



**HAL**  
open science

## Application of critical path analysis to streaming potential coupling coefficient in porous media

Luong Duy Thanh, van Nghia Nguyen, Phan van Do, Tran Thi Chung Thuy,  
Damien Jougnot

► **To cite this version:**

Luong Duy Thanh, van Nghia Nguyen, Phan van Do, Tran Thi Chung Thuy, Damien Jougnot. Application of critical path analysis to streaming potential coupling coefficient in porous media. *Geophysical Journal International*, 2022, 168, pp.104289. 10.1016/j.advwatres.2022.104289 . hal-03796437

**HAL Id: hal-03796437**

**<https://hal.sorbonne-universite.fr/hal-03796437>**

Submitted on 4 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Application of critical path analysis to streaming potential coupling coefficient in porous media

Luong Duy Thanh<sup>1</sup>, Nguyen Van Nghia <sup>1\*</sup>, Phan Van Do<sup>1</sup>, Tran Thi Chung  
Thuy <sup>1</sup> and Damien Jougnot <sup>2</sup>

<sup>1</sup> *Thuyloi University, 175 Tay Son, Dong Da, Ha Noi, Vietnam*

<sup>2</sup> *Sorbonne Université, CNRS, EPHE, UMR 7619 Metis, F-75005, Paris, France*

(February 26, 2022)

## ABSTRACT

Streaming potential is a passive hydrogeophysical method that can be used to monitor water flow in porous media. However, a quantitative use of this method requires a good understanding the signal generation through electrokinetic coupling. In this work, we use the Critical Path Analysis (CPA) method to propose a new model to predict the streaming potential coupling coefficient in porous media. This CPA-based model is expressed in terms of the effective excess charge density, viscosity, pore water electrical conductivity and critical pore radius. The proposed model is successfully validated for heterogeneous porous media with both broad and narrow pore size distributions. As a result from uniform grain packings, we obtain a relationship between the critical pore radius and grain diameter that may be useful for applications of CPA based models for unconsolidated samples. Additionally, this model is also successfully compared with simulated data available in literature. We believe that the CPA-based models are very useful to describe the transport properties, including electrokinetic coupling. Therefore the CPA approach may help us to better characterize transport properties in porous media.

1  
2  
3  
4  
5 Streaming potential; Zeta potential; Critical Path Analysis; Porous media; Electroki-  
6 netic  
7  
8  
9

## 10 INTRODUCTION

11  
12 Streaming potentials arise by electrokinetic coupling from water flow in porous media and is  
13 due to the existence of the electrical double layer (EDL). Constitutive minerals of geologic  
14 porous media normally exhibit electrostatic charges at their surfaces when they are in  
15 contact with pore water leading to the charge distribution known as an EDL developed at  
16 these interfaces. The EDL consists of the Stern layer, where the charges are adsorbed on  
17 solid mineral surfaces and can be considered as immobile, and the diffuse layer, where the  
18 charges are mobile. Among geophysical methods, the self-potential (SP) method is the only  
19 one that is directly sensing the water flow due to the electrokinetic contribution to this signal,  
20 the so-called streaming potential. For example, SP measurements can be used to monitor  
21 water flow in aquifers (e.g., Jouniaux et al., 1999; Fagerlund and Heinson, 2003; Titov et al.,  
22 2005; Aizawa et al., 2009) or in the vadose zone (e.g., Doussan et al., 2002; Jougnot et al.,  
23 2015; Voytek et al., 2019). In hydrogeological application, SP measurements can also be  
24 utilized to predict hydrogeological parameters such as the hydraulic conductivity, the depth  
25 and thickness of the aquifer (e.g., Jardani et al., 2007; Revil and Jardani, 2013).  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41

42 In electrokinetics, the streaming potential coupling coefficient (SPCC) is a key factor  
43 as it relates the difference in water pressure (i.e., the water flux) and the difference in  
44 electrical potential (i.e., the self-potential). In the literature, there are two main expressions  
45 to characterize the SPCC in porous media. The more classical approach is the Helmholtz-  
46 Smoluchowski (HS) equation that relates the SPCC to properties of pore water-solid systems  
47 as (Smoluchowski, 1903)  
48  
49  
50  
51  
52

$$53 C_{\text{SP}} = \frac{\epsilon_r \epsilon_0 \zeta}{\mu \sigma_w}, \quad (1)$$

54 where  $\epsilon_r$  (no units) is the relative permittivity,  $\epsilon_0$  (F/m) is the dielectric permittivity in  
55 vacuum,  $\zeta$  (V) is the zeta potential describing properties of solid-water interfaces and  $\sigma_w$   
56 (S/m) is the pore water electrical conductivity. A large amount of experimental data on the  
57  
58  
59  
60  
61  
62  
63  
64  
65

SPCC is reported in the literature using Eq. (1) for porous media such as glass beads (e.g., Li et al., 1995; Pengra et al., 1999; Glover and Dery, 2010) and sandstones (e.g., Ishido and Mizutani, 1981; Li et al., 1995; Jouniaux and Pozzi, 1997; Lorne et al., 1999; Pengra et al., 1999; Jaafar et al., 2009; Vinogradov et al., 2010). When taking into account the electrical surface conductivity of solid surfaces, one has to apply a modified HS equation (e.g., Morgan et al., 1989; Glover et al., 2012). Note that conceptualizing a porous medium as a bundle of tortuous capillary tubes has been successfully applied to obtain an expression for the SPCC in porous media (e.g., Ishido and Mizutani, 1981; Jackson, 2010; Thanh et al., 2018, 2020a,b; Vinogradov et al., 2021).

Beside the HS equation, one can determine the SPCC via the effective excess charge density dragged by the pore water  $\widehat{Q}_v$  (C/m<sup>3</sup>), the permeability  $k$  (m<sup>2</sup>) and electrical conductivity  $\sigma$  (S/m) of water saturated porous media (e.g., Kormiltsev et al., 1998; Revil and Leroy, 2004; Cerepi et al., 2017; Jougnot et al., 2020) as

$$C_{\text{SP}} = -\frac{k\widehat{Q}_v}{\mu\sigma}. \quad (2)$$

The effective excess charge density  $\widehat{Q}_v$  (C/m<sup>3</sup>) can be estimated using an empirical expression with the knowledge of  $k$  (m<sup>2</sup>) as proposed in the literature (e.g., Jardani et al., 2007; Cherubini et al., 2018). For example, Jardani et al. (2007) used a large set of published data for various lithologies and different electrolyte concentrations to propose an empirical relationship as

$$\log(\widehat{Q}_v) = -9.23 - 0.82\log(k). \quad (3)$$

Recently, Guarracino and Jougnot (2018) proposed an analytical model for  $\widehat{Q}_v$ , which takes into account (1) the properties of porous media such as porosity  $\phi$  (no units), permeability  $k$  (m<sup>2</sup>), and tortuosity  $\tau$  (no units) and (2) the electro-chemical properties such as ionic concentration  $C_f$  (mol/L), Debye length  $\lambda_d$  (m), and Zeta potential  $\zeta$  (V), that is given by

$$\widehat{Q}_v = 1000N_AeC_f\lambda_D^2 \left[ -\frac{2e\zeta}{k_B T} - \left( \frac{e\zeta}{3k_B T} \right)^3 \right] \frac{\phi}{\tau^2 k}, \quad (4)$$

1  
2  
3  
4  
5 where  $N_A$  is the Avogadro number ( $\text{mol}^{-1}$ ),  $e$  (C) is the elementary charge,  $k_B$  ( $\text{JK}^{-1}$ )  
6  
7 is the Boltzmann constant and  $T$  (K) is the temperature. The Debye length is given by  
8  
9

$$\lambda_D = \sqrt{\frac{\epsilon_r \epsilon_0 k_B T}{2000 N_A C_f e^2}}. \quad (5)$$

