# Degenerate Band Edge Resonances in Air-filled Substrate Integrated Waveguide

Tianyu Zheng, Massimiliano Casaletti, Ahmed Abdelshafy, Filippo Capolino, Zhuoxiang Ren, Guido Valerio

## To cite this version:

Tianyu Zheng, Massimiliano Casaletti, Ahmed Abdelshafy, Filippo Capolino, Zhuoxiang Ren, et al.. Degenerate Band Edge Resonances in Air-filled Substrate Integrated Waveguide. Conference EuCAP 2020, Mar 2020, Copenhague, Denmark. 10.23919/EuCAP48036.2020.9135497 . hal-03169533

# Degenerate Band Edge Resonances in Air-filled Substrate Integrated Waveguide

Tianyu Zheng*, Massimiliano Casaletti*, Ahmed F. Abdelshafy†,
Filippo Capolino†, Zhuoxiang Ren*, Guido Valerio*
* Laboratoire d'Electronique et Electromagnétisme, Sorbonne Université, Paris, France, guido.valerio@sorbonne-universite.fr
† Dept. of EECS, University of California-Irvine, Irvine, CA 92697, USA, f.capolino@uci.edu

***Abstract*—The degenerate band edge (DBE) is a special fourth-order degenerate point in a dispersion diagram, where four eigenmodes coalesce to a single degenerate eigenmode. It leads to field enhancement of the Bloch mode and to high quality factors, which are useful for high-Q resonators, oscillators and ultra-sensitive sensors. The air-filled substrate integrated waveguide (AFSIW) is a novel form of SIW which is low cost and low loss. We propose a design of an AFSIW supporting a degenerate band edge (DBE). We show the occurrence of the so-called "giant resonance" associated to the DBE and we study how losses influence the DBE.**

***Index Terms*—propagation, exceptional point, degenerate band edge, air-filled substrate integrated waveguide, periodic structures.**

## I. INTRODUCTION

In recent years, exceptional points of degeneracy (EPDs) in electromagnetic systems have attracted interest because of their interesting characteristics related to non-Hermitian systems [1]. Here we specify that EPD are interesting because they are degenerate points therefore we deem appropriate to include the "degeneracy" term in their term definition. EPDs manifest themselves as singularities in the wavenumber-frequency dispersion diagram. Not only eigenvalues but also their corresponding eigenvectors coalesce [2]. A fourth-order EPD occurring at a band edge of the dispersion diagram of modes in a waveguide without losses and gain is called degenerate band edge (DBE); it was firstly studied in photonic crystals made of stacks of anisotropic layers [3]. The theory shows that in one-dimensional (1D) periodic structures, the DBE is associated to a strong resonance and a high quality factor, and these properties are more remarkable with respect to those associated with a regular band edge (RBE) which is a second-order degeneracy [3]. The RBE appears in every periodic structure and its dispersion relation is locally approximated as [4]

$$f - f_g = -\alpha_g (k - k_d)^2, \tag{1}$$

where $f_g$ is the RBE frequency, $k_d = \pi/p$ is the edge of the Brillouin zone, $p$ is the spatial period and $\alpha_g$ is a constant based on the structure's geometry. Instead, the DBE requires a specific design with at least two coupled transmission lines and it is generally more difficult to achieve. Its dispersion relation is locally approximated as a quartic curve [4]

$$f - f_d = -\alpha (k - k_d)^4, \tag{2}$$

where $f_d$ is the DBE frequency. In (1), two degenerate eigenmodes coalesce at the RBE frequency, whereas, in (2), four eigenmodes (two propagating and two evanescent modes) coalesce at the DBE frequency.

The DBE condition has been achieved in various microwave devices, such as, coupled microstrip lines [5], [6], circular waveguides [4] and multilayer dielectric antenna [7]. The experimental proof of the DBE occurrence in microwave structures is shown in [4], [6]. The DBE provides a field enhancement, a high quality factor and a dramatic reduction of the group velocity in the vicinity of the DBE frequency. These properties can be used in high power oscillators [8] based on slow-wave structures, circuit-based oscillators [9], microstrip-based oscillators [10], antenna miniaturization [7], and ultra-sensitive sensors.

In this work we propose a design of an air-filled substrate-integrated waveguide (AFSIW) supporting a DBE. The AFSIW has recently been proposed as a novel class of SIW [11] to decrease dielectric losses by replacing the dielectric substrate with air. Since losses limit the quality factor and deteriorate DBE resonances [4], a low-loss AFSIW with DBE is a good candidate to mitigate this problem. First, we show the design of the unit cell of periodic AFSIW with DBE. Then, we demonstrate the DBE's contribution to the field enhancement and quality factor comparing to a similarly shaped AFSIW resonator with RBE. At last, we estimate the closeness to the DBE for a lossy AFSIW and for a lossy SIW design, and compare their performances, using a metric recently introduced [6].

