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Maths in Action in Contemporary Archaeology: numerical simulation of fire propagation in Roman buildings

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ABSTRACT. This short paper explores the possibility of conducting high-performance computing simulations of complex fire propagation in buildings of archaeological interest. The simulation protocol described here involves several steps: i) the geometric modelling of the buildings, ii) the mathematical modelling of combustion and fire propagation, iii) the numerical simulation using a Large Eddy Scale approach on parallel systems and iv) the real time rendering of the simulation data. Numerical examples are provided to emphasize the efficiency of the approach and its importance in supporting research in archaeology and validating hypotheses through simulation.

1. Archaeological context and scientific issues

Fire in Antiquity has been a historical and archaeological issue for many years. How to imagine cities, such as Rome and its one million inhabitants in the 2nd century AD, where fire ravaged the urban landscape on a daily basis ? Archeology, epigraphy and ancient authors count 88 fires for the thousand years of history of this city [9]. It has been shown that these fires have been brought to our knowledge through the centuries only because of their exceptional, divine, or political character. The anonymous fire, ravaging a *insula* [1], after the fall of an oil lamp or starting from a hearth of kitchen having escaped any vigilance, is not or very little documented. Yet it was one of the main threats in the ancient city: an enemy of urban planning, very heavily sanctioned as we learn from the Digest, written under Justinian I, which condenses Roman jurisprudence. Fire was omnipresent.

Although we can study its implications on the evolution of ancient urbanism or technological advances in firefighting, the analysis and understanding of fire itself in the ancient urban scheme is difficult to grasp. While the archaeological reflection faces obvious limits, it is through mathematics and simulation that the problem can be approached from a new angle. The transposition of the problems in three dimensions before being submitted to simulation allows us to study this phenomenon that we can no longer observe today in our cities with a new approach.

Models for solving fire propagation problems have existed for many years. However, they apply to modern architecture. The aim is to propose a mathematical answer, taking into account constraints linked to antiquity, to an archaeological problem. To do this, our object of study will be one of the *insulae* located in the "suburbs" of Rome, the *Casa di Diana* [6]. Presenting an exceptional state of conservation, it is part of the large buildings of the city of Ostia. The width of its walls, its masonry partitions and the distribution of its space make it a high status

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Math. classification: 00X99.

¹Latin translation of the word building. The *insulae* constituted the great majority of dwellings in the city of Rome. They could reach up to 7 floors, for a height of approximately 30 meters.

dwelling place (at least until its third phase, after which the addition of a stable seems to show that it was converted into a hotel) [1]. It is a good example of the best built *insulae*, as opposed to the buildings of the Roman working-class districts, which were often poorly built due to the high pressure of real estate. The latter had a very dense plan with numerous wooden or wattle and daub partitions: apart from the external walls and partitions, everything was inflammable. The *Casa di Diana* thus allowed us to reflect on the resistance to fire of a building almost entirely made of masonry.

The simulation protocol described in this note involves the three-dimensional modelling of this test building using Blender [2], with the coarsest possible mesh. The mathematical modelling of combustion and fire propagation is based on the Navier-Stokes equations appropriate for low-speed, thermally driven flow on smoke and heat transport from fires, described Section 2. The numerical resolution is performed using Fire Dynamics Simulator (FDS) [7] and uses an Large Eddy Scale approach for computational efficiency, Section 3. Various time-dependent numerical solutions will be presented, to emphasize the efficiency and relevance of the proposed high-performance computing approach.

2. Model description

Our protocol involves the resolution of combustion and fire propagation using a dedicated open-source software. Fire Dynamics Simulator (FDS) is a computational fluid dynamics (CFD) model that describes the flow of smoke and hot gases from a fire [7]. This Large Eddy Scale (LES) reactive flow software has been aimed at solving practical fire propagation problems and has been extensively tested and validated by a wide engineering community. It is an efficient and versatile tool, which can deal with complex geometries as well as with a large range of physical phenomena.

2.1. Equation modelling in FDS

FDS solves the Navier-Stokes equations appropriate for low-speed, thermally driven flow on smoke and heat transport from fires. LES approach allows to overcome the shortcomings of poor accuracy related to the averaging of the flow field, and to reduce the computational effort by using a relatively coarse mesh. The computational domain is subdivided into small sub-grids in which conservation equations are solved to provide relevant fire data such as pressure, temperature, speed and flow of smoke. The LES equations are derived by applying a low-pass filter based on the grid-size on the governing equations and are then closed with sub-grid models. LES simulation of fire dynamics is the simplified low-Mach number formulation of the reactive Navier-Stokes equation. The ideal gas pressure is approximately decomposed into ambient pressure \bar{p} and fluctuating pressure \tilde{p} : $p(x, y, z, t) = \bar{p}(z, t) + \tilde{p}(x, y, z, t)$. The simplified governing equations for mass, species, momentum and energy, expressed as a velocity divergence condition, write as follows:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \\ \frac{\partial \rho Z_\alpha}{\partial t} + \nabla \cdot (\rho Z_\alpha u) = \nabla \cdot (\rho D_\alpha \nabla Z_\alpha) + \dot{m}_\alpha''' \\ \frac{\partial u}{\partial t} - u \times \omega + \nabla H - \tilde{p} \nabla \left(\frac{1}{\rho} \right) = \frac{1}{\rho} ((\rho - \rho_0)g + \nabla \cdot \tau) \\ \nabla \cdot u = \frac{1}{\rho h_s} \left(\frac{D}{Dt} (\bar{p} - \rho h_s) + \dot{q}''' + \dot{q}_r''' + \nabla \cdot \dot{q}'' \right) \end{cases} \quad (2.1)$$