10  
11  
12  
13  
14  
15 Dependence of  $\zeta$  (V) with the ionic concentration  $C_f$  (mol/L) for silica-based samples  
16  
17 in NaCl brine can be expressed as (e.g., Pride and Morgan, 1991; Vinogradov et al., 2010):  
18  
19

$$\zeta = \{a + b \log_{10}(C_f)\} \times 10^{-3}, \quad (6)$$

20  
21  
22  
23  
24 where  $a = -9.67$  (mV) and  $b = 19.02$  (mV).  
25

26  
27 Critical path analysis (CPA) is a very powerful approach that has been applied to study  
28 fluid flow and transport phenomena in heterogeneous media. For example, the CPA ap-  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

proach has been applied to determine permeability  $k$  in porous media with broad pore size distribution (PSD) such as soils or rocks (e.g., Katz and Thompson, 1986; Hunt, 2001; Ghanbarian et al., 2016; Daigle, 2016; Hunt and Sahimi, 2017). It is shown that the CPA also works well to predict  $k$  for uniform grain packings, known as homogeneous porous media with relatively narrow PSDs (e.g., Ghanbarian, 2020a,b). Very recently, the CPA has also been applied to estimate hydraulic conductivity in dual-porosity soils under unsaturated conditions (e.g., Ghanbarian, 2021). Additionally, the CPA has been applied for the electrical conductivity  $\sigma$  (S/m) in porous media (e.g., Ewing and Hunt, 2006; Ghanbarian et al., 2015; Daigle, 2016; Ghanbarian and Sahimi, 2017). Consequently, one can apply the CPA to find the  $k - \sigma$  relationship for porous media (e.g., Friedman and Seaton, 1998; Skaggs, 2011; Daigle, 2016). As indicated in Eq. (2), along with  $\hat{Q}_v$ ,  $k$  and  $\sigma$  are very important parameters affecting the SPCC and they are linked to each other as shown by the CPA. To the best of our knowledge, the CPA has not yet been applied to study the streaming potential in porous media. Therefore, in this work, we use the CPA to obtain a model for the SPCC. Then, we validate the model for porous media with both broad and narrow PSDs. From obtained results for uniform grain packings, we obtain the relationship between the critical pore radius and grain diameter that may be useful for applications of

1  
2  
3  
4  
5 CPA based models for unconsolidated samples. Additionally, the model is also compared  
6  
7 with simulated data available in literature.  
8  
9

## 10 11 THEORY

### 12 13 14 Critical path analysis

15  
16  
17 Based on the CPA, a heterogeneous porous medium is assumed to be made up of different  
18 flow pathways (pores) with different abilities of transmitting flow, that are different con-  
19 ductances. The pore conductance depends on the pore size in such a way that larger pores  
20 have larger conductances and vice versa. Most flow (either electrical or hydraulic flow)  
21 through media happens on pores with high conductances. Consequently, the macroscopic  
22 flow is controlled by pores whose conductances are larger than a certain value and low  
23 conductance pores have minor contribution to the overall flow through porous media (e.g.,  
24 Ambegaokar et al., 1971; Daigle, 2016; Ghanbarian, 2020b).  
25  
26  
27  
28  
29  
30  
31  
32

33 To better understand the CPA, we consider a pore network made of pores with five  
34 different apertures (i.e., 1, 1.5, 2, 2.5 and 3 with arbitrary units) as indicated in Fig. 1  
35 (modified from Ghanbarian, 2020b). Then, the pores in their original positions are removed  
36 following an order from the largest pores to the smallest pores. When the first two largest  
37 pores (3 and 2.5) are kept in the original location and four smaller pores (2, 1.5 and 1) are  
38 removed from the network, no percolating cluster exists as shown in Fig. 1(b). Nevertheless,  
39 when keeping the three largest pore sizes (3, 2.5 and 2) and removing smaller pores (1.5 and  
40 1), a sample-spanning cluster starts to be formed and the system percolates as shown in Fig.  
41 1(c). The smallest pore size (2) that is required to set up a conducting sample spanning  
42 cluster is defined as the critical pore size  $r_c$  (e.g., Daigle, 2016; Ghanbarian, 2020b). The pore  
43 conductance corresponding to  $r_c$  is defined as the critical conductance  $g_c$  (e.g., Ambegaokar  
44 et al., 1971). We assume that the volumetric probability density function of pore sizes  
45 in media is  $f(r)$ . For a fully-saturated medium, the minimum fraction of water volume  
46 must be filled for a sample spanning cluster that is called the percolation threshold  $p_c$  and  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

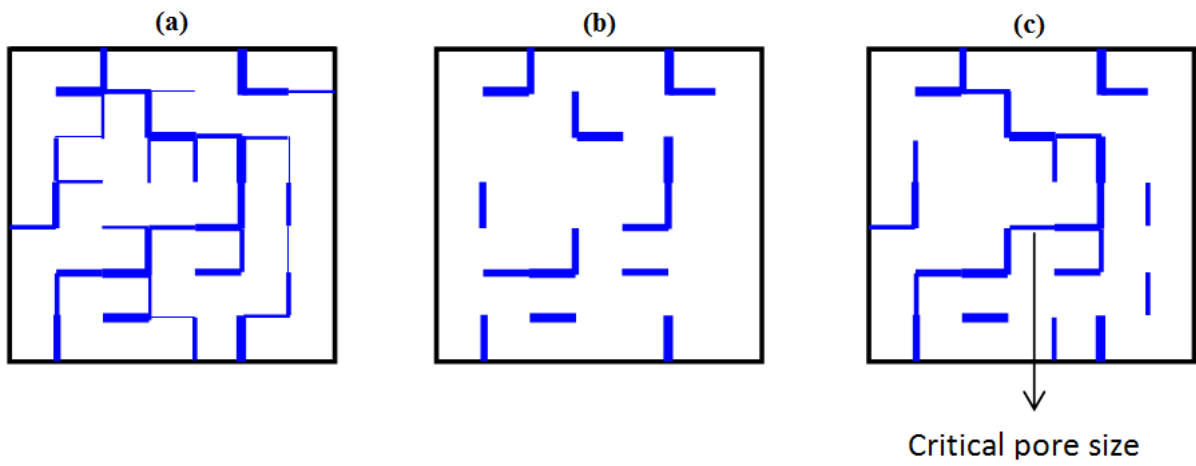


Figure 1: Two-dimensional scheme of the critical path analysis. (a) A pore network composed of five different pore sizes (i.e., 1, 1.5, 2, 2.5 and 3 with arbitrary units) randomly distributed in the medium. (b) The same network with only the first two largest pores (2.5 and 3) in their original locations. Pores smaller than 2.5 were removed from the pore network. As can be seen, the medium does not percolate. (c) The network after adding the third largest pores with size 2 (critical pore size). The sample-spanning cluster is first formed and the network starts percolating (modified from Ghanbarian, 2020b).

1  
2  
3  
4  
5 calculated as (e.g., Daigle, 2016)  
6  
7

$$p_c = \int_{r_c}^{r_{max}} f(r) dr, \quad (7)$$

8  
9  
10  
11 where  $r_{max}$  (m) is the largest pore size of a porous medium. If  $p_c$  and  $f(r)$  are known, one  
12 can find  $r_c$  and therefore can predict the permeability  $k$  and electrical conductivity  $\sigma$  of  
13 porous media as presented in the following subsections. Note that there have been different  
14 volumetric probability density functions  $f(r)$  that are applicable for porous media such  
15 as lognormal, fractal and power law distributions (e.g., Daigle, 2016; Ghanbarian, 2020b;  
16 Skaggs, 2011). For example, using the fractal distribution for  $f(r)$ , Daigle (2016) obtained  
17 an expression for  $r_c$  as  
18  
19  
20  
21  
22  
23  
24

$$r_c = r_{max} \left( 1 - \frac{\phi p_c}{\beta} \right)^{\frac{1}{3-D}}, \quad (8)$$

25  
26 where  $\phi$  is the porosity of media,  $D$  is the fractal dimension and  $\beta$  is a constant defined as  
27 the ratio of pore volume to the sum of the pore and solid volumes of media.  
28  
29  
30  
31  
32  
33

### 34 Streaming potential coupling coefficient from CPA

35  
36  
37 We consider a pore of radius  $r$  (m) and length  $L$  (m) that is occupied by water with an  
38 electrical conductivity  $\sigma_w$  (S/m), a dynamic viscosity  $\mu$  (Pa.s) and a density  $\rho_w$  (kg/m<sup>3</sup>).  
39 According to Ohm's law, the relationship between the electrical current  $j^e$  (A) and electrical  
40 potential difference  $\Delta V$  (V) across a water filled pore of radius  $r$  is given by  
41  
42  
43  
44  
45  
46

$$j^e(r) = g^e(r) \Delta V, \quad (9)$$

47  
48  
49 where  $g^e(r)$  (S) is the electrical conductance of the water-filled pore.  
50

