Newton's gravitational law in 2D and the MOND theory.

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Abstract

The efficiency of the MOND theory of Mordehai Milgrom might be unrelated to distance, but instead connected to flatness of galaxies and the fact that the large scale distribution of matter is essentially two-dimensional. A combination of the pushing gravity paradigm with essential flatness of large luminous matter clusters leads to an approximate MOND theory which would allow to avoid the dark matter hypothesis.

Key words: gravitation, MOND theory, galactic velocity curves.

1 Introduction

While preparing a pedagogical document to explain the pushing gravity of De Duillier-Lesage for a general public, all of a sudden, the author wondered what could be a 2 dimensional version of Newton's law of gravitation. A possible connection then appeared between the laminar distribution of matter at high scale and the slow decay of gravitational attractive forces between very distant objects proposed by Mordehai Milgrom in his MOND model. This may lead to a new explanation of the strange velocity curves far from the center of galaxies, as well as Zwicky's paradox in the Coma galaxy cluster. In this case, there would be no need for dark matter to explain the divergence from Newtonian attraction law. The inverse square law may turn out to be an effect of spatial dimension three. And in 2D , the inverse square would be simply replaced by...the inverse law without square! After recalling the missing mass enigma and the history of the De Duillier-Lesage theory, we explain how it may connect with a variant of Milgrom's MOND hypothesis. Moreover, although the effect of laminar distribution leads naturally to overcoming the paradox in the framework of the De Duillier-Lesage theory, it might even appear natural without making any assumption on the cause of gravity. Our conclusion is that the strange velocity curves observed in spiral galaxies may be a consequence of their flatness, both globally and locally near the boundary.

2 Recalling the missing mass enigma.

2.1 The Coma cluster

The missing mass problem follows from very interesting observations made by the americano-swiss astronomer Fritz Zwicky around 1930 and reported in the two basic papers [13, 14]. While examining the Coma galaxy cluster in 1933, Zwicky was the first to use the virial theorem to discover the existence of a gravitational anomaly, consisting in an excessive rotational velocity of the luminous matter compared to the calculated gravitational attraction within the cluster. He calculated the dynamical gravitational mass of the galaxies within the cluster from the observed rotational velocities and obtained a value at least 400 times greater than expected from the total luminosity of the cluster The same calculation today shows a smaller factor, based on greater values for the mass of luminous material. The factor can be made even smaller by taking account of the hot invisible gaz inside the cluster which was unknown at the time of Zwicky's observations, but even like that the "dynamical mass" appears much larger than the total mass of ordinary matter, either luminous or dark (gas, planets, dark dwarfs, black holes), found in the cluster until now. Since no other explanation was found, and because Zwicky himself mentioned the possible existence of unseen "dark matter" as a possible explanation of the discrepancy, some astronomers, after some problem with rotation of galaxies were pointed out in the seventies, started to investigate the possible existence of a different kind of matter (of non-baryonic nature) and to look for new particles called WIMPS or AXIONS. But no such exotic particle has been found until now.

2.2 About Zwicky's papers.

It is quite interesting to read with some attention the two papers of F. Zwicky [13, 14] and even to compare their approaches. In [13] the paradox appears as a rather short remark (Section 5 of [13]) because most of the paper is devoted to promoting the redshift as a measurement tool for distant universe, and the calculations are somewhat intricated. In [14], the calculations are more detailed and the contradiction comes from comparing the total luminous mass (evaluated from a statement on the average luminosity of the galaxies, without specifying if it is in the cluster or in general) and the dynamical mass calculated from the average relative radial velocities and the average distances in the cluster. A crucial hypothesis is that the cluster is essentially spherical with uniform distribution of galaxies (an hypothesis that might be wrong and could be very difficult to confirm or infirm) at least in the more central part, this allows to fix the average distance and the relation between average squared radial velocity (the only accessible component) and average squared actual velocity. To compute the "dynamical mass" of the cluster, the final formula, assuming that the shape of the cluster is stationary or not too far from it, is of the form

$$M \sim a\frac{\overline{v^2}R}{G}$$

where R is the radius of the cluster and a is around 1, with two possible values depending on the hypotheses. From this formula, Zwicky concludes that the rate Mass over Absolute luminosity of the average galaxy in the cluster overpasses the usual value (currently admitted to be around 4, with what we know on the average mass repartition of stars in neighboring galaxies) by a factor 125. This is already less than the factor 400 claimed in [13]. In addition, we would like to underline 2 facts :

1) It is now estimated that classical dark objects (clouds, brown dwarfs, black holes...) multiply the real weight of galaxies by a factor around 4, this reduces the factor 125 to around 30, more than 10 times less than the initial claim. And there might exist other "black" objects made of "normal" matter which were not found yet.

