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Abstract

It is suggested that the efficiency of the MOND theory of Mordehai Milgrom might be, instead of large distances, related to flatness of galaxies and the fact that the large scale distribution of matter is essentially two-dimensional. A simple continuous model of planar circular galaxies relying on a two dimensional variant of Newton's gravitational law allows to dispense for the need of dark matter outside the disk for the explanation of the galactic rotational velocity curve paradox .

Key words: gravitation, MOND theory, galactic velocity curves.

1 Introduction

This paper concerns an alternative potential explanation for the so called galaxy rotation problem which is usually associated to the hidden mass hypothesis (HMH).

While examining the Coma galaxy cluster in 1933, the astronomer Fritz Zwicky discovered the existence of a gravitational anomaly. He estimated the global dynamical gravitational mass of the galaxies within the cluster from the observed rotational velocities and obtained a value much larger than expected from the total luminosity of the cluster. This was reported in the two basic papers [15, 16]. In [6], we pointed out the accumulation of circumstances which led to an exageration of the discrepancy, but the discordance remains and presently many specialists, following the HMH hypothesis made by Zwicky himself, are looking for unseen "dark matter", possibly of non-baryonic nature.

The HMH track became popular after the discovery of the so-called flat galactic rotation curves by Vera Rubin (cf. [12, 13]), also called the galaxy rotation problem. Quoting wikipedia: "Vera Rubin investigated the rotation curves of spiral galaxies, beginning with Andromeda, by looking at their outermost material, and observed flat rotation curves: the outermost components of the galaxy were moving as quickly as those close to the center. This was an early indication that spiral galaxies might be surrounded by dark matter haloes. She further uncovered the discrepancy between the predicted angular motion of galaxies based on the visible light and the observed motion. Her research showed that spiral galaxies rotate quickly enough that they should fly apart, if the gravity of their constituent stars was all that was holding them together. Because they stay intact, a large amount of unseen mass must be holding them together, a conundrum that became known as the galaxy rotation problem..." In the recent years, observations seem to confirm the existence of a huge halo of classical dark matter around the galaxy M 31 and even around the milky way. But the arguments used by the experts to infer this important potential solution are quite indirect, so that it not absurd to pursue alternative tracks towards an explanation of the galaxy rotation problem.

In 1983, in [11], M. Milgrom introduced an alternative way of solving the galaxy rotation problem without assuming the existence of dark matter. He made the hypothesis that Newton's law might not be correct for very remote objects and in this case the inverse square law should be replaced by an inverse law. However, it is not easy to find a physical interpretation of this divergence from Newton's law. Then a modified-inertia MOND approach was proposed as a change in Newton's second law at small accelerations. This questions the fondations of dynamics, as a matter of fact, even the classical Newton's second law is difficult to understand in a completely empty space, cf.[8] and also the works of T. Van Flandern, for instance [14].

Recently, the author wondered what could be a 2 dimensional version of Newton's law of gravitation. This leads, as we shall see, to a new potential explanation of the flat velocity curves far from the center of galaxies. Actually, the paradigm of the Fatio-Lesage theory naturally leads to replace in 2D the inverse square law of Newton by an inverse law which in a sense represents a variant of Milgrom's MOND hypothesis, replacing the influence of large distances by the quasi-flatness property of galaxies and more generally large structures. The plan of this paper is as follows: In Section 2, we briefly recall the pushing gravity paradigm of

Fatio de Duillier - Lesage. In Section 3 we introduce a 2D variant of Newton's theory which is purely mathematical since all real objects are 3D. In Section 4, we recall more precisely the results of Vera Rubin and what is meant by "flat velocity curves". Then in Section 5, we show that by applying a 2D variant of Newton's law to circular 2D galaxies, the paradox on velocity curves disappears without any need for dark matter. Sections 6 and 7 are devoted to some remarks and the conclusion.