where ρ, u, Z_α represent the density, the velocity vector and the mass fraction of a species α , respectively; ω is the vorticity, h_s is the sensible enthalpy of the gas mixture and $H = |u|^2/2 + \tilde{p}/\rho$

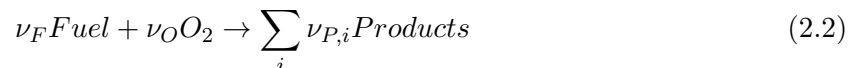
represents the stagnation energy per unit mass. The net mass production of species α due to chemical reactions is denoted \dot{m}'''_{α} . The term \dot{q}''' is the heat release rate per unit volume from combustion and \dot{q}'' represents the conductive, diffusive, and radiative heat fluxes. Finally, the gravity vector and the viscous stress tensor are denoted g and τ , respectively.

FDS solves the governing equations using an explicit predictor-corrector scheme for the time-stepping approximation and second order accurate finite-differences on uniform, structured and staggered meshes for spatial discretization.

2.2. Simplified combustion model

Fire propagation is a tremendously complex dynamical combustion process that may involve thousands of species and chemical reactions. Combustion is usually modelled using mass fractions which represent ratios of combustible gasses originated in given place corresponding to all main reactants and products which can be derived from mass fractions of mixtures by analysis or measurements.

FDS uses an empirical model to evaluate the mixing time scale, and incorporates a two-steps reaction scheme to account for CO production in fires. The simplified approach, based on single-fuel combustion mechanisms, requires to track at least six gaseous species (Fuel, O_2 , CO_2 , H_2O , CO , N_2) throughout the resolution of six transport equations. The introduction of a "lumped" species, which can be defined as a mixture of gas that transport and react together, allows to dramatically decrease the computational cost of the simulation. Thus, the single-step combustion reaction has the general form:



where the stoichiometric coefficients for products are either established by using data from bench-scale tests or specified by users.

Combustion is taken into account into the governing equations of the fluid motion (2.1) through the source terms \dot{q}''' , where $\Delta h_{f,\alpha}$ is the heat of formation of species α :

$$\dot{q}''' = - \sum_{\alpha} \dot{m}'''_{\alpha} \Delta h_{f,\alpha}. \quad (2.3)$$

Two FDS combustion models have been considered in this study, in order to deal with small and large scale phenomena, respectively:

- i) An accurate pyrolysis-based combustion of wood: applied to simulate small-scale experiments under varied forced ventilation velocities, where wood elements in the floor/ceiling are used as the fuel source. Notice however that the pyrolysis model used is mainly controlled by temperature; it does not take into account the degradation process of the solid. After a threshold temperature is reached, wood is decomposed according to the following reaction: $Wood \rightarrow Cellulose + Char$. And in turn, cellulose is reacting with air in a one-step combustion mechanism: $Cellulose + Air \rightarrow Products$.
- ii) A simplified model: in lieu of solving the species conservation equation, once wood has reached its auto-ignition temperature, it will burn during a limited time t_c while releasing a constant amount of heat per time per surface unit \dot{q}'' . Hence, \dot{q}'' can be considered as a boundary condition that approximates the combustion of wood. This approach drastically decreases the numerical cost of the simulation.

In the second approach, there is not transport of species, which makes this model more suitable to deal with large scale simulations of buildings, while the first approach will be more efficient and accurate to deal with small scale fire sources in restricted areas (i.e., within a room for instance).

Once the self-ignition temperature of the material is reached, the material (here wood) is supposed to burn during:

$$t_c = \frac{\rho \delta H_c}{\dot{q}''}$$

where δ is the thickness of the material, ρ is the density, H_c is the enthalpy of combustion and \dot{q}'' is the rate of heat released per unit area. This is therefore a simplified version of material combustion.

Radiation heat transport is modelled by the Finite Volume Method. Thermal boundary conditions and reliable information about fire properties of materials must be entered for solid surfaces and domain boundaries. Moreover, the radiation model is a large-scale approximation. It does not take into account the temperature in the reactive zone.

2.3. Geometric modelling

To help understand how a domestic incident could have triggered a devastating fire in an *insula*, which would have rapidly propagated through the building and to the neighbouring constructions, a digital replica of a Roman *insula* has been created. This model makes the connection between archaeological data and 3D model understandable, can be used for numerical simulations and consequently serves the purpose of validating hypotheses by archaeologists.

