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Ballistics, Fluid Mechanics, and Air Resistance at Gâvre, 1829-1915 : Doctrine, Virtues, and the Scientific Method in a Military Context

David Aubin*

Abstract

In this paper, we investigate the way in which French artillery engineers met the challenge of air drag in the nineteenth century. This problem was especially acute following the development of rifled barrels, when projectile initial velocities reached values much higher than the speed of sound in air. In these circumstances, the Newtonian approximation according to which the drag was a force proportional to the square of the velocity (v^2) was not nearly good enough to account for experimental results. This prompted a series of theoretical and experimental investigations aimed at determining the correct law of air resistance. Throughout the nineteenth century, contrary to what happened before or after, ballisticians were—with very rare exceptions—alone in trying to tackle the problem of air resistance. This was a complex problem where theoretical considerations, experimental results and computational algorithms intermingled with one another, as well as with the development of new materials and doctrine in artillery. By carefully studying the reasons why ballisticians finally opted for a complex empirical law at the end of the nineteenth century, we show that military procedures for evaluating materials became a yardstick for assessing the worth of mathematical theories as well. In conclusion, we try to assess why military specialists were not able to face the challenges posed by World War I and required the help of civilian scientists and mathematicians.

Keywords: Ballistics, Fluid Mechanics, Air Drag, Military Science, France.

Introduction

In August 1788, the young Napoléon Bonaparte, still a student at the Artillery School of Auxonne, was asked to set up the shooting range in preparation for the trial of guns of different calibers [Teil 1897, 71]. Overseeing the experiments was his mathematics professor Jean-Louis Lombard, who had recently published a translation of Benjamin Robins' experiments in ballistics together with Leonhard Euler's extensive comments [Robins 1783]. During the following winter, the future Emperor attended Lombard's course and some of the handwritten notes he took on that occasion have survived. They

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bear witness to the way a good artilleryman received the professor's lessons on the eve of the French Revolution [Masson & Biagi 1895, 252–261]. For Bonaparte, it seemed clear that Robins' experiments had established that the Galilean parabola definitely was an unrealistic approximation of a projectile trajectory. While projectile velocities remained relatively small, one could apply Newtonian theory according to which resistance was proportional to the square of the velocity (v^2). At higher velocities (above 1,200 feet per second), air drag was three times as much as computed with Newton's law. In his notebook, Bonaparte underscored that these scientific considerations led to practical recommendations regarding the quantity of powder, length of the bore, and the usefulness of rifling for maximum efficiency in artillery.

Historically, ballistics has been a concern of great interest not only to professionals but also to scientists and mathematicians [Long & Weiss 1999]. By showing that a projectile trajectory in the absence of a resisting medium was a parabola, Galileo had come up with a powerful tool for computing range tables, a long elusive goal of artillerymen [Aubin & Tournès forthcoming]. Resistance to projectile motion through air, some thought, could be neglected for practical purposes [Halley 1686]. Even though it had long been recognized, by Isaac Newton no less (book II, prop. X of the *Principia*), that the parabolic trajectory was unrealistic due to air drag, an extended book of tables for the use of bombardiers was published based on the parabolic assumption [Belidor 1731]. Soon after, however, the British mathematical scientist Robins invented an instrument called the ballistic pendulum and experimentally showed that projectile trajectories were far from parabolic, at least in the case of light firearms.[1] Following Robins' experimental work, Euler developed a theoretical analysis of ballistic trajectories that was based on the calculus. A few decades later, a series of experiments were performed by Charles Hutton, one of Robins' followers at Woolwich, an account of which was translated into French [Hutton 1802}.[2] These experiments definitely established the need for a new air drag law, even for big guns.

In the late eighteenth and early nineteenth century, all over Europe a new type of technician emerged—the military engineer who received an increasingly sophisticated mathematical and scientific education [Steele 1994, Alder 1997, Alder 1999, Bret 2002]. The aim of this article is to show how this new type of professional introduced a distinctive experimental approach to rational kinematics. Throughout the nineteenth century ballistics was an area of research mostly pursued through the work of military engineers, most of them in France trained at the École polytechnique.[3] Even though ballistics may have been the only area of science that dealt with air drag at high velocities, academic mathematicians and physicists all but deserted the question during

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[1] On the history of ballistics in general, see the classical studies establishing its crucial role in the development of mechanics at the time of the Scientific Revolution [Charbonnier 1928, Hall 1952]. On Robbins and ballistics in the eighteenth century, see also [Barkla 1973, Steele 1994, Barnett 2009]. On the problem of air resistance more specifically, see [Guilbaud 2012].

[2] For a description of Woolwich at that time, see Dupin [1824], 1:85–96.

[3] In the following, we systematically underline the polytechnical training of the scientists and military officers we discuss by recalling, as is traditional, their year of admission preceded by the letter "X," like, e.g., Augustin-Louis Cauchy (X 1805) or Henri Poincaré (X 1873). For research on polytechnicians, I used the database "Famille polytechnicienne" of the Bibliothèque centrale de l'École polytechnique (`https://bibli-aleph.polytechnique.fr/`).

this period. In this paper, we shall be especially interested in the problem of finding a law for the resistance of the air acting on a projectile shot at supersonic velocities. Only after the outbreak of World War I, confronted with the development of in- direct fire, of anti-aircraft gunnery, and of heavy artillery—not to mention the need to compute quickly a great number of firing tables for new weapons—did military ballisticians finally sought academic scientists' help once again [Aubin 2014].

Military engineers' success in mathematizing the science of artillery in the nineteenth century was no small feat, especially in view of the fact that they were able to keep up with—and indeed often triggered–tremendous technological change. From 1855 to the end of the 1880s alone, according to one study [Anon. 1889], initial velocities doubled while the kinetic energy developed by artillery pieces was multiplied by ten (see fig. 4 below). Technological change was met by military engineers with intense research and innovation on both theoretical and experimental grounds. What is greatly interesting about this type of research in a military context is the fact that criteria used to assess the value of scientific results might be more complex than just accounting for observed behaviors. A striking peculiarity of the military context was the insistence on tradition—on "virtues" indeed. Various scientific approaches were not only assessed against one another; they were often officially "adopted" by bodies able to enforce strict obedience to the methods by whole teams of people working on a problem. In this context, debates about the scientific method became very heated, since any innovation had to be persuasively presented to officials if it had any chance of being implemented. Because of this, military experimental stations, the many committees in charge of adopting new procedures or new materials, and the military journals are revealing observatories for watching the development of the experimental method and the way in which it relied more and more on precise measurement and no less precise mathematical treatment of the data.

< Insert figure 1 here, with caption: The Gâvre experimental station: engraving by Godefroy Durand representing an accident on 30 June 1865. Source: *Le monde illustré* (22 July 1865): 49. >

The problems faced by artillerymen were indeed very complex: to understand them they not only had to rely on several branches of science but often needed to push their boundaries further. But pure knowledge was never the ultimate objective of the military scientists. In 1899, the so-called "Gâvre function" was adopted by French Navy engineers as the best representation for air drag. This was a complex empirical law for which there was no adequate theoretical basis. But in practice, to compute projectile trajectories, it seemed to do the trick. In this paper, we explore the way this pragmatic choice was finally made, after nearly a century of intense investigations. By carefully studying the reasons why ballisticians finally opted in favor of a complex empirical law, we show that military procedures for evaluating materials became a yardstick for assessing the worth of mathematical theories as well.

1. Gâvre and the Problem of Air Resistance in the 1830s

On June 22, 1829, the French Minister of Navy Jean-Guillaume Hyde de Neuville (1776–1857) established the *Commission d'expériences de Gâvre*} [hereafter, Gâvre Commission; fig 1] and explained his decision as such:

> Part of the fire pieces recently adopted by the services of the Royal Navy has only been the object of preparatory tests, especially with regards to the span of the ranges, the accuracy of shooting, the initial speed of solid and hollow projectiles, and the effects they can produce on the broadsides of warships.
>
> I wish that measures be taken so that the data already gathered on these various points be completed and corrected by means of a series of experiments whose program will be written down under the care of the M. the Inspector of the Material of Artillery, and I have chosen for these experiments the port of Lorient, because at the same time as one finds there more officers and artillery troops than in other ports there seems to be in the neighborhood of the city some places suitable for the trials in question.[4]

For three years, in the summers of 1830, 1831, and 1832, the Ministry assigned five naval officers and as much as seventy men to carry out experimental trials with cannons. Experiments took place on a beach on the Gâvre peninsula, near Lorient, Brittany (fig. 2). Covered by the sea at each tide, the fine sand naturally registered the impact of projectiles falling on the beach. In those experiences, powder, guns, and projectiles were precisely calibrated, and meteorological data were recorded. One of the goals of the first sets of experiments carried out in Gâvre was to determine the initial velocities of projectiles. The only means experimenters had at their disposal however was to try and determine initial speed on the basis of measured ranges. Obviously, results crucially hinged on the air drag law that was assumed.

To get a better sense of the state of knowledge concerning air drag at the time, we may look at a book published in 1832 by Lieutenant Prosper Coste, titled *Recherches balistique sur les vitesses initiales, le recul et la résistance de l'air* [Coste 1832].[5] As noted above, the relationship between experimental and mathematical approaches was an important issue as far as military scientists were concerned and Coste's book was no

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[4] "Une partie des bouches à feu récemment adoptées pour les services de la Marine Royale n'a encore été l'objet que d'essais préparatoires, particulièrement en ce qui concerne l'étendue des portées, l'exactitude du tir, la vitesse initiale des projectiles pleins et creux et les effets qu'ils peuvent produire sur les murailles des bâtiments de guerre.
Je désire qu'il soit pris des mesures pour que les données déjà obtenues sur ces différents points soient complétées et rectifiées au moyen d'une série d'expériences dont le programme va être rédigé par les soins de M. l'inspecteur du matériel de l'Artillerie, et j'ai choisi pour l'exécution de ces expériences le port de Lorient parce qu'en même temps que les officiers et les troupes d'artillerie s'y trouvent réunies en plus grand nombre que dans les autres ports, il paraît exister à proximité de cette ville des localités favorables aux épreuves dont il s'agit." Quoted in [Crémieux 1930, 146]. On the history of the Gâvre Commission, see the following publications: [Poyen-Bellisle 1889–1893], [Charbonnier 1906], [Crémieux 1930], [Patard 1930], and [Aubin 2014]. Experimental results were published in [Gâvre 1841}, [Gâvre 1844], [Gâvre 1847], and [Hélie 1865], as well as in the *Mémorial de l'artillerie de marine*, founded in 1873.