2 The Fatio de Duillier - Lesage theory.

According to Wikipedia: "Le Sage's theory of gravitation is a kinetic theory of gravity originally proposed by Nicolas Fatio de Duillier in 1690 and later by Georges-Louis Le Sage in 1748. The theory proposed a mechanical explanation for Newton's gravitational force in terms of streams of tiny unseen particles (which Le Sage called ultra-mundane corpuscles) impacting all material objects from all directions. According to this model, any two material bodies partially shield each other from the impinging corpuscles, resulting in a net imbalance in the pressure exerted by the impact of corpuscles on the bodies, tending to drive the bodies together." In [6], we describe this paradigm in a rather detailed manner and we recall the main usual objections against the Lesage theory. On the other hand, we observe that the pushing gravity model opens the door to a possible variability of the "gravitational constant" G at very large spatial (or time) scale. And we point out that different local gravitational constants might fill the gap in Zwicky's estimate. For detailed modern developments about the Fatio-Lesage theory of gravitation and closely related topics, we refer to [1, 2, 3, 4] and [14] .

3 Newton's law in 1D and 2D.

3.1 Starting point

In the previous preprint [7], the author tried to understand the simplest case of Lesage's pushing gravity, namely the mutual attraction of two nucleons. He found out that the theory, contrary to what was claimed until now, can work even with purely elastic shocks, because the gravitons transfer a part of their kinetic energy even in the elastic case. The argument according to which rebounding gravitons can cancel the effect of incoming ones is also answered by this "toy model" since rebounding gravitons have less kinetic energies than directly incoming corpuscles.

3.2 A peculiar situation

After that study, the author tried to imagine how to recover Newton's law of gravitation for massive objects by summing the vector fields corresponding to atoms. But he readily realized that starting from punctual corpuscules makes it impossible to take account of the distance! Because in the calculations, the distance of the nucleons has no effect. Which means that in 1D, the inverse square factor just disappears.

3.3 What happens in 2D?

It is only when trying to picture out the situation for a teaching purpose that the author realized something: the inverse square law in Newton's formula is related to dimension 3. It comes from the fact that the proportion of gravitons eclipsed by one body seen from a distant point at distance d is proportional to the solid angle, varying like $1/d^2$. In one dimension, the distance has no effect, and in 2D, the angle of vision is proportional to $1/d$. Therefore in 2 dimensions, Newton's law should become

$$F = -k\frac{mm'u}{||u||^2}$$

where u is the vector difference of positions between two quasi-punctual flat coplanar objects. In other terms, in the case of small 2D masses confined in a plane the force is radial directed towards the attracting object, with norm

$$||F|| = k\frac{mm'}{d}$$

where $d = ||u||$. Here the gravitational potential becomes logarithmic, which may look counterintuitive, but it was already the case for the MOND model. To conclude this Section, we note that in this 2D variant of Newton's theory, the acceleration field generated in the plane by a quasi-punctual flat mass m located at point M is given, at any other point A of the plane, by the formula

$$\vec{\gamma}(A) = km\frac{\overrightarrow{AM}}{AM^2} \tag{3.1}$$

Remark 3.1. It is natural to wonder whether formula (3.1) really depends on Fatio-Lesage's paradigm. Actually what is involved here is geometry, and this seems to be rather independent of the hypotheses made on the cause of gravity. Newton did not need that to write his formula in 3D. Hence the formula in 2D might finally be more intrinsic than initially imagined. And by the way, it even gives rise to a 2D variant of the Gauss Theorem, cf. Remark 6.1.

Remark 3.2. The dimensionality of k is of course different from that of the gravitational constant G. More precisely we have, with respect to the usual dimensional analysis notation

$$[G] = M^{-1}L^3T^{-2}; \quad [k] = [G]L^{-1} = M^{-1}L^2T^{-2}$$

We have no idea about the possible universality, even locally, of the "constant" k which is just introduced here as an abstract tool to understand the gravitational field generated by a flat object. It is clear that if this tool becomes useful, a more detailed analysis will have to be done.