2) We must also analyse the effect of the wrong value of H_0 at the time of Zwicky. Actually, the Coma cluster happens to be about 7 times farther from us than what was thought in the thirties. As a consequence, the total absolute luminosity has been underestimated by a factor 49, and R by a factor 7. This reduces the discrepancy from the factor 30 to less than 5.

That factor 5 is not really negligible, especially since Zwicky's estimate of the total mass is an estimate from below. But it might be explained much more easily than the initial factor 400 by the combination of potential methodological biases recalled in [5] and perhaps some other ideas which will be investigated in the next sections.

2.3 Flat galactic rotation curves

The missing mass enigma, who left many astronomers indifferent at the period where it was pointed out by Zwicky, reappeared in another form around 1970 as a consequence of some measurements by Vera Rubin (cf. [10, 11]). Quoting wikipedia: "wishing to avoid controversial areas of astronomy, including quasars and galactic motion, Rubin began to study the rotation and outer reaches of galaxies. She investigated the rotation curves of spiral galaxies, beginning with Andromeda, by looking at their outermost material, and observed flat rotation curves: the outermost components of the galaxy were moving as quickly as those close to the center. This was an early indication that spiral galaxies might be surrounded by dark matter haloes. She further uncovered the discrepancy between the predicted angular motion of galaxies based on the visible light and the observed motion. Her research showed that spiral galaxies rotate quickly enough that they should fly apart, if the gravity of their constituent stars was all that was holding them together. Because they stay intact, a large amount of unseen mass must be holding them together, a conundrum that became known as the galaxy rotation problem. Rubin's calculations showed that galaxies must contain at least five to ten times as much dark matter as ordinary matter. Rubin's results were confirmed over subsequent decades and became the first persuasive results supporting the theory of dark matter, initially proposed by Fritz Zwicky in the 1930s..."

3 The MOND theory.

In 1983, in [9], M. Milgrom made the hypothesis that Newton's law might not be correct for very remote objects and in this case the inverse square law should be replaced by an inverse law. It seems that his idea allows to understand the motion of the most external parts of galaxies and clusters and might give an alternative to the dark matter hypothesis. However, it is not easy to find a physical interpretation of this divergence from Newton's law. Then a modified-inertia MOND approach was proposed as a change in Newton's second law at small accelerations. This questions the fondations of dynamics, as a matter of fact even the classical Newton's second law is difficult to understand in a completely empty space, cf. [7] .

4 The Fatio de Duillier - Lesage theory.

In this section, we recall the main ideas of the theory of "pushing gravity" which was an attempt to explain by a simple mechanism the characteristics of the gravitational field. This theory, appealing to some kind of infinitesimal mechanism involving ultra-microscopic particles is in a sense the exact opposite of Einstein's general relativity which relies on macroscopic local deformations due to presence of matter of a preexisting global curvature of that ultra-macroscopic substratum usually called universe.

4.1 Basic principle of the model.

At the time of Newton, nothing was known about atoms and molecules. These two authors (with a very different personality and almost opposite cognitive profile) thus imagined independently that the cohesion of matter and gravity might share the same origin. According to Wikipedia: "Le Sage's theory of gravitation is a kinetic theory of gravity originally proposed by Nicolas Fatio de Duillier in 1690 and later by Georges-Louis Le Sage in 1748. The theory proposed a mechanical explanation for Newton's gravitational force in terms of streams of tiny unseen particles (which Le Sage called ultra-mundane corpuscles) impacting all material objects from all directions. According to this model, any two material bodies partially shield each other from the impinging corpuscles, resulting in a net imbalance in the pressure exerted by the impact of corpuscles on the bodies, tending to drive the bodies together. The theory posits that the force of gravity is the result of tiny particles (corpuscles) moving at high speed in all directions, throughout the universe. The intensity of the flux of particles is assumed to be the same in all directions, so an isolated object A is struck equally from all sides, resulting in only an inward-directed pressure but no net directional force. With a second object B present, however, a fraction of the particles that would otherwise have struck A from the direction of B is intercepted, so B works as a shield, i.e. from the direction of B, A will be struck by fewer particles than from the opposite direction. Likewise B will be struck by fewer particles from the direction of A than from the opposite direction. One can say that A and B are "shadowing" each other, and the two bodies are pushed toward each other by the resulting imbalance of forces. Thus the apparent attraction between bodies is, according to this theory, actually a diminished push from the direction of other bodies, so the theory is sometimes called push gravity or shadow gravity, although it is more widely referred to as Lesage gravity."