## II. AIR-FILLED SIW WITH DEGENERATE BAND EDGE

### A. Unit Cell Design

An ideal DBE point exists when four eigenmodes perfectly coalesce. However, in reality, losses affect the degeneracy, so real structures can only approximate ideal DBE structures. If losses are decreased, the performance of a DBE structure will enhance. The air-filled substrate-integrated waveguide is a suitable solution to minimize dielectric losses, relatively easy to fabricate and allows the use of a low-cost substrate [11].

The unit cell of the periodic AFSIW supporting a DBE is shown in Fig. 1(a). The DBE is a degeneracy of four eigenmodes, so one can achieve it by the proper coupling of two waveguides, so that four modes (two in each direction) are coupled in a suitable way while propagating or decaying along the $z$ direction. In Fig. 1(b), the two waveguides are AFSIW 1 and AFSIW 2. AFSIW 1 and AFSIW 2 have the same width $w$, and the period is $p$. Between the two adjacent waveguides, the modes couple through a long gap in each unit cell whose length is $g$. In AFSIW 1, an oblique line of metallic posts is the key to obtain the DBE. It provides an anisotropic characteristic to the periodic AFSIWs and its function is similar to the elliptical rings in [4]. By adjusting the misalignment angle $\varphi$, a DBE is achieved. Furthermore, the distance $d$ between the oblique posts also determines the amount of coupling between two AFSIWs. The substrate of the AFSIW is FR4 whose relative permittivity is $\epsilon_r = 4.4$, loss tangent of $\tan\delta = 2 \times 10^{-2}$ and height of 1.28 mm. In the AFSIW waves propagate mainly in the air-filled area so a low-cost material with a relative high loss tangent can be used for the substrate. The diameter of each metallic post is 0.6 mm and the distance between posts for the lateral post line is 1.2 mm.

In Fig. 1(c), the dispersion relation of the periodic AFSIWs is calculated in the absence of any source of loss using a coupled transmission line analysis [5] and a Bloch analysis for periodic structures [12]. A full-wave simulation based on the finite element method (FEM) (implemented in Ansys HFSS) of the 4-port unit cell gives its scattering matrix, which is converted into a transfer matrix to calculate the dispersion diagram. First, we consider the structure to be lossless to prevent the influence of losses to the degeneracy. After a proper design choice of parameters the DBE is obtained at 8.638 GHz. The method using the FEM-based Eigensolver of HFSS to calculate the dispersion relation agrees with the Bloch analysis's result. Furthermore, in Fig. 1(c), a fourth-order polynomial fitting validates the existence of the DBE. We then consider the presence of losses, including conductor losses, dielectric losses and radiation losses. The radiation losses is negligible when the diameter of the lateral posts and distance between posts are well-designed [13] so the two main contributions to losses are conductor losses and dielectric losses. When considering losses the dispersion curve of the Bloch eigenmodes shows that the perfect degeneracy is lost (see Fig. 1(d)). In next section, we investigate how the presence of losses impacts on the parameters of a DBE-based cavity formed by a AFSIW with finite length.

### *B. Resonance in Finite-length AFSIW with DBE*

Field enhancement in an AFSIW resonator made of a finite number of unit cells is studied here. Field-magnitude enhancement and a higher quality factor ($Q$) compared to another AFSIW resonator supporting an RBE are expected. An 8-unit-cell AFSIW resonator is shown in Fig. 2(a). In the simulation, two ports (bottom left and top right) are set as excitation ports and the other two ports (top left and bottom right) are shorted by walls of metallic posts. In order to compare two AFSIW resonators, one supporting a DBE and one an RBE, we only move the position of the oblique line of posts to obtain an RBE at 8.728 GHz. The other parameters remain the same in order to get a fair comparison between the two structures. In Fig. 2(b), we compare the magnitude of electric field between a lossless AFSIW-DBE and a lossless AFSIW-RBE with same excitation at Port 1 of 1 W. The data of electric field are taken along the red dotted line shown in Fig. 2(a), which is at the center of the lower AFSIW. The 8-cell AFSIW resonator with DBE has a maximum E-field magnitude of 32.93 kV/m, while the AFSIW with RBE only has 25.58 kV/m. The periodic AFSIW resonator works as a Fabry-Perot resonator, so when the number of cell increases, the resonance frequency approaches the band-edge frequency and the $Q$ factor increases [4]. Furthermore, the DBE resonator's $Q$ factor increases faster than that of the RBE resonator when increasing the number of unit cells [4]. The comparison of the $Q$ factor between the AFSIW-DBE and the AFSIW-RBE resonators is shown in Fig. 2(c). This confirms the improvement of the $Q$ factor due to a DBE regime, and

