Scattering of ultrashort laser pulses on plasmons in a Maxwellian plasma

V. A Astapenko, F. B Rosmej, E. S Khramov

Matter and Radiation at Extremes

Scattering of ultrashort laser pulses on plasmons in a Maxwellian plasma

Cite as: Matter Radiat. Extremes **6**, 054404 (2021); https://doi.org/10.1063/5.0065835
Submitted: 05 August 2021 . Accepted: 23 August 2021 . Published Online: 17 September 2021

V. A. Astapenko, F. B. Rosmej, and E. S. Khramov

Matter and Radiation at Extremes | RESEARCH ARTICLE | scitation.org/journal/mre

Scattering of ultrashort laser pulses on plasmons in a Maxwellian plasma

Cite as: Matter Radiat. Extremes 6, 054404 (2021); doi: 10.1063/5.0065835
Submitted: 5 August 2021 • Accepted: 23 August 2021 •
Published Online: 17 September 2021

V. A. Astapenko,[1] F. B. Rosmej,[1,2,3,4] and E. S. Khramov[1,a)]

AFFILIATIONS

[1]Moscow Institute of Physics and Technology (National Research University), Institutskii per. 9, 141701 Dolgoprudny, Russia
[2]Sorbonne University, Faculty of Science and Engineering, UMR 7605, Case 128, 4 Place Jussieu, F-75252 Paris Cedex 05, France
[3]LULI, Ecole Polytechnique, CEA, CNRS, Laboratoire pour l'Utilisation des Lasers Intenses, Physique Atomique dans les Plasmas Denses, F-91128 Palaiseau, France
[4]National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe sh. 31, 115409 Moscow, Russia

[a)]**Author to whom correspondence should be addressed:** Egor.Khramov@phystech.edu

ABSTRACT

On the basis of equations obtained in the framework of second-order quantum-mechanical perturbation theory, the standard approach to the calculation of scattering radiation probability is extended to the case of ultrashort laser pulses. We investigate the mechanism of the appearance of plasmon peaks in the spectrum of the plasma form factor for different parameters of the problem. For the case in which scattering on plasmons dominates over scattering on electron density fluctuations caused by chaotic thermal motion, we derive analytical expressions describing the scattering probability of ultrashort laser pulses on plasmons. Together with this, we obtain a simple expression connecting the frequency of scattered radiation and the energy transmitted from the incident pulse to plasmon, and vice versa. In considering the scattering probability, our emphasis is on the dependence on the pulse duration. We assess in detail the trends of this dependence for various relations between pulse carrier frequency and plasmon energy.

I. INTRODUCTION

One of the most common plasma diagnostics methods is based on radiation scattering on electron density fluctuations.[1] The fluctuation phenomenon involved here relates to both chaotic thermal motion and the collective excitations[1] that are the focus of the present paper. Collective radiation scattering on electron density fluctuations was predicted by Akhiezer *et al.*[2] A few years later, Salpeter[3] obtained a fluctuation spectral characteristic of a nonmagnetic equilibrium plasma that is integral to the description of the plasma. In the same decade, a number of authors developed both classical as well as quantum mechanical approaches for describing fluctuation spectra for various types of plasmas.[4–8]

Along with the advances in the theory of radiation and plasma interaction, experimental applications of scattering theory were actively developed. As the scattering radiation spectra contain information on plasma temperature and density,[9] many papers devoted to scattering plasma diagnostics have appeared since the 1960s. Thus, Hughes[10] proposed a method for measuring plasma temperature and density based on Thomson scattering. Decades later, this approach became a classical method of plasma diagnosis in tokamaks.[11–13]

In a series of papers, attention was drawn to the interaction of radiation with plasma particles involved in collective motion. For instance, Mooradian and McWhorter[14] experimentally investigated radiation scattering on plasmons in a solid-state plasma. Glenzer *et al.*[15] demonstrated a new x-ray scattering technique for studying the properties of warm dense matter. The theory of coherent radiation collective scattering in plasmas is described and supported by experimental data in the book by Froula *et al.*[1]

The last two decades have been marked by significant advances in techniques for generating ultrashort laser pulses (USLPs). One of the most significant achievements in this area is the development of attosecond extreme ultraviolet (XUV) lasers, which have now became a widely used experimental tool.[16–19] With the aid of high-harmonic generation (HHG) spectroscopy,[20,21] it is now possible to observe the dynamics of atomic-scale systems with sub-angstrom spatial and attosecond time resolution.[22,23] This makes it possible to measure such fundamental characteristics as phases of quantum-mechanical wave-packets.[24] Along with this, there are more practical applications of pulses with sub-femtosecond duration, such as chemical reaction control at the molecular level[25] and study of charge migration in biologically relevant molecules.[26] As lasers generating pulses with duration of the order of hundreds of attoseconds are no longer exotic research instruments and, moreover, methods have been proposed for the generation of x-ray free electron laser (XFEL) pulses with duration of ~40 as,[27] we are faced with the challenge of reconsidering standard models of the interaction of radiation and matter, which were developed for pulse durations much greater than the characteristic time of atomic processes. Following this paradigm, we focus on the interaction of ultrashort laser pulses (USLPs) with a Maxwellian plasma and, in particular, on electron collective effects, which are a rather important aspect of experimental plasma diagnosis.