For our study, the input geometric model of the *Casa di Diana insula* shown in Figure 1 was created by the open-source 3D creation suite Blender [2], which enables interactive modelling of complex structures geometry. Blender is an interactive modelling environment that integrates numerous tools and add-ons for creating complex structure geometries, with the added benefits of a multi-platform.

The archaeological model of one of the last standing *insulae* in Ostia, Italy, has been generated based on measurements extracted from available archaeological 2D floor plans, Fig. 1 top left [8]. The dimensions of the building are $34m \times 48m \times 12.6m$. In our model, not only the visible structure but also the hidden architectural elements, such as attics, wooden frames of the ceilings and the roof, were integrated in order to carry out the simulation of fire propagation, Fig. 1 bottom.

Only three materials are taken into account in the current model: clay and stone for the walls, clay for the roof and wood for the floors and for the roof frame. However, only wood contributes to the one or two-steps combustion process. Notice that in the current stage of development of the model, furniture is not present, but will be included in further studies. Once added, it will allow to study the fire propagation across different rooms, at a smaller scale. In addition, and contrary to the situation in modern buildings, floors were not airtight and combustion can rapidly propagates through the small gap between the planks to adjacent rooms. However, these gaps have a width of the order of the millimeter and are therefore well under the mesh resolution used in simulations. Nonetheless, their effect is taken into account by adding a vent boundary condition on each side of the floor in FDS simulations (see Section 3 for more details on the procedure).

2.4. Real-time rendering

Over the last decades, computer-based visualization has made enormous progress in terms of graphics rendering and realism. Nonetheless, existing software lack the ability to integrate new algorithms to provide cutting-edge rendering. On the other hand, modern game engines are sophisticated pieces of middleware that provide convenient abstractions for a variety of complex functionality. The features of such frameworks deemed well suited for research purposes, offering open-source code, blueprint scripting language that can be use in combination with C/C++ for

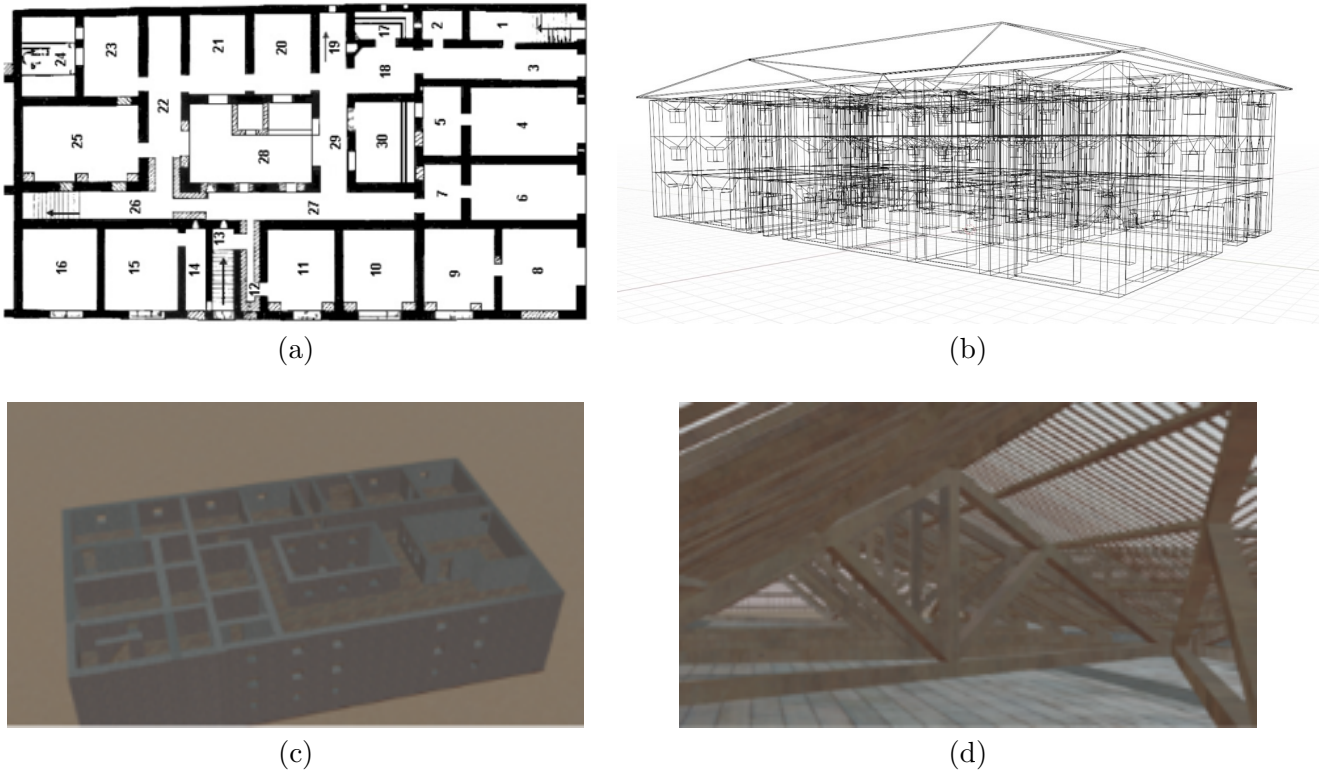


FIGURE 1. *Geometrical model of the Casa di Diana using Blender: (a) Original plan [8] (b) wireframe model in Blender (c)-(d) views of the building and the ceiling.*

rapid prototyping, interoperability with external software, cross platform support as well as regular maintenance and improvement.

