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# **A unified model for the permeability, electrical conductivity and streaming potential coupling coefficient in variably saturated fractured media**

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1           **A unified model for the permeability, electrical conductivity**  
2           **and streaming potential coupling coefficient in variably**  
3           **saturated fractured media**

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9           (October 13, 2022)

10           **ABSTRACT**

11 We present a new unified model for the permeability, electrical conductivity, and streaming  
12 potential coupling coefficient in variably saturated fractured media. For those, we con-  
13 ceptualize the fractured medium as a partially saturated bundle of parallel capillary slits  
14 with varying sizes. We assume that the fracture size distribution of the corresponding  
15 medium follows a fractal scaling law, which allows us establish a pressure head-saturation  
16 relationship based on the Laplace equation. We first describe the flow rate, the conduc-  
17 tion current, and the electrokinetic streaming current within a single fracture. Then, we  
18 upscale these properties at the scale of an equivalent fractured media partially saturated  
19 in order to obtain the relative permeability, the electrical conductivity and the streaming  
20 potential coupling coefficient. The newly proposed model explicitly depends on pore water  
21 chemistry, interface properties, microstructural parameters of fractured media, and water  
saturation. Model predictions are in good agreement with both experimental and simulated  
data and with another model from the literature. The results of this work constitute a useful  
framework to estimate hydraulic properties and monitor water flow in fractured media.

22 Keywords: Fractured media; Streaming potential; Electrical conductivity; Permeability;  
23 Fractal

## INTRODUCTION

24 The streaming potential (SP) is a contribution to the self-potential signal that is generated  
25 by water flow in porous media. Due to the sensitivity of the SP method to subsurface water  
26 flow, the SP technique has drawn an increasing attention to find or track underground  
27 water in aquifers or reservoirs (e.g., Revil et al., 2012; Parsekian et al., 2015; Binley et al.,  
28 2015). This technique has been used for identifying and monitoring subsurface water flow  
29 (e.g., Jouniaux et al., 1999; Fagerlund and Heinson, 2003; Titov et al., 2005; Aizawa et al.,  
30 2009), monitoring geothermal and volcanic areas (e.g., Corwin and Hoover, 1979; Finizola  
31 et al., 2004; Mauri et al., 2010; Soueid Ahmed et al., 2018; Grobbe and Barde-Cabusson,  
32 2019), mapping areas influenced by a contaminant plume (e.g., Martinez-Pagan et al., 2010;  
33 Naudet et al., 2003; Roy, 2022), monitoring water flow in the vadose zone (e.g., Doussan  
34 et al., 2002; Jougnot et al., 2015; Hu et al., 2020) or eco-hydrology (e.g., Voytek et al.,  
35 2019). The SP technique can be applied to estimate hydrogeological parameters of the  
36 aquifer (e.g., Jardani et al., 2007; Straface et al., 2010; Revil and Jardani, 2013).

37 Fractured rocks are ubiquitous in the environment and they play a major role in a wide  
38 range of geoscience issues, such as groundwater flow and contaminant transport (e.g., Neu-  
39 man, 2005; Medici et al., 2019), hydraulic fracturing (e.g., Osiptsov, 2017; Peshcherenko  
40 et al., 2022), storage of CO<sub>2</sub> and nuclear waste (e.g., Bodvarsson et al., 1999; Wang and  
41 Hudson, 2015; Ren et al., 2017), geothermal production (e.g., Murphy et al., 1981; Patter-  
42 son et al., 2020). Geophysical methods offer a variety of tools to obtain information on  
43 subsurface structure and physical properties of fractured rocks. Examples of those methods  
44 include the electrical conductivity imaging (e.g., Stesky, 1986; Shen et al., 2009; Roubinet  
45 and Irving, 2014), seismic technique (e.g., Herwanger et al., 2004; Li, 1997; Clair et al.,  
46 2015), or the self potential technique (e.g., Fagerlund and Heinson, 2003; Wishart et al.,  
47 2006; Mainault et al., 2013). It is shown that numerical approaches are effective to char-  
48 acterize fractured media (e.g., Roubinet and Irving, 2014; Roubinet et al., 2016; Demirel

49 et al., 2018; Haas et al., 2013; DesRoches et al., 2018; Jougnot et al., 2020). However, to  
50 the best of our knowledge, there are only few analytical models for fractured rocks in the  
51 literature. For example, Thanh et al. (2021) proposed a model for the electrical conductivity  
52 and streaming potential coupling coefficient in fractured media under saturated conditions  
53 using a capillary bundle model following the fractal scaling law. Guarracino and Jougnot  
54 (2022) presented a model to predict the effective excess charge density for fully and partially  
55 water saturated fractured media that are described by the fractal Sierpinski carpet.