There have been a number of previous works devoted to USLP interaction with plasmas. For example, Glover *et al.*[28] experimentally obtained spectra of sub-picosecond x-ray pulses scattered from the plasma. Later work by Ta Phuoc *et al.*[29] was devoted to experimental investigation of nonlinear Thomson scattering of femtosecond x-ray pulses on relativistic electrons in a helium plasma. Nevertheless, in our literature review, we did not find any descriptions of models predicting USLP scattering probability during all the time of pulse action. Thus, the development of such a model is the main purpose of the current paper. In Sec. II we describe the basic relationships between the spectral characteristics of plasma fluctuations and the scattering cross-section. Section III presents our extension of Fermi's golden rule to USLPs and provides the bridge between the conventional theory of plasma fluctuations and the scattering probability of USLPs. In Sec. IV, we apply our new approach to the description of USLP scattering on plasmons and demonstrate some particular features of this interaction. In this work, we consider plasmas with temperature in the range of 1–20 eV and electron densities of 10^{16}–10^{18} cm^{-3}, which are typical conditions found in industrial plasmas.[1]

II. SPECTRAL SCATTERING CHARACTERISTICS OF MAXWELLIAN PLASMAS

The character of radiation interaction with plasmas depends on the relation between the incident light wavelength λ and the Debye radius λ_D. In the case when $\lambda < \lambda_D$, the incident photons interact with individual electrons.[30,31] Otherwise, when $\lambda \geq \lambda_D$, the situation is different. The incident pulse interacts with all electrons constituting the Debye sphere and photoprocesses have a collective character.[32]

The scattering process in a plasma is due to the existence of electron density fluctuations. Otherwise, in a plasma with uniformly distributed electron density, for each scattered component the opposite component could always be found, and their superposition would give zero field strength. The fluctuations are caused by electron motion.[30,33] Besides thermal motion, plasma elections are involved in collective oscillations. It is possible to identify two types of oscillations in unmagnetized plasmas: transverse and longitudinal. Transverse oscillations are represented by waves that are similar to electromagnetic waves in vacuum, but, in contrast to the vacuum case, these waves correspond to a dispersion relation that makes the plasma opaque for waves with frequencies lower than the plasma frequency ω_{pe}.[34] Longitudinal waves are represented by ion-acoustic and electron oscillations. Ion-acoustic wave generation requires the electron temperature to be much higher than the ion temperature,[35] which is not the case for the plasma parameters in the present work, and so we leave aside the issue of ion-acoustic waves. Longitudinal electron waves occur owing to oscillations of the electrons shielding each electron and each ion.

As mentioned above, the process of radiation scattering in a plasma is governed by the character of the electron density fluctuations. For a detailed spectral description of the whole system of interacting particles, we use a dynamic plasma form factor that is defined as the time and space Fourier transform of the electron density pair correlation function $S(\mathbf{k}, \omega)$ (for details, see Refs. 1, 30, and 36).

In the framework of the Salpeter approximation,[3] a general expression for the dynamic form factor of a plasma with charge number Z is[37]

$$S(\omega, \mathbf{k}) = S_e + S_i = \frac{2\pi}{k}\left|1 - \frac{\chi_e}{\varepsilon}\right|^2 f_{e0}\left(\frac{\omega}{k}\right) + \frac{2\pi Z}{k}\left|\frac{\chi_e}{\varepsilon}\right|^2 f_{i0}\left(\frac{\omega}{k}\right). \tag{2.1}$$

The four-vector $(\omega/c, \mathbf{k})$ indicates the changes in the radiation frequency and the wave vector as a result of the scattering process. The difference between the frequencies ω_i and ω' of the incident and scattered field components, respectively, is denoted by ω.

As we consider a Maxwellian plasma, the electron (ion) velocity distribution function $f_{e(i)0}$ has the form[1,38]

$$f_{e(i)0}\left(\frac{\omega}{k}\right) = \sqrt{\frac{m_{e(i)}}{2\pi T_{e(i)}}}\exp\left(-\frac{m_{e(i)}\omega^2}{2T_{e(i)}k^2}\right), \tag{2.2}$$

where $m_{e(i)}$ and $T_{e(i)}$ are the electron (ion) mass and temperature in eV. The following dimensionless values are used:

$$x_{e(i)} = \frac{\omega}{k}\sqrt{\frac{m_{e(i)}}{2T_{e(i)}}}, \qquad \alpha = \frac{1}{k\lambda_D}.$$

The plasma dielectric permittivity $\epsilon = 1 + \chi_e + \chi_i$ contains the electron and ion components of the electric susceptibility, given respectively by[1]

$$\chi_e = \alpha^2 F(x_e), \tag{2.3}$$

$$\chi_i = \alpha^2 \frac{ZT_e}{T_i} F(x_i), \tag{2.3$'$}$$

where

$$F(x) = 1 - \sqrt{\pi}\, x \exp(-x^2)\mathrm{erfi}(x) + i\sqrt{\pi}\, x \exp(-x^2). \tag{2.4}$$

The contributions of the electron and ion components to the scattering process described by (2.1) are significantly different in various frequency ranges. Thus, at frequencies much greater than the ion plasma frequency ($\omega \gg \omega_{pi}$), ions cannot respond to the excitation, and the ion component in (2.1) can be neglected. In other words, the incident radiation is completely scattered at plasmon and thermal fluctuations, as described in more detail in Sec. IV. In the opposite case, when $\omega \ll \omega_{pi}$ and the plasma is almost in equilibrium, the ion component dominates,[1] and the initial radiation is scattered by the electrons shielding the ions. In the present work, we do not consider this case and focus only on the parts of the $S(\mathbf{k}, \omega)$ spectrum that respond to plasmon excitations. Furthermore, for simplicity, we consider $Z = 1$.

III. EXTENSION OF FERMI'S GOLDEN RULE TO ULTRASHORT LASER PULSES. TOTAL PROBABILITY DURING THE ENTIRE TIME OF PULSE ACTION

According to Fermi's golden rule, for monochromatic radiation, the differential scattering probability per unit time can be expressed through the scattering cross-section σ_{sc} and the incident photon flux j_{ω_i} with frequency ω_i:[37]

$$\frac{d^2 w_{sc}}{d\Omega' \, d\omega'} = \frac{d^2 \sigma_{sc}}{d\Omega' \, d\omega'} j_{\omega_i}. \tag{3.1}$$

The following expression relates the scattering cross-section per electron and the dynamical form factor of the plasma:[30]

$$\frac{d^2 \sigma_{sc}(\mathbf{k}, \omega)}{d\Omega' \, d\omega} = \frac{1 + \cos^2 \theta}{2} r_e^2 S(\omega, \mathbf{k}), \tag{3.2}$$

where θ and Ω' are the plane and solid scattering angles, respectively.