[5] For a complementary presentation of the question in the same period, see also a preliminary study of the ballistic pendulum by a professor of the Naval School in Toulon Jean Pierre Louis Antide Roche (X 1805) [Roche 1834].

exception to this. On the title page, Coste significantly put a quote from Denis Diderot, which underscored the importance of the experimental methods in rational mechanics:

> without experimentation, mathematics, with its essentially transcendental approach, leads to nothing precise; it amounts to a kind of generalised metaphysics, in which bodies are stripped of their individual qualities; and there would still be a need for someone to write a great work with the title *The Application of experimentation to geometry* or *A Treatise on the aberration of measurements* [Diderot 1753, 36, original emphasis].[6]\footnote{}

< Include Figure 2 here with following caption:
Map of the harbor of Lorient on the southern coast of Brittany, showing on the top left, the Gâvre peninsula in 1825, just before ballistic experiments started to be conducted on this site. Note that the map in inverted with the North direction pointing to the bottom right corner. Source: Plan général de la place de Port-Louis avec le terrain jusqu'à Lorient et la cote depuis la presqu'il [sic] de Gavres, jusqu'à l'embouchure de la rivière de Quimperlé, by the artillery Captain De Rison. Bibliothèque nationale de France, département Cartes et plans, GE D-14768. >

Born in 1793, admitted at the École polytechnique in 1812, like Michel Chasles and Sadi Carnot, Coste (X 1812) belonged to a new generation of artillery officers whose training stressed methodologies. In 1811, the Artillery School of Metz had complained that the students they got from the École polytechnique ignored too much of the approximate solutions to applied problems [Fourcy 1828, 294]. Like most French engineers concerned with ballistics in the nineteenth century, Coste, who had meanwhile become engineer-geographer [*ingénieur-géographe*] for the Army, insisted on the caution needed to ground hypotheses on empirical evidences.

> I do not believe, I must say, that I have reached the most exact, the simplest, and the most useful formulas in practice: they have often been deduced from too small a number of facts; . . . I only consider my work as an attempt geared at attracting the attention of artillery officers on this matter and at showing that one can, by means of formulas, represent various experiments on initial velocities with a precision to which one was until now far from expecting [Coste 1832, viii].[7]

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[6] "Les mathématiques transcendantes, surtout, ne constituent rien de précis sans l'expérience […] ; c'est une espèce de métaphysique générale, où les corps sont dépouillés de leurs qualitées individuelles. […] Il resterait à faire un grand ouvrage, qu'on pourrait appeler l'APPLICATION DE L'EXPÉRIENCE À LA GÉOMÉTRIE, ou TRAITÉ DE L'ABERRATION DES MESURES."

[7] "Je ne crois pas être parvenu, je l'avoue, aux formules les plus exactes, les plus simples et les plus commodes pour la pratique : souvent elles ont été déduites d'un trop petit nombre de faits, ou elles en diffèrent d'une manière trop sensible [...] ; je ne considère mon travail que comme un essai destiné a' attirer sur ce sujet l'attention des officiers d'artillerie, et à montrer que l'on peut parvenir à représenter, par des formules, les différentes expériences sur les vitesses initiales, avec une exactitude à laquelle on était loin de s'attendre jusqu'ici."

By attracting the attention of artillery officers to the value of ballistic studies, Coste wished that experiments would be carried out in full scale and that the model provided by astronomy (that is, celestial mechanics) be followed more closely:

> My goal would be achieved . . . if I was able to bring about new full-scale experiments worthy of the artillery corps; which would gibe to others the means to follow other ways and apply to artillery . . . the methods used in astronomy in order to perfect [range] tables and increase acquired knowledge [Coste 1832, viii–ix].[8]

But, Coste added, ballistics was more difficult than astronomy because the laws of motion for projectiles were not known with exact precision. Contrary to astronomers, ballisticians were not merely concerned with the computation of coefficients but also with the deduction of fundamental laws. In ballistics, moreover, errors stemmed not only from the imperfection of instruments, but also from the great irregularity of the phenomena under study. It is to be noted that Coste used the theory of errors of Laplace–Gauss to assess the value of the laws he deduced from experimentation. The only sound strategy one had to follow therefore was experimentation: “it is only through numerous and often repeated experiments . . . that one may hope to get closer to . . . the limit of perfection where human intelligence may be forced to stop” [Coste 1832, ix].

Concerning the problem of air drag, Coste introduced little that differed from the knowledge developed in the second half of the eighteenth century. Following Robins’ and Euler’s work on ballistics, other scientists like Jean-Charles Borda [1769], had significantly explored the theoretical domain, while a large series of experiments were performed by Hutton, as recalled above. Using the same elements, Lombard had already summarized the question in his *Traité du mouvement des projectiles, appliqué aux bouches à feu*} [Lombard 1797], which Coste quoted at length. Wishing to counter the widespread feeling that mathematical knowledge was of little use when maneuvering with artillery, Lombard had introduced his translation of Euler’s discussions of Robins’ experiment with a plea for the usefulness of science in the artilleryman’s practice: “His art now belongs to the class of the sciences” [Lombard 1797, xi]. The artilleryman, he argued, was altogether a chemist, a physicist, a metallurgist, a mechanician and a geometer. “It is commonly thought,” he acknowledged, “that shooting fire pieces requires no transcendent lights, that it is even easy with a little bit of experience to reach all the accuracy that is required.” Some officers had indeed voiced opinions stressing the unreliable variability of shooting and, to some extent, the uselessness of the mathematical approach to ballistics [Urtubie 1787, 70]. To Lombard, however, Robins’ experiments and Euler’s computations established that the “physico-mechanical sciences” were the true foundation of the rules of thumb used by the practitioners and that “differential calculus is the instrument that was necessary to use for the discovery

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[8] “Mon but serait rempli [...] si je parvenais à provoquer de nouvelles expériences, faites en grand, et dignes du corps de l’artillerie ; ce qui donnerait le moyen de suivre d’autres routes et d’appliquer à l’artillerie [...] les méthodes en usage dans l’astronomie, pour perfectionner de plus en plus les tables, et augmenter le nombre des connaissances acquises.”

of useful truths." In consequence, he wrote, "with these resources, artillery can only be more constantly, more uniformly, and less uncertainly successful."[9]

As far as air drag was concerned, Lombard's discussion was however far from precise. It was summarized by a few simple, but rather vague, laws. Considering the distinct behaviors of a plane orthogonal to the flow and an oblique plane, Lombard stated that resistance was proportional to the area of surfaces perpendicular to fluid motion and to the square of the sine of the angle α of the inclined plane. In other words, $R \propto S \sin^2 \alpha$ [Lombard 1797, 87–88]. Concerning the proportionality to velocity, Lombard recalled, as we have seen, that following Newton, it was often taken to be proportional to the square of the velocity. There was a theoretical justification for this law, since resistance was assumed to be proportional to the number of particles displaced per unit of time and to the force with which each particle hit the projectile. Both being proportional to the velocity, resistance naturally was proportional to the square of the velocity [Lombard 1797, 89].[10] But he explained that these assumptions were only valid for relatively slow motions. "In all other cases, the density of the fluid in front of the mobile varying with velocity, it is as if the moving body was successively crossing variously dense media" [Lombard 1797, 91]. He refrained from speculating about the form of the formula for resistance applicable to all velocities, but set it up as the ultimate goal for ballistics. Unless this unknown formula led to overly complicated computation, theory would then have "all the perfection one may desire" [ibid.].

Discussing the causes for variations in the range of projectiles sent with the same gunpowder load and at the same angle, Hutton's translator Pierre-Laurent de Villantroys was pessimistic about ever being able to reduced uncertainties caused by the imperfect knowledge of the air resistance:

> The direct resistance opposed by air to the motion of projectile is unknown. Perhaps is there ground to believe that the law it truly follows will never be [known]. In the mean time, it is impossible to doubt its inconstancy, its specific force depending on causes that are seen continuously varying . . . without taking into consideration those about which we have no idea and whose existence can be suspected due to the constant discoveries made by the chemistry of the gases found in the atmosphere or accidentally present.[11]

Discussing Lombard's, Hutton's, and Borda's opinions about air drag, Coste concluded in 1832 that at high speed the Newton law was inaccurate. Like his predecessors, he suggested that the exponent of the velocity might be higher than 2.

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[9] All quotations above come from Lombard's preface to [Robins 1783, i–ii]; quoted in [Buat 1911, 57].

[10] On the history of the square-velocity law, see [Calero 2008].

[11] "La résistance directe que l'air oppose au mouvement des projectiles n'est point connue. Peut-être y a-t-il lieu de croire que la loi qu'elle suit réellement ne le sera jamais. En attendant, il est impossible de douter de son inconstance, sa force spécifique dépendant de causes que l'on voit dans une variation continuelle, le ressort, la densité, le mouvement, etc., sans compter celles dont nous n'avons aucune idée, et dont les découvertes continuelles que fait la chimie des gaz qui concourent à la formation de l'atmosphère, ou s'y rencontrent accidentellement, nous font soupçonner l'existence." P.-L. Villantroys, "Discours préliminaire" [Hutton 1802, xi]. On Villantroys, an artillery officer with some mathematical training, see [Marion 1845].

Supposing that motion occurred in a straight line, he wrote that $v = dx/dt$. Denoting what he called the “elementary speed” by $f = (1/dt)dv$, he derived the equation $vdv = fdx$. In such case, f therefore became equal to the “elementary air resistance,” that is, Nv^2 [Coste 1832, 209]. Then, he supposed that $f = Nv^t$, where $t > 2$, yielding $dv/v^{t-1} = -Ndx$.[12] Assuming that the velocity could be measured at two different positions on the trajectory giving two values V (at $x = 0$) and v (at a certain point x of the trajectory), this equation could be integrated and the coefficient N would be given by the formula [Coste 1832, 227]:

$$N = \frac{1}{(t-2)x}\left(\frac{1}{v^{t-2}} - \frac{1}{V^{t-2}}\right).$$

The method used by Coste to assess the value of the law for various values of the exponent t was efficient. It consisted in computing the value of the coefficient N corresponding to experiments performed under various circumstances and, on this basis, determined in which case it seemed to remain stable. Hutton had shot 1–, 3–, and 6–pound cannon balls with various powder loads. Computing the value of N corresponding to different exponents t, he found that, for lighter balls, the coefficient appeared most stable when the exponent was set at 2.3, while for heavier ones, the corresponding exponent was closer to 2.5 [Coste 1832, 218–232]. Summarizing his results, Coste gave a law of resistance where the exponent was experimentally fixed at 2.3 (known however to be inaccurate for larger balls) and where the coefficient varied as a function of size and weight [Coste 1832, 308–309]. Moreover, he also provided a table of coefficients as a function of the velocity if the main dependence was taken to be quadratic. The air resistance “law” given by Coste was therefore rather peculiar, since (1) no definite conclusion was given about the exponent and (2) the coefficient was moreover assumed to have a complex dependence on not only size and weight but also speed itself.