56 The aim of this study is to develop a unified model for the permeability, electrical  
57 conductivity, and streaming potential coupling coefficient in fractured media by extend-  
58 ing the work proposed by Thanh et al. (2021) to partially saturated conditions. For this  
59 purpose, we conceptualize a fractured medium as a partially saturated bundle of parallel  
60 capillary slits following the fractal scaling law. This conceptualization allows us to de-  
61 termine the capillary pressure-saturation relationship and later deduce expressions for the  
62 electrical conductivity and permeability of fractured media under partially saturated con-  
63 ditions. From the electrokinetic streaming current and conduction current within a single  
64 slit, we obtain an upscaled expression for the streaming potential coupling coefficient. The  
65 new obtained model explicitly depends on properties of fracture water, interface properties,  
66 microstructural parameters of fractured media and water saturation. Model predictions are  
67 then compared with experimental data, simulated data as well as another previous model  
68 in the literature.

## THEORETICAL BACKGROUND OF STREAMING POTENTIAL

69 The streaming current is caused by electrokinetic coupling, that is the drag of electrical  
70 charge by water flow in porous media conceptualized as a bundle of cylindrical capillary  
71 tubes or a bundle of capillary slits. This phenomenon is directly related to the presence  
72 of an electric double layer (EDL) that exists at the solid-water interface of the tubes or  
73 fractures (e.g., Overbeek, 1952; Hunter, 1981). This EDL contains an excess of charge in  
74 water to compensate the charge deficit of the capillary inner surface. The EDL is composed  
75 of the Stern and diffuse layers. The Stern layer only contains counter-ions, i.e., the ions

76 with opposite sign to the charged surface. The ions in the Stern layer can be considered  
77 as immobile due to strong electrostatic attraction. The diffuse layer contains both the  
78 counter-ions and co-ions, i.e., the ions with same sign as the charged surface. The ions in  
79 the diffuse layer are free to move but with a net excess of charge (e.g., Hunter, 1981; Jougnot  
80 et al., 2020). The interface between the Stern layer and the diffuse layer corresponds to the  
81 shear plane or slipping plane that separates the stationary fluid and the moving fluid. The  
82 electrical potential at this plane is called the zeta potential  $\zeta$  (V) that mostly depends on  
83 mineral composition of porous media, ionic strength, temperature and pH of water (e.g.,  
84 Hunter, 1981; Vinogradov et al., 2022b). The generated streaming current is, in turn,  
85 balanced out by an electrical conduction current in the opposite direction, leading to a so-  
86 called streaming potential. At the steady state condition, the streaming potential coupling  
87 coefficient (SPCC) is defined as (e.g., Smoluchowski, 1903; Morgan et al., 1989):

$$C_S = \frac{\Delta V}{\Delta P}, \quad (1)$$

88 where  $\Delta V$  (V) and  $\Delta P$  (Pa) are the measured streaming potential and the imposed pres-  
89 sure difference across a probed medium, respectively. There have been two approaches to  
90 determine the SPCC at saturated conditions in the literature. For the first approach, the  
91 classical one, the SPCC that is expressed in terms of the zeta potential  $\zeta$  is given by (e.g.,  
92 Smoluchowski, 1903)

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \sigma_w}, \quad (2)$$

93 where  $\epsilon_r$  (no units) is the relative permittivity,  $\epsilon_0$  (F/m) is the dielectric permittivity in  
94 vacuum,  $\eta$  (Pa s) is the dynamic viscosity and  $\sigma_w$  (S/m) is the electrical conductivity of  
95 water. Eq. (2) is called the Helmholtz-Smoluchoski (HS) equation. Note that the surface  
96 electrical conductivity  $\sigma_s$  is not considered in Eq. (2). If  $\sigma_s$  is taken into consideration,  
97 Eq. (2) can be replaced by the following equation (e.g., Hunter, 1981; Ishido and Mizutani,  
98 1981)

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta(\sigma_w + 2\frac{\Sigma_s}{\Lambda})}, \quad (3)$$