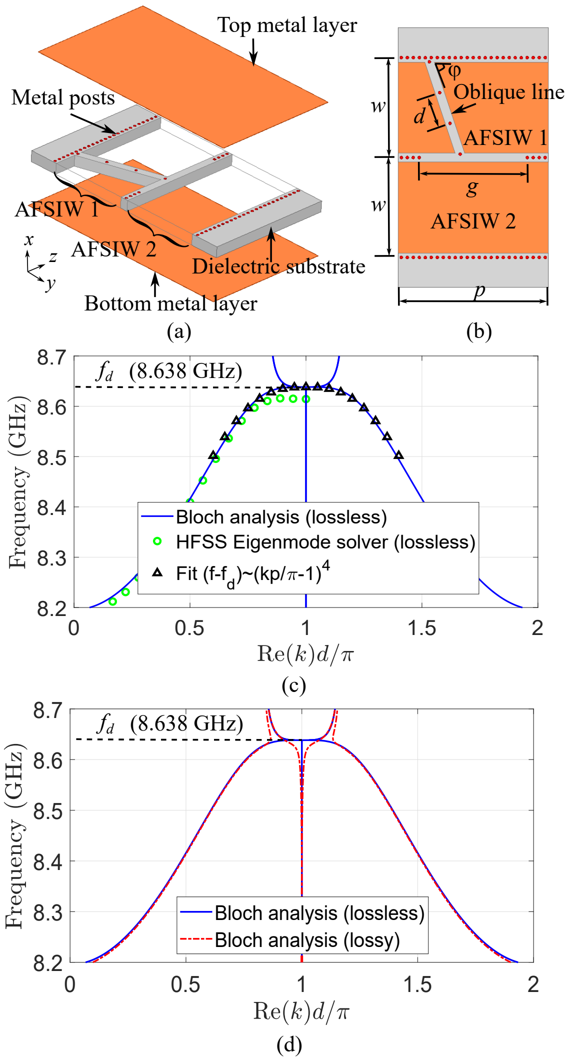

Fig. 1. Unit cell of AFSIW supporting the DBE. (a) Exploded-view of the AFSIW with DBE. (b) Top view of the $yz$-plane. ($p$=30 mm, $w$=20 mm, $g$=21.6 mm, $d$=6.5mm, $\varphi = 71.5°$) (c) The dispersion diagram with a DBE point at 8.638 GHz. (d) The dispersion diagram under the influences of losses.

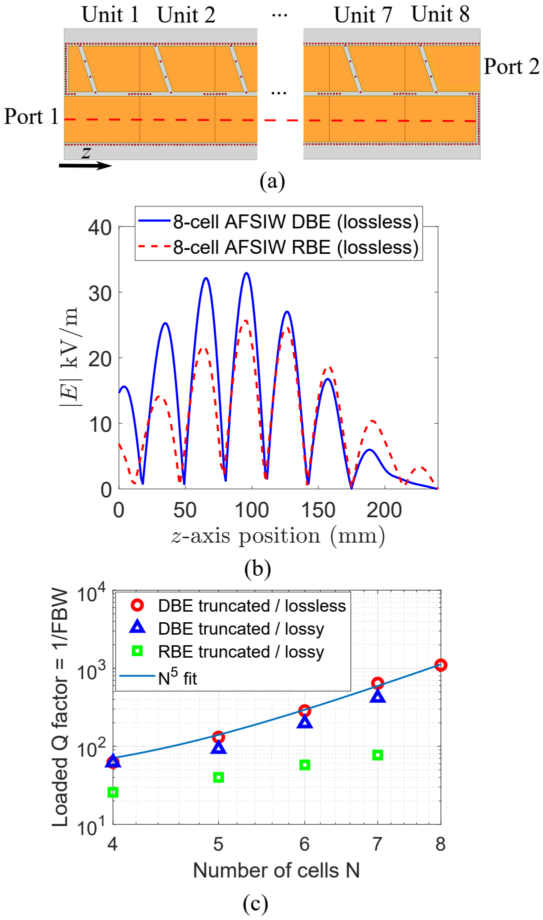

Fig. 2. (a) The 8-cell AFSIW resonator. The electrical field's magnitude is calculated along the red dashed line. (b) Comparison of the magnitudes of the E-field in a 8-cell AFSIW-DBE resonator and in a 8-cell AFSIW-RBE resonator. (c) Loaded $Q$ factor growing with the number of cells. The quality factor is calculated by observing the FBW (fractional bandwidth), which is the -3db bandwidth relative to the reflection coefficient $S_{11}$ [4].