In this context, it is hardly surprising that the Gâvre Commission decided to adopt a strictly empirical approach to the problem. A shooting range was established, staffed and endowed with material and buildings in order to follow systematic courses of experimentation approaching combat situations. In 1834, the professor of hydrography at the Lorient naval school, Félix Hélie (X 1813), joined the Commission.[13]\footnote{ }.} In 1814, as a student at the École polytechnique, Hélie took part in the defense of Paris. He later attended the Metz Artillery School and served for a few years in the artillery before leaving the army. At Gâvre, he was the only civilian allowed to take part in the experimental work. But he quickly came to assume a leading role in the Commission. Each summer, extensive sets of trials were carried out after which most officers went back to their various duties. Remaining in Lorient to teach at the School of Hydrography, Hélie was in charge of all the computations required to analyze the data and drafted all the reports issued by the Gâvre Commission until his death in 1885. The first fifteen

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[12] There is a mistake here in Coste’s text [Coste 1832, 227]. The correct expression should have been: $dv/v^{t-1} = Ndx$. Since the rest of the development follows the mistaken formula, I go with Coste here. I would like to thank Umberto Bottazzini and Jed Buchwald for pointing this out to me

[13] See the composition of the Commission in the first years in [Gâvre 1844, 3–4]; see also \cite{Poyen-Bellisle 1889,Delauney1892

years of work were summarized in three publications
\cite{Gavre1841,Gavre1844,Gavre1847}.

A staunch empiricist, Hélie distrusted theories developed from first principles, which gave poor results when tested on the proving ground. In his mind, every new range table had to be established through extensive experimental work, and the laws and formulas derived from this massive work could hope for no more than ephemeral existences. In his \textit{Traité de balistique expérimentale} \cite{Helie1865}, he developed his method: summarize experimental results by a mathematical formula as simple as possible that should not be applied outside of the experimental limits used to derive it. Like most people commenting Hélie's work, his biographer Captain Julien Delauney (X 1867) insisted:

\begin{quote}
The method followed by Hélie for the study of various questions was always the same: experimental results were represented by the simplest formula possible to summarize them. It was understood that to apply it outside of the limits of the experiments was risky. No theory was built upon the experiment and the empirical formulas, which can sometimes be seductive but most often leads to bitter disappointments.
\cite[6]{Delauney1892}.\footnote{"La méthode suivie par Hélie pour l'étude des diverses questions était toujours la m\^{e}me: les résultats expérimentaux étaient représentés par une formule aussi simple que possible qui les condensaient. Il était entendu qu'il y avait imprudence \'{a} l'appliquer en dehors des limites des expériences. On n'édifiait sur l'expérience ou les formules empiriques aucune de ces théories qui peuvent séduire quelquefois, mais qui, le plus souvent, conduisent \'{a} d'amers mécomptes."}

\end{quote}

A future president of the Gâvre Commission, Prosper-Jules Charbonnier (X 1884) who as we shall had an opposite opinion, wrote that in his work Hélie showed more patience than originality. "Having the superstition of experiments," he added, "he only wanted to retain their brute results and professed a reasoned distrust of theory"
\cite[413]{Charbonnier1906}. It is indeed striking that Hélie opened his book with the following preliminaries:

\begin{quote}
All the questions relating to the firing of projectiles and their destructive effects belong to ballistics. The principles of rational mechanics are not sufficient to solve them: the forces and resistances that are at play can only be appreciated through observation. A treatise on ballistics must therefore be composed in large part of the descriptions and discussions of experiments whose results often constitute the only possible demonstration of the proposition one feels confident enough to establish
\cite[1]{Helie1865}.\footnote{"Toutes les questions relatives au tir des projectiles et \'{a} leurs effets destructeurs sont du ressort de la balistique. Les principes de la mécanique rationnelle ne suffisent point pour les résoudre ; les forces et les résistances qui se trouvent en jeu ne peuvent \^{e}tre appréciées que par l'observation. Un traité de balistique doit donc se composer en grande partie de descriptions et de discussions

d'expériences dont les résultats constituent souvent la seule démonstration possible des propositions que l'on se croit en droit d'établir"}

\end{quote}
He went on:

\begin{quote}
The search for general laws that rule over the phenomena offers great difficulties. Artillery experiments surely are many, but they almost always are determined by particular circumstances and to satisfy the need of the moment. . . . [I]t folows that the observed laws can only be verified within rather narrow ranges and that carefully constructed formulas only have an ephemeral existence \cite[2]{Helie1865}.\footnote{"La recherché de lois générales qui régissent les phénom\'{e}nes offre de grandes difficultés. Les expériences d'artillerie sont certainement fort nombreuses, mais elles sont presque toujours déterminées par des circonstances particuli\'{e}res et pour satisfaire aux besoins du moment. [...] [P]ar suite les lois observées ne peuvent \^{e}tre vérifiées qu'entre des limites assez resserrées, et les formules construites avec le plus grand soin n'ont qu'une existence éphém\'{e}re."}
\end{quote}
Before we examine further Hélie's attempts at drafting a general account of air resistance, it is useful to review another series of experiments, which took place at about the same time at the Artillery School in Metz.

\section{Didion's Experiments in Metz, 1839–1840}

\begin{figure}[t]
\center
\includegraphics[width=.6\textwidth]{Rennes1833.png}
\caption{Schematics of the ballistic pendulum used by Normand in the Rennes trials of 1833. Source: \cite[pl. \textsc{viii}]{Rennes1833}.}
\label{fig:rennes1833}
\end{figure}

Just a few years after the Ministry of Navy established the Gâvre Commission, the War Minister also set up a study group with the specific goal of improving knowledge about ballistics. The ballistic pendulum used bu Hutton with large guns was still rather unknown in France, except for a brief mention in Persy's lithogragh course at the Artillery and Engineering School of Metz \cite[190]{Roche1834}. In 1826, a series of experiments with a ballistic pendulum was undertaken in the gunpowder factory of Esquerdes, Pas-de-Calais \cite{Anon1837}. At the Artillery School in Rennes, a team directed by Lieutenant-Colonel Pierre-François Hubert Normand (X 1800) was set up under the auspices of the Minister of War. It also used a ballistic pendulum (figure \ref{fig:rennes1833}) to study the trajectories of projectiles under various conditions \cite{Rennes1833}.

At the same time, the \textit{Commission des principles du tir} was established at the Metz School.\footnote{In the following, I have used documentation that was provided to me by Claudine Fontanon to whom I want to express my thanks for her generous collaboration. See also \cite{Didion1839}, \cite{Piobert1842}, \cite{Bru1996},

\cite{Belhoste1996} and \cite[1248ff]{Grattan-Guiness1990}.} This school was famous for the high level of its scientific teachers among whom Guillaume Piobert, Arthur Morin, and Isidore Didion taught mechanics, artillery, and ballistics.\footnote{See their military files at the Service historique de la défense (SHD): resp., GB1326 (Morin), GB3461 (Didion), and GD1288 (2nd ser.) (Piobert).} Graduates of the École polytechnique (the first two were X 1813, like Hélie, and Didion was X 1817), all three perceived themselves as the followers of Jean-Victor Poncelet who had previously taught a course of industrial mechanics at Metz. Starting in 1834, the Metz Commission devised a diversified program of experiments. They investigated the initial velocities of projectiles, as well as the resistance of water and air opposed to the penetration of projectiles. These studies have attracted attention because of their innovative use of probability theories \cite{Bru1996}. To us, they are especially notable for the fact they involved the first attempt at using a ballistic pendulum with full-size projectiles.

Experiments were directed by Piobert and Morin, who were assisted by Didion. When the former two left Metz, respectively, in 1836 and 1839, Didion took the lead of the experimental effort. Sent to the ministry, the reports of the Metz Commission dealt, among other things, with resistance in air (Piobert) and water (Morin).\footnote{Original reports are found at the SHD-Terre, 2 W 296, 297, and 286.} Contrary to the reports of the gunpowder commission in Esquerde, the Rennes Commission, or the Gâvre Commission, these results concerning air resistance were widely diffused. They were published in the \textit{Mémorial de l'artillerie}; integrated in the various editions of the \textit{Aide-mémoire \'{a} l'usage des él\'{e}ves artilleurs} after 1836; and were submitted to prize competitions at the Academy of Sciences. Piobert included some of the results in his \textit{Traité d'artillerie théorique et pratique} \cite{Piobert1836}\footnote{This hardly changed the simple recommendations Piobert gave to artillerymen with regards to aiming: "If one wants to hit an object further than point blank [that is, when fired horizontally], one must aim above the target, or raise the line of sight by aiming an elevated point . . . by a certain quantity called the backside adjustment [\textit{hausse}]. If the target is nearer, one must aim below [that point]" \cite[178]{Piobert1836}; quoted in \cite[72]{Buat1911}.}; Didion published a \textit{Traité de balistique} \cite{Didion1848}; Poncelet discussed the results of the Commission in his \textit{Introduction \'{a} la mécanique industrielle} \cite{Poncelet1841}; Morin published his experiments in his \textit{Leçons de mécanique pratique} \cite[404–417]{Morin1855}; and nearly twenty-five years later, a second, much updated edition of Didion's treatise appeared \cite{Didion1860}. Partly as a result of this, both Morin and Piobert were elected to the Academy of Sciences.

Concerning air resistance, more precisely, a memoir by Piobert, Morin, and Didion was submitted to the \textit{Grand prix des sciences mathématiques} of the Academy of Sciences on 15 June 1836. Using Hutton's published results, Piobert concluded that resistance increased more quickly than the square of the velocity and instead of tinkering with the exponent suggested that the resistance law was composed of two terms: V^2+BV^3 (V being the speed), where the coefficient took the value $B=0.0017$ \cite[21]{Didion1857}. In 1839–1840, a further set of experiments was carried out by Didion at Metz. Very detailed, this series of trials examined the trajectories and initial velocities of projectiles of various calibers (from 8 to 22 cm) with various gunpowder loads. Each time, the state of the atmosphere was systematically registered \cite[3]{Didion1857}. In a report sent to the war ministry in September 1840, the experimenter concluded that the law suggested by Piobert gave results that best fitted experimental data. In May 1844, he computed coefficients derived from both Hutton's and the Metz experiments. For large-caliber projectiles, Didion established the following law for air resistance (taking 1.174 for air density and R for the radius of the projectile):

$$\rho=\pi R^2 \thinspace 0.024(1+0.0023V) V^2$$.