99 where  $\Sigma_s$  (S) is the specific surface conductance and  $\Lambda$  (m) is a characteristic length scale  
 100 of porous media (Johnson et al., 1986). For the second approach, the SPCC is expressed in  
 101 terms of the effective excess charge density  $\widehat{Q}_v$  (C/m<sup>3</sup>) dragged by water (e.g., Kormiltsev  
 102 et al., 1998; Revil and Leroy, 2004; Jougnot et al., 2020)

$$C_S = -\frac{k\widehat{Q}_v}{\eta\sigma}, \quad (4)$$

103 where  $k$  (m<sup>2</sup>) and  $\sigma$  (S/m) are the permeability and electrical conductivity of fully saturated  
 104 porous media, respectively.

105 One can note that Thanh et al. (2021) developed a model for the SPCC using the zeta  
 106 potential  $\zeta$  for fully saturated fractured media, whereas Guarracino and Jougnot (2022)  
 107 proposed a model to predict the  $\widehat{Q}_v$ , for fully and partially water saturated fractured media,  
 108 that also permits to determine the SPCC.

## THEORETICAL DEVELOPMENT

### 109 Description of fractured media

110 Fractures in geological media exist over a wide range of scales, from microns to thousands of  
 111 kilometers, and fractal patterns for fractured rocks have been reported in published works  
 112 (e.g., Okubo and Aki, 1987; Bonnet et al., 2001; Kruhl, 2013). To derive the SPCC in  
 113 fractured media, we regard the geometrical description reported in the literature for frac-  
 114 tured media which are assumed to be made up of the fractures and the surrounding matrix  
 115 (e.g., Tyler and Wheatcraft, 1990; Miao et al., 2015; Roubinet et al., 2016; Guarracino and  
 116 Jougnot, 2022). The matrix permeability is usually much smaller than that of the fractures  
 117 and thus the matrix can be considered as impermeable and no fluid exchange through the  
 118 fracture walls. Note that, for consideration of fluid transfer from the matrix to the fractures,  
 119 we refer readers to the work reported by Miao et al. (2019), for example. The representative  
 120 elementary volume (REV) is assumed to be a cuboid of length of  $L_o$  (m) and cross-section  
 121 area  $A$  (m<sup>2</sup>) as shown in Fig. 1. We conceptualize the fractures of the REV as a bunch

122 of parallel tortuous slits of varying aperture  $a$  (m) and width  $w$  (m) following the fractal  
 123 scaling law (e.g., Tyler and Wheatcraft, 1990; Miao et al., 2015, 2019):

$$f(w) = D_f w_{\max}^{D_f} w^{-D_f-1}, \quad w_{\min} \leq w \leq w_{\max}, \quad (5)$$

124 where  $D_f$  (no units) is the fractal dimension that is between 1 and 2 in two-dimensional  
 125 spaces and it can be determined by a box-counting method (e.g., Miao et al., 2015, 2019),  
 126  $w_{\min}$  (m) and  $w_{\max}$  (m) are the smallest and largest fracture widths in the REV, respectively,  
 127 representing the lower and upper bounds of the fractal distribution. Therefore, the number  
 128 of fractures whose widths in the range from  $w$  to  $w+dw$  is given by  $f(w)dw$  (e.g., Majumdar  
 129 and Bhushan, 1990; Miao et al., 2015). The total number of fractures, from  $w_{\min}$  to  $w_{\max}$ ,  
 130 is given by

$$N_t = \int_{w_{\min}}^{w_{\max}} f(w)dw \approx \left( \frac{w_{\max}}{w_{\min}} \right)^{D_f}. \quad (6)$$

131 Dividing Eq. (5) by Eq. (6), one is able to obtain the probability density function  $f_r(w)$

$$f_r(w) = D_f w_{\min}^{D_f} w^{-D_f-1}. \quad (7)$$

132 It is shown that the aperture  $a$  is normally related to the width  $w$  by a linear scaling law  
 133 (e.g., Torabi and Berg, 2011; Miao et al., 2015):

$$a = \beta w, \quad (8)$$

134 where  $\beta$  (unitless) is the proportionality coefficient called the fracture aspect ratio.