51 According to Poiseuille's law, the relationship between the volumetric flow rate  $j^h$  (m<sup>3</sup>/s)  
52 and pressure head difference  $\Delta h$  (m) across the pore is given by  
53  
54  
55  
56

$$j^h(r) = g^h(r) \Delta h, \quad (10)$$



1  
2  
3  
4  
5 where  $g^h(r)$  (m<sup>2</sup>/s) is the hydraulic conductance of the water-filled pore. For cylindrical  
6  
7 pores, the electrical and hydraulic conductances, respectively, are given by  
8  
9

$$10 \quad g^e(r) = \frac{\pi r^2 \sigma_w}{L}, \quad (11)$$

11  
12  
13  
14 and

$$15 \quad g^h(r) = \frac{\rho_w g \pi r^4}{8 \mu L}, \quad (12)$$

16  
17  
18 where  $g$  (m/s<sup>2</sup>) is the acceleration due to gravity. Note that the electrical surface conduc-  
19  
20 tivity of the pore has not yet been considered in Eq. (11).  
21

22  
23 Consequently, the critical electrical conductance and the critical hydraulic conductance are  
24  
25 given by

$$26 \quad g_c^e(r_c) = \frac{\pi r_c^2 \sigma_w}{L}, \quad (13)$$

27  
28  
29 and

$$30 \quad g_c^h(r_c) = \frac{\rho_w g \pi r_c^4}{8 \mu L}, \quad (14)$$

31  
32  
33 respectively. It should be noted that the critical pore radii for electrical and hydraulic  
34  
35 conductances are assumed to be the same in Eq. (13) and Eq. (14) (see Friedman and  
36  
37 Seaton, 1998; Daigle, 2016). When electrical surface conductivity of pores is not negligible,  
38  
39 especially at low electrical conductivity  $\sigma_w$ , the percolation thresholds for electrical and  
40  
41 hydraulic conductivities may be different. The reason is that electrical current can occur  
42  
43 at the pore surfaces through electric double layer even when it can not happen in the bulk  
44  
45 pore space. Consequently, the critical pore radius for electrical conductance may be smaller  
46  
47 than that for hydraulic conductance (e.g., Ewing and Hunt, 2006; Daigle, 2016; Ghanbarian,  
48  
49 2020b). As shown by Friedman and Seaton (1998) and Hunt (2001), the macroscopic con-  
50  
51 ductance (either electrical or hydraulic conductance) of porous media  $g_m$  is approximately  
52  
53 equal to the critical conductance  $g_c$ . Therefore, the ratio of hydraulic conductivity  $K$  (m/s)  
54  
55 to electrical conductivity  $\sigma$  (S/m) of porous media under fully saturated conditions is ap-  
56  
57 proximately equal to the ratio of the critical hydraulic conductance  $g_c^h(r_c)$  to the critical  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5 electrical conductance  $g_c^e(r_c)$ :

$$\frac{K}{\sigma} = \frac{g_c^h(r_c)}{g_c^e(r_c)} = \frac{\rho_w g r_c^2}{8\mu\sigma_w}. \quad (15)$$

6  
7  
8  
9  
10 Using the relationship between the hydraulic conductivity  $K$  and the permeability of porous  
11 media  $k$  as  $K = \rho_w g k / \mu$ , the following is obtained

$$\frac{k}{\sigma} = \frac{r_c^2}{8\sigma_w}. \quad (16)$$

12  
13  
14  
15  
16  
17  
18  
19 Eq. (16) can be written as

$$k = \frac{\sigma r_c^2}{8\sigma_w} = \frac{\sigma r_c^2}{c\sigma_w}, \quad (17)$$

20  
21  
22  
23 where  $c$  (no units) is a constant coefficient, that is equal to 8 in this work. Note that  $c$   
24 would be 12 if thin cracks are considered instead of cylindrical pores (e.g., Bernabé et al.,  
25 2010). Eq. (16) is similar to those reported in the literature with a difference for the value  
26 of  $c$ . For example, Thompson (1991) and Ghanbarian (2020a) used  $c = 56.5$  for accurate  
27 permeability predictions in consolidated samples such as sandstone, carbonate rocks and  
28 unconsolidated samples such as grain packings, respectively. Skaggs (2011) used  $c \approx 13.3$   
29 for cylindrical pores and  $c \approx 6.7$  for slit-shaped pores. Daigle (2016) found  $c = 8$  for  
30 permeability prediction in natural porous media.  
31  
32  
33  
34  
35  
36  
37  
38  
39

40 We remark that the electrical formation factor is a scale invariant parameter characteriz-  
41 ing pore space topology of porous media. For porous media in which the surface conductivity  
42 can be neglected, it is defined as (e.g., Revil et al., 1999; Bernabé and Mainault, 2015)

$$F = \lim_{\sigma_s \rightarrow 0} \left( \frac{\sigma_w}{\sigma} \right). \quad (18)$$

43  
44  
45  
46  
47  
48  
49  
50 Combining Eq. (17) and Eq. (18) yields the following equation

$$k = \frac{r_c^2}{cF}. \quad (19)$$

51  
52  
53  
54  
55  
56  
57 Among permeability models, Kozeny-Carman (KC) models are most commonly used and

one of them is given by (e.g., Paterson, 1983; Bernabé and Maineult, 2015)

$$k = \frac{C_{\text{KC}} r_h^2}{F}, \quad (20)$$

where  $C_{\text{KC}}$  (no units) is a geometric factor which depends on geometry of pores,  $r_h$  (m) is the hydraulic radius (i.e.,  $r_h = 2V_p/S_p$ , where  $V_p$  (m<sup>3</sup>) is the total pore volume and  $S$  (m<sup>2</sup>) is the area of the interface between the pores and the solid matrix). It is also shown that the KC model, given by Eq. (20), can be improved by replacing the hydraulic radius  $r_h$  by the characteristic length scale  $\Lambda$  (m), that can be viewed as a dynamically weighted version of  $r_h$ . Namely, permeability  $k$  can be given by (e.g., Johnson et al., 1987; Schwartz et al., 1989; Revil et al., 1999)

$$k = \frac{C_{\text{KC}} \Lambda^2}{F}, \quad (21)$$

where  $C_{\text{KC}}$  is taken as 1/8 by Johnson et al. (1987), or 1/4 by Schwartz et al. (1989) or 1/2 by Revil et al. (1999). Obviously, there is a correspondence between  $r_c$ ,  $r_h$  and  $\Lambda$ .

Substituting Eq. (16) into Eq. (2), we obtain an expression for the SPCC for water saturated porous media based on the CPA framework as

$$C_{\text{SP}} = -\frac{\widehat{Q}_v r_c^2}{8\mu\sigma_w}. \quad (22)$$

We want to emphasize that this simple equation is a new CPA-based model that describe the SPCC for porous media when the surface conductivity can be neglected.

## RESULTS AND DISCUSSION

Bolève et al. (2007) measured the properties of seven glass packs with different sizes (grain diameter  $d$ , porosity  $\phi$ , permeability  $k$ ) and the corresponding SPCC as a function of the pore water electrical conductivity  $\sigma_w$  (Table 1). Additionally, Bolève et al. (2007) also measured the electrical conductivity of these samples for a larger range of pore water electrical conductivity  $\sigma_w$  (Table 2). In the following, we use the sets of experimental data reported by Bolève et al. (2007) to validate the proposed model.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

Table 1: Measured values for seven samples at different values of  $\sigma_w$  reported by Bolève et al. (2007). Symbols  $d$ ,  $\phi$  and  $k$  stand for the glass bead diameter, porosity and permeability, respectively. Note that the SPCC value for sample S6 at  $\sigma_w = 10^{-4}$  (S/m) was not reported by Bolève et al. (2007).