Reviewing Hutton's experiments, Piobert found the same formula with the different multiplicative factor of 0.030586 \cite[22]{Didion1857}. The tenth report of the Metz Commission summarized results on air resistance. The conclusion drawn from all this by Didion was that the law could be applied with great confidence for all shots of spherical projectile if one used the factor 0.027 \cite[79]{Didion1857}). In his treatise first published in 1848, Didion therefore reached a position diametrically opposed to Hélie's and insisted that the problems of ballistics could not be solved by experiments alone:

\begin{quote}
It is in vain, for that matter, that one should try to solve the [ballistic problem] by experimentation alone. In some circumstances [that is, direct fire], it seems that one can do without the knowledge of projectile motion, and that a small number of experiments may suffice to determine the elevation that allows to hit a target at a given distance; that, moreover, one might succeed by trial and error when this distance is but imperfectly known. But one cannot dispense from the knowledge of velocities, flight times, and angles of fall; there are indispensable in a great number of cases \cite[vi]{Didion1848}.\footnote{" C'est en vain d'ailleurs qu'on voudrait essayer de la résoudre [la question balistique] par l'expérience seule. Dans quelques circonstances [tir de plein fouet], il semble qu'on peut se passer de la connaissance du mouvement des projectiles, et qu'un petit nombre d'expériences doit suffire pour déterminer l'angle de projection qui permet de frapper un objet \'{a} une distance donnée ; que, de plus, on pourrait y arriver par tâtonnement lorsqu'on ne conna\^{i}t qu'imparfaitement cette distance. Mais on ne saurait se passer de conna\^{i}tre les vitesses, les durées et les angles de chutes ; il sont indispensables dans un grand nombre de cas."}
\end{quote}

With its two terms, Piobert's law unfortunately led to nonintegrable equations. According to Didion, however, the main problem for the applicability of mathematical ballistics was not computation and integration methods \textit{per se} but the air resistance law. With the Piobert formula, Didion felt satisfied that he could compute accurate approximations by using introduced by Euler, that is, piecewise integration using the average resistance on the portion of trajectory computed.\footnote{Didion's analytic solution of the hodograph need not concern us here. Let us note that his method was approved by the war ministry on November 16, 1845, and then published in the \textit{Recueil des savants étrangers} of the Academy of Sciences, vol. 10. About his integration method, Didion wrote: "The differential equation if the trajectory is not integrable: but one reaches the approximate equation of an arc of some length when, on this length, one replaces the variable value of the ratio of an element to its projection by its mean value. For any point of this arc and as a function of the x-axis, one thus gets the y-coordinate, the slope of the tangent, the time of flight, and the velocity of the projectile" \cite[vii]{Didion1848}; quoted in \cite[75]{Buat1911}.}

\section{The Pitton-Bressant Formula}
\label{sec:3}

At Gâvre also, experimenters were now well aware that the Newtonian drag in v^2 was fairly inaccurate for high velocities. To counterbalance this inaccuracy, they (rather artificially) usually assumed that initial firing speeds grew with shooting angle.\footnote{See \cite[23]{Piton-Bressant1892} and \cite[419]{Charbonnier1906}.} Instead of the Piobert law, which led to nonintegrable equations, the Gâvre Commission opted for a simpler resistance law in v^4 suggested by Alexandre-Hippolyte Piton-Bressant (1820–1867). Admitted as a student at the École polytechnique, Piton-Bressant (X 1840) was expelled after just one just year due to his “bad” conduct. He joined the Navy and travelled overseas. A disciple of the Utopian socialist Charles Fourier, he resigned in protest against the coup d’état of Louis-Napoléon Bonaparte (the future Napoléon III) in 1852. Becoming a teacher and science popularizer for \textit{L’Ami des sciences} \cite[186 and 199–200]{Pinet1898}, he was allowed to join the Navy again in 1861. He published many fundamental papers in the \textit{Mémorial de l’artillerie de la marine} (a journal founded by him). Assigned to Lorient in 1846, after his first trip to the colonies, Piton-Bressant interacted with Hélie and soon wrote a memoir on ballistics that enchanted him. Studying various explicit solutions of the hodograph for quadratic, cubic and biquadratic resistance laws, Piton-Bressant adopted the drag function proportional to v^4 which led to results for the range of the projectiles that were both closer to experimental data and simpler in their expression \cite{Piton-Bressant1892}. He gave the exact solution of the trajectory:

$$ y=x\tan\alpha-g\left[\frac{x^2}{2u_0^2}+\frac{Kx^3}{3}\right],$$
where α was the initial firing angle; u_0 the horizontal component of the velocity of the projectile out of the mouthpiece; g the standard gravity; and K an experimentally determined number, called the air resistance coefficient.

Carefully erasing its theoretical origin, Hélie started to use Piton-Bressant’s formula. In 1850, this law was officially adopted by the Navy and printed in the \textit{Aide-Mémoire d’artillerie navale} for the practical use of officers \cite[526–536]{Lafay1850}. From the Piton-Bressant solution (reprinted with various errors), the range X for a given firing angle was easily found by equating $y=0$:

$$\tan\alpha=g\left(\frac{X}{2V^2} + \frac{KX^2}{3}\right).$$
Conversely, the experimental determination of the range allowed Gâvre officers to determine the air resistance coefficient:

$$K=3 \left(\frac{\tan \alpha}{gX^2} - \frac{1}{2V^2 X}\right).$$

Eighteen years of experimentation had shown that the air resistance coefficient K was independent of initial shooting velocity and angle. Experiments in Metz \cite[294]{Didion1848} were used to determine that one could use the value \cite[53]{Lafay1850}:

$$
K=\frac{73,902}{10^{12}}.

$$
Navy artillerymen could therefore summarize by a single number and formula all that was needed to express the results of twenty years of experimentation—the so-called Piton-Bressant law valid for small angles (from 0° to 40°). But the factor K varied according to the model of the canon used, the projectile, and the gunpowder load. Nevertheless, an approximate formula was given to try and evaluate its value in a few cases:

$$K=\frac{b\delta}{ad},$$
where δ represented air density taken to be 0.0012; a the diametrer of the projectile or the caliber; d its density, on average 7.15, and b a constant number usually taken to be $\frac{55}{10^7}$.

To complicate matters, for larger angles $\alpha>12^\circ$, the value of K was assumed to be not a constant, but rather to be a function of the range X. Most frequently in the end, the Commission would assume that K was a polynomial in X:

$$ 10^{10} K=A+BX+CX^2.$$

Note that this double law for air drag where the coefficient varies as a function of range (itself a function of velocity) comes up rather naturally with the method of stabilizing coefficients already used by Coste. By tabulating K for various values of the range X to check whether it remained stable under various circumstances, one was in position to discern whether there was an additional dependency to factor out. As Charbonnier later wrote :

\begin{quote}
This is a true obesession for the mind to know that numbers unfolding indefinitely in a table have among them sure, mathematical relations and not to know what these relations are. One wants to know the laws governing these numbers and when exact and complete knowledge seem impossible efforts are geared toward an approximate solution \cite[434]{Charbonnier1906}.\footnote{“C’est une véritable obsession pour l’esprit de savoir que des nombres qu’on voit se dérouler indéfiniment dans une table ont entre eux des relations certaines et mathématiques et ne pas savoir quelle sont ces relations. On veut conna\^{i}tre les lois qui gouvernent ces nombres, et quand la connaissance exacte et enti\'{e}re en para\^{i}t impossible, les efforts se portent vers une solution approchée.”}

\end{quote}
But in Hélie’s hands, this approach remained completely empirical: never any attempt was made to derive the coefficients A, B, and C from observable parameters such as the initial elevation α or speed u_0.

Based on the integration of the hodograph with a given, hypothetical form of the air drag law, this approach was fairly theoretical for someone like Hélie. The \textit{Aide-mémoire} also provided a different formula valid for larger angles (from 10° to 30°) relating initial angle and range:

\begin{equation}

\sin3\alpha= \left(\frac{X}{P}\right)^M,
\label{eq2}
\end{equation}

where P was the range at 30° considered as the angle of greatest range and M an exponent “which seems to increase as the caliber decreases” \cite[536]{Lafay1850}. Without precise experimental data, one just assumed that $M=3/2$.

As opposed to the above formula derived from Piton-Bressant’s law (equation \ref{eq1}), this last formula (equation \ref{eq2}) was solely based on observations. The fact that many different formulas coexisted hardly bothered Hélie. On the contrary, he wrote: “It is a good thing to have many formulas, to avoid attaching too exclusive a confidence in one of them, with which data could later be in disagreement”
%Il est bon d’avoir plusieurs formules, afin de ne pas attacher une confiance trop exclusive \'{a} l’une d’elles, avec laquelle les faits ultérieurs pourraient bien \^{e}tre en désaccord”
\cite[72]{Helie1865}. Later, this situation was felt to be rather unsatisfactory:

\begin{quote}
all firing tables remained independant from one another, without mutual oversights and without logical links. The Piton-Bressant equation has thus falllen into the purest of empiricisms—that which painfully follows experiments and which heavily reduces formulas to mediocre instruments of numerical compensation. Artillery however made new progress every day, dragging behind itself a despised science of ballistics whose formulas were tortured and that now seemed up to only the basest tasks \cite[425]{Charbonnier1906}.\footnote{“toutes les tables de tir rest\'{e}rent indépendantes les unes des autres, sans contr\^{o}le mutuel et sans lien logique. L’équation de Piton-Bressant était donc tombée dans le plus pur empirisme, celui qui suit péniblement les expériences et, lourdement, réduit les formules \'{a} n’\^{e}tre plus que de médiocres instruments de compensation numérique. L’artillerie cependant faisait chaque jour de nouveaux progr\'{e}s, tra\^{i}nant apr\'{e}s elle une Balistique méprisée dont on torturait les formules et qui ne paraissait apte qu’aux plus vulgaires besognes.”}
\end{quote}
But this type of criticism may not have been voiced had artillery not undergone major technological changes in the second half of the nineteenth century.

\section{A Changing Technological Context}
\label{sec:4}

When they discuss the ballistic revolution of the eighteenth century, few historians insist on the fact that it went hand in hand with the standardization of artillery materials with the adoption of the so-called “Gribeauval system” of 1765 \cite{Alder1997}. Similarly, the surge of interest in ballistic studies that we have noted in the 1830s was correlated with the adoption of the Valée system in 1827. At that time, the French artillery underwent important reorganization processes. Building on Gribeauval, the system adopted in Republican Year \textsc{ix} (1802–1803) added some complexity by developing new calibers that were not always produced in large enough quantities. In 1827, the Army of the Restoration in many ways decided to go back to the Gribeauval system. But the insistence put on mobility had indeed changed the ballistic properties of the firearms used. Hence the emergence of new experimental commissions, as noted above.