All of the theory presented in Sec. II is sufficient for the description of the interaction of quasi-monochromatic pulses. The situation is entirely different, however, in the case of ultrashort pulses, because their spectrum is rather broad ($\Delta\omega \sim 1/\tau$). Thus, the formula (3.1) no longer provides an adequate description.

A formula describing the scattering probability of ULSPs by atoms in second-order quantum-mechanical perturbation theory is derived in Ref. 39, and the following extension of this formula to scattering in a plasma is presented in Ref. 40:

$$\frac{d^2 W_{sc}}{d\Omega' \, d\omega'} = \frac{c}{4\pi^2} \int_0^{+\infty} \frac{d^2 \sigma_{sc}}{d\Omega' \, d\omega} \frac{|E(\omega_i)|^2}{\hbar \omega_i} d\omega_i, \tag{3.3}$$

where c is the speed of light and $E(\omega_i)$ is the Fourier transform of the incident pulse.

To explain the meaning of the formula (3.3), it is convenient to refer to the phenomenological Fermi-equivalent photon concept.[41] For calculation of the USLP scattering probability, the contribution of each spectral component of the incident pulse should be taken into account. Thus, a USLP could be presented as a composition of photon fluxes, the densities of which are proportional to the spectral density of the incident pulse. To calculate the full USLP scattering probability during the entire time of pulse action, it is necessary to integrate over all spectral components of equivalent photons fluxes multiplied by the corresponding cross-section, and this is reflected in the formula (3.3).

To model the spectral and time dependences of the laser pulse electric field strength, a Gaussian approximation could be used. Nevertheless, this gives an adequate description only for quasi-monochromatic pulses. In the case when $\omega_c \tau < 1$, which corresponds to a USLP, $E(\omega_i = 0) \neq 0$. The presence of the constant field component is physically incorrect, and, moreover, such an approximation could cause artifacts.[17] For the description of generated laser pulses, we use the model of a corrected Gaussian pulse,[42] which does not suffer from these drawbacks. The Fourier transformation of such a pulse is

$$E(\omega_i, \omega_c, \tau) = iE_0 \tau \sqrt{\frac{\pi}{2}} \frac{\omega_i^2 \tau^2}{1 + \omega_c^2 \tau^2} \left\{ \exp\left[-(\omega_i - \omega_c)^2 \frac{\tau^2}{2} \right] - \exp\left[-(\omega_i + \omega_c)^2 \frac{\tau^2}{2} \right] \right\}, \tag{3.4}$$

where E_0 is the electric field magnitude of the incident pulse and φ is the initial phase of the pulse. Hereinafter, we consider that $E_0 = 1$ a.u. (corresponding to the strength of the field at the first Bohr orbit of the hydrogen atom).

The formula (3.3) is at the core of the present work. From this formula, we have a model for the calculation of the USLP scattering probability in a Maxwellian plasma, and this is what Sec. IV is devoted to.

IV. SCATTERING ON PLASMONS

In as much as $x_i/x_e = \sqrt{m_i/m_e}$, it follows that $x_i \gg x_e$. According to (2.2), the function $S_e(x_e)$ has an exponential decay, and we can consider values of $x_e \sim 1$. Thus, taking into account the fact that $F(x) \approx 0$ for $x \gg 1$, we can neglect the ion component of the dielectric permittivity, χ_i. The ion part of the dynamic form factor according to (2.2) can also be neglected. The following simplified expression for the dynamical form factor for an equilibrium plasma then follows from (2.1)–(2.4):

$$S(\omega, \mathbf{k}) = \frac{1}{e} \sqrt{\frac{m_e}{\overline{n}_e}} \frac{\alpha \exp(-x_e^2)}{(1 + \alpha^2 \mathrm{Re}\{F(x_e)\})^2 + (\alpha^2 \mathrm{Im}\{F(x_e)\})^2}, \tag{4.1}$$

where $\overline{n}_e$ is the mean electron density and e is the elementary charge. The condition for plasmon resonance excitation is

$$\mathrm{Re}\{\varepsilon\} = 0 \Leftrightarrow 1 + (\alpha^{\mathrm{res}})^2 - (\alpha^{\mathrm{res}})^2 \sqrt{\pi} x^{\mathrm{res}} \exp[-(x^{\mathrm{res}})^2] \mathrm{erfi}(x^{\mathrm{res}}) = 0, \tag{4.2}$$

where α^{res} and x^{res} are the values of α and x_e corresponding to the resonance conditions.

Figures 1(a)–1(c) demonstrate how the dependence of the plasma form factor on the frequency of scattered radiation changes with changes in the parameters that influence the value of α. It can be seen that for some changes in the problem parameters, the peak related to the thermal fluctuations is split into two plasmon peaks.

For a clearer understanding of the dependency dynamics, it is useful to consider the spectrum as a function of the dimensionless parameter x_e. Figure 1(d) shows the dependence of the dynamic form factor on x_e on a logarithmic scale for different plasma densities. It can be seen that the dynamic form factor has a bell-shaped form, with peaks corresponding to plasmon resonances. With increasing density, the plasmon peaks appear to slide down the side of the bell. Thus, at low concentration, only thermal motion peaks are seen. As the density increases, the order of magnitude of the plasmon peaks becomes similar to the peak magnitude of thermal motion peak, and it becomes possible to observe dense fluctuation spectra of different types: only a thermal peak, only plasmon peaks, and simultaneous thermal and plasmon peaks. However, with further increase in density, the effect of thermal motion completely suppresses the plasmon resonance. This description of the form-factor spectral dynamics $S_e(x_e)$ can be generalized: any changes in the parameters that lead to growth in α (an increase in density or a decrease in temperature or scattering angle) will lead to shifts of the plasmon peaks from zero. It should be noted that the magnitudes of the two plasmon peaks are not equal in the general case.