Our team has been working to process the archaeological data on *insulae* to a 3D interactive model of an Ancient Rome district ² using Unreal Engine, by Epic Game ³, that allows new immersive outputs and virtual reality experiences, Fig. ². To achieve real-time rendering, it is critically important to design an efficient data-bridging method between the FDS combustion simulation code and the game engine. Obviously, even if FDS can run on high-performance computer architectures, simulation data can't be produced in real-time (see Section ^{3.2}). Hence, we are developing a plugin to feed combustion data from FDS simulations into the Unreal Engine, using code and blueprints and to visualise it with particle systems and volumetric fog. This will allow the user have an immersive experience in a district of an ancient city while visualising the real-time the propagation of the fire ignited inside a building room and propagating to the adjacent rooms ⁵.

3. Simulation issues and numerical results

In this section, we will present various numerical results produced by the simulation protocol described in the previous Section. At first, we describe the simulation setup, Section ^{3.1}, then we detail the parallel implementation of the simulation on a high performance distributed memory architecture, Section ^{3.2}. Finally, simulation results are depicted in Section ^{3.3}.

²The *Casa di Diana* building has been integrated in this virtual district for this simulation purpose, and does not reflect reality.

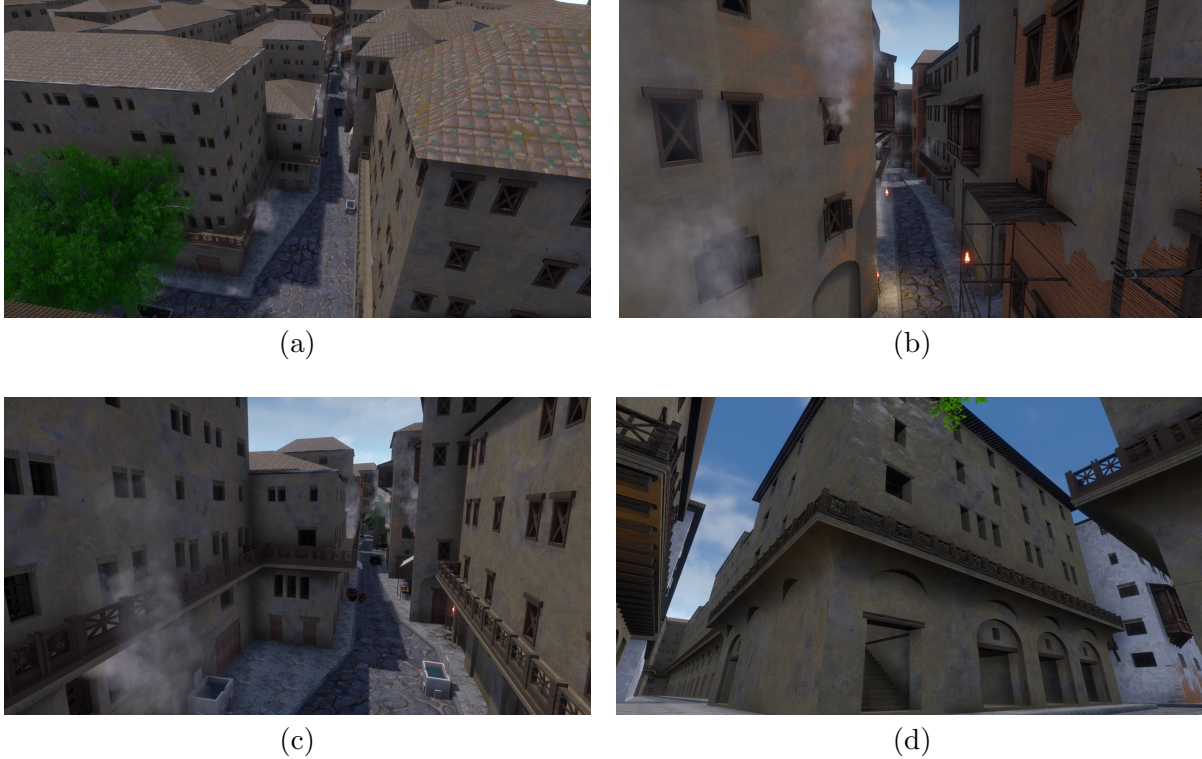


FIGURE 2. *Virtual rendering of a small district in Ancient Rome, top, bottom-left. Rendering of the Casa di Diana building using Unreal Engine 4, bottom-right.*

3.1. Fire propagation simulation setup

The discretisation of the *Casa di Diana* building, $34m \times 48m \times 12.6m$, with a grid size of $\delta = 10cm$, led to a mesh containing $20.5M$ vertices. The geometric quality of the isotropic mesh has been assessed by cross-validating the ratio D^*/δ , where D^* is the characteristic size of the simulated fire [10].