However, it is safe to say that until 1853 artillery received no crucial technological innovation that changed the bases of ballistic science. In the second half of the nineteenth century, on the other hand, shrapnel, oblong projectiles, long, rifled barrel loaded by the breech and cast in steel, among other innovations, revolutionized the materials of artillery. From then on, initial velocities as well as the weight of projectiles increased (figure \ref{fig:Artilleryrange}). In 1846 in Turin, experiments by the Piedmontese captain Giovanni Cavalli compared the range of projectiles shot under the same conditions by traditional fire pieces and by rifles guns. The results were striking: range increasing from 2,700 to 4,000 meters in one case and from 3,000 to 4,200 in another. In France, such innovation were discussed by inventors like Henri-Gustave Delvigne (1800–1876) or military engineers like Antoine-Hector-Thésée Treüille de Beaulieu (1809–1886; X 1829). The system was tried in combat during the Crimean war in 1854, as well as in Algeria in 1857. This was, according to a military historian, a "revolution comparable, as far as the intensity of effects are concerned, to that which occurred following the invention of gunpowder" %évolution comparable, pour ce qui est de l'intensité des effets, \'{a} celle qui s'est produite du fait de l'invention de la poudre" \cite[77]{Hennebert1887}. Delvigne's design led to intensive trials at Gâvre in 1845–1846 and again in 1849, but to no useable weapons \cite{Anon1873}. As a result, rifled barrels were not as widely adopted in France as they were in Great Britain and, especially, Germany, which relied on the famous Krupp workshops in Essen.

From the ballisticians' standpoint, rifled barrels and oblong projectiles brought about crucial changes. Beyond the significant increase in initial velocities, these innovations also modified the characteristics of the trajectory. While spherical bodies rotated erratically in the air leading to uncontrollable deviations, the spinning impelled by rifled barrels stabilized the trajectory (provided the axis of rotation coincided with the trajectory—a problem engineers sought to solve at the time). The stability of the trajectory made it possible to design projectiles with pointed tips that considerably lessened air resistance by a factor of about 1.5 (a fact known for a long time at low speeds, and used for example in ship designs). At constant speed, thinner projectiles had a smaller cross section that also lessened the drag. The fact that projectile could have different shapes therefore needed to be taken into account in the air resistance law.\footnote{Rotating projectiles were however deported sideways. But this effect, the so-called Magnus effect, also known as the \textit{derivation}, was found to be quite stable under various shooting circumstance and relatively simple to model theoretically. We shall not be concerned by it in this article.}

\begin{figure}[t]
\includegraphics[width=1.0\textwidth]{Artillery-range.png}
\caption{Evolution of the maximum initial velocities and range in naval artillery, 1855–1889. Source of data: \textit{Mémorial de l'artillerie de la marine} 17 (1889): 513–535.}
\label{fig:Artilleryrange}
\end{figure}

In 1860, Didion published the second edition of his \textit{Traité de balistique} \cite{Didion1860}. Five years later, Hélie published his \textit{Traité de balistique expéri\-men\-tale}, the results of thirty years of hard work at Gâvre \cite{Helie1865}. The adjective he added to Didion's title was, as we have seen, very significant:

\begin{quote}
The permanent reporter at the Commission of Gâvre saw in theoretical formulas established by various scientists—and notably in General Didion's famous \textit{Traité de balistique}—nothing but a curious exercise useless for artillery \cite[149]{Cremieux1930}.\footnote{"Le rapporteur permanent de la Commission de Gâvre ne voyait dans les formules théoriques établies par divers savants, et notamment dans le fameux Traité de balistique du général Didion, qu'un curieux sujet d'exercice, sans utilité pour l'artillerie."}

\end{quote}
At the Academy of Sciences, the illustrious naval writer, Admiral Edmond Jurien de la Gravière, presented Hélie's distrust for theory in a rather more positive light:

\begin{quote}
The distinctive character of this first edition [Hélie's] was the systematic abstention from all hypotheti[cal though], the patient study, and the rigorous discussion of facts—the scientific method that alone is suited to the search for natural laws. Not that theory was banished; it there to be found, at times quite adavanced. But most of the time formulas only are for M. Hélie . . . a commodious means for provisionally grouping some observed facts, [which was] useful to check them and to foresee their consequences. This was empiricism, if you want, but scientifically-led empiricism \cite[1316]{Jurien1885}.\footnote{"Le caract\'{e}re distinctif de cette premi\'{e}re édition était l'abstention systématique de toute hypoth\'{e}se, l'étude patiente et la discussion rigoureuse des faits ; méthode scientifique qui seule convient dans la recherche des lois naturelles. Ce n'est pas que la théorie en fût bannie ; on l'y rencontre, au contraire, parfois tr\'{e}s savante. Mais, pour M. Hélie, les formules ne sont, le plus souvent, [...] qu'un moyen commode de grouper provisoirement les faits constatés, utile pour en faciliter la vérification et en prévoir les conséquences. C'est de l'empirisme, si l'on veut, mais de l'empirisme scientifiquement dirigé."}

\end{quote}

To show the impact innovation in the artillery system had on ballistics, we may note that Hélie's treatise was divided in two parts of equal importance, respectively devoted to traditional and rifled gunnery. Let us focus here on the second part where the question of air drag was discussed. Generally speaking, for both types of gunnery, air drag R was proportional to air density, cross section of projectiles and a certain function of the velocity:

$$
R=\frac{\pi a^2 \delta}{4 \phi(v)}.
$$

For rifled guns, this function $\phi(v)$ was now dependent on the projectile: “it can only be determined experimentally” %elle ne peut \^{e}tre déterminée que par l’expérience”
\cite[404]{Helie1865}. According to Hélie, experiments with cone-nosed projectiles led to the following law (which was different from both Piobert’s and Piton-Bressant’s laws):

$$
\phi(v)=Hv^3.
$$
Replacing the factor H by c defined as:

$$
c=\delta \frac{\pi gH}{4} \frac{ a^2}{p},
$$
easily computable using velocity measurements at two nearby positions, Hélie was able to determine its value for several cases, but refrained from generalizing it further than for projectiles keeping the same relative proportions. The main conclusion he drew from series of experiments carried out from 1859 to 1861 was that this coefficient could vary in a proportion of 5 to 2 depending on whether the projectile was cylindrical or with pointed nose \cite[416]{Helie1865}.

In the next chapter (that dealt with initial velocities), Hélie drew conclusions about the problems faced by artillery if the whole system was changed in favor of rifled barrels and pointed-nose projectiles.

\begin{quote}
One would no doubt wish for the knowledge of formulas in which all the causes exerting some influence on initial speeds would be separately put in evidence, but they could not be established without long series of experiments whose decision to undertake would be difficult to make \cite[433]{Helie1865}.\footnote{“Il serait sans doute \’{a} désirer que l’on possédât des formules dans lesquelles les effets de toutes les causes qui exercent quelque influence sur les vitesses initiales seraient séparément mise en évidence, mais elles ne pourraient \^{e}tre établies qu’\’{a} la suite de nombreuses expériences qu’on se déciderait difficilement \’{a} entreprendre.”}

\end{quote}
This conclusion also applied to all other elements needed for establishing accurate ballistic tables. In the absence of reliable ballistic theory, especially for air resistance, only extensive—and costly—experimentation would make the adoption of the new artillery system worthwhile. Or, as Charbonnier later wrote, firing circumstances constantly led one outside of earlier bounds and “broke the mould of formulas that were always to reestablish on new bases and infinitely complicated.”\footnote{“les conditions nouvelles du tir des bouches \’{a} feu sortaient \’{a} chaque instant des limites antérieures et brisaient les moules des formules qui toujours étaient \’{a} reprendre sur de nouvelles bases et se compliquaient \’{a} l’infini” \cite[415]{Charbonnier1906}.} This may explain why land and naval artilleries were reluctant to the widespread adoption of

the new rifled steel technology. This however was a reluctance the French would soon pay dearly.

\section{A Changing Geopolitical Context}

During the French-German War of 1870–1871, the French artillery’s unpreparedness had disastrous effects. In a letter, Prince Karl August zu Hohen\-lohe-Ingelfingen recalled:

\begin{quote}
In the campaign of 1870, the Prussian artillery found itself, at the beginning of the war, in a much better position than was the enemy’s artillery, as regarded the question of matériel; since all the Prussian guns were rifled, and were, besides, constructed in conformity with our technical progress, and with the most recent inventions. The French guns, on the other hand, were still the old bronze [smoothbore] guns, which had been altered on Lahitte’s system. This alteration was a sort of half measure.
\cite[29]{Hohenlohe-Ingelfingen}; French ed. quoted by \cite[116–117]{Hennebert1887}
%
%Dans la campagne de 1870, l’artillerie prussienne se trouva, au début, dans une situation matérielle tr\’{e}s favorable vis-\’{a}-vis de l’artillerie ennemie. En effet, toutes les pi\’{e}ces prussiennes étaient rayées et, en outre, construites conformément aux progr\’{e}s techniques et aux invention les plus récentes. Les pi\’{e}ces françaises, au contraire, étaient encore ces vieux canons de bronze \’{a} âme lisse, transformés d’apr\’{e}s le syst\’{e}me Lahitte. Cette transformation constituait, en quelque sorte, un palliatif ; on avait fait, avec ces pi\’{e}ces, des canons qui n’étaient ni chair ni poisson [Lettres sur l’artillerie, letter 2 ; quoted by (Hennebert 1887, 116-117)].

\end{quote}
In France, the shock of the defeat against Germany was tremendous, especially for its armed forces. It also questioned the efficiency of its educational and research systems, as well as their relationship with the army and the industry \cite{Charle1994,Gispert2002}.