4 More details on the flat velocity curve paradox.

In 1970, in the paper [12], Vera Rubin and Kent Ford published a very detailed analysis of the velocity curves, i.e. the relation between the distance r to the center and the rotational velocity of stars for the Andromeda Galaxy M31. By making measurements in several regions and taking some kind of average $v(r)$, they found a velocity curve very different from what was expected considering the evolution of the density of luminous matter as a function of r. To be more precise, there was a theoretical relation, replacing the actual shape of M31 by a flat disc, allowing to recover the density $\rho(r)$ from the knowledge of $v(r)$. The theoretical and measured curves had completely different shapes.

In 1980, in the paper [13], Vera Rubin, Ken Ford and Norbert Thonnard extended the study to a sample of 21 late spiral (Sc) galaxies. The observations and measurements confirmed what was found on M31. They displayed the velocity curves for the 21 galaxies in pictures 5 and 6, most of the curves did not decrease at all when approaching the exterior boundary. Moreover, picture 7 page 480 from [13] which displays the average curve is not even flat: it is clearly increasing, although in a globally concave way, probably reflecting the quick decay of local density near the boundary.

As we mentionned in the introduction, after these findings there was a consensus that the discrepancy is due to the presence of dark matter in the disk and probably outside the visible part of the galaxy. Some authors also tried to relate the strange velocity curves to the shape of the galaxy which may be more complicated than just lenticular, there are so many works on the subject that it would be impossible to cite all of them, cf. e.g. the recent papers [5, 10] and their references. In a very different direction, we recalled in the introduction the important attempt of M. Milgrom ([11]) postulating a modification of Newton's law for weak accelerations which implies a different form of the gravitational force for very remote objects.

5 The limiting case of flat circular galaxies.

The very flat character of galaxies suggests that we could try to use a 2D version of Newton's law to see which behavior we obtain for the velocity curves. In this section, we construct an entirely computable 2 dimensional simulation for galactic dynamics. Of course, real galaxies are neither flat nor circular, but an explicit model may help to understand what is going on for real galaxies. In order to do that, we consider, for some $R > 0$, a rotationally symmetric distributed mass with area density μ supported in the disk centered at the origin with radius R:

$$\mu(z) = \mu(|z|), z \in \mathbb{C}, |z| \leq R$$

To make the calculations, we shall use polar coordinates and consider in $D = B(O, R)$ the generic point

$$z = se^{i\theta}, \quad 0 \leq s \leq R.$$

Since the galaxy is represented by a 2D distribution of matter, it is natural to make use of the 2D variant (3.1) of Newton's formula.

5.1 The acceleration field produced by the mass density at a generic point

Due to rotational symmetry, the acceleration at the generic point $re^{i\alpha}$ is given by $e^{i\alpha}\gamma(r)$ where, as a consequence of (3.1),

$$\gamma(r) = -k \int_D \frac{s\mu(s)(r - se^{i\theta})}{|r - se^{i\theta}|^2} ds d\theta. \tag{5.2}$$

We shall prove that this integral is absolutely convergent and give its value. To this end the preliminary calculation of some simple integrals is necessary.

5.2 Some integrals

We shall need to evaluate, for any $\alpha \in (0,1)$, the integrals

$$J = J(\alpha) = \int_0^\pi \frac{d\theta}{1 - \alpha\cos\theta}$$

and

$$K = K(\alpha) = \int_0^\pi \frac{\cos\theta d\theta}{1 - \alpha\cos\theta}$$

For the first integral the change of variable $t = \tan\frac{\theta}{2}$ gives after standard manipulations

$$J = 2\int_0^\infty \frac{dt}{1 - \alpha + (1+\alpha)t^2} = \frac{2}{1-\alpha}\int_0^\infty \frac{dt}{1 + \beta^2 t^2} = \frac{2}{\beta(1-\alpha)}\int_0^\infty \frac{d\tau}{1+\tau^2}$$

with $\beta = (\frac{1+\alpha}{1-\alpha})^{1/2}$ and $\tau = \beta t$. We conclude that

$$J(\alpha) = \int_0^\pi \frac{d\theta}{1 - \alpha\cos\theta} = \frac{\pi}{\sqrt{1-\alpha^2}} \tag{5.3}$$