4.2 The criticism from the scientific community.

At the time of Newton, and later until the beginning of the last century, many scientists have been informed of Lesage theory and criticized it. Rather interestingly, as reported on Wikipedia, some major scientists as different as Newton, Euler, Kelvin had essentially the same reaction: at first happy that an explanatory mechanism was found and sometimes even saying that it was the only possible, then concluding that the theory had too many weak points to be considered reasonable. Then they did not try to find alternative theories, somehow confirming that no other system could be imagined. Maybe Poincaré was an exception in the sense that he never showed any belief in the model, and confirmed some arguments of his predecessors implying that the theory is not viable (communication in 1908). Moreover, this theory was essentially abandoned after the discovery of atomic structure of the matter, maybe because the cohesion of matter and the behavior of solids were considered as definitely understood while Lesage theory cannot really explain at the same time the weak gravitational forces and the important cohesion forces leading to the stability of matter. For some details of the main usual objections against the Lesage theory, cf. eg [5] .

4.3 Interesting aspects of the model.

If we assume that matter is made of very tiny grains separated by vacuum or any substance which does not interact with the corpuscles, it is not difficult to understand that the intensity of the pushing gravitational force so created varies as the inverse of the square of the distance and is approximately proportional to the product of "masses", defined as the number of "material grains" constitutive of the objects A and B. Interestingly enough, to recover Newton's gravitational law, the creators of that model had to imagine the existence of atoms with about 2 centuries of anticipation, although, by our modern view of matter (thinking about metals with their electronic sea) the grains should probably refer to the atomic nuclei rather than atoms. Lesage also estimated the velocity of " ultra-mundane corpuscles" to be about 10^5 times the speed of light, but after all who ever said that Einstein's theory applies to "ultra-mundane corpuscles" which are somehow immaterial?

Although this is a source of difficult problems from the point of view of geometry, the pushing gravity model opens the door to a possible variability of the "gravitational constant" G at very large spatial (or time) scale. This is one of the main reasons for our present interest in this theory.

4.4 The possibility of an inhomogeneous gravity

Let us forget for a moment the fact that Lesage's theory of pushing gravity has been rejected (with quite relevant arguments) by most physicists and try to understand whether this type of model might help to solve the missing mass paradox. Since we do not wish to invent a new word for something which might not exist, let us call "gravitons" the " ultra-mundane corpuscles" although this term has been used with a different meaning, for other things whose existence have not been established either so far. Then gravitons can be thought as forming a gas of immaterial particles (like photons) possessing however a linear momentum which can be transferred partially to material particles after a shock. Then the gravitational force can be thought as a pressure, the difference with usual gases that we meet in physics being that the graviton gas can cross the matter and the force is proportional to the volume of matter struck by gravitons rather than any kind of surface. And actually this is only approximative (cf. e.g. [2]) since, among other complicated phenomena, successive layers of matter reduce the number of incoming particles by a small proportion. In this model, the gravitational constant G can be seen as representing a local pressure per volume of space. And the final formula for the gravitational force will be sharp only when a large number of nuclei are involved and the distances are not large enough to imply a big variation of the gravitational pressure, which is usual in our macroscopic but not extra galactic familiar world.

4.5 A different local gravitational constant might fill the gap in Zwicky's estimate.

Let us therefore start from the working hypothesis that gravitation is produced by gravitons. The Coma system is located more than 300 millions of light years apart from our galaxy. It does not look completely absurd to imagine that the pressure of gravitons is only locally constant, permitting around Coma a value of G four or five times greater than in our local group of galaxies. A variation of G with respect to time (and what we see from Coma is 300 millions years old!) will not surprise all physicists, especially if one thinks about "Big Bang" which implies a violent evolution at the "beginning". Some authors already imagined a variation of G with respect to time in their models. A spatial variation might be considered more improbable by most physicists convinced that things tend to homogenize naturally, but this is valid for systems near equilibrium, and according to what we see in Astronomy, we are rather living in an ever evolving universe, with subsystems that rotate or oscillate without any kind of rest. Very large subsystems have very small rotation speeds with respect to their size which might, seen from a distance, make us believe that they are static and invariable, but we know that this is an illusion.