The defeat also had important effects on ballisticians. Because Metz was annexed to the German Empire, the Artillery School moved to Fontainebleau where ballistics was generally held in disregard \cite[325–326]{Aubin2014}. To address some of the problems, \textit{ad hoc} methods were developed by Professor Pierre Henry (1848–1907; X 1868) from the Fontainebleau School, or by General Alfred-Nicolas Duch\^{e}ne (1826–1905; X 1845) in the Commissions set up by the Army. But they seemed inelegant, laborious, and idiosyncratic. At Fontainebleau, Henry, Jules Challéat (1867–1936; X 1888), and later Henri Batailler (1871–1915; X 1890) were in charge of ballistics courses. But Army ballisticians deplored the general lack of interest for their trade. According to Henry, the relative success of the empirical formulas provided by the Gâvre commission led to a widespread belief in the inefficiency of theory in ballistics \cite{Henry1894}. In 1904, Challéat’s course barely consisted in four lessons devoted to ballistics and the making of firing tables \cite[2:259 and 64]{Challeat1935}. To him, in 1870 the French artillery still mostly relied on empirical practice to adjust shooting.\footnote{“en 1870, l’Artillerie fran\c caise s’en remettait encore, pour régler

son tir, \’{a} l’habileté de ses pointeurs et au coup d’\oe il de ses officiers, ceux-ci rectifiant, \’{a} leur idée, la hausse et la dérive d’apr\’{e}s l’observation des points de chute, quand elle était possible” \cite[2:236]{Challeat1935}.}
For popular writers, such as Eugène Hennebert (X 1845; 1826–1896), it was the skill of the good artillerymen to know by instinct or experience how to adjust shooting when only an average trajectory could be computed in advance.\footnote{“Ce n’est donc qu’\’{a} une trajectoire moyenne entre toutes les trajectoires que peuvent se rapporter les éléments que l’on détermine \’{a} l’avance avec soin, pour fournir au praticien le moyen de tirer bon parti de sa bouche \’{a} feu. Quant aux causes de déviation générale qui se prononcent indépendamment de toutes les causes d’irrégularité du tir, c’est \’{a} l’opérateur qu’il appartient de les bien reconna\^{i}tre et d’y remédier par des procédés convenablement choisis. Ce ne sont point l\’{a} des éléments d’infériorité réelle pour un tireur intelligent et adroit ; si ces déviations sont bien déterminées, il lui est possible d’obtenir de son arme un sérieux effet utile. L\’{a} est le talent de l’artilleur” \cite[111]{Hennebert1887}.}

Whatever the official position about the use of firing tables and ballistics in artillery, the shock of the defeat produced other important effects on the landscape of French ballistics. In 1874, the \textit{Commission d’expériences de Calais} (an experimental station, similar to Gâvre set up by the Army) produced a comprehensive report on the present state of artillery \cite[“Documents relatifs à l’artillerie,” separate pagination]{Calais1874}. It addressed all current issues from use of steel for making cannons, various system for loading by the breech, the rifling of guns, powders, projectiles, carriages, strategic and tactical use of artillery in campaign, etc. It also devoted special attention to Krupp’s German guns and, finally, to ballistics, conducting an extensive review of the state of theoretical and experimental ballistics.

Concerning air resistance, the Calais commissioners noted that this problem had been recognized for almost three centuries by now and that physicists, mathematicians and ballisticians had progressed very little in their understanding of it. “Should one conclude from this statement that theory is absolutely powerless and that ballistics, à wholly random sort of science,’ is condemned to perpetual empiricism?”\footnote{“voil\’{a} trois si\’{e}cles bient\^{o}t que l’influence de l’air est reconnue et que physiciens et analystes, s’associant aux études des artilleurs, se sont appliqués \’{a} la solution des probl\’{e}mes balistiques. Faut-il donc conclure de cet énoncé que la théorie est absolument impuissante et que la balistique “science toute aléatoire” est vouée \’{a} un perpétuel empirisme ?” \cite[“Balistique”, p. 1]{Calais1874}.} Making use of the extensive documentation available in Calais and of “the experiments carried out daily under pour eyes,” they meant to “determine as much as possible the cause of the failure” of theory [ibid.].

In their report, the Calais commissioners therefore carefully reviewed theoretical and experimental approaches dealing with low velocities, including the work of mathematical physicists and hydrodynamicists like Poisson, Navier, Barré de Saint-Venant, etc. For higher velocities, they recognized that the situation was now verging on the ridiculous as the number of empirical laws suggested by experiments multiplied:

\begin{itemize}
\item av^2+bv^4 \qquad (attributed to Euler, after Robins’ experiments);

\item av^2+bv^3 \qquad (attributed to colonel Duchemin and Didion, after the Metz experiments of 1840);
\item av^2+bv^4 \qquad (Saint-Robert);
\item av^3 \qquad \qquad (Metz experiments, 1956–1858);
\item $av^{5/2}$ \qquad \qquad (Hélie).\footnote{Strangely, the expressions in the first and third lines are indeed identical in the report \cite["Balistique extérieure," 4]{Calais1874}.}
\end{itemize}

The Calais commissioners went on to discuss the most recent experiments in Saint-Petersburg by Nikolai Vladimirovich Maievski (in 1868–1869) and in England by Francis Bashforth (in 1867–1869).\footnote{Both experimental results were published in France in the 1870s: see \cite{Maievski1872} and \cite{Sebert1874}.} In both sets of experiments air resistance on pointed-nose projectiles was determined by measuring speed at two point not too distant from one another (Bashforth famously used electric chronographs). Both pointed out that resistance seemed to change quite suddenly around the speed of sound (340–360 m/s). From this observation, Maievski made an interesting suggestion well in tune with Eulerian piecewise integration methods. Instead of adopting a single function for air resistance, he suggested that the law (for pointed-nose projectiles) was a piecewise-defined function as such:

```
\[
f(v)= \left\{
   \begin{array}{@{  }l@{\thinspace}l}
     av^2+bv^4 &\textrm{  up to }v=280 \textrm{ m,}\\
     a^\prime v^6  &\textrm{  from } v=280\textrm{ to } v=360 \textrm{ m,}\\
     a^{\prime\prime} v^2 & \textrm{  from } v=360\textrm{ to } v=510 \textrm{ m.}\
   \end{array}
   \right.
\]
```

The Calais commissioners doubted the value of such law, arguing at length that no evidence had ever been found for sudden changes in the derivative. They explained that this was contrary to the tradition of mathematical physics and could also be an artifact of the means of analysis. They concluded that the question required more much theoretical and experimental studies—"and above all the special study of the physical law of air resistance"
%et surtout l'étude intime de la loi physique de la résistance de l'air"
\cite["Balistique", p. 27]{Calais1874}. However, the complexity of the question perhaps had a paralyzing effect on the Calais Commission which refused to undertake such studies.

At the end of the nineteenth century, therefore, for a given weapon, once range tables were experimentally established, land artillery was however able to solve most of the practical problems it encountered on the field. It was even able, for some new equipment, to compute its range tables using the formulae of the \textit{Aide-mémoire} (4th ed., 1883, chap. 15). But it did not have theoretical formulae allowing it to predict the ballistic potential of a projected weapon nor the means to establish at small cost its future range tables \cite[2:261]{Challeat1935}.

\section{Rifled Gunnery and the Gâvre Function, 1892–1895}

At Gâvre, changing contexts impacted the extremely empiricist approach favored by Hélie. In 1873, the Navy launched a new series of the journal called \textit{Mémorial de l’artillerie de la marine} with the aim of rekindling innovative research in the field. In 1876, the aging professor of hydrology Hélie published a new study on air resistance in this journal. In the early 1880s, a young polytechnician was sent to Lorient: Pierre-Henry Hugoniot (1851–1887; X 1870), whose inclination for mathematical theories was much greater than Hélie’s. This divergence of method notwithstanding, the two teamed up to publish a second edition of Hélie’s treatise. Mostly written by Captain Hugoniot and published in two volumes in 1884, this greatly expanded edition is a testimony to the evolution of the Gâvrais doctrines where theoretical developments were now gaining momentum. Hugoniot was “an exceptional mathematician, a remarkable physicist, [and] an ample theoretical mind”
%un mathématicien hors ligne, un physicien remarquable, un esprit théorique de large envergure”
\cite[413]{Charbonnier1906}. But he died in 1887 when he was barely 36 years old.\footnote{On the life and work of Hugoniot, see also \cite{Cheret1990} and \cite[148–160]{Johnson1998}. A clear offshoot of his ballistic studies, his seminal paper on shock compression was published in the \textit{Journal de l’École polytechnique} \cite{Hugoniot1887,Hugoniot1889}. Both papers have been translated in \cite[161–358]{Johnson1998}. Hugoniot’s theory of shock waves entered mainstream mathematical physics and was discussed by Paul Appell, in his \textit{Traité de mécanique rationnelle}, vol. 3; by Jacques Hadamard, in his \textit{Le\c cons sur la propagation des ondes}; by Pierre Duhem, in his \textit{Le\c cons sur l’hydrodynamique}, etc.}

With rifled barrels and rotating pointed-nose projectiles, the problem of air resistance addressed by Hélie and Hugoniot in their joint work became more intricate since the drag clearly was dependant on the shape of the projectile. Hélie and Hugoniot now wrote a formula for air drag as such:

$$
R=\frac{\Delta}{g} a^2 f(v) v^2,
$$
that is, the product of the mass of a cubic meter of air, the square of the projectile’s diameter and an unknown function of velocity $f(v)$ departing from the standard Newtonian approximation. Up to that time, form factors had never been considered (although the Calais Commission, following Maievski’s and Bashforth’s conclusions, estimated that the drag on pointed-nose projectile were about $2/3$ that exerted on spheres). Hélie’s experiments showed that the ogival angle γ was a significant variable in the problem, at least for values of this angle between 40° and 90°. In this case, $f(v)$ was proportional to $\sin\gamma$. In their book, Hélie and Hugoniot provided tables for $\frac{f(v)}{\sin\gamma}$ for velocities between 100 and 400 m/s, which were in use at Gâvre until 1893.

In addition to the work of Maievski and Bashforth there was new evidence from Colonel Willem Christoph Hojel from the Netherlands (in 1884) and from the Krupp Company on the Meppen shooting range (various trials from 1879 onward) that the function $f(v)$ was decreasing again at higher speeds. At Gâvre, after Hélie's death, those trials where followed with interest. In 1887, new materials were adopted by the French Navy. Used with smokeless powder (or \textit{poudre B}), those guns propelled their projectiles at much higher velocities. On this occasion, advances since 1855 were summarized in the \textit{Mémorial de l'artillerie de la marine} \cite{Anon1889}. Experimental data available at Gâvre concerning pointed-nose projectiles increased greatly. In 1893, it was decided that a new formula for $f(v)$ had to be adopted.

Foreign trials, it was felt, exhibited "flagrant uncertainties" \cite[570]{Gibert1900}, especially as far as the exact shape of the projectiles were concerned. In the \textit{Revista d'artigliera e genio} (1896, vol. 1), Italian colonel Frederigo Siacci had attributed a "shape index" i to the projectiles and recent work tried to determine the ogival angle of the projectiles used in Russian, British, Dutch and German trials.\footnote{Later adopted by Charbonnier, the approaches put forward by the Italian officer were extremely influential in France; see the French translation of his treatise \cite{Siacci1892}.} Based only on experiments done at the Gâvre shooting range for which ogival angles were known exactly, the Gâvre Commission endeavored to establish the form of this function.