Both of the spectral maxima result from satisfaction of the Bohm–Gross relation[43,44] for Langmuir plasma waves. Let us define the plasmon energy as $\hbar\omega_{pl} = \hbar(\omega_i - \omega') = \hbar\omega$. The peak with lower frequency corresponds to photon absorption by the plasmon with transition to the virtual state, after which photons are re-radiated with

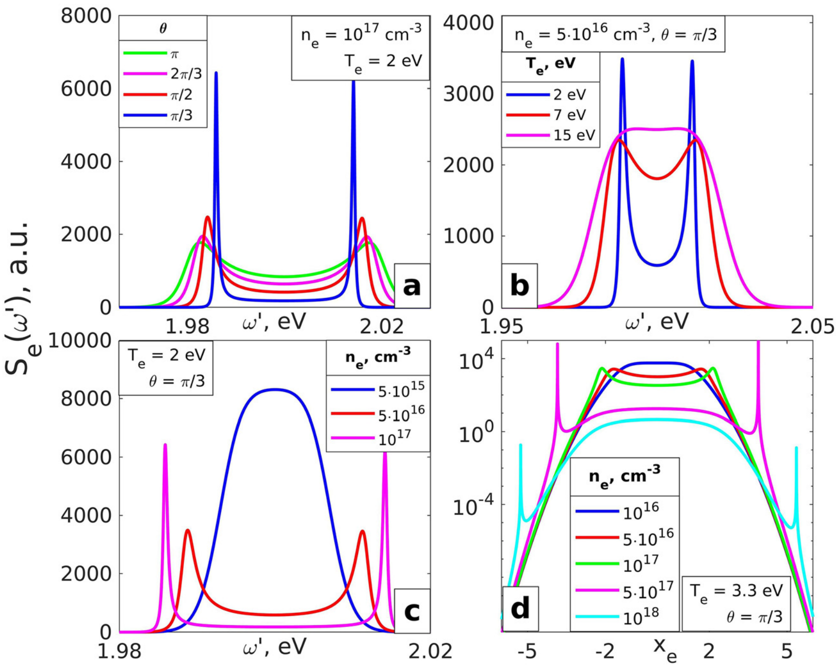

FIG. 1. Variation of the electron part of the dynamic plasma form-factor spectrum with changes in parameters that influence α: (a) scattering angle; (b) plasma temperature; (c) and (d) plasma density. The incident radiation frequency $\omega_i = 2$ eV.

lower frequency, transferring part of their energy to the plasmon. The peak with higher frequency corresponds to the reverse process: on reradiations, photons take away part of the plasmon energy.

In the case when the plasmon peaks are narrow enough and do not overlap with maxima of the thermal fluctuations, the Lorenz approximation can be used for description of a particular peak $\Gamma_m(x_e, \alpha)$:[1,3]

$$\Gamma_m(x_e,\alpha) = \frac{1}{2k}\sqrt{\frac{\pi m_e}{2T_e}}\,\frac{(x_m^{\rm res})^2\exp[-(x_m^{\rm res})^2]}{(x_e - x_m^{\rm res})^2 + \left\{\frac{1}{2}\sqrt{\pi}\,\alpha^2(x_m^{\rm res})^2\exp[-(x_m^{\rm res})^2]\right\}^2} \to \sqrt{\frac{\pi m_e}{8n_e}}\,\frac{\delta(x_e - x_m^{\rm res})}{\alpha e}, \tag{4.3}$$

where $m = \{1, 2\}$. The full width at half maximum (FWHM) of the peak, γ_m, is given by

$$\gamma_m = \frac{\sqrt{\pi}}{4}(\alpha_m^{\rm res}x_m^{\rm res})^2\exp[-(x_m^{\rm res})^2]. \tag{4.4}$$

The resonance condition is

$$x_e = x_m^{\rm res} \Leftrightarrow \frac{\omega_i - \omega'}{a\sqrt{\omega_i^2 + \omega'^2 - 2\omega_i\omega'\cos\theta}} = \frac{\omega_{im}^{\rm res} - \omega'}{a\sqrt{(\omega_{im}^{\rm res})^2 + \omega'^2 - 2\omega_{im}^{\rm res}\omega'\cos\theta}}. \tag{4.5}$$

The equality (4.5) leads to the following relation between the two resonant frequencies and the frequency of scattered radiation:

$$\omega_{i1}^{\rm res}\omega_{i2}^{\rm res} = \omega'^2. \tag{4.6}$$

When the peak magnitude of thermal fluctuations is negligibly small in comparison with the magnitude of the plasmon peaks, the dynamic form factor tends to the sum of two delta functions:

$$S(\omega,\mathbf{k}) \to \frac{1}{\alpha e}\sqrt{\frac{\pi m_e}{8n_e}}[\delta(x_e - x_1^{\rm res}) + \delta(x_e - x_2^{\rm res})] = G_1\delta(\omega_i - \omega_{i1}^{\rm res}) + G_2\delta(\omega_i - \omega_{i2}^{\rm res}), \tag{4.7}$$

with

$$G_m = \frac{\pi\lambda_D}{\alpha_m^{\rm res}c}\left[\frac{1}{k_m^{\rm res}c} - \frac{(\omega_{im}^{\rm res} - \omega')(\omega_{im}^{\rm res} - \omega'\cos\theta)}{(k_m^{\rm res}c)^3}\right]^{-1}. \tag{4.8}$$

On combining (3.2), (3.3), and (4.7) we derive a simple formula for the scattering probability:

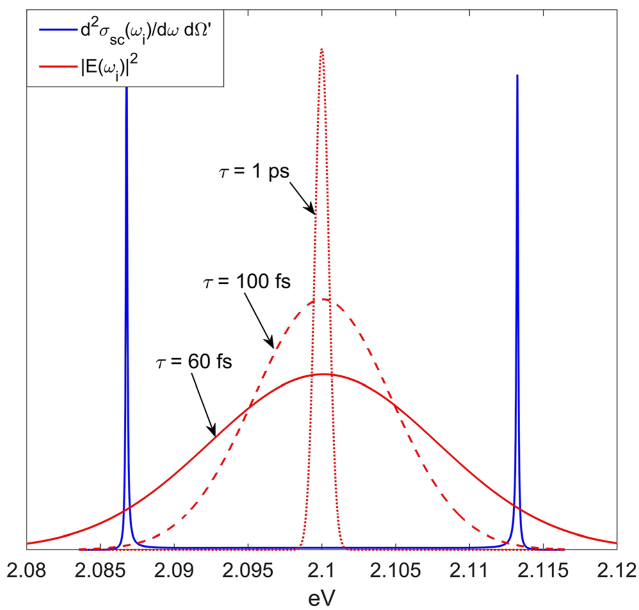

FIG. 2. Spectra of incident pulses with various durations (red curves) and differential scattering cross-section at $T_e = 2$ eV, $n_e = 10^{17}$ cm^{-3}, $\omega' = 2.1$ eV, and $\theta = \pi/4$ (blue curves).

$$\frac{d^2W_{sc}}{d\Omega'\,d\omega'} = \frac{cr_e^2(1+\cos^2\theta)}{8\pi^2} \times \left[\frac{G_1|E(\omega_i=\omega_{i1}^{\text{res}})|^2}{\hbar\omega_{i1}^{\text{res}}} + \frac{G_2|E(\omega_i=\omega_{i2}^{\text{res}})|^2}{\hbar\omega_{i2}^{\text{res}}}\right]. \tag{4.9}$$

Let us consider a scattering process with parameters $T_e = 2$ eV, $n_e = 10^{17}$ cm^{-3}, $\omega' = 2.1$ eV, and $\theta = \pi/4$. The scattering cross-section spectrum for this case is illustrated in Fig. 2, together with the spectra of incident pulses with various durations. For the parameter values considered here, the plasmon peaks do not overlap, and their width is small enough that the approximations (4.7) and (4.9) apply. The resonant frequencies in this case are $\omega_{i1}^{\text{res}} = 2.087$ eV and $\omega_{i2}^{\text{res}} = 2.113$ eV. These frequency values correspond to the Bohm–Gross dispersion relation with high accuracy:[44]

$$(\omega_i^{\text{res}} - \omega')^2 = \omega_{pe}^2 + \frac{3T_e(k^{\text{res}})^2}{m_e}. \tag{4.10}$$

Figure 3 presents the dependence of the differential scattering probability on the pulse duration (the τ dependence) for various values of the carrier frequency. The τ dependence is calculated by two methods: numerical integration using the formula (3.3) and the analytical approximation (4.9). The τ dependence shows different trends at different carrier frequencies. Its shape is determined by the relative positions of the pulse spectrum and the scattering cross-section spectrum. Figure 4 illustrates how the type of curve depends

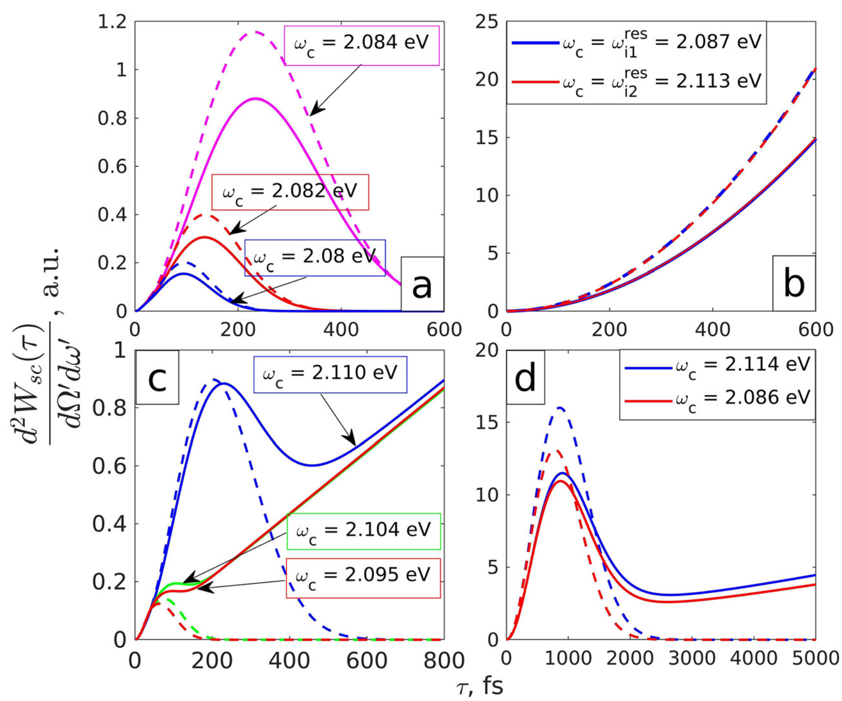

FIG. 3. Dependence of the differential scattering probability on the pulse duration for different values of the carrier frequency. Different trends are illustrated in the different panels: (a) a bell-shape maximum with an asymptote tending to zero; (b) a monotonic rise; (c) and (d) a nonmonotonic rise, tending to the linear regime. The parameter values are $T_e = 2$ eV, $n_e = 10^{17}$ cm^{-3}, $\omega' = 2.1$ eV, and $\theta = \pi/4$. Solid lines correspond to calculation using the formula (3.3) and dashed lines to calculation using the approximations (4.7) and (4.9).