4.6 A connection with the large scale structure of universe.

Is it really a coincidence that 300 millions of light years is about the size of the "bubbles of matter" discovered in the pioneering paper of Valérie de Lapparent and co-authors in [8]? Later this repartition of luminous matter has been confirmed by many large scale observations and called "Cosmic Tapestry", which is a very suggestive term. It does not look entirely absurd to imagine that, whatever be, either temporal or spatial, the character of the variation, the gravitational pressure might vary at the same scale as this fundamental inhomogeneity of universe. More than that, such a variation could explain why matter concentrates in such a strange way to display complicated configurations qualified as "bubbles" or 'matter filaments" in the literature. We might even conjecture that large galactic clusters are always situated where the local gravitational pressure is highest (The Coma cluster is precisely very large).

5 Newton's law in 1D and 2D.

5.1 Starting point

In the previous preprint [6], the author tried to understand the simplest case of Lesage's pushing gravity, namely the mutual attraction of two nucleons. He found out that the theory, contrary to what was claimed until now, can work even with purely elastic shocks, because the gravitons transfer a part of their kinetic energy even in the elastic case. The argument according to which rebounding gravitons can cancel the effect of incoming ones is also answered by this "toy model" since rebounding gravitons have less kinetic energies than directly incoming corpuscles.

5.2 A peculiar situation

After that study, the author tried to imagine how to recover Newton's law of gravitation for massive objects by summing the vector fields corresponding to atoms. But he readily realized that starting from punctual corpuscules makes it impossible to take account of the distance! Because in the calculations, the distance of the nucleons has no effect. Which means that in 1D, the inverse square factor just disappears.

5.3 What happens in 2D?

It is only when trying to picture out the situation for a teaching purpose that the author realized something: the inverse square law in Newton's formula is related to dimension 3. It comes from the fact that the proportion of gravitons eclipsed by one body seen from a distant point at distance d is proportional to the solid angle, varying like $1/d^2$. In one dimension, the distance has no effect, and in 2D, the angle of vision is proportional to $1/d$. Therefore in 2 dimensions, Newton's law should become

$$F = -k\frac{mm'u}{||u||^2}$$

where u is the vector difference of positions between two quasi-punctual flat coplanar objects. In other terms, in the case of small 2D masses confined in a plane the force is radial directed towards the attracting object, with norm

$$||F|| = k\frac{mm'}{d}$$

where $d = ||u||$. Here the gravitational potential becomes logarithmic, which may look counterintuitive, but it was already the case for the MOND model.

5.4 Does it really depend on Lesage's paradigm?

What is involved here is geometry, and this seems to be rather independent of the hypotheses made on the cause of gravity. Newton did not need that to write his formula in 3D. Hence the formula in 2D might finally be more intrinsic than initially imagined.

6 Recovering a MOND like effect in the case of galaxies.

It is interesting to try to compare the effect of gravity for the rotation of stars which are close to the center and those who are far from it. In the first case, the gravitational effect is basically 3D, since nearby central stars, the thickness is comparable to the distance from the center, so that the most important contribution follows the 3D Newton law. On the other hand, far from the center, the global contribution is more and more comparable to 2D gravity because the thickness becomes small compared with the average distance of attracting stars. This way, we recover a velocity curve very similar to what can be computed using the MOND paradigm. This is all qualitative, but we must also point out that since thickness tends to 0 near the boundary of the galaxy, the predominance of 2D effects is even reinforced for the most external part of the galaxy.

7 Conclusion.

The situation of the matter in galaxies is close to a 2D setting. It might be the same for galaxy clusters, contrary to the assumption made by Zwicky at a time when serious studies about accretion disks did not start. It is very strange that professionals do not seem to have suspected that flatness of galaxies could play an important role in the overall gravitational effect. Large structures, for a reason which is not clarified yet, tend to organize in laminar structures, as shown by the discoveries of Lapparent & alt [8]. This might be the reason why the MOND model has some success not only in predicting the velocity curve in galaxies near the boundary, but also to solve Zwicky's paradox. Finally, we understand that it could be difficult to justify this at the quantitative level by calculations involving classical methods such as the virial theorem. A limiting continuous model does not seem much easier to handle, and passing to the limit when the thickness tends to zero seems to be a non-obvious mathematical challenge requiring the invention of new methods.

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