After a moment of doubt (report no. 1193 by Charbonnier), Hélie's law was confirmed and it was established that the ratio $\frac{f(v)}{\sin\gamma}$ was constant for values of the velocity as high as 1200 m/s (reports nos. 1414 and 1429, March 14 and July 21, 1896, written by Eugène Louis Marie Gibert [1857–1909; X 1877], giving $\log f(v)$ for values of v from 0 to 1200 m/s). For lower velocities, however, Colonel Louis Frédéric Gustave Jacob (1857–1930; X 1878), also from Gâvre, suggested that it might be better to use instead the expression \cite{Jacob1899}:

$$R(v)=\frac{f(v)v^2}{\gamma+\frac{1}{3}}.$$

\begin{figure}[t]
 \includegraphics[width=1.0\textwidth]{Gavre1899.png}
\caption{A comparison between resistance laws. \cite[foldout]{Gibert1899}.}
\label{fig:Gavre}
\end{figure}

Although Gibert's conclusion certainly was that a form index i independent of the velocity, as suggested by Siacci, was impossible, the signification of the function $f(v)$ was changed so as to get rid of everything that was affected by shape. The Gâvre function therefore was therefore redefined as the multiplicative factor in the resistance that summarized all the dependencies on velocity. Thousands of numerical results, derived from firing tests with initial speeds from 400 to 1200 m/s with various calibers and ogival angles were used to determine this function which was presented as a table, as a graph (fig. \ref{fig:Gavre}).\footnote{One finds a graphic representation of the laws and the various experimental trials that led to it in \cite[17]{Cranz1913}. See Table I1 in \cite{Gavre1916}; "Tableaux nécessaires au calcul des trajectoires par arcs successifs," SHD–Terre 2W292; and \cite[20–22]{Bingen}. Cf. \cite[152]{Cremieux1930}. See also \cite{Gibert1896}, \cite{Gibert1899}, and \cite{Gibert1900}.} It also, perhaps more surprisingly, received two analytical expressions. In Italy, Siacci suggested the following expression \cite[16]{Cranz1913}:

$$
F(v)=0.2002 v - 48.05 + \sqrt{(0.1648v - 47.95)^2 + 9.6}+\frac{0.442v(v-300)}{371+\left(\frac{v}{200} \right)^{10}}.
$$

In Gâvre, an analytic expression was introduced by the \textit{chef d'escadron}Julien Eugène Demogue (1867–1965; X 1885). The function used during the First World War had the following form where the exponential term was introduced by Maurice Garnier during the war \cite[p; 3]{Haag1921}.\footnote{Note that there is a mistake in this function. To reproduce the numerical data in published tables, the coefficient of the arctangent, and this one alone, should be multiplied roughly by a factor of 3500. See \cite{Aubin2014}.}

$$
F(v)=\left[0.255+\frac{\sqrt[4]{1+0.0392\left(\frac{v-300}{500}\right)^8}}{27,226+494\left(\frac{v-330}{50}\right)^2}\arctan\frac{v-330}{50}\right] \exp\left(\frac{v-600}{10^6}\right).
$$

The purpose served by such expressions is not clear, but for some mathematicians during WWI it was useful in ensuring that approximations used to solve the hodograph were well founded analytically.

\section{The "Brisky Wind of Theory"}

Even though the Gâvre function was, in the end, empirically determined, the period that followed Hélie's death in 1885 witnessed the revival of theoretical approaches at Gâvre. The "struggle engaged between traditionalism and progress was vigorously fought" \cite[416]{Charbonnier1906}. In 1899–1900, however, the fight was over and General Charbonnier (1862–1936), who was also a polytechnician (X 1884), published in the \textit{Mémorial de l'artillerie de marine} a full treatise of exterior ballistics \cite{Charbonnier1899}. Based on Henry's course mentioned above and on the work of several of his predecessors, like Hippolyte Sebert (1839–1930; X 1858), Magnus de Sparre (1849-1932; X 1868) and others from abroad, and, in turn, was the basis for his monumental later work \cite{Charbonnier1907,Charbonnier1927}.

After years of empiricism, the "brisky wind of \index{theory}theory" [\textit{l'air vivifiant de la théorie}] \cite[415]{Charbonnier1906} was now blowing over the Gâvre peninsula. "Theory," Charbonnier added as a caveat:

\begin{quote}
has no pretense to substitute for experimentation nor to confront it as a disdainful adversary. A union of both operations of the mind in a general rule for the search for truth constitutes the essence of the method
\cite[416]{Charbonnier1906}.\footnote{"n'a point la prétention de se substituer \'{a} l'expérience ni de se poser en face d'elle en adversaire dédaigneux. C'est l'union de ces

deux opérations de l'esprit dans une r\'{e}gle générale pour la recherche de la vérité qui constitue l'essence de la méthode."}
\end{quote}
Charbonnier furthermore argued that the most current physical theory was now needed for ballistics. The field of ballistics could lead to bitter disappointment, he underscored. Ballistics was a graveyard of theories, where simple explanations and audacious analogies had led nowhere: "we now know that we must call to the rescue the most profound analysis, in almost unexplored areas of science, to look for the physical laws of exterior ballistics" \cite[440]{Charbonnier1906}.\footnote{"aujourd'hui le découragement para\^{i}t général et définitif et de si nombreuses tentatives il ne reste absolument rien d'utilisable ni d'important. La foi dans les explications simples en l'audace des raisonnements par analogie ont été déçues encore une fois et on sait maintenant que c'est \'{a} la plus profonde analyse, dans des régions presque inexplorées de la science, qu'il faut s'adresser pour chercher les lois physiques de la Balistique Extérieure." }

Theory however had an crucial advantage. Perhaps surprisingly for a domain so intimately tied with national defense, theoretical exterior ballistics as a disinterested branch of science could be pursued at an international level by specialists that were in contact with one another. Together with Bashforth's and Greenhill's efforts in Britain, Siacci's in Italy and Carl Cranz's in Germany, the work of the Gâvre ballisticians completely renewed the field of exterior ballistics. While access to Gâvre was strictly forbidden for anyone outside of the Commission before 1871, most results were now published openly, while knowledge of foreign artillery was widespread. As the Baron Raymond de La Rocque (X 1861), who was an artillery officer, explained, this situation was a mixed blessing:

\begin{quote}
publicity without limits, which has many proponents now in the world, seem to us ruinous for the peoples in general, and particularly funeste to that the most laborious nation, [the one that is] the most fertile in practical ideas [and] in trained and disinterested officers. This publicity will however have the advantage of helping the progress of artillery considered as a universal, international art. All the nations will contribute to this progress by enormous expanses of intelligence and money, without any particular profit for any of them.\footnote{"la publicité sans limites, qui a beaucoup de partisans aujourd'hui dans le monde, nous para\^{i}t ruineuse pour les peuples en général, et particuli\'{e}rement funeste \'{a} la nation la plus laborieuse, la plus fertile en idée pratiques, en officiers instruits et désintéressés. Cette publicité aura, par contre, l'avantage de faire progresser l'artillerie, considérée comme un art universel et international. Tous les peuples contribueront \'{a} ces progr\'{e}s par des dépenses énormes d'intelligence et d'argent, sans aucun profit particulier pour aucun." \cite[9]{LaRocque1885}. About the value of disinterestedness in turn-of-the century French science, \cite[136]{Aubin2014b}.}
\end{quote}

Becoming international, the science of ballistics indeed went through tremendous transformations. At Gâvre, Hippolyte Gossot (1853–1935; X 1874) adopted a method for computing trajectories a priori based on Euler's piecewise integration method, which was first used at Gâvre by him in 1887–1888 to compute the firing tables of new 34–cm and 90–mm cannons.\footnote{See \cite[vol. 2, chap. 8]{Helie1884} and \cite{Gossot1890}. Reports were published in the \textit{Mémorial de l'artillerie de la marine} 16 (1888).} At first, Gossot used an air resistance function of the Maievski type, that is defined piecewise using with various powers of the velocity. In other words, $f(v) \propto v^n$, where n took different values according to the value of the velocity v:

\begin{itemize}
\item from $v=0$ to 240: \qquad\quad $n=2$,
\item from $v=240$ to 295: \qquad $n=3$,
\item from $v=295$ to 375: \qquad $n=5$,
\item from $v=375$ to 419: \qquad $n=3$,
\item and for v above 419: \qquad $n=2$.

\end{itemize}
For Gossot, this method both the most efficient and as precise as the present knowledge of the law of air resistance allowed it to be: "a more simple solution to the ballistic problem cannot be given" \cite[305]{Gossot1888}. His younger colleague Charbonnier however had a different opinion. According to him, "the Commission accepted deliberately and for many years to submit itself to the boredom of the very lengthy, very fastidious, and very inelegant computations of a piecewise trajectory" \cite[427]{Charbonnier1906}, because this was the price to pay for buying its freedom from empiricism. This "natural" approach had the additional advantages of being much cheaper since only a few shots were required for computing a trajectory from first principles. Unobserved parameters of the trajectory could moreover be computed a priori \cite[240–241]{Patard1930}.\footnote{"Si la Commission consentit ainsi \'{a} se soumettre délibérément et pour de longues années aux ennuis des tr\'{e}s longs, tr\'{e}s fastidieux et tr\'{e}s inélégants calculs d'une trajectoire par arcs, c'est qu'elle comprenait bien quelles étaient les conséquences pratiques de son affranchissement des méthodes empiriques et du retour de la Balistique \'{a} sa source naturelle : possibilité de calculer a priori une trajectoire quelconque d'un canon quelconque ; réduction \'{a} un extr\^{e}me minimum des tirs balistiques nécessaires pour établir une table de tir ; facilité de multiplier en quelque sorte ces tirs par le calcul ; obtention d'une valeur exacte des éléments non observés de la trajectoire \cite[427]{Charbonnier1906}.}

Although "slow and painful," Euler's method was adapted to the new resistance law established experimentally. On this basis, the Gâvre Commission was in position to compute all range tables required by artillerymen "with certainty, accuracy, swiftness, and economy" \cite[430]{Charbonnier1906}. But to him, Euler's piecewise integration method could only be "provisional and incomplete"; this merely was "a computing algorithm [\textit{procédé de calcul}] and not . . . a theory" \cite[431]{Charbonnier1906}. Nominated as president of the Gâvre Commision in 1906, Charbonnier opted instead for the Siacci direct fire approximation and without changing the general form just tinkered a little with the Gâvre function $f(v)$ for air resistance. Charbonnier's method was however only valid for small angles $\alpha<15^\circ$, a fact that will come back to haunt the Commission during WWI \cite{Charbonnier1906}.\footnote{For a discussion of Siacci's method, see \cite[321–322]{Aubin2014}.}