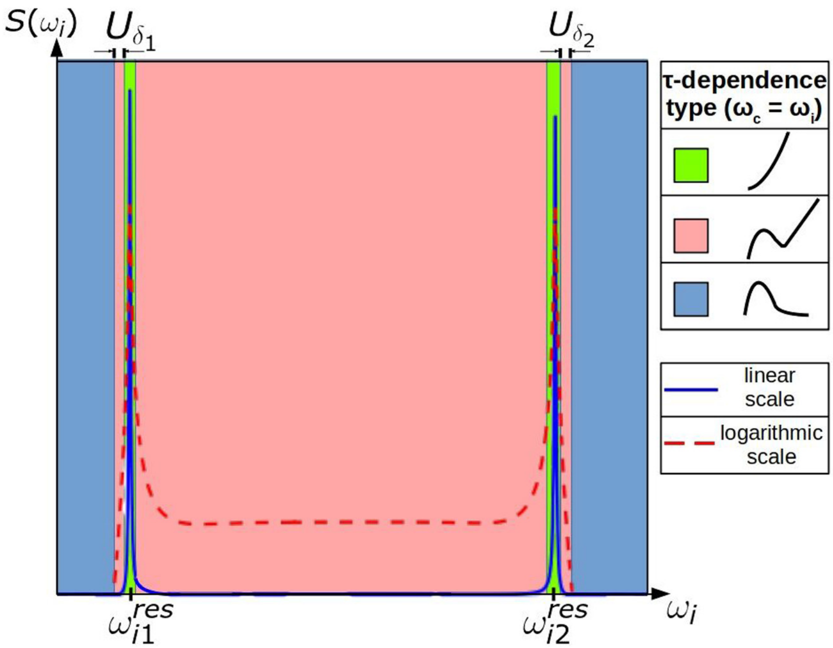

FIG. 4. Types of τ dependence for different values of the carrier frequency.

on the relative positions of the carrier frequency ω_c and the plasmon resonance frequency ω_i.

When the carrier frequency is close to the plasmon resonance frequency (the green areas in Fig. 4), the τ dependence is monotonically increasing and becoming linear as the pulse duration growths [Fig. 3(b)]. If $\omega_c < \omega_{i1}^{\rm res}$ and $\omega_c > \omega_{i2}^{\rm res}$ (the blue areas in Fig. 4), the curves are bell-shaped [Fig. 3(a)]. With increasing τ, the probability becomes almost zero, because the pulse spectrum becomes narrow and does not overlap with the peaks of the cross-section (see the red curves in Fig. 2). Thus, this effect is inherent in pulses with rather broad spectrum. There have been a number of studies describing the interaction of nonmonochromatic pulses with plasmas in terms of probability. For example, Gorbunov and Salikhov[45] considered quasi-stationary nonmonochromatic radiation. Thus, it is correct to describe the problem in terms of probability per unit time assuming incident energy flux to be constant. Hence, the interaction can be described in terms of the scattering cross-section corrected by the spectral width of the nonmonochromatic radiation. Nevertheless, such considerations are not related to the pulse duration τ. In our approach, the stationary radiation flux is absent, and we therefore deal with the scattering probability for the whole time, which is the novel aspect of our approach. The revealed nonlinear dependence is an effect that appears when the ratio of the spectral width to the carrier frequency of the pulse is of the order of 10^{-2} and more.

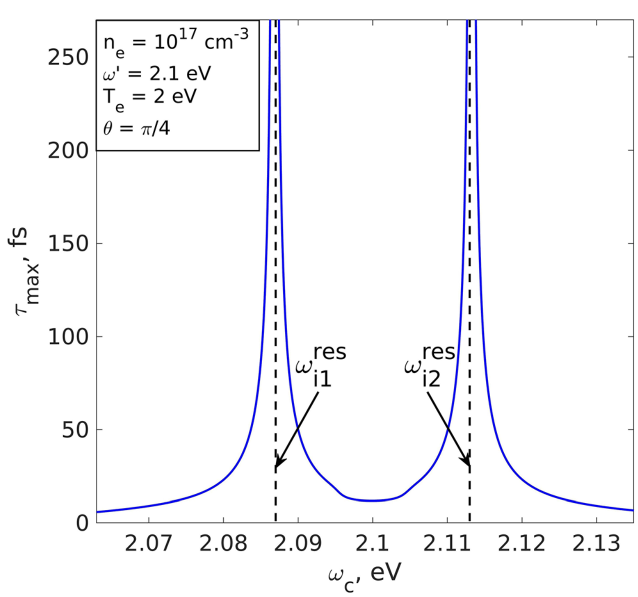

FIG. 5. Carrier-frequency dependence of the duration corresponding to the maximum scattering probability.

When $\omega_{i1}^{\rm res} < \omega_c < \omega_{i2}^{\rm res}$ [Fig. 3(c)], the τ dependence is not monotonic for low durations, a clear maximum and minimum appear. As the pulse duration increases, the τ dependence becomes linear. A similar trend is found in some regions $\omega_c \in U_{\delta_1}(\omega_{i1}^{\rm res} - \Delta\omega_1) \cup U_{\delta_2}(\omega_{i2}^{\rm res} + \Delta\omega_2)$ [Fig. 3(d)], where $\Delta\omega_{1,2}$ are small detunings from the resonant frequencies (the pink areas in Fig. 4). In contrast to the case illustrated in Fig. 3(a), as τ increases, the probability grows. This is due to scattering on the thermal fluctuations. In spite of the fact that in the case considered here, the magnitude of this process is several orders lower than the plasmon peaks, it is nevertheless not zero, and the integral (3.3) provides linear growth.

The same applies for $\omega_c \in U_{\delta_1} \cup U_{\delta_2}$, owing to the tails of the plasmon peaks.

With regard to the approximations (4.7) and (4.9), this approach does not predict linear growth of the τ dependence for long durations for the cases illustrated in Figs. 3(c) and 3(d), because it does not take scattering on thermal fluctuations into account. In spite of the magnitude discrepancy, the probability values obtained in both ways are of the same order. The values of $\tau_{\max}$ corresponding to the probability maximum have a discrepancy of ~10%.