ID	$d$ ( $\mu\text{m}$ )	$\phi$ (-)	$k$ ( $\text{m}^2$ )	SPCC (in $10^{-7}$ V/Pa)					
				$\sigma_w=3\times 10^{-2}$ (S/m)	$10^{-2}$ (S/m)	$3\times 10^{-3}$ (S/m)	$10^{-3}$ (S/m)	$3\times 10^{-4}$ (S/m)	$10^{-4}$ (S/m)
S1a	56	0.4	$2.0\times 10^{-12}$	12.5	22	75	159	454	647
S1b	72	0.4	$3.1\times 10^{-12}$	8.5	36	142	245	748	1944
S2	93	0.4	$4.4\times 10^{-12}$	8.1	24	87	224	477	3215
S3	181	0.4	$2.7\times 10^{-11}$	7.6	30.5	137	319	1219	4793
S4	256	0.4	$5.6\times 10^{-11}$	7.5	23	82	317	1132	4502
S5	512	0.4	$1.2\times 10^{-10}$	11.1	36	107	331	1451	3483
S6	3000	0.4	$1.4\times 10^{-8}$	17.2	43	159	510	1014	-

Table 2: Measured values for the electrical conductivity of the samples  $\sigma$  (in  $10^{-4}$  S/m) saturated by different electrical conductivity of electrolytes  $\sigma_w$  as reported by Bolève et al. (2007)

ID	$\sigma_w=10^{-1}$	$6\times 10^{-2}$	$3\times 10^{-2}$	$10^{-2}$	$3\times 10^{-3}$	$10^{-3}$	$3\times 10^{-4}$	$10^{-4}$
	(S/m)	(S/m)	(S/m)	(S/m)	(S/m)	(S/m)	(S/m)	(S/m)
S1a	289.5	199.0	100.9	38.4	12.61	6.59	3.38	2.11
S1b	291.4	199.0	101.3	38.6	12.30	6.62	2.54	1.94
S2	291.4	195.6	91.6	32.6	11.77	6.20	2.29	1.78
S3	290.9	193.0	86.8	32.7	11.74	5.11	1.81	1.19
S4	288.0	188.7	86.5	31.9	11.47	3.78	1.33	1.08
S5	283.1	187.3	84.6	31.5	11.14	3.31	1.14	0.94
S6	279.8	182.0	75.6	31.6	9.61	3.59	2.28	0.91

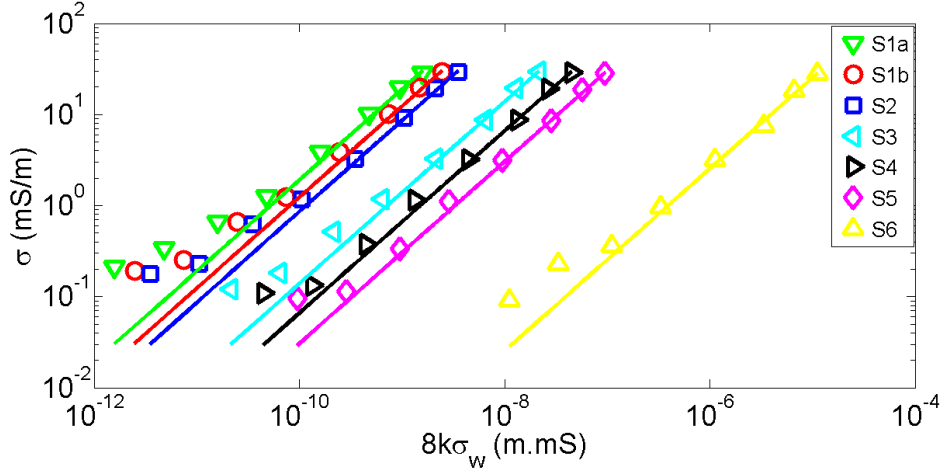


Figure 2: Variation of measured electrical conductivity of the samples  $\sigma$  as a function of  $8k\sigma_w$  for the different samples (symbols) reported by Bolève et al. (2007). The colored lines correspond to the fitting line using the “fminsearch” function in MATLAB to seek a minimum value for root-mean-square error (RMSE).

Figure 2 shows the variation of  $\sigma$  with  $8k\sigma_w$  for the samples measured by Bolève et al. (2007). Note that  $k$  is taken from Table 1,  $\sigma_w$  and corresponding  $\sigma$  are taken from Table 2. It is clearly shown that  $\sigma$  linearly depends on  $8k\sigma_w$  with the slope of  $\alpha = 1/r_c^2$  as indicated by Eq. (16) for all samples when the electrolyte electrical conductivity  $\sigma_w$  is larger than  $10^{-2}$  S/m. The reason is that the electrical surface conductivity of the glass beads can be neglected for these values of  $\sigma_w$ , hence Eq. (16) becomes valid. However, for small values of  $\sigma_w$ , one can not ignore the electrical surface conductivity and Eq. (16) does not hold as explained in the previous section. Therefore, there is no linear relationship between  $\sigma$  and  $8k\sigma_w$  for  $\sigma_w < 10^{-2}$  S/m. Using the linear part of the graph, the slope of  $\alpha$  and hence the critical pore radius  $r_c$  can be obtained. Namely,  $\alpha$  is optimized using the “fminsearch” function in MATLAB to seek a minimum value for root-mean-square error (RMSE). The straight lines corresponding to the minimum RMSE are shown by solid lines in Fig. 2. Obtained values for  $\alpha$  and  $r_c$  for all samples are shown in Table 3.

From the critical pore radius  $r_c$ , shown in Table 3, and the corresponding grain diameter  $d$ , shown in Table 1, the variation of  $r_c$  with  $d$  is shown in Fig. 3 (symbols). Clearly,

Table 3: Obtained values for  $\alpha$  ( $1/\text{m}^2$ ) and  $r_c$  (m) corresponding to the minimum RMSE for the samples reported by Bolève et al. (2007).

ID	S1a	S1b	S2	S3	S4	S5	S6
$\alpha$	$1.89 \times 10^{10}$	$1.23 \times 10^{10}$	$8.55 \times 10^9$	$1.38 \times 10^9$	$6.57 \times 10^8$	$3.02 \times 10^8$	$2.54 \times 10^6$
$r_c$	$7.26 \times 10^{-6}$	$9.02 \times 10^{-6}$	$10.8 \times 10^{-6}$	$26.9 \times 10^{-6}$	$38.9 \times 10^{-6}$	$57.5 \times 10^{-6}$	$628 \times 10^{-6}$

there is a strong correlation between  $r_c$  and  $d$ . Ng et al. (1978) proposed a relationship between the average pore throat radius and the grain diameter for simple cubic packing of mono-sized bead packs as  $r_p = 0.21d$ . Subsequently, Ghanbarian (2020a) followed Ng et al. (1978) and suggested a relationship between the critical pore radius  $r_c$  and grain diameter as  $r_c = 0.21d$ . They showed that this approximation can provide good estimation of permeability for mono-sized glass and sand packs. Additionally, it was shown that it is possible to translate mean particle diameters into a mean pore radius using  $r_p = 0.15d$  (e.g., Hamamoto et al., 2011; Sakaki et al., 2014). Based on the above arguments, we obtain a linear relationship  $r_c = 0.14d$  as shown by a solid line in Fig. 3. The obtained  $r_c$ - $d$  relationship is rather relevant to the suggestions proposed by Ghanbarian (2020a) or Hamamoto et al. (2011). Using Eq. (19) in combination with  $r_c = 0.14d$ , one can predict the permeability of unconsolidated samples from the mean grain diameter  $d$  and formation factor  $F$ . Fig. 4 shows the comparison between measured and predicted permeability for samples made up of glass beads or sand reported by a compilation of literature data (Glover et al., 2006; Glover and Walker, 2009; Glover and Dery, 2010; Kimura, 2018; Biella et al., 1983; Moghadasi et al., 2004; Chauveteau and Zaitoun, 1981). Note that  $F$  was not reported for the samples of Chauveteau and Zaitoun (1981), Moghadasi et al. (2004) and Glover and Dery (2010). Therefore, we obtain  $F$  from porosity  $\phi$  using  $F = \phi^{-m}$  (Archie, 1942) with  $m = 1.5$  for spherical beads (e.g., Sen et al., 1981). One can clearly see that the permeability prediction using  $r_c = 0.14d$  is in good agreement with measured data for unconsolidated samples. Revil et al. (1999) also proposed a linear relationship between  $\Lambda$

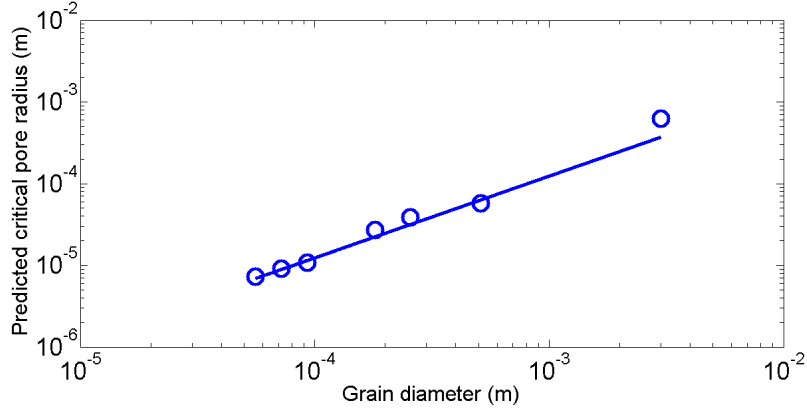


Figure 3: Variation of measured electrical conductivity of the samples  $\sigma$  with  $8k\sigma_w$  for different samples (symbols) reported by Bolève et al. (2007).

and  $d$  as  $\Lambda=d/(2m(F-1))$  where  $m$  is the cementation exponent of porous media. For sand packs, for example,  $m$  is reported to be 1.5 (Sen et al., 1981) and one approximately obtains a relationship  $\Lambda = 0.12d$  that is close to our finding  $r_c = 0.14d$ .

