_2) \\ & + 2\text{Re}[f(\omega_1, \omega_2) f^*(\omega_1 + \Omega, \omega_2) e^{i\omega_1 \tau}] \}. \quad (2) \end{aligned}$$

The joint spectral phase difference,

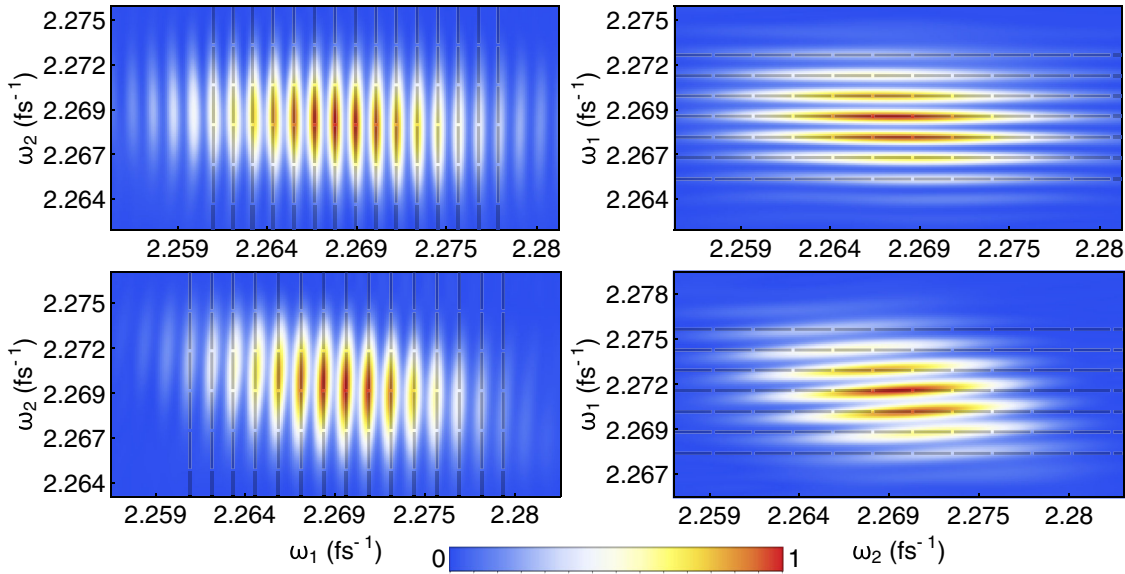
$$\phi(\omega_1 + \Omega, \omega_2) - \phi(\omega_1, \omega_2) = \text{Arg}[f(\omega_1, \omega_2) f^*(\omega_1 + \Omega, \omega_2)],$$

can be extracted from the spectral interference term in Eq. (2) using Fourier transform filtering techniques in [26] and Supplement 1. From this phase difference measurement, it is possible to reconstruct  $\phi(\omega_1, \omega_2)$  up to an additive term that is a function of  $\omega_2$  only (specifically, the terms with coefficients  $\phi_{0j}$ ). This term can be determined by repeating the measurement with the paths of the signal and idler photons exchanged. The JSI and the relative phase of the complex JSA  $f(\omega_1, \omega_2)$  can thus be determined from these measurements.

## 2. EXPERIMENTAL METHODS

To demonstrate our method for measuring the complex JSA, we prepare photon pairs with nearly factorable (uncorrelated)





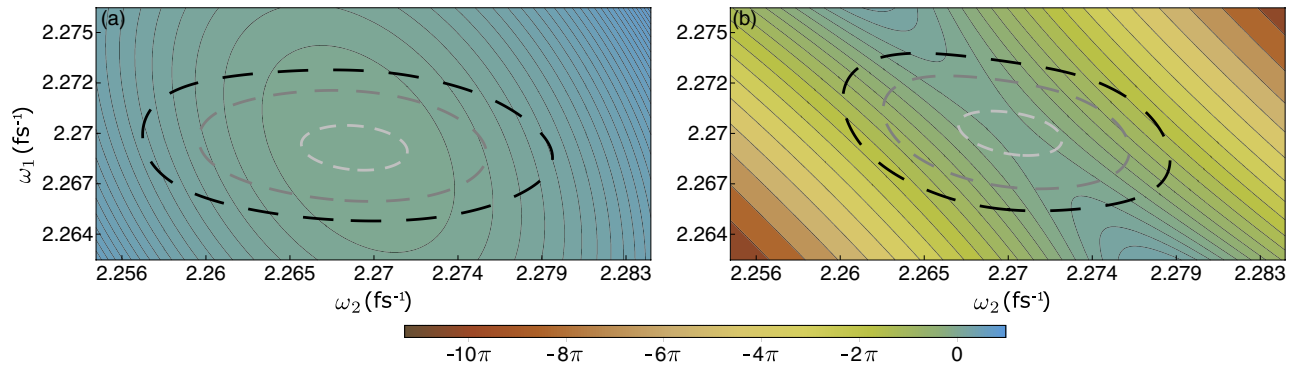
**Fig. 3.** Experimental low pass filtered joint spectral interferograms for both daughter photons of the SPDC in the case of an unchirped (top) and chirped (bottom) pump, representing a typical 6 min acquisition. A color map shows JSI normalized to the maximum. Diagonal fringes in the chirped pump case are indicative of spectral phase correlations between the two photons. Dashed lines perpendicular to the axis of the phase-sensitive measurement serve as a visual aid to emphasize the slant in the chirped case.