On the other hand we have

$$J - \alpha K = \int_0^\pi \frac{1 - \alpha\cos\theta}{1 - \alpha\cos\theta} d\theta = \pi$$

whence

$$K(\alpha) = \frac{J(\alpha) - \pi}{\alpha} \tag{5.4}$$

5.3 Preliminary estimates

First of all we check that formula (5.2) defines an absolutely convergent integral under a reasonable assumption on μ. This will allow us to use Fubini's theorem. So we want to prove that

$$\int_D \frac{s\mu(s)}{|r - se^{i\theta}|} ds d\theta < \infty. \tag{5.5}$$

As a preliminary step we compute

$$\int_0^{2\pi} \frac{d\theta}{|r - se^{i\theta}|^2} = \int_0^{2\pi} \frac{d\theta}{|r^2 + s^2 - 2rs\cos\theta|} = \frac{1}{r^2+s^2}\int_0^{2\pi} \frac{d\theta}{1 - \alpha\cos\theta}$$

with

$$\alpha = \alpha(r,s) = \frac{2rs}{r^2+s^2}.$$

It is immediate to check that

$$1 - \alpha^2 = 1 - \frac{4r^2s^2}{(r^2+s^2)^2} = \left(\frac{r^2 - s^2}{r^2+s^2}\right)^2$$

which gives, by the previous section

$$\int_0^{2\pi} \frac{d\theta}{|r - se^{i\theta}|^2} = \frac{1}{r^2+s^2}\int_0^{2\pi} \frac{d\theta}{1 - \alpha\cos\theta} = \frac{\pi}{|r^2 - s^2|}$$

Then we obtain first

$$\int_0^{2\pi} \frac{s\mu(s)}{|r - se^{i\theta}|} d\theta \leq \frac{2^{1/2}\pi s\mu(s)}{|r^2 - s^2|^{1/2}}$$

For any r fixed the RHS is integrable with respect to r on $(0, R)$, so that already

$$\int_D \frac{s\mu(s)}{|r - se^{i\theta}|} dsd\theta \leq 2^{1/2}\pi \int_0^R \frac{s\mu(s)}{|(r+s)(r-s)|^{1/2}} ds < \infty$$

as soon as μ is bounded. But integrability still clearly holds true if for instance

$$\mu(s) \leq C(1 + s^{-\lambda}) \tag{5.6}$$

for some $\lambda < 2$, This allows rather strong **local singularities at the center.**

5.4 Exact formula for the acceleration field produced by the mass density at a generic point

We shall prove the following

Theorem 5.1. *Under condition* (5.6)*, the value of the acceleration produced by the rotationally invariant mass density $\mu(z) = \mu(|z|)$ at the point $y = re^{i\psi} \in D$ is given by the formula*

$$\gamma(y) = -\frac{2k\pi e^{i\psi}}{r} \int_0^r s\mu(s)ds = -\frac{2k\pi y}{r^2} \int_0^r s\mu(s)ds \tag{5.7}$$

Proof. Forgetting for the moment the constant k and μ, we compute

$$I_1(s) = \int_0^{2\pi} \frac{rs\, d\theta}{|r - se^{i\theta}|^2} = \frac{2rs}{r^2+s^2} J(\frac{2rs}{r^2+s^2})$$

$$I_2(s) = \int_0^{2\pi} \frac{e^{i\theta}s^2\, d\theta}{|r - se^{i\theta}|^2} = \int_0^{2\pi} \frac{\cos\theta s^2\, d\theta}{|r - se^{i\theta}|^2} = \frac{2s^2}{r^2+s^2} K(\frac{2rs}{r^2+s^2})$$

$$\frac{2s^2}{r^2+s^2} \times \frac{r^2+s^2}{2rs}\left[J(\frac{2rs}{r^2+s^2}) - \pi\right] = \frac{s}{r}\left[J(\frac{2rs}{r^2+s^2}) - \pi\right]$$

where we used that by symmetry of the mass distribution, I_2 has to be real. By subtraction we find

$$I_1 - I_2 = \frac{\pi s}{r} + [\frac{2rs}{r^2+s^2} - \frac{s}{r}]J(\frac{2rs}{r^2+s^2}) = \frac{\pi s}{r} + \frac{s(r^2-s^2)}{r(r^2+s^2)} J(\frac{2rs}{r^2+s^2})$$