From Eq. (22), we can deduce  $\hat{Q}_v$  for the samples reported by Bolève et al. (2007) from known values of  $C_{SP}$ ,  $\sigma_w$  (see Table 1) and  $r_c$  (see Table 3). Similarly, we also obtain  $\hat{Q}_v$  for glass packs reported by Glover and Dery (2010) and Pengra et al. (1999) where we estimate  $r_c$  from  $d$  using  $r_c = 0.14d$ . From obtained  $\hat{Q}_v$ , the relationship between  $\hat{Q}_v$  and permeability  $k$  of the samples at different electrolyte electrical conductivities is shown Fig. 5 (symbols). We also use Eq. (3) to reproduce experimental data in Fig. 5 (solid line). It is seen that the variation of  $\hat{Q}_v$  with  $k$  predicted from the CPA based model is in good agreement with the empirical relationship proposed by Jardani et al. (2007).

Figure 6 shows the variation of  $\sigma$  with  $8k\sigma_w$  for a Fontainebleau sample obtained from Vinogradov et al. (2010). The properties of the sample is shown in Table 4. Note that the experimental data for  $\sigma$  and  $\sigma_w$  are extracted from a linear part of the  $\sigma_w$ - $\sigma$  graph shown in Fig. 6 of Vinogradov et al. (2010) and  $\tau$  is estimated using a relation  $\tau=\sqrt{F\phi}$  (Winsauer et al., 1952). Applying the same approach as previously mentioned, we obtain  $r_c = 3.11 \mu\text{m}$ .

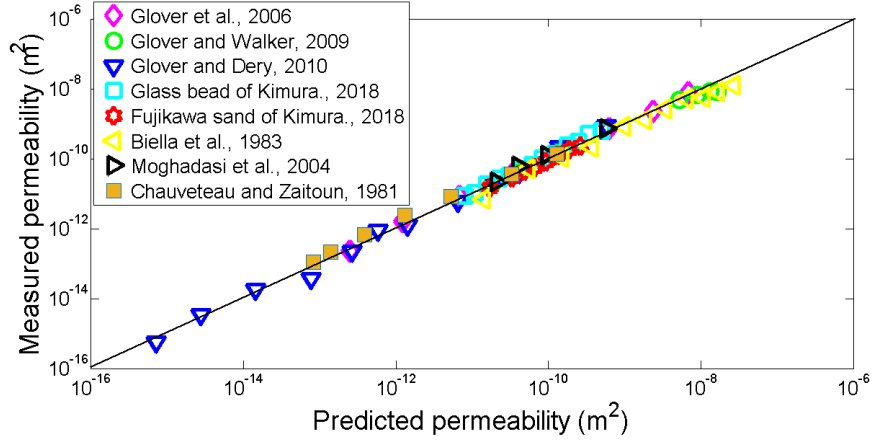


Figure 4: Comparison between measured permeability and the proposed model. The solid line represents the 1:1 line.

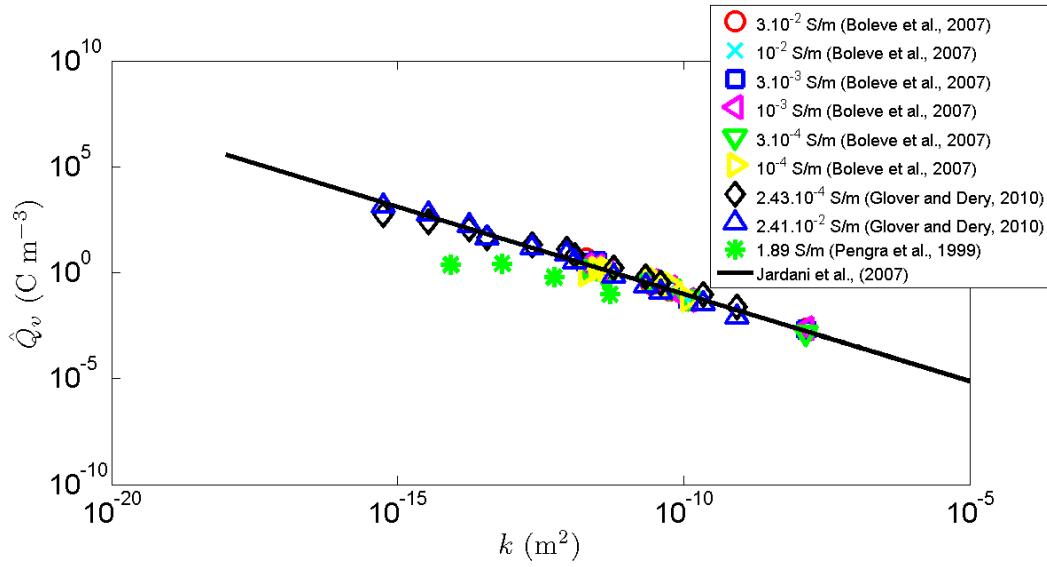


Figure 5: Variation of  $\hat{Q}_v$  with  $k$ . Symbols are data deduced from Eq. (22) with knowledge of  $C_{SP}$ ,  $\sigma_w$  and  $r_c$ . The solid line is predicted from Eq. (3) proposed by Jardani et al. (2007).



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

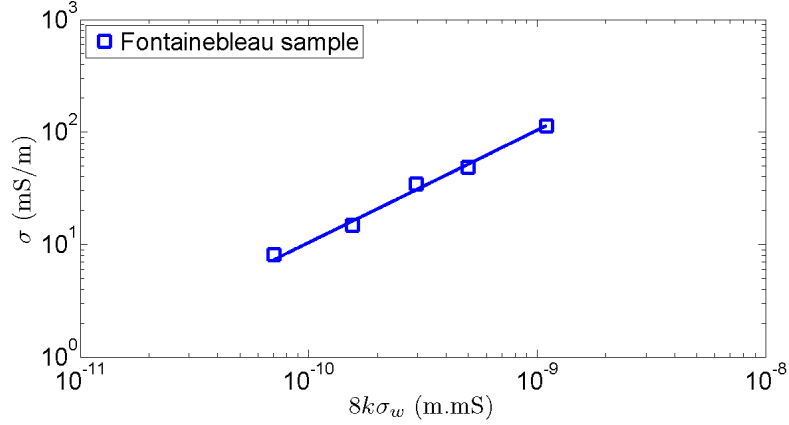


Figure 6: Variation of  $\sigma$  with  $8k\sigma_w$  for the Fontainebleau sample (symbols) reported by Vinogradov et al. (2010). The solid line is the fitting line with minimum RMSE.