### 135 **Hydraulic properties**

136 The porosity of the REV is defined as

$$\begin{aligned} \phi &= \frac{V_p}{V_t} = \frac{\int_{w_{\min}}^{w_{\max}} (aw)(L_\tau)f(w)dw}{L_o A} = \frac{\beta \tau D_f w_{\max}^{D_f}}{A} \int_{w_{\min}}^{w_{\max}} w^{1-D_f} dw \\ &= \frac{\beta \tau D_f w_{\max}^2}{A(2-D_f)} (1 - \alpha^{2-D_f}), \end{aligned} \quad (9)$$

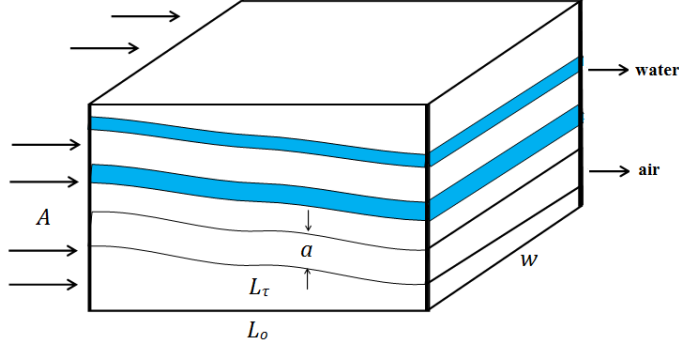


Figure 1: Schematic view of a fractured medium conceptualized as of a bunch of parallel fractures.

137 where  $V_p$  and  $V_t$  are the pore volume and total volume of the REV, respectively,  $L_o$  is the  
 138 length of the REV,  $L_\tau$  is the real length of the fracture,  $\tau=L_\tau/L_o$  is the dimensionless  
 139 hydraulic tortuosity of the fracture and  $\alpha=w_{\min}/w_{\max}$ . For the purpose of simplification,  
 140 the fracture length is assumed to be unchanging with its width, hence  $\tau$  is considered to be  
 141 constant over the REV and therefore independent from the water saturation.

142 We suppose that the REV is initially filled by water and dewatered by the application  
 143 of a pressure head  $h$  (m). For a capillary slit, the relationship between the fracture width  
 144  $w_h$  and the pressure head  $h$  is approximately given by (e.g., Bullard and Garboczi, 2009)

$$h = \frac{2T_s \cos\theta}{\rho_w g \beta w_h}, \quad (10)$$

145 where  $T_s$  (N/m) is the surface tension of water,  $\theta$  ( $^\circ$ ) is the contact angle,  $\rho_w$  ( $\text{kg}/\text{m}^3$ ) is the  
 146 water density and  $g$  ( $\text{m}/\text{s}^2$ ) is the gravitational acceleration. A fracture is fully desaturated  
 147 when its width  $w$  is greater than value  $w_h$  given by Eq. (10). We assume that each fracture  
 148 is filled by either water or air. Therefore, in water-wet systems, water fills the fractures of  
 149 the smallest widths, while air occupies fractures of the largest widths when all the fractures  
 150 are simultaneously accessible. In other words, fractures with widths  $w$  comprised between  
 151  $w_{\min}$  and  $w_h$  will be occupied by water while those with widths comprised between  $w_h$  and  
 152  $w_{\max}$  will be filled by air under application of the pressure head  $h$ . The contribution of

153 water in the REV depends on the effective water saturation  $S_e$  (unitless), that is defined as

$$S_e = \frac{S_w - S_{wr}}{1 - S_{wr}}, \quad (11)$$

154 where  $S_w$  (unitless) is the water saturation and  $S_{wr}$  (unitless) is the residual water saturation  
 155 that represents the water held as films on the fracture walls which can not be drained by  
 156 the pressure head  $h$  or in non-connected fractures which cannot be taken into account in  
 157 the present conceptual model.

158 Following a similar approach to what is reported in the literature (e.g., Guarracino,  
 159 2006; Thanh et al., 2020; Guarracino and Jougnot, 2022),  $S_e$  is expressed in terms of  $w_h$  as  
 160 follows:

$$S_e = \frac{\int_{w_{\min}}^{w_h} (aw)L_\tau f(w)dw}{\int_{w_{\min}}^{w_{\max}} (aw)L_\tau f(w)dw} = \frac{w_h^{2-D_f} - w_{\min}^{2-D_f}}{w_{\max}^{2-D_f} - w_{\min}^{2-D_f}}. \quad (12)$$

161 Combining Eq. (10) and Eq. (12), the capillary pressure curve for fractured media can be  
 162 obtained as

$$S_e = \frac{h^{D_f-2} - h_{\max}^{D_f-2}}{h_{\min}^{D_f-2} - h_{\max}^{D_f-2}}, \quad h_{\min} \leq h \leq h_{\max}, \quad (13)$$

163 where  $h_{\min} = \frac{2T_s \cos\theta}{\rho_w g \beta w_{\max}}$  and  $h_{\max} = \frac{2T_s \cos\theta}{\rho_w g \beta w_{\min}}$ .