Since

$$J(\frac{2rs}{r^2+s^2}) = \frac{\pi(r^2+s^2)}{|r^2-s^2|}$$

we finally obtain

$$I_1 - I_2 = \frac{\pi s}{r}\left(1 + \frac{r^2-s^2}{|r^2-s^2|}\right)$$

this expression is equal to 0 if $r < s$, and to $2\frac{\pi s}{r}$ if $s \leq r$. This means that the symmetric mass distribution outside the disk of radius r will have no global gravitational effect at the boundary. By integrating on $(0, R)$ in s after multiplication by $k\mu(s)$, we find easily

$$\gamma(r) = -\frac{2k\pi}{r} \int_0^r s\mu(s)ds = -\frac{2k\pi y}{r^2} \int_0^r s\mu(s)ds$$

The case of general y follows immediately. $\square$

5.5 A circular galactic motion and the associate velocity curves

Theorem 5.2. *Under condition* (5.6)*, the function defined for all* t *and all* $y = re^{i\psi} \in D$ *by the formula*

$$y(t, re^{i\psi}) = re^{i(\psi+\omega(r)t)}$$

$$\text{with } \omega(\mathrm{r}) := \left[\frac{2\mathrm{k}\pi}{\mathrm{r}^2}\int_0^{\mathrm{r}} \mathrm{s}\mu(\mathrm{s})\mathrm{ds}\right]^{1/2} \tag{5.8}$$

is a solution of

$$y'' = \gamma(y)$$

Proof. We immediately find $y'' = -\omega^2 y$ and this implies the result. □

As a conclusion, we were able to exhibit a rotating continuum medium with essentially arbitrary rotation invariant mass density, which can be considered as a model of circular galaxy. The "particles", which we could identify with stars or groups of stars, follow circular orbits, and in the case of a constant density, the angular rotation density does not depend on r. In that case, of course different from reality, the rotation curve is linearly increasing.

5.6 Qualitative and quantitative consequences

As we shall see, formula 5.8 is very different from the formulas previously used in the litterature and allow to understand the shape of rotation curves without any need for dark matter. More precisely

- If $\mu(r) \equiv \mu_0$, we have $\omega(r) = \sqrt{2k\pi\mu_0}$. More generally if $\mu(s) \geq \mu_0$ for all s, we find

  $$\forall r \in (0, R), \quad \omega(r) \geq \sqrt{2k\pi\mu_0}$$

  In that case, the rotation curve $v(r) = r\omega(r)$ is super-linear.

- (5.8) provides the following general formula for the rotational velocity curve

  $$v(r) = \left(2k\pi \int_0^r s\mu(s)ds\right)^{1/2} \tag{5.9}$$

  This shows that whenever μ remains positive, the velocity curve is increasing. It never decreases and can be stationary on some interval of values of r only if the density vanishes in the corresponding annulus. This behavior is completely consistent with what was observed by Rubin & alt [12, 13], for which all averaged velocity curves were increasing.

- It is of interest to compare (5.9) with the formulas previously considered in the litterature. On Wikipedia, we find a formula allowing to compute the so-called "density profile" $\rho(r)$ as a function of the velocity curve for a rotation invariant circular-like galaxy of the form

  $$\rho(r) = \frac{v^2(r)}{4\pi G r^2}\left(1 + \frac{d(logv(r))}{dlogr}\right) = \frac{1}{4\pi G r^2}\left(v^2(r) + rv(r)v'(r)\right) \tag{5.10}$$

  while from (5.9) we find

  $$\mu(r) = \frac{1}{r}\frac{(v^2(r))'}{2k\pi} = \frac{1}{k\pi r^2}\left(rv(r)v'(r)\right) \tag{5.11}$$