The value of $\tau_{\max}$ tends to infinity in the neighborhoods of the resonant frequencies (Fig. 5), which corresponds to monotonic growth of the τ dependence [Fig. 3(b)].

V. CONCLUSION

We have generalized the theory of radiation scattering on electron collective excitations in a Maxwellian plasma to the case of USLPs. Drawing on formulas obtained in the framework of second-order quantum-mechanical perturbation theory, we have extended Fermi's golden rule and derived a formula for the scattering probability that takes into account the contribution of each spectral component of an incident USLP.

Scattering of a USLP on Langmuir excitations (plasmons) in an equilibrium plasma has been described in terms of the differential scattering probability during all the time of pulse action. We have studied the trends according to which plasmon components appear in the plasma form-factor spectrum and have assessed the behavior of the components corresponding to plasmon excitation and thermal motion fluctuations for different parameters of the problem. For the case in which the plasmon components dominate over the thermal motion and there is no overlap between the two plasmon peaks, an expression connecting the plasmon energy and the incident radiation frequency has been obtained. Moreover, in the framework of this approximation, we have derived an analytical expression for the scattering probability.

The way in which the scattering probability depends on the pulse duration (the τ dependence) has been investigated for different values of the carrier frequency by examining the spectrum of the plasma form factor $S(\omega_i)$. We have demonstrated that τ dependence can be nonmonotonic and that, in particular, for some values of the carrier frequency it can exhibit a maximum and a minimum. The value of $\tau_{\max}$ corresponding to the local maximum has been studied as a function of the carrier frequency. It has been shown that $\tau_{\max}$ tends to infinity in the vicinity of the plasma resonant frequencies, which corresponds to monotonic growth. An important feature of USLPs has been demonstrated, namely, that in contrast to quasi-monochromatic pulses, the scattering probability of USLPs can be nonzero for a carrier frequency lying beyond the scattering cross-section band, owing to the broad pulse spectrum.

ACKNOWLEDGMENTS

The study reported here was funded by MIPT in the framework of the 5-Top-100 Program. This work has also been supported by RFBR, Project No. 19-32-90016.

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

REFERENCES

[1] D. H. Froula, N. C. Luhmann, Jr., J. Sheffield, and S. H. Glenzer, *Plasma Scattering of Electromagnetic Radiation: Theory and Measurement Techniques* (Elsevier, 2011).

[2] A. Akhiezer, I. Prokhoda, and A. Sitenko, "Scattering of electromagnetic waves in a plasma," Sov. Phys. JETP **6**, 576 (1958).

[3] E. E. Salpeter, "Electron density fluctuations in a plasma," Phys. Rev. **120**, 1528 (1960).

[4] T. Hagfors, "Density fluctuations in a plasma in a magnetic field, with applications to the ionosphere," J. Geophys. Res. **66**, 1699–1712, https://doi.org/10.1029/jz066i006p01699 (1961).

[5] D. Farley, J. Dougherty, and D. Barron, "A theory of incoherent scattering of radio waves by a plasma II. Scattering in a magnetic field," Proc. R. Soc. London, Ser. A **263**, 238–258 (1961).

[6] A. Akhiezer, I. Akhiezer, and A. Sitenko, "Contribution to the theory of plasma fluctuations," Sov. Phys. JETP **14**, 462–468 (1962).

[7] D. F. DuBois and V. Gilinsky, "Incoherent scattering of radiation by plasmas. I. Quantum mechanical calculation of scattering cross sections," Phys. Rev. **133**, A1308 (1964).

[8] D. F. Dubois and V. Gilinsky, "Incoherent scattering of radiation by plasmas. II. Effect of Coulomb collisions on classical plasmas," Phys. Rev. **133**, A1317 (1964).

[9] H. Röhr, "A 90° laser scattering experiment for measuring temperature and density of ions and electrons in a cold dense theta pinch plasma," Z. Phys. **209**, 295–310 (1968).

[10] T. P. Hughes, "A new method for the determination of plasma electron temperature and density from Thomson scattering of an optical maser beam," Nature **194**, 268–269 (1962).

[11] T. N. Carlstrom, G. L. Campbell, J. C. DeBoo, R. Evanko, J. Evans, C. M. Greenfield, J. Haskovec, C. L. Hsieh, E. McKee, R. T. Snider *et al.*, "Design and operation of the multipulse Thomson scattering diagnostic on DIII-D," Rev. Sci. Instrum. **63**, 4901–4906 (1992).

[12] R. Behn, D. Dicken, J. Hackmann, S. A. Salito, M. R. Siegrist, P. A. Krug, I. Kjelberg, B. Duval, B. Joye, and A. Pochelon, "Ion temperature measurement of tokamak plasmas by collective Thomson scattering of D_2O laser radiation," Phys. Rev. Lett. **62**, 2833 (1989).

[13] H. Murmann, S. Götsch, H. Röhr, H. Salzmann, and K. H. Steuer, "The Thomson scattering systems of the ASDEX upgrade tokamak," Rev. Sci. Instrum. **63**, 4941–4943 (1992).

[14] A. Mooradian and A. L. McWhorter, "Light scattering from plasmons and phonons in ga as," in *Light Scattering Spectra of Solids* (Springer, 1969), pp. 297–308.

[15] S. H. Glenzer, O. L. Landen, P. Neumayer, R. W. Lee, K. Widmann, S. W. Pollaine, R. J. Wallace, G. Gregori, A. Höll, T. Bornath *et al.*, "Observations of plasmons in warm dense matter," Phys. Rev. Lett. **98**, 065002 (2007).

[16] J. R. Lawrence and D. Waugh, *Laser Surface Engineering: Processes and Applications* (Elsevier, 2014).