164 Under laminar flow conditions, the average velocity in a single fracture of aperture  $a$  is  
 165 given by (e.g., Chung, 2010)

$$\bar{v} = \frac{\rho_w g a^2 \Delta h}{12\eta\tau L_o}, \quad (14)$$

166 where  $\Delta h$  is the pressure head drop across the REV.

167 The flow rate in a single fracture follows the well-known cubic law as (e.g., Neuzil and Tracy,  
 168 1981; Klimczak et al., 2010)

$$q = \bar{v} \cdot (aw) = \frac{\rho_w g a^3 w \Delta h}{12\eta\tau L_o}. \quad (15)$$

169 The total volumetric flow through the REV under unsaturated conditions is given by

$$q^{\text{REV}} = \int_{w_{\min}}^{w_h} qf(w)dw. \quad (16)$$

170 Combining Eq. (5), Eq. (8), Eq. (15) and Eq. (16), one obtains

$$\begin{aligned} q^{\text{REV}} &= \int_{w_{\min}}^{w_h} \frac{\rho_w g a^3 w}{12\eta\tau} \frac{\Delta h}{L_o} [D_f w_{\max}^{D_f} w^{-D_f-1} dw] \\ &= \frac{\rho_w g \beta^3 D_f}{12\eta\tau} w_{\max}^{D_f} \frac{w_h^{4-D_f} - w_{\min}^{4-D_f}}{4-D_f} \frac{\Delta h}{L_o}. \end{aligned} \quad (17)$$

171 Combining Eq. (12) and Eq. (17), one can express  $q^{\text{REV}}$  in terms of  $S_e$  as

$$q^{\text{REV}} = \frac{\rho_w g \beta^3 D_f}{12\eta\tau} w_{\max}^4 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{4-D_f} \frac{\Delta h}{L_o}. \quad (18)$$

172 Following Darcy's law for Newtonian fluid flow in fractured media,  $q^{\text{REV}}$  is given by

$$q^{\text{REV}} = \frac{kA}{\eta} \frac{\rho_w g \Delta h}{L_o}, \quad (19)$$

173 where  $k$  and  $A$  are the permeability and the cross sectional area of the REV.

174 Combining Eq. (18) and Eq. (19), we obtain an expression for  $k$  under unsaturated  
175 conditions as

$$k(S_e) = \frac{\beta^3 D_f}{12\tau A} w_{\max}^4 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{4-D_f}. \quad (20)$$

176 Hence, the permeability under fully saturated conditions ( $S_e = 1$ ) is given by

$$k_s = \frac{\beta^3 D_f}{12\tau A} w_{\max}^4 \frac{1 - \alpha^{4-D_f}}{4-D_f}. \quad (21)$$

177 The relative permeability of fractured media, that is defined as  $k_r = k(S_e)/k_s$ , is given by

$$k_r(S_e) = \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{1 - \alpha^{4-D_f}}. \quad (22)$$

178 It is remarked that using a classical fractal object known as the Sierpinski carpet for the  
 179 fracture network in combination with the Burdine model, Guarracino (2006) obtained an  
 180 expression for  $k_r$  as a function of  $S_e$  as follows:

$$k_r(S_e) = S_e^2 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{1 - \alpha^{4-D_f}}. \quad (23)$$

181 The only difference between our proposed model given by Eq. (22) and the one proposed  
 182 by Guarracino (2006) given by Eq. (23) for  $k_r$  is a prefactor  $S_e^2$ .