The 3D geometric model is converted into a FDS readable input model thanks to the open-source Python based BlenderFDS add-on³. It allows the conversion of complex 3D geometrical model into FDS readable geometry through a voxelization procedure, Figure 3, while maintaining a complete authority on the other FDS parameters.

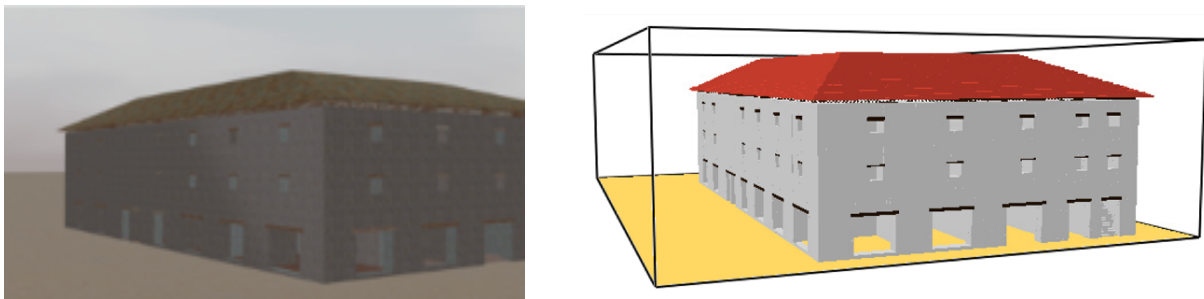


FIGURE 3. *Conversion of a geometrical blender into a computational structured mesh. Blender model, left and computational voxel mesh, right.*

³www.blenderfds.org

3.2. Parallel implementation issues

The fire propagation simulation stage is based on an explicit time-stepping algorithm. Consequently, the value of the time step per iteration is bounded by the following CFL condition:

$$CFL = \delta t \frac{\|u\|}{\delta x} < 1.$$

In this study, the physical time-step size is of the order of 5ms. Moreover, the characteristic time-scales of fire propagation in a large building like the *Casa di Diana* is of the order of one hour, which implies approximatively 720,000 iterations. As the CPU time per time-step is directly related to the number of grid points, we resorted to a single-threaded, domain-splitting parallelization using Message Parsing Interface (MPI), as advocated in FDS User’s Guide [7], to achieve a *reasonable* simulation wall-time.

To assess and calibrate the efficiency of the parallelization stage, we performed a strong-scaling study. We introduce the parallelization efficiency parameter $E_s = t_1/nt_n$, where t_1 and t_n represent wall-time of simulation using 1 and n processors, respectively. Results obtained on a SGI ICE-XA system available at Sorbonne Université are shown in Figure 4. The high performance computing architecture is a distributed cluster which offers 3,456 high performance computing cores, distributed in 144 nodes of 24 cores. It is associated with a fast storage system based on a Lustre file system. It is an extremely energy efficient server, and is equipped with a water cooling distribution unit (CDU).

It can be seen in Fig. 4 that efficiency decreases rapidly compared to the ideal $E_s = 1$ objective. Based on this analysis, we kept the number of MPI process to 192 as it provides a good trade-off between the amount of numerical resources involved and the parallel efficiency attained.

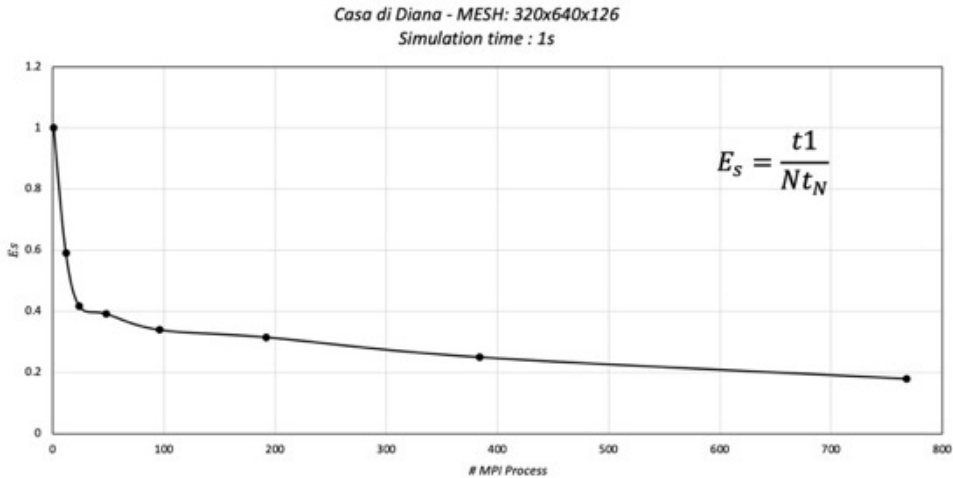


FIGURE 4. Strong scaling analysis of the *Casa di Diana* model, performed on a SGI ICE-XA parallel computer.