the expected increase of the $Q$ factor with a fifth-order power law with respect to the number of cells $N$ in the lossless case. Furthermore, a similar trend is observed in the lossy DBE case.

## III. COMPARISON BETWEEN DBE IN AFSIW AND IN SIW DESIGNS

We aim here at quantify how close are the four eigenmodes to each other; indeed they should coalesce onto a single degenerate eigenmode at the DBE, in absence of losses, but this is prevented by the presence of losses. The aim is to decide whether the approximate DBE obtained in a lossy structure is acceptable and how far is from the ideal DBE case. By calculating the hyperdistance of four eigenvectors in complex vector space, we can measure the degeneracy of the DBE point [6] and the equation is

$$D_H = \frac{1}{6}\sum_{\substack{m=1,n=1\\ m\neq n}}^{4} |\sin\theta_{mn}|, \tag{3}$$

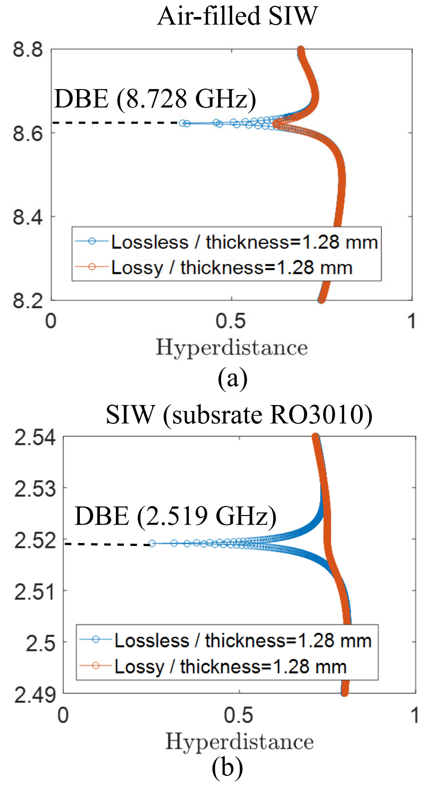

Fig. 3. (a) Hyperdistance $D_H$ relative to the air-filled SIW with and without considering losses. At the DBE frequency the AFSIW's hyperdistance for the lossy case has a dip indicating that the four eigenvectors are "close" to each other. (b) Hyperdistance relative to the SIW whose substrate material is Rogers RO3010. At the DBE frequency, considering the losses, the hyperdistance does not really decrease very much.

where $\theta_{mn}$ is the angle between two complex vectors $\mathbf{\Psi}_m$ and $\mathbf{\Psi}_n$. The definition and calculation of the angles of complex vectors are summarized in [14]. The value of $D_H$ measures the closeness of the four eigenvectors. When the hyperdistance $D_H$ in (3) vanishes, all four eigenvectors perfectly coalesce. The smaller $D_H$ is, the closer the structure is to an ideal DBE. By observing the values of $D_H$ we can study how losses influence the DBE and compare the performance of different DBE structures.

In Fig. 3, we calculate and compare the hyperdistance $D_H$ between an AFSIW-DBE and a SIW-DBE using the dispersion diagram of their unit cell. The substrate material of the SIW is Rogers RO3010 whose relative permittivity ($\epsilon_r$) is 10.2 and loss tangent is $3\times10^{-3}$. The SIW's geometry are the same as AFSIW-DBE structure shown in Fig. 1 and its DBE manifests at 2.519 GHz. From the comparison, the hyperdistance in the two lossless structures are similar. However, after considering the losses, the hyperdistance for the SIW is 0.75 whereas for the AFSIW is just 0.62. Therefore, AFSIW's DBE is more robust to the influence of losses and its substrate material (FR4) is cheaper than the SIW (RO3010).