For comparison, the experiment was repeated without a chirped pump by removing the prism stretcher. A pulse shaper was used to select the bandwidth of the idler mode for the unchirped pump measurement to ensure matched JSI with the chirped pump case. The pulse shaper was fiber-coupled, adding additional dispersion to the light before the EOSI. This phase is well characterized and is compensated by the pulse shaper itself, as described in Refs. [23,24]. The joint spectral interference fringes depicted in Fig. 3 are not slanted as they are in the chirped pump case, pointing to the absence of phase correlations. We recovered a small but significant positive correlated phase  $\phi_{11} = (7.4 \pm 1) \cdot 10^3 \text{ fs}^2$  and  $(5.5 \pm 2) \cdot 10^3 \text{ fs}^2$  for the idler and signal passing through the EOSI, respectively. While small, these values are statistically significant and can be attributed to the pump pulses being slightly chirped at the point of the second harmonic generation process. The reconstructed local spectral phases were  $\phi_{20} = (1.4 \pm 0.5) \cdot 10^4 \text{ fs}^2$  for the idler and  $\phi_{02} = (1.2 \pm 0.2) \cdot 10^4 \text{ fs}^2$  for the signal, showing that the pulse

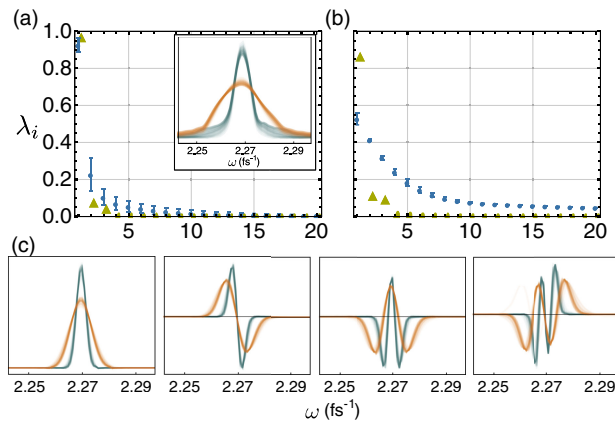
shaper was close to compensating the large amount of dispersion caused by the optical fiber leading to the interferometer.

The complete JSA  $f(\omega_1, \omega_2)$  for the cases of a chirped and a nearly transform-limited pump pulse was estimated from the joint EOSI data and is shown in Fig. 4. While the JSI distribution is nearly uncorrelated in both cases, the two-photon state for the case of a chirped pump exhibits significant spectral phase correlations encapsulated by the asymmetric saddle behavior. This distribution is not possible for a separable state and reveals entanglement from the spectral phase only. The magnitude of the JSAs differs between both cases because the prism compressor modifies the spatial mode of the pump and hence the interaction within the SPDC crystal, inducing weak amplitude correlations.

Since  $f(\omega_1, \omega_2)$  is a function of two variables, it is possible to represent it in terms of a singular-value or Schmidt decomposition [29]. This represents  $f(\omega_1, \omega_2)$  as a weighted sum of products of normalized Schmidt mode functions  $\psi_i(\omega_1)$  and  $\chi_i(\omega_2)$  (Supplement 1), where modes are numbered in decreasing



**Fig. 4.** Full phase-sensitive reconstruction of a two-photon joint spectral-temporal wave function,  $f(\omega_1, \omega_2)$ , for (a) an unchirped and (b) a chirped pump field. Background color indicates the phase of the complex JSA,  $\text{Arg}[f(\omega_1, \omega_2)]$  (plotted on an unwrapped scale), whilst the dotted contours show lines of constant amplitude  $|f(\omega_1, \omega_2)|^2$ . The state shown in (a) is nearly separable, whereas the nonlocal phase of the pump in (b) induces multimode entanglement.