3.3. Simulation results

In this section, we gather a few numerical results to assess our protocol to model the fire propagation in an (ancient) building of archeological interest. Most following plots have been obtained using the open source Smokeview software [4]. Smokeview has an emphasis on realistic smoke and fire visualization, but contains more classical method such as 2D or 3D contours plots of flow field data.

Archaeological specifications. In order to ignite properly the fire, a square patch generating $2MW$ of heat has been added in a room on the second floor of the *Casa di Diana* model. This setting may be a good approximation of the accidental departure of a fire from a stove, for instance.

A preliminary stage was to investigate which model was more suitable for fire propagation: with or without vents in the ceiling and floor of rooms. Figure 5 shows that the temperature field below the first floor ceiling is far hotter without vents (left) than with vents (right), thus inducing the fire to start at the ceiling. However, the high temperature field cannot be conveyed through convection nor conduction to the second floor without vents. This makes the no vent scenario less realistic for such fire propagation.

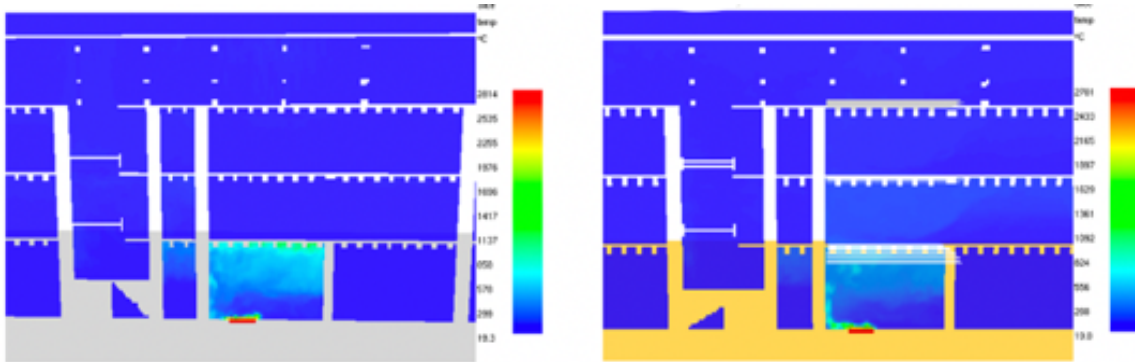


FIGURE 5. Influence of vent boundary conditions on fire propagation: visualization of temperature distribution in a room at $t = 3,600s$.

Pyrolysis vs. simple combustion models. Next, we studied the influence of the combustion model on the propagation. It shall be reminded here that two models are available for testing in FDS: a pyrolysis model, applied to simulate small-scale experiments under varied forced ventilation velocities, where wood elements in the floor/ceiling are used as the fuel source, and a simple combustion model (cf. Section 2.2). The thermal properties of the materials making up the *insulae* (stone, wood and terracotta) can be found in [10] and are summarized in Table 1. Figure 6 shows the resulting temperature field with the pyrolysis model. We observe that the fire does not propagate across the building in one hour $t = 3,600s$. This drawback could be explained by the fact that the only combustible is constituted by wood elements having large dimensions and thus offering a substantial thermal inertia. Therefore, these materials are rather difficult to light up without smaller, intermediate energy sources, such as the furniture. In addition, one has to notice the deprivation of oxydizer to sustain combustion. Heat is only exchanged through the vents and the sole source of oxydizer is through the building’s openings i.e., through windows and doors.

The results obtained with the simple combustion model are drastically different, cf. Figure 7. Looking at the temperature field evolution, it is quite obvious that the fire propagated across the second floor at the end of the simulation, at $t = 3,600s$. Indeed, after one hour, we observe that the fire hit the staircase on the left of the room in which the fire ignited. It seems thus safe to assume that the fire will eventually reach the rest of the building and especially the roof. And the roof being set to fire, the fire will most likely propagate to the nearby *insulae*, actually leading to the devastation of the whole neighbourhood.

Fire propagation. We can observe the 3D dimensionality and the extend of fire propagation through two orthogonal temperature slices in X and Y and an extra one at a fixed Z altitude, Figure 8. At $t = 6,500s$, the fire propagated to the entirety of the second floor with temperature