183 If one invokes Eq. (9),  $k_s$  can be expressed as

$$k_s = \frac{\beta^2 w_{\max}^2 \phi}{12\tau^2} \frac{1 - \alpha^{4-D_f}}{1 - \alpha^{2-D_f}} \frac{2 - D_f}{4 - D_f} = \frac{a_{\max}^2 \phi}{12\tau^2} \frac{1 - \alpha^{4-D_f}}{1 - \alpha^{2-D_f}} \frac{2 - D_f}{4 - D_f}. \quad (24)$$

184 For  $w_{\max} \gg w_{\min}$  ( $\alpha \rightarrow 0$ ) that is normally reported to be satisfied for the fractal fractured  
 185 media (e.g., Guarracino, 2006; Miao et al., 2015, 2019; Thanh et al., 2021), Eq. (24) reduces  
 186 to

$$k_s = \frac{\beta^2 w_{\max}^2 \phi}{12\tau^2} \frac{2 - D_f}{4 - D_f} = \frac{a_{\max}^2 \phi}{12\tau^2} \frac{2 - D_f}{4 - D_f}. \quad (25)$$

## 187 **Electrical conductivity**

188 Following Thanh et al. (2021), the total electrical conductivity in a single fracture with  
 189 consideration of the surface conductivity is given by

$$\sigma_f(w) = \sigma_w \frac{\beta w^2}{A\tau} + \Sigma_s \frac{2(1 + \beta)w}{A\tau}. \quad (26)$$

190 Recall that  $\sigma_w$  (S/m) and  $\Sigma_s$  (S) are the electrical conductivity of water and specific surface  
 191 conductance at the solid–water interface as previously mentioned.

192 The total electrical conductivity of considered porous media under unsaturated conditions  
 193 is therefore obtained by

$$\sigma = \int_{w_{\min}}^{w_h} \sigma_f(w) f(w) dw. \quad (27)$$

194 Combining Eq. (5), Eq. (26) and Eq. (27) yields the following:

$$\begin{aligned}\sigma &= \frac{1}{A\tau} \left\{ \sigma_w \beta D_f w_{\max}^{D_f} \frac{w_h^{2-D_f} - w_{\min}^{2-D_f}}{2 - D_f} + 2\Sigma_s(1 + \beta) D_f w_{\max}^{D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{1 - D_f} \right\}. \\ &= \frac{\beta D_f w_{\max}^{D_f} (w_h^{2-D_f} - w_{\min}^{2-D_f})}{A(2 - D_f)\tau} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta} \frac{2 - D_f}{1 - D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right\}.\end{aligned}\quad (28)$$

195 Invoking Eq. (12), the  $\sigma$  under unsaturated conditions is expressed in terms of  $S_e$  as

$$\sigma = \frac{\beta D_f w_{\max}^2 S_e (1 - \alpha^{2-D_f})}{A(2 - D_f)\tau} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta w_{\max}} \frac{2 - D_f}{1 - D_f} \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1 - \alpha^{2-D_f})} \right\}.\quad (29)$$

196 Substituting  $A$  from Eq. (9) into Eq. (28), the  $\sigma$  is obtained as

$$\sigma = \frac{\phi S_e}{\tau^2} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta w_{\max}} \frac{2 - D_f}{1 - D_f} \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1 - \alpha^{2-D_f})} \right\}.\quad (30)$$

197 Eq. (30) shows the dependence of  $\sigma$  under unsaturated conditions on microstructural  
198 parameters of the fractured media ( $D_f$ ,  $\phi$ ,  $\alpha$ ,  $\beta$ ,  $w_{max}$ ,  $\tau$ ), water electrical conductivity  $\sigma_w$ ,  
199 specific surface conductance  $\Sigma_s$ , and effective water saturation  $S_e$ .

## 200 Streaming potential coupling coefficient

201 *Streaming current through the REV*

202 Under the thin EDL assumption in which the Debye length is small compared to fracture  
203 widths and the Debye-Hückel approximation that is applicable for small values of  $\zeta$  (i.e., 50  
204 mV) for a binary symmetric 1:1 electrolyte, for example (e.g., Rice and Whitehead, 1965;  
205 Pride, 1994), the electrokinetic streaming current in a single fracture due to transport of  
206 excess charge in the EDL by water flow is given by (e.g., Thanh et al., 2021)

$$i_s(w) = -\frac{\epsilon_r \epsilon_o \zeta}{\eta} \frac{w a}{\tau} \frac{\rho_w g \Delta h}{L_o} = -\frac{\epsilon_r \epsilon_o \zeta}{\eta} \frac{\beta w^2}{\tau} \frac{\rho_w g \Delta h}{L_o}.\quad (31)$$

207 The total streaming current through the REV under unsaturated conditions is determined  