Computing methods at Gâvre were thus submitted to the same kind of scrutiny as experimentally-derived laws. Like the Gâvre function for air drag, computing algorithms were officially adopted by the hierarchy. To use a different algorithm, Euler's piecewise integration instead of Siacci's direct fire approximation, amounted to disobey an order. With these computing methods, theoretical ballistics however fully satisfied the artillerymen's prewar needs. "Exterior ballistics was, before the war, brought by the Gâvre Commission to a degree of perfection which fully satisfied all practical requirements" \cite[i]{Garnier1918}. In view of the fact that ballistics was shown to be lacking so much in the spring of 1915 and for several months after, that they had to revise all of their computing methods, and massively enroll civilian mathematicians as never before, this statement by a member of the Gâvre Commission may seem self-apologetic. We may however found traces of a similar sentiment before the war:

\begin{quote}
while it is true that from a purely scientific point of view, the ballistic problem still remains to be solved, since its fundamental equation can only be integrated at the cost of [additional] hypotheses, . . . it is nevertheless the case that approximate solutions due to the successors of the Didions and Pioberts satisfy all practical needs \cite[122–123]{Buat1911}.\footnote{"s'il est vrai qu'au point de vue purement scientifique, le probl\'{e}me balistique reste encore \'{a} résoudre, attendu que son équation fondamentale ne peut \^{e}tre intégrée qu'au prix d'hypoth\'{e}ses, [...] il n'en subsiste pas moins que les solutions approchées dues aux continuateurs des Didion et des Piobert satisfont \'{a} tous les besoins de la pratique."}
\end{quote}

In August 1914, military ballisticians were indeed so confident that the job required of them had been done that the Gâvre personnel volunteered en masse to serve on active duty. Only five officers remained at the shooting range \cite[274]{Patard1930}. Commandant Batailler who, as we have seen, was in charge of the ballistics course at the Fontainebleau School was killed on the Marne front on 9 June 1915. Four months later, an overworked Charbonnier suggested the Minister of the Navy to call on mathematicians to assist him with the computations they were ask to do.\footnote{The story of Gâvre in World War I has been studied recently \cite{Aubin2014}.}

\section{Conclusion}

On January 21, 1931, an official ceremony was organized on the beach of Gâvre to mark the centennial of the Commission. In their speeches, officials insisted on the scientific traditions embodied by the military organization. The recently appointed president of the Commission, Maxime Crémieux (1869–1932; X 1889) said that he perceived "a true intellectual and moral character" at Gâvre, a specific character that spanned its whole history \cite[145]{Cremieux1930}. This of course was a consensual occasion where rough debates about the precedence of theory over empiricism wer diplomatically erased. But insisting of the specific character of the Gâvre Commission, Crémieux's speech reflected something profound about the scientific practice that was able to solve the air drag problem:

\begin{quote}

The gestation of these doctrines was arduous, because the respect for the original, purely experimental tradition was sometimes detrimental to the development of nascent theories due to innovators [that were] judged too audacious. However, it imprinted on the Gâvrais charac- ter a mark that cannot be erased—that of the absolute respect for experimental results dutifully recorded in experimental reports. To the scientific righteousness of the Gâvre Commission, the hiding of disagreements between experiments and a fashionable theory has indeed always felt despicable. On the contrary, [such disagreements] were clearly put in evidence \cite[145]{Cremieux1930}.\footnote{“La gestation de ces doctrines fut ardue car le respect de la tradition originelle purement expérimentale fut parfois nuisible au développement des théories naissantes dues \'{a} ces novateurs jugés trop audacieux. Toutefois, il a imprimé au caract\'{e}re “gâvrais” une marque indélébile, celle du respect absolu du résultat expérimental dûment consigné dans les proc\'{e}s-verbaux. La droiture scientifique de la Commission a toujours répugné en effet \'{a} masquer les désaccords entre l’expérience et la théorie du moment créée par elle. Elle les a mis au contraire nettement en évidence’.’}

\end{quote}
As the strong moral connotations in the above shows, science carried out in the military context answered to ethical demands. In practical terms, one may note that it gave rise to a wealth of written reports that were sometimes published, but most often sent to one’s superiors and stored in the archives. Did this particular way of writing science lead to special methods? In his speech, Crémieux claimed that:

\begin{quote}
a general conclusion can be drawn from the study of the reports of the Commission. Beyond these innumerable studies that have dealt with all the sides of artillery: ballistics, gunpowder, fire pieces, . . . shooting at sea, aerial shooting, underwater shooting, shrapnell, fuses, rockets, observation methods and instruments, range tables, etc.; beyond all experimental and theoretical investigations, one senses a working method that presided over the whole: . . . submission to experimental facts, scientific probity, respect of tradition, minute consideration of details, clarity of deductions, freedom in technical opinions \cite[145]{Cremieux1930}.\footnote{“une conclusion générale se dégage de l’étude des rapports de la Commission. Au-dessus de ces essais innombrables qui ont porté sur toutes les parties de l’Artillerie : balistique, poudres, bouches \'{a} feu, affûts, perforation des cuirasses, projectiles de rupture, tir en mer, tir aérien, tir sous-marin, obus explosifs, amorçage, fusées, méthodes et instruments d’observation, tables de tir, etc... ; au-dessus de toutes ces recherches expérimentales et théoriques, on sent qu’une méthode de travail a dominé tout l’ensemble et guidé, \'{a} chaque époque, la Commission : soumission aux faits expérimentaux, probité scientifique, respect de la tradition, netteté des exposés, minutieuse considération des détails, clarté des déductions, liberté des opinions techniques .”}
\end{quote}

The same brand of military scientific research ethos had been underscored by Charbonnier earlier, when he praised the “Gâvrais virtues”: “respect for the continuity of doctrines, independence of thoughts, the faith in experiments, the patience for long computations and careful verifications” \cite[425–426]{Charbonnier1906}.\footnote{“le respect de la continuité des doctrines,

l’indépendance des idées, la foi dans l’expérience, la patience des longs calculs et des vérifications minutieuses.”}
Science as military-report writing followed the trends set up by academic science, but to these men, at least, it seemed to embody a special moral character. The linchpin of the Gâvrais character was the relationship between theory and experiment, between practice and computing: this was “a very practical conception” of the scientific method, according to which a theory was considered satisfactory “only when it has been numerically experimented . . . often at the cost of considerable off-putting work that can only be understood by those who have carried out similar work”
\cite[432]{Charbonnier1906}.\footnote{“une vue tr\’{e}s pratique des questions qui ne tient une théorie comme satisfaisante que lorsqu’elle a été vérifiée numériquement [...] souvent au prix d’un travail considérable et rebutant dont ne peuvent avoir idée que ceux qui en ont, eux-m\^{e}mes, exécutés de semblables.”}
In a footnote, Charbonnier explained that when an author presented a theory to a commission, he had to pay attention to a few special things:

\begin{quote}
For theoretical considerations\index{theory!vs. practice} to be put in practice, three precautions were required:
\begin{enumerate}
\item compute all necessary tables in full, without disdainfully leaving this vulgar care to technicians [\textit{praticiens}];
\item provide full and detailed \index{numerical results}numerical examples;
\item prepare with great care \index{computing}computation skeletons, which can be lithographed and whose columns are merely left so to speak to be filled out numerically
\cite[432]{Charbonnier1906}.\footnote{“Quand un auteur quelconque présente une théorie quelconque relative aux sciences appliquées \’{a} une commission et plus généralement au public savant, il y a les plus grandes chances de ne jamais la voir entrer en pratique et d’obtenir tout au plus un pur succ\’{e}s d’estime, s’il ne prend pas les trois précautions suivantes : (1) Calculer compl\’{e}tement toutes les tables nécessaires, sans laisser dédaigneusement ce soin vulgaire aux praticiens ; (2) Donner des exemples numériques détaillés et complets ; (3) Préparer, avec le plus grand soin, des mod\’{e}les de calculs qu’on puisse lithographier et dont il n’y a plus qu’\’{a} remplir, pour ainsi dire numériquement, les colonnes préparées pour recevoir les chiffres.”}
\end{enumerate}
\end{quote}

The military engineer’s rigor and ascetics as Hélie, Charbonnier, and Crémieux saw them, each in their ow way, were crucial resources for solving the problems raised by ballistics. While these procedures ensured the high international regard in which results coming out of the Gâvre Commission were held, “doctrinal” thinking, the requirements of efficiency, and respect for hierarchy also rigidified processes. There was a”correct” way to approach problems, one that had received the sanction of time and was explicitly approved by the hierarchy. It was this approach that had allowed the Gâvre ballisticians to provide a practical solution to complex scientific problems like air drag.

During World War I, to train civilians to follow the same routine would be no simple task. The president of the Gâvre Commission from 1925 to 1931, Léon Patard (1872?1964; X 1890) testified to that difficulty: “A success of the Naval Artillery during this war, and not the least, was the use of the good will of all these men, some of whom [were] eminent savants but little prepared for their new role." While perhaps not their equal in pure science, naval engineers showed them that they were able to understand and discuss their ideas. Some tactfulness was required to convince savants to undertake long training courses in routine computing, "to make them comply with the strict rules of experimentation and observation, and finally to lead them progressively from the role of auxiliaries to that of collaborators."\footnote{"Ce ne fut pas une des moindres réussites de l'Artillerie navale pendant cette guerre que l'utilisation des bonnes volontés de tous ces hommes, dont quelques uns étaient des savants éminents, mais peu préparés \'{a} leur nouveau r\^{o}le. Il fallait gagner leur confiance, leur montrer que les ingénieurs d'Artillerie navale, s'ils ne les égalaient pas par la science pure, savaient comprendre leurs idées, discuter leurs théories, répondre \'{a} leurs objections. On devine le doigté qu'il fallut pour les astreindre \'{a} un long stage de calculateurs mécaniques, enrayer leurs tentatives professionnelles de perfectionner d\'{e}s leur arrivée les méthodes de calculs, les plier aux r\'{e}gles strictes de l'expérience et de l'observation, enfin les amener progressivement du r\^{o}le d'auxiliaires \'{a} celui de collaborateurs" \cite[279]{Patard1930}. } At the same time, scientists showed that they were able to solve problems that military ballisticians were ill-prepared to tackle on their own. After the war, they would join the Gâvre Commission on a permanent basis and contribute to impulse even greater changes to the science of ballistics.\footnote{The history of ballistics at Gâvre during the First World War is the topic of another article; see \cite{Aubin2014}.} Once again in their long history, military and academic sciences moved closer to one another.

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