**Fig. 5.** Top: comparison of first 20 Schmidt coefficients of the measured complex JSA functions (a) with a nearly transform-limited pump and (b) with a chirped pump, illustrating the introduction of correlations. Green triangles represent Schmidt coefficients assuming flat phase distribution. Blue dots represent Schmidt coefficients calculated using the reconstructed phase distributions. Error bars are determined by the variance of Schmidt coefficients across the ensemble of reconstructions. (a) Inset: spectra of the dominant pair of normalized Schmidt modes with nearly transform-limited pump for signal (orange) and idler (cyan) photons. (c) Real part of the first four pairs of normalized Schmidt modes in the chirped-pump case (reconstructed from experimental data) for signal (orange) and idler (cyan) photons.

magnitude of the corresponding Schmidt coefficients. We use the effective Schmidt rank  $K$  as our measure of entanglement, with  $K > 1$  indicating spectral-temporal entanglement (Supplement 1). The first 20 Schmidt coefficients are depicted in Fig. 4 for both the chirped and unchirped pump cases. As a comparison to illustrate the importance of spectral phase correlations, the Schmidt decomposition of the modulus of the JSA,  $|f(\omega_1, \omega_2)|$ , was also calculated in both cases.

The distribution of Schmidt coefficients when assuming a constant spectral phase is dominated by only a single term in both the transform-limited and chirped pump cases, with Schmidt number  $K = 1.016$  and  $K = 1.059$ , respectively. When the measured joint spectral phase is taken into account, for the transform-limited pump case, the dominant Schmidt mode pair consists of the marginals of the JSA,  $f(\omega_1, \omega_2) \approx \psi(\omega_1)\chi(\omega_2)$ , with the Schmidt coefficients of the higher-order modes negligible, resulting in a Schmidt number  $K = 1.2 \pm 0.2$ . The measured nonlocal phase in the case of a chirped pump results in a highly multimode decomposition with the Schmidt number increased to  $K = 5.0 \pm 0.5$ . There are significant contributions from many Schmidt mode pairs for this state. The first four Schmidt modes are shown in Fig. 5, and their functional form resembles the first four Hermite–Gauss modes, as predicted when analyzing the output of SPDC [30]. These results confirm that when generating photon pairs using SPDC, controllable manipulation of the pump phase introduces spectral phase correlations in the daughter photons that influence the degree of two-photon entanglement. They furthermore highlight that to reveal this high-dimensional entanglement requires characterization of the joint spectral phase and not simply the JSI. We also note that given a well-known down-conversion process, the nonlocal phase of the daughter photon pair encodes the phase of the pump pulse and is invariant under local phase operations, suggesting a potential classical application in the characterization of ultraviolet pump pulses. Lastly, in directly

measuring the full effect of pump chirp on the wave function of an SPDC photon pair for the first time, to the best of our knowledge, our work carries a result of direct interest to developers of PDC-based sources of quantum light.

#### 4. CONCLUSION

To conclude, we have applied the technique of EOSI to demonstrate an original, highly flexible, and reference-free method to measure the global wave function of nonlocal quantum systems that exhibit entanglement across a high-dimensional basis. Since our method depends only on linear optical processes and has no need for an external optical reference, device implementations could be made quite compact and could characterize a larger ensemble of test states than other proposed approaches.

This demonstration, where control of the parameters of the source was shown to have a subtle but profound effect on the quantum properties of the output state, exemplifies the importance of a full characterization scheme in understanding the properties of quantum devices. In providing a tool for understanding and hence controlling the behavior of the elements of a quantum network, this approach has the potential to lead to transformative changes in high-dimensional quantum optical technologies such as TF-encoded cryptography, simulation, metrology, and communications.

**Funding.** National Science Foundation (1620822); Horizon 2020 Framework Programme (665148); Defence Science and Technology Laboratory (DSTLX-100092545).

**Acknowledgment.** We are grateful to B. Brecht and M. Karpiński for fruitful discussions and insight on the results.

**Disclosures.** The authors declare no conflicts of interest.

See Supplement 1 for supporting content.

†These authors contributed equally to the paper.

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