Table 4: Properties of the Fontainebleau sample reported by Vinogradov et al. (2010) in which  $\phi$ ,  $d$ ,  $k$ ,  $F$ ,  $r_c$  and  $\tau$  stand for porosity, grain diameter, permeability, formation factor, critical pore radius and tortuosity of the sample, respectively. Superscript \* refers to measured quantities by Vinogradov et al. (2010),  $r_c$  is obtained by fitting with minimum RMSE and  $\tau$  is estimated using the relation  $\tau = \sqrt{F\phi}$

$\phi^*$ (no units)	$d^*$ ( $\mu\text{m}$ )	$k^*$ (mD)	$F^*$ (no units)	$r_c$ ( $\mu\text{m}$ )	$\tau$ (no units)
0.072	250	25	157	3.1	3.3

Figure 7 shows the variation of the SPCC in magnitude with the NaCl electrolyte concentration ( $C_f$ ) measured by Vinogradov et al. (2010) for the Fontainebleau sample (symbols). We also use Eq. (22) in combination with Eq. (3) and Eq. (4) to reproduce experimental data (solid and dashed lines). Note that, in Eq. (22),  $\sigma_w$  is obtained using an approximation  $\sigma_w = 10C_f$  that was stated to be valid for  $C_f$  between  $10^{-6}$  mol/L and 1 mol/L (Sen and Goode, 1992). The solid and dashed lines correspond to the predictions using Eq. (3) and Eq. (4), respectively. One can see that the proposed model using Eq. (4) provides a better result in reproducing data than that using Eq. (3). The reason is that the  $\widehat{Q}_v$ - $k$  relationship, given by Eq. (3), proposed by Jardani et al. (2007) is limited by the fact that it does not take into consideration other properties of porous media like porosity and chemical composition of the pore water, especially the pore water electrical conductivity (see discussion in Jougnot et al., 2012). Therefore, predicted values of  $\widehat{Q}_v$  may deviate from the experimental data up to two order of magnitude for a given value of  $k$  (e.g., Jougnot et al., 2015, 2020). Eq. (4) proposed by Guarracino and Jougnot (2018) considers both the geometrical properties of porous media ( $\phi$ ,  $k$ ,  $\tau$ ) and the electro-chemical properties ( $C_f$ ,  $\lambda_d$ ,  $\zeta$ ) and is therefore more relevant for a single sample rather than a large set of samples with various lithologies and different electrolyte concentrations collected by Jardani et al. (2007) for fitting.

Jougnot et al. (2019) performed 2-D pore network simulations to investigate the influence of PSD on the streaming potential mechanism. Based on the numerical data reported by Jougnot et al. (2019) (see their appended Table S1) for four different PSDs (fractal, exponential symmetric, lognormal, double lognormal distributions) in a various range of pore radii at different ionic concentrations, the variation of  $\sigma/\phi$  with  $8k\sigma_w/\phi$  is shown in Fig. 8 (symbols) for a radii range between 1  $\mu\text{m}$  and 100  $\mu\text{m}$ . Applying the same approaches as previously described, we obtain  $r_c$  for the fractal, exponential symmetric, lognormal, double lognormal distributions as 1.65, 10.2, 10.2 and 9.0  $\mu\text{m}$ , respectively. Those values are quite close to values of  $\Lambda$  reported by Jougnot et al. (2019) (1.56, 9.4, 9.4 and 6.2  $\mu\text{m}$ , respectively). Consequently, we can predict  $\widehat{Q}_v$  for four different PSDs with the knowledge of  $C_{\text{SP}}$ ,  $\sigma_w$ , which are presented in Table S1 by Jougnot et al. (2019), and obtained values

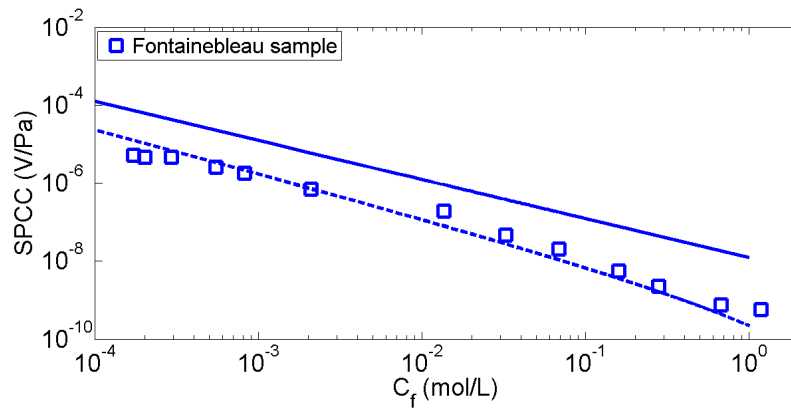


Figure 7: Variation of the SPCC with electrolyte concentration measured by Vinogradov et al. (2010) for the Fontainebleau sample (symbols). The solid line is based on Eq. (22) using Eq. (3) (empirical model of Jardani et al., 2007). The dashed line is based on Eq. (22) using Eq. (4) (mechanistical model of Guarracino and Jougnot, 2018).

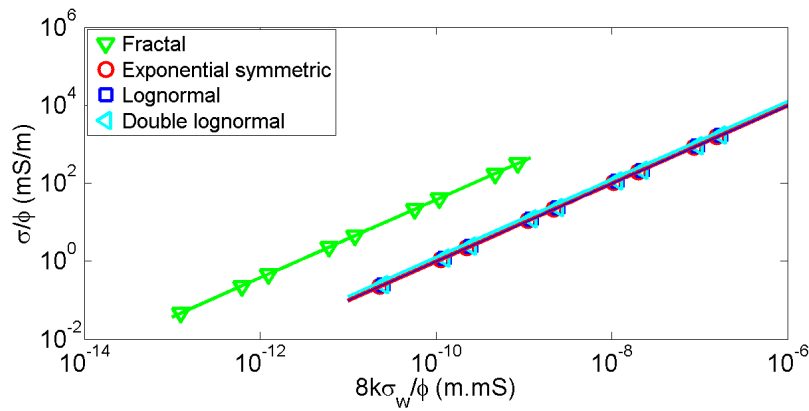


Figure 8: Variation of simulated values of  $\sigma/\phi$  with those of  $8k\sigma_w/\phi$  for different PSDs (symbols) reported by Jougnot et al. (2019) for a radii range between 1  $\mu\text{m}$  and 100  $\mu\text{m}$ . The solid lines are fitting ones.

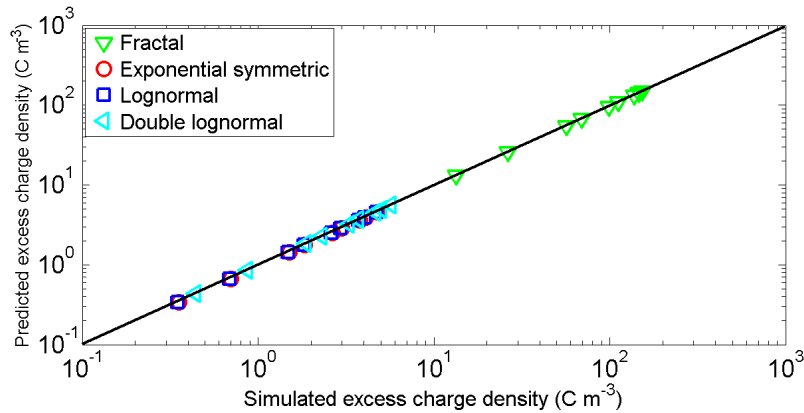


Figure 9: Comparison between the predicted  $\widehat{Q}_v$  from the CPA based model and simulated  $\widehat{Q}_v$  reported by Jougnot et al. (2019) for different PSDs (symbols) for a radii range between 1  $\mu\text{m}$  and 100  $\mu\text{m}$ . The solid black line corresponds to the 1:1 line.

of  $r_c$ .

Comparison between the predicted  $\widehat{Q}_v$  from the newly proposed CPA-based model and the simulated  $\widehat{Q}_v$  reported by Jougnot et al. (2019) is then shown in Fig. 9. The predicted results from the proposed model are in very good agreement with simulated data by Jougnot et al. (2019) for four different PSDs.

## CONCLUSIONS

In this work, the Critical Path Analysis (CPA) approaches for permeability and electrical conductivity are combined to propose a model for the streaming potential coupling coefficient (SPCC) in water saturated porous media. We show that this new CPA-based model is expressed in terms of the effective excess charge density, viscosity, electrical conductivity of pore water, and critical pore radius  $r_c$  (i.e., a typical parameter of the CPA framework). We remark that the parameter  $r_c$  corresponds to the characteristic length scale  $\Lambda$ , also known as the Johnson length (Johnson et al., 1986), that can be viewed as a dynamically weighted version of hydraulic radius. The proposed model is successfully validated for heterogeneous porous media with both narrow (uniform grain packing) and broad (sandstone)

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

PSDs. From the results on the uniform grain packings, we obtain the relationship between the critical pore radius and grain diameter that may be useful for applications of CPA based models for unconsolidated samples. Additionally, the model is also successfully compared with simulated data available in literature. We believe that the CPA-based models are very useful to describe the transport properties, including electrokinetic coupling. Therefore the CPA approach may help us to better characterize transport properties in porous media.