Formula (5.10), probably coming from the standard 3D Newton gravitation law, has an additional positive term, while the second term is similar to the RHS of (5.11). However we must observe here that ρ is a volumetric density and μ is an area density, so that the formulas are not really comparable. In practice we have

$$\mu(r) \sim \rho(r)\varepsilon(r)$$

where $\varepsilon(r)$ is the thickness (or rather the average thickness) of the galaxy along the circle of radius r. So if we identify the second term from (5.10) to the RHS of (5.11), this would provide

$$k :\sim 4G\varepsilon$$

where ε stands for the average thickness of the galaxy. The result, depending on ε hence on the galaxy, shows that either k is not a universal constant, or the identification is irrelevant (and most probably both).

- The absence of the positive term in our formula (5.11) results in the impossibility for any velocity curve to be decreasing at any radius r **in sharp contrast with the standard 3D analysis**, since this would require a negative local density. Actually, formula (5.10) is in fact easily invertible and gives

$$v^2(r) = \frac{4\pi G}{r} \int_0^r s^2 \rho(s) ds$$

implying a decay of v as $r^{-1/2}$ for large r if ρ is rapidly decreasing, to be compared with

$$v^2(r) = 2k\pi \int_0^r s\mu(s) ds$$

indicating an increasing character of $v = v(r)$ for any area density function μ.

6 Some remarks.

Remark 6.1. *We might have derived the formula* (5.7) *from a 2D version of Gauss's theorem, by applying formally Green's formula to the gradient and the Laplacian of a weighted integral involving a logarithmic potential. From this we can understand why the acceleration does not depend on the part of the symmetric distribution exterior to the disk of radius r, But this does not seem completely obvious to justify rigorously.*

Remark 6.2. *It is worth noting that the usual (3D) formulation of Newton's law cannot be used for a uniformly distributed mass in a flat disc. As a matter of fact, the integral*

$$\Gamma(r) = -G \int_D \frac{s\mu(s)(r - se^{i\theta})}{|r - se^{i\theta}|^3} ds d\theta \tag{6.12}$$

is not absolutely convergent even when μ is a constant. More precisely ,

$$\forall r \in (0, R), \int_D \frac{s}{|r - se^{i\theta}|^2} ds d\theta = \infty$$

as a consequence of the formula

$$\int_0^{2\pi} \frac{d\theta}{|r - se^{i\theta}|^2} = \frac{\pi}{|r^2 - s^2|}$$

It does not seem likely that we can give any reasonable meaning to the integral defined by (6.12)*.*

Remark 6.3. *On the other hand, if we replace the completely flat disc D by the thin 3D domain $D_\varepsilon = D \times (-\varepsilon, \varepsilon)$ on introducing the vertical coordinate $z \in (-\varepsilon, \varepsilon)$, it is not difficult to check that the vector integral*

$$\Gamma_\varepsilon(r) = -G \int_{D_\varepsilon} \frac{s\mu(s)(r - se^{i\theta}, z)}{||(r - se^{i\theta}, z)||^3} dsd\theta dz \tag{6.13}$$

becomes normally convergent for any bounded function μ. But realistic models of "almost circular" galaxies probably require a different shape in the vertical direction, with a thickness depending on r.

7 Conclusion.

The situation of the matter in galaxies seems to be close to a 2D setting. It might be the same for galaxy clusters, contrary to the assumption made by Zwicky at a time when serious studies about accretion disks did not start. Large structures, for a reason which is not clarified yet, tend also to organize in laminar structures, as shown by the discoveries of Lapparent & alt [9]. This might be the reason why the MOND model has some success not only in predicting the velocity curve in galaxies near the boundary, but also to solve Zwicky's paradox. The above toy model may help explaining the flat and even slightly increasing velocity curves observed on real galaxies, but it is a very simplified situation, even for dimension 2. Moreover, passing to the limit for real 3D galaxies when the thickness tends to zero seems to be a non-obvious mathematical challenge, requiring rather sophisticated methods. The result may indicate that for very thin domains, using the usual 3D Newton's law becomes irrelevant.

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