<pre>&REAC FUEL='CELLULOSE' SOOT_YIELD=0.01/ &SURF ID='Bois' RGB=38,20,6 MATL_ID='WOOD' THICKNESS=0.05 / &MATL ID='WOOD' DENSITY=750 N_REACTIONS=1 CONDUCTIVITY=0.1 SPECIFIC_HEAT=2.4 HEAT_OF_COMBUSTION=20000 HEAT_OF_REACTION=7500 NU_SPEC=0.92 SPEC_ID='CELLULOSE' NU_MATL=0.08 MATL_ID='char' REFERENCE_TEMPERATURE=380 /</pre> <p style="text-align: center;">pyrolysis</p>	<pre>&MATL ID='char' DENSITY = 71. CONDUCTIVITY = 0.1 SPECIFIC_HEAT = 1.0 / &SPEC ID='CELLULOSE' FORMULA='C6H10O5' CONDUCTIVITY = 0.04 DIFFUSIVITY = 0 ENTHALPY_OF_FORMATION = 484 SPECIFIC_HEAT = 1.4 VISCOSITY = 10000 /</pre> <p style="text-align: center;">pyrolysis</p>	<pre>&SURF ID='Bois_plancher' RGB=38,20,6 HRRPUA=30.000 TAU_Q=0.0 MATL_ID='WOOD' IGNITION_TEMPERATURE=350.0 THICKNESS=0.05 RAMP_Q='fire_plancher'/ &RAMP ID='fire_plancher', T=0, F=0 / &RAMP ID='fire_plancher', T=10, F=1 / &RAMP ID='fire_plancher', T=2490.0, F=1 / &RAMP ID='fire_plancher', T=2500.0, F=0 /</pre> <p style="text-align: center;">simple combustion</p>
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TABLE 1. Setting materials properties for modelling wood in FDS [10].

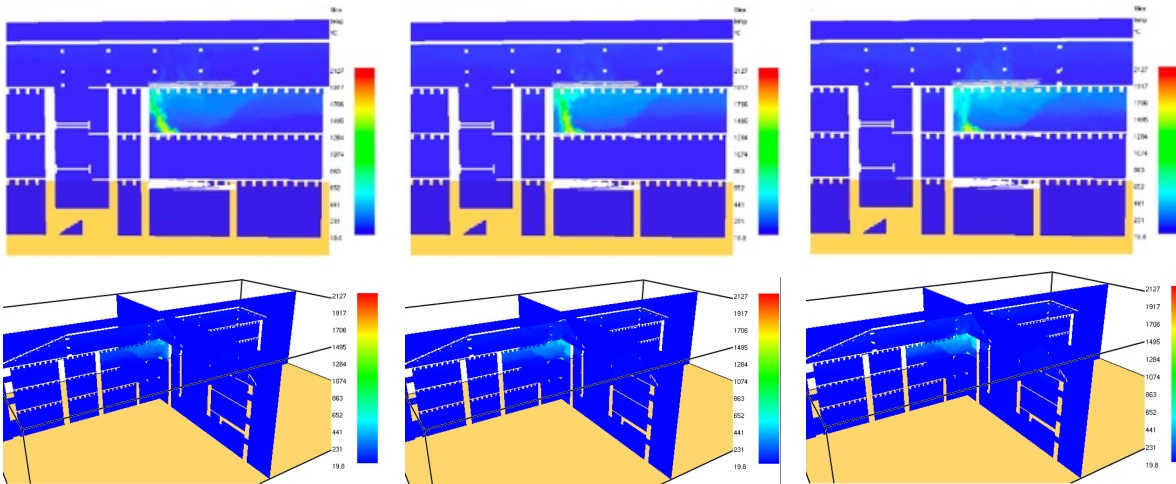


FIGURE 6. Time evolution of a 2MW fire inside Casa di Diana: pyrolysis model. 2D view (top) and orthogonal cutting planes (bottom) at various times: $t = 133s$, $t = 1,382s$ and $t = 3,600s$.

reaching 900K on the opposite side of the building, across the atrium. This temperature is sufficiently hot to instantaneously ignite wood with the simple combustion model. One can also notice that temperature is much lower inside the staircase and in the atrium. Actually, the hot temperature gas is freely convected upward due to its lighter density, without a ceiling or roof blocking the way. In turn, the roof elements start to ignites just above the staircase. Therefore, it is safe to assume that the whole roof will burn. From there, the fire will probably spread to the closest *insulae* and might devastate a whole neighbourhood.

On the contrary, the fire did not reach the first floor as temperature are much lower there, 24°C maximum. In fact, as hot air moves upward, the temperature does not build up enough on the first floor to ignite wood with the simple combustion model.

3.4. Conclusions

In this short note, we have shown that 3D models have played an important role in supporting research in archaeology, allowing discussion and comparison of interpretative hypotheses and their verification through simulations and visualisation. Regarding this specific study, we have shown that it seems difficult, if not impossible, to burn a whole building mostly made of stone