## REFERENCES

- 1  
2  
3  
4  
5  
6  
7  
8 Aizawa, K., Y. Ogawa, and T. Ishido, 2009, Groundwater flow and hydrothermal systems  
9 within volcanic edifices: Delineation by electric self-potential and magnetotellurics: Jour-  
10 nal of Geophysical Research, **114**.  
11  
12  
13 Ambegaokar, V., B. I. Halperin, and J. S. Langer, 1971, Hopping conductivity in disordered  
14 systems: Physical Review B, **4**, no. 8, 2612–2620.  
15  
16  
17 Archie, G. E., 1942, The electrical resistivity log as an aid in determining some reservoir  
18 characteristics: Petroleum Transactions of AIME, **146**, 54–62.  
19  
20  
21 Bernabé, Y., M. Li, and A. Maineult, 2010, Permeability and pore connectivity: A new  
22 model based on network simulations: Journal of Geophysical Research: Solid Earth,  
23 **115**.  
24  
25  
26  
27 Bernabé, Y., and A. Maineult, 2015, 11.02 - physics of porous media: Fluid flow through  
28 porous media: Treatise on Geophysics (Second Edition), Elsevier, 19–41.  
29  
30  
31 Biella, G., A. Lozej, and I. Tabacco, 1983, Experimental study of some hydrogeophysical  
32 properties of unconsolidated porous media: Groundwater, **21**, 741–751.  
33  
34  
35 Bolève, A., A. Crespy, A. Revil, F. Janod, and J. L. Mattiuzzo, 2007, Streaming potentials  
36 of granular media: Influence of the dukhin and reynolds numbers: Journal of Geophysical  
37 Research, **B08204**.  
38  
39  
40  
41 Cerepi, A., A. Cherubini, B. Garcia, H. Deschamps, and A. Revil, 2017, Streaming potential  
42 coupling coefficient in unsaturated carbonate rocks: Geophysical Journal International,  
43 **210**, 291–302.  
44  
45  
46  
47 Chauveteau, G., and A. Zaitoun, 1981, Basic rheological behavior of xanthan polysaccharide  
48 solutions in porous media: Effect of pore size and polymer concentration: Enhanced Oil  
49 Recovery, edited by F. J. Fayers, pp. 197–212.  
50  
51  
52  
53 Cherubini, A., B. Garcia, A. Cerepi, and A. Revil, 2018, Streaming potential coupling  
54 coefficient and transport properties of unsaturated carbonate rocks: Vadose Zone Journal,  
55 **17**, 180030.  
56  
57  
58  
59 Daigle, H., 2016, Application of critical path analysis for permeability prediction in natural  
60 porous media: Advances in Water Resources, **96**, 43–54.  
61  
62  
63  
64  
65

- 1  
2  
3  
4  
5 Doussan, C., L. Jouniaux, and J.-L. Thony, 2002, Variations of self-potential and unsatu-  
6 rated water flow with time in sandy loam and clay loam soils: *Journal of Hydrology*, **267**,  
7 173 – 185.  
8  
9  
10  
11 Ewing, R. P., and A. G. Hunt, 2006, Dependence of the electrical conductivity on saturation  
12 in real porous media: *Vadose Zone Journal*, **5**, 731–741.  
13  
14  
15 Fagerlund, F., and G. Heinson, 2003, Detecting subsurface groundwater flow in fractured  
16 rock using self-potential (sp) methods: *Environmental Geology*, **43**.  
17  
18  
19 Friedman, S., and N. Seaton, 1998, Critical path analysis of the relationship between perme-  
20 ability and electrical conductivity of three-dimensional pore networks: *Water Resources*  
21 *Research*, **34**, 1703–1710.  
22  
23  
24 Ghanbarian, B., 2020a, Applications of critical path analysis to uniform grain packings  
25 with narrow conductance distributions: I. single-phase permeability: *Advances in Water*  
26 *Resources*, **137**, 103529.  
27  
28  
29 ———, 2020b, Applications of critical path analysis to uniform grain packings with narrow  
30 conductance distributions: II. water relative permeability: *Advances in Water Resources*,  
31 **137**, 103524.  
32  
33  
34 ———, 2021, Unsaturated hydraulic conductivity in dual-porosity soils: Percolation theory:  
35 *Soil and Tillage Research*, **212**, 105061.  
36  
37  
38 Ghanbarian, B., A. G. Hunt, T. E. Skinner, and R. P. Ewing, 2015, Saturation Dependence  
39 of Transport in Porous Media Predicted by Percolation and Effective Medium Theories:  
40 *Fractals*, **23**, 1540004–42.  
41  
42  
43 Ghanbarian, B., and M. Sahimi, 2017, Electrical conductivity of partially saturated packings  
44 of particles: *Transport in Porous Media*, **118**, 1–16.  
45  
46  
47 Ghanbarian, B., C. Torres-Verdín, and T. H. Skaggs, 2016, Quantifying tight-gas sandstone  
48 permeability via critical path analysis: *Advances in Water Resources*, **92**, 316–322.  
49  
50  
51 Glover, P., I. I. Zadjali, and K. A. Frew, 2006, Permeability prediction from micp and nmr  
52 data using an electrokinetic approach: *Geophysics*, **71**, F49–F60.  
53  
54  
55 Glover, P. W. J., and N. Dery, 2010, Streaming potential coupling coefficient of quartz glass  
56 bead packs: Dependence on grain diameter, pore size, and pore throat radius: *Geophysics*,  
57 **75**, F225–F241.  
58  
59  
60  
61  
62  
63  
64  
65

- 1  
2  
3  
4  
5 Glover, P. W. J., and E. Walker, 2009, Grain-size to effective pore-size transformation  
6 derived from electrokinetic theory: *Geophysics*, **74(1)**, E17–E29.  
7  
8  
9 Glover, P. W. J., E. Walker, and M. Jackson, 2012, Streaming-potential coefficient of reser-  
10 voir rock: A theoretical model: *Geophysics*, **77(2)**, D17–D43.  
11  
12  
13 Guarracino, L., and D. Jougnot, 2018, A physically based analytical model to describe  
14 effective excess charge for streaming potential generation in water saturated porous media:  
15 *Journal of Geophysical Research: Solid Earth*, **123**, 52–65.  
16  
17  
18 Hamamoto, S., P. Moldrup, K. Kawamoto, L. Wollesen de Jonge, P. Schjønning, and T.  
19 Komatsu, 2011, Two-region extended archie’s law model for soil air permeability and gas  
20 diffusivity: *Soil Science Society of America Journal*, **75**, 795–806.  
21  
22  
23 Hunt, A., 2001, Applications of percolation theory to porous media with distributed local  
24 conductances: *Advances in Water Resources*, **24**, 279–307.  
25  
26  
27 Hunt, A. G., and M. Sahimi, 2017, Flow, transport, and reaction in porous media: Per-  
28 colation scaling, critical-path analysis, and effective medium approximation: *Reviews of*  
29 *Geophysics*, **55**, 993–1078.  
30  
31  
32  
33 Ishido, T., and H. Mizutani, 1981, Experimental and theoretical basis of electrokinetic phe-  
34 nomena in rock-water systems and its applications to geophysics: *Journal of Geophysical*  
35 *Research*, **86**, 1763–1775.  
36  
37  
38  
39 Jaafar, M. Z., J. Vinogradov, and M. D. Jackson, 2009, Measurement of streaming potential  
40 coupling coefficient in sandstones saturated with high salinity nacl brine: *Geophysical*  
41 *Research Letters*, **36**, doi:10.1029/2009GL040549.  
42  
43  
44  
45 Jackson, M. D., 2010, Multiphase electrokinetic coupling: Insights into the impact of fluid  
46 and charge distribution at the pore scale from a bundle of capillary tubes model: *Journal*  
47 *of Geophysical Research: Solid Earth*, **115**.  
48  
49  
50  
51 Jardani, A., A. Revil, A. Boleve, A. Crespy, J.-P. Dupont, W. Barrash, and B. Malama,  
52 2007, Tomography of the darcy velocity from self-potential measurements: *Geophysical*  
53 *Research Letters*, **34**.  
54  
55  
56  
57 Johnson, D. L., J. Koplik, and L. M. Schwartz, 1986, New pore-size parameter characterizing  
58 transport in porous media: *Phys. Rev. Lett.*, **57**, no. 20, 2564–2567.  
59  
60  
61 Johnson, D. L., T. J. Plona, and H. Kojima, 1987, Probing porous media with 1st sound,  
62  
63  
64  
65