[17] U. Keller, "Recent developments in compact ultrafast lasers," Nature **424**, 831–838 (2003).

[18] A. H. Zewail, "Femtochemistry: Atomic-scale dynamics of the chemical bond," J. Phys. Chem. A **104**, 5660–5694 (2000).

[19] J. Shah, *Ultrafast Spectroscopy of Semiconductors and Semiconductor Nanostructures* (Springer Science & Business Media, 2013), Vol. 115.

[20] M. Fushitani and A. Hishikawa, "Single-order laser high harmonics in XUV for ultrafast photoelectron spectroscopy of molecular wavepacket dynamics," Struct. Dyn. **3**, 062602 (2016).

[21] R. Al-Obaidi, M. Wilke, M. Borgwardt, J. Metje, A. Moguilevski, N. Engel, D. Tolksdorf, A. Raheem, T. Kampen, S. Mähl *et al.*, "Ultrafast photoelectron spectroscopy of solutions: Space-charge effect," New J. Phys. **17**, 093016 (2015).

[22] R. Torres, T. Siegel, L. Brugnera, I. Procino, J. G. Underwood, C. Altucci, R. Velotta, E. Springate, C. Froud, I. C. E. Turcu *et al.*, "Extension of high harmonic spectroscopy in molecules by a 1300 nm laser field," Opt. Express **18**, 3174–3180 (2010).

[23] R.-F. Lu, P.-Y. Zhang, and K.-L. Han, "Attosecond-resolution quantum dynamics calculations for atoms and molecules in strong laser fields," Phys. Rev. E **77**, 066701 (2008).

[24] V. S. Yakovlev, J. Gagnon, N. Karpowicz, and F. Krausz, "Attosecond streaking enables the measurement of quantum phase," Phys. Rev. Lett. **105**, 073001 (2010).

[25] E. Lindroth, F. Calegari, L. Young, M. Harmand, N. Dudovich, N. Berrah, and O. Smirnova, "Challenges and opportunities in attosecond and XFEL science," Nat. Rev. Phys. **1**, 107–111 (2019).

[26] F. Calegari, A. Trabattoni, A. Palacios, D. Ayuso, M. C. Castrovilli, J. B. Greenwood, P. Decleva, F. Martín, and M. Nisoli, "Charge migration induced by attosecond pulses in bio-relevant molecules," J. Phys. B: At., Mol. Opt. Phys. **49**, 142001 (2016).

[27] Y. Kida, R. Kinjo, and T. Tanaka, "Synthesizing high-order harmonics to generate a sub-cycle pulse in free-electron lasers," Appl. Phys. Lett. **109**, 151107 (2016).

[28] T. E. Glover, T. D. Donnelly, E. A. Lipman, A. Sullivan, and R. W. Falcone, "Subpicosecond Thomson scattering measurements of optically ionized helium plasmas," Phys. Rev. Lett. **73**, 78 (1994).

[29] K. Ta Phuoc, A. Rousse, M. Pittman, J. P. Rousseau, V. Malka, S. Fritzler, D. Umstadter, and D. Hulin, "X-ray radiation from nonlinear Thomson scattering of an intense femtosecond laser on relativistic electrons in a helium plasma," Phys. Rev. Lett. **91**, 195001 (2003).

[30] P. M. Platzman and P. A. Wolff, *Waves and Interactions in Solid State Plasmas* (Academic Press, New York, 1973), Vol. 13.

[31] A. Sitenko, *Electromagnetic Fluctuations in Plasma* (Elsevier, 2012).

[32] B. S. Tanenbaum, "Continuum theory of Thomson scattering," Phys. Rev. **171**, 215 (1968).

[33] D. H. Froula, J. S. Ross, L. Divol, and S. H. Glenzer, "Thomson-scattering techniques to diagnose local electron and ion temperatures, density, and plasma wave amplitudes in laser produced plasmas," Rev. Sci. Instrum. **77**, 10E522 (2006).

[34] A. Dinklage, T. Klinger, G. Marx, and L. Schweikhard, *Plasma Physics: Confinement, Transport and Collective Effects* (Springer Science & Business Media, 2005), Vol. 670.

[35] A. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, *Plasma Electro-Dynamics* (Nauka, Moscow, 1975).

[36] H.-J. Kunze, *The Laser as a Tool for Plasma Diagnostics* (North-Holland Publishing Company, 1968).

[37] F. B. Rosmej, V. A. Astapenko, and V. Lisitsa, *Plasma Atomic Physics* (Springer International Publishing, 2020).

[38] V. A. Astapenko, F. B. Rosmej, V. S. Lisitsa, and E. S. Khramov, "Thomson scattering in plasmas: Theory generalization for ultrashort laser pulse effects," Phys. Plasmas **27**, 083301 (2020).

[39] V. A. Astapenko, "Scattering of an ultrashort electromagnetic radiation pulse by an atom in a broad spectral range," J. Exp. Theor. Phys. **112**, 193–198 (2011).

[40] V. A. Astapenko, "Scattering of an ultrashort electromagnetic pulse in a plasma," Plasma Phys. Rep. **37**, 972–977 (2011).

[41] E. Fermi, "Über die theorie des stosses zwischen atomen und elektrisch geladenen teilchen," Z. Phys. **29**, 315 (1924).

[42] V. A. Astapenko and V. S. Lisitsa, "On the theory of hydrogen atom ionization by ultra-short electromagnetic pulses," Contrib. Plasma Phys. **55**, 522–528 (2015).

[43] L. Tonks and I. Langmuir, "Note on 'oscillations in ionized gases'," Phys. Rev. **33**, 990 (1929).

[44] N. Maafa, "Dispersion relation in a plasma with arbitrary degeneracy," Phys. Scr. **48**, 351 (1993).

[45] L. M. Gorbunov and D. K. Salikhov, "Scattering of nonmonochromatic radiation," Radiophys. Quantum Electron. **24**, 759–763 (1981).