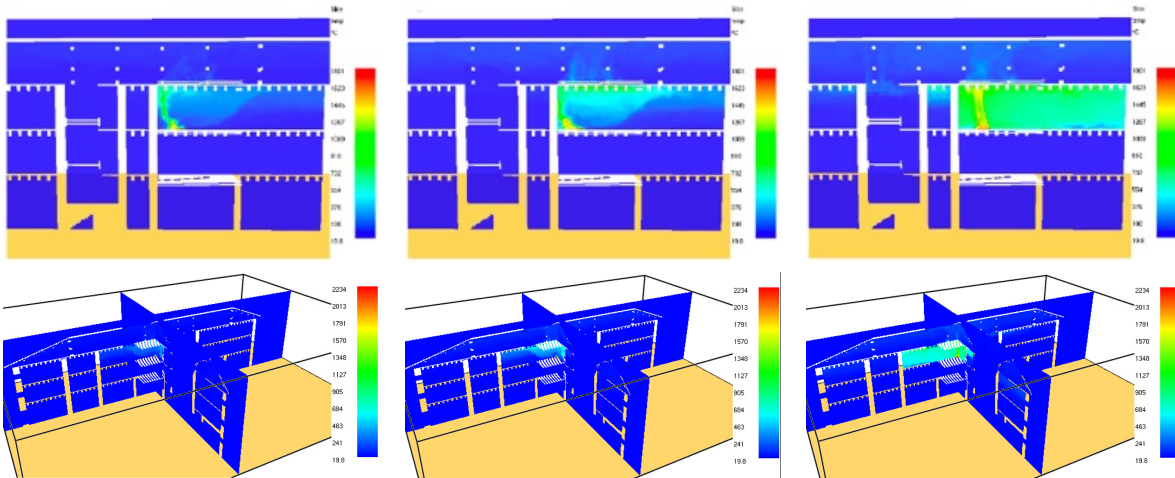


FIGURE 7. Time evolution of a 2MW fire inside *Casa di Diana*. simple combustion model. 2D view (top) and orthogonal cutting planes (bottom) at times $t = 133s$, $t = 1,382s$ and $t = 3,600s$.

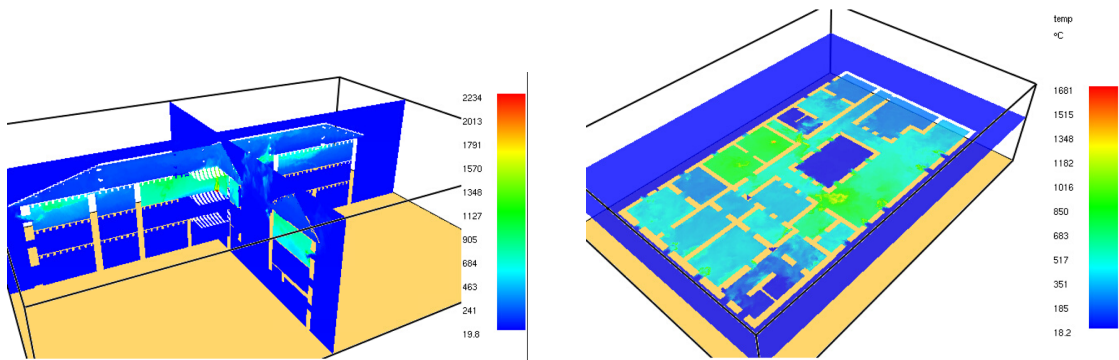


FIGURE 8. Fire propagation using the simple combustion model at $t = 6,500s$.

with the pyrolysis model. Furniture of the room need to be incorporated for more efficiency. On generic *insulae*, made essentially of wood, this simulation protocol may be more efficient and accurate.

Regarding the methodology, we have presented a robust and generic procedure to study fire propagation in buildings of archeological interest. This protocol will allow to conduct parametric study for fire start and investigate historical repercussions.

The next stage will consist in incorporating the combustion result in an interactive and immersive visualisation environment to provide a virtual reality experience. The dynamic of the fire (flames, smokes, etc.) will be rendered in real time using a graphics engine. First experiments have been conducted using Unreal Engine and are shown Figure 9.

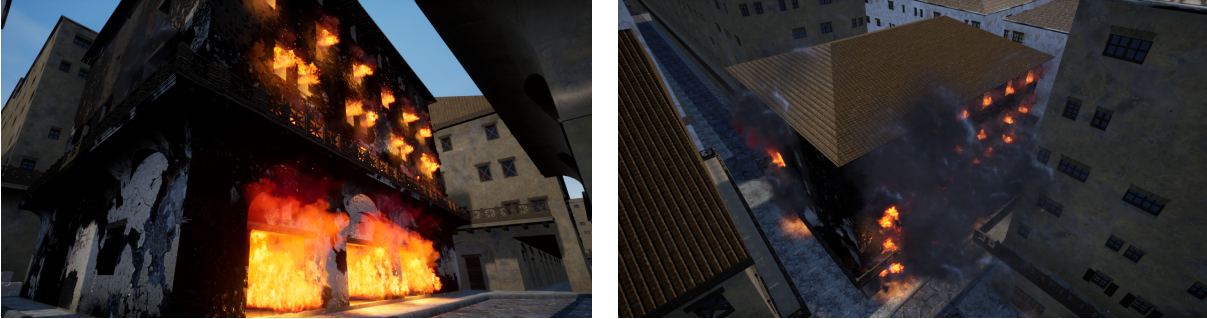


FIGURE 9. *Example of virtual rendering of the fire in the Casa di Diana building using Unreal Engine.*

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