## IV. CONCLUSION

We have proposed a periodic air-filled SIW that exhibits a DBE in the dispersion diagram. The AFSIW is low-loss, low-cost and easy to fabricate, and is a suitable structure for achieving the DBE in integrated technology. A commercial simulation has been used to calculate the scattering matrix of the unit cell and the dispersion diagram of the periodic AFSIW lines has been calculated using a multimodal Bloch analysis. The DBE is achieved easily by adjusting the angle of the oblique posts in the AFSIW. An 8-cell resonator shows field enhancement and high quality factor due to the DBE. A hyperdistance has been used to calculate the closeness of the eigenmodes, i.e., the quality of the DBE. The presence of losses prevents the hyperdistance from vanishing and hence degrades the performance of DBE structures. We have observed that the AFSIW suffers less from the presence of losses than a similarly shaped SIW, since the former shows a smaller hyperdistance than the latter.

Future work will focus on the fabrication and the experimental verification of the phenomena presented here, which are currently under progress.

## ACKNOWLEDGMENT

This work was partially supported by the French government under the ANR grant HOLeYMETA ANR JCJC 2016 ANR-16-CE24-0030 and by the USA National Science Foundation under award NSF ECCS-1711975.

## REFERENCES

[1] W. D. Heiss, "Exceptional points of non-Hermitian operators," *Journal of Physics A: Mathematical and General*, vol. 37, no. 6, pp. 2455–2464, Jan. 2004.

[2] ——, "The physics of exceptional points," *Journal of Physics A: Mathematical and Theoretical*, vol. 45, no. 44, p. 444016, Oct. 2012.

[3] A. Figotin and I. Vitebskiy, "Gigantic transmission band-edge resonance in periodic stacks of anisotropic layers," *Physical Review E*, vol. 72, no. 3, Sep. 2005.

[4] M. A. K. Othman, X. Pan, G. Atmatzakis, C. G. Christodoulou, and F. Capolino, "Experimental Demonstration of Degenerate Band Edge in Metallic Periodically Loaded Circular Waveguide," *IEEE Transactions on Microwave Theory and Techniques*, vol. 65, no. 11, pp. 4037–4045, Nov. 2017.

[5] C. Locker, K. Sertel, and J. L. Volakis, "Emulation of Propagation in Layered Anisotropic Media With Equivalent Coupled Microstrip Lines," *IEEE Microwave and Wireless Components Letters*, vol. 16, no. 12, pp. 642–644, Dec. 2006.

[6] A. F. Abdelshafy, M. A. K. Othman, D. Oshmarin, A. T. Almutawa, and F. Capolino, "Exceptional points of degeneracy in periodic coupled waveguides and the interplay of gain and radiation loss: Theoretical and experimental demonstration," *IEEE Transactions on Antennas and Propagation*, 2019.

[7] S. Yarga, K. Sertel, and J. Volakis, "Multilayer Dielectric Resonator Antenna Operating at Degenerate Band Edge Modes," *IEEE Antennas and Wireless Propagation Letters*, vol. 8, pp. 287–290, 2009.

[8] A. F. Abdelshafy, M. A. K. Othman, F. Yazdi, M. Veysi, A. Figotin, and F. Capolino, "Electron-Beam-Driven Devices With Synchronous Multiple Degenerate Eigenmodes," *IEEE Transactions on Plasma Science*, vol. 46, no. 8, pp. 3126–3138, Aug. 2018.

[9] D. Oshmarin, F. Yazdi, M. A. K. Othman, J. Sloan, M. Radfar, M. M. Green, and F. Capolino, "New oscillator concept based on band edge degeneracy in lumped double-ladder circuits," *IET Circuits, Devices and Systems*, 2019.

[10] A. F. Abdelshafy, T. Mealy, H. Kazemi, M. A. K. Othman, and F. Capolino, "Novel regime for antenna array oscillators based on exceptional point of degeneracy," in *2019 13th European Conference on Antennas and Propagation (EuCAP)*, March 2019, pp. 1–3.

[11] F. Parment, A. Ghiotto, T. Vuong, J. Duchamp, and K. Wu, "Air-filled substrate integrated waveguide for low-loss and high power-handling millimeter-wave substrate integrated circuits," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 4, pp. 1228–1238, April 2015.

[12] D. M. Pozar, *Microwave engineering; 3rd ed.* Hoboken, NJ: Wiley, 2005.

[13] D. Deslandes and Ke Wu, "Accurate modeling, wave mechanisms, and design considerations of a substrate integrated waveguide," *IEEE Transactions on Microwave Theory and Techniques*, vol. 54, no. 6, pp. 2516–2526, Jun. 2006.

[14] K. Scharnhorst, "Angles in Complex Vector Spaces," *Acta Applicandae Mathematica*, vol. 69, no. 1, pp. 95–103, Oct. 2001.