- 1  
2  
3  
4  
5 2nd sound, 4th sound, and 3rd sound: AIP Conference Proceedings, **154**, 243–277.  
6  
7 Jougnot, D., N. Linde, E. Haarder, and M. Looms, 2015, Monitoring of saline tracer move-  
8 ment with vertically distributed self-potential measurements at the hobe agricultural test  
9 site, vouldund, denmark: Journal of Hydrology, **521**, 314 – 327.  
10  
11 Jougnot, D., N. Linde, A. Revil, and C. Doussan, 2012, Derivation of soil-specific streaming  
12 potential electrical parameters from hydrodynamic characteristics of partially saturated  
13 soils: Vadose Zone Journal, **11**, 272–286.  
14  
15 Jougnot, D., A. Mendieta, P. Leroy, and A. Mainault, 2019, Exploring the effect of the  
16 pore size distribution on the streaming potential generation in saturated porous media,  
17 insight from pore network simulations: Journal of Geophysical Research: Solid Earth,  
18 **124**, 5315–5335.  
19  
20 Jougnot, D., D. Roubinet, L. Guarracino, and A. Mainault, 2020, Modeling streaming  
21 potential in porous and fractured media, description and benefits of the effective excess  
22 charge density approach: In: Biswas A., Sharma S. (eds) Advances in Modeling and  
23 Interpretation in Near Surface Geophysics. Springer Geophysics. Springer, Cham.  
24  
25 Jouniaux, L., J. Pozzi, J. Berthier, and P. Masse', 1999, Detection of fluid flow variations at  
26 the nankai trough by electric and magnetic measurements in boreholes or at the seafloor:  
27 Journal of Geophysical Research, **104**, 29293–29309.  
28  
29 Jouniaux, L., and J.-P. Pozzi, 1997, Laboratory measurements anomalous 0.1–0.5 hz stream-  
30 ing potential under geochemical changes: Implications for electrotelluric precursors to  
31 earthquakes: Journal of Geophysical Research: Solid Earth, **102**, 15335–15343.  
32  
33 Katz, A. J., and A. H. Thompson, 1986, Quantitative prediction of permeability in porous  
34 rock: Physical Review B, **34**, no. 11, 8179–8181.  
35  
36 Kimura, M., 2018, Prediction of tortuosity, permeability, and pore radius of water-saturated  
37 unconsolidated glass beads and sands: The Journal of the Acoustical Society of America,  
38 **143**, 3154–3168.  
39  
40 Kormiltsev, V. V., A. N. Ratushnyak, and V. A. Shapiro, 1998, Three-dimensional modeling  
41 of electric and magnetic fields induced by the fluid flow movement in porous media:  
42 Physics of the Earth and Planetary Interiors, **105**, 109 – 118.  
43  
44 Li, S. X., D. B. Pengra, and P.Z.Wong, 1995, Onsager's reciprocal relation and the hydraulic  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

- 1  
2  
3  
4  
5 permeability of porous media: *Physical Review E*, **51**, 5748–5751.
- 6  
7 Lorne, B., F. Perrier, and J. P. Avouac, 1999, Streaming potential measurements: 1. prop-  
8 erties of the electrical double layer from crushed rock samples: *Journal of Geophysical*  
9 *Research*, **104**, 17.857–17.877.
- 10  
11 Moghadasi, J., H. Müller Steinhagen, M. Jamialahmadi, and A. Sharif, 2004, Theoretical  
12 and experimental study of particle movement and deposition in porous media during  
13 water injection: *J. Pet. Sci. Eng.*, **43**, 163–181.
- 14  
15 Morgan, F. D., E. R. Williams, and T. R. Madden, 1989, Streaming potential properties of  
16 westerly granite with applications: *Journal of Geophysical Research*, **94**, 12.449–12.461.
- 17  
18 Ng, K., H. Davis, and L. Scriven, 1978, Visualization of blob mechanics in flow through  
19 porous media: *Chemical Engineering Science*, **33**, 1009–1017.
- 20  
21 Paterson, M., 1983, The equivalent channel model for permeability and resistivity in fluid-  
22 saturated rock—a re-appraisal: *Mechanics of Materials*, **2**, 345 – 352.
- 23  
24 Pengra, D., S. X. Li, and P. Wong, 1999, Determination of rock properties by low frequency  
25 ac electrokinetics: *Journal of Geophysical Research*, **104**, 29485–29508.
- 26  
27 Pride, S. R., and F. D. Morgan, 1991, Electrokinetic dissipation induced by seismic waves:  
28 *Geophysics*, **56**, 914–925.
- 29  
30 Revil, A., L. M. Cathles III, and P. D. Manhardt, 1999, Permeability of shaly sands: *Water*  
31 *Resources Research*, **3**, 651–662.
- 32  
33 Revil, A., and A. Jardani, 2013, *The self-potential method: Theory and applications in*  
34 *environmental geosciences*: Cambridge University Press.
- 35  
36 Revil, A., and P. Leroy, 2004, Constitutive equations for ionic transport in porous shales:  
37 *Journal of Geophysical Research: Solid Earth*, **109**. (B03208).
- 38  
39 Sakaki, T., M. Komatsu, and M. Takahashi, 2014, Rules-of-thumb for predicting air-entry  
40 value of disturbed sands from particle size: *Soil Science Society of America Journal*, **78**,  
41 454–464.
- 42  
43 Schwartz, L. M., P. N. Sen, and D. L. Johnson, 1989, Influence of rough surfaces on elec-  
44 trolytic conduction in porous media: *Phys. Rev. B*, **40**, no. 4, 2450–2458.
- 45  
46 Sen, P., C. Scala, and M. H. Cohen, 1981, A self-similar model for sedimentary rocks with  
47 application to the dielectric constant of fused glass beads: *Journal of the Soil Mechanics*  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

- 1  
2  
3  
4  
5 and foundations Division, **46**, 781–795.  
6  
7 Sen, P. N., and P. A. Goode, 1992, Influence of temperature on electrical conductivity on  
8 shaly sands: *Geophysics*, **57**, 89–96.  
9  
10 Skaggs, T., 2011, Assessment of critical path analyses of the relationship between perme-  
11 ability and electrical conductivity of pore networks: *Advances in Water Resources*, **34**,  
12 1335–1342.  
13  
14 Smoluchowski, M., 1903, Contribution à la theorie de l’endosmose electrique et de quelques  
15 phenomènes correlatifs: *Bulletin international de l’Academie des Sciences de Cracovie*,  
16 **8**, 182 – 200.  
17  
18 Thanh, L., D. Jougnot, P. Do, N. Ca, and N. Hien, 2020a, A physically based model for the  
19 streaming potential coupling coefficient in partially saturated porous media: *Water*, **12**,  
20 1588.  
21  
22 Thanh, L., D. Jougnot, P. Do, M. Mendieta, N. Ca, T. P. Hoa, V.X., and N. Hien, 2020b,  
23 Electroosmotic coupling in porous media, a new model based on a fractal upscaling pro-  
24 cedure: *Transport in Porous Media*, **134**, 249–274.  
25  
26 Thanh, L. D., P. Van Do, N. Van Nghia, and N. X. Ca, 2018, A fractal model for streaming  
27 potential coefficient in porous media: *Geophysical Prospecting*, **66**, 753–766.  
28  
29 Thompson, A. H., 1991, Fractals in rock physics: *Annual Review of Earth and Planetary*  
30 *Sciences*, **19**, 237–262.  
31  
32 Titov, K., A. Revil, P. Konosavsky, S. Straface, and S. Troisi, 2005, Numerical modelling  
33 of self-potential signals associated with a pumping test experiment: *Geophysical Journal*  
34 *International*, **162**, 641–650.  
35  
36 Vinogradov, J., R. Hill, and D. Jougnot, 2021, Influence of pore size distribution on the  
37 electrokinetic coupling coefficient in two-phase flow conditions: *Water*, **13**.  
38  
39 Vinogradov, J., M. Z. Jaafar, and M. D. Jackson, 2010, Measurement of streaming potential  
40 coupling coefficient in sandstones saturated with natural and artificial brines at high  
41 salinity: *Journal of Geophysical Research*, **115**.  
42  
43 Voytek, E. B., H. R. Barnard, D. Jougnot, and K. Singha, 2019, Transpiration- and  
44 precipitation-induced subsurface water flow observed using the self-potential method:  
45 *Hydrological Processes*, **33**, 1784–1801.  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

Winsauer, W. O., H. M. Shearin, P. H. Masson, and M. Williams, 1952, Resistivity of  
brine saturated sands in relation to pore geometry: American Association of Petroleum  
Geologists Bulletin, **36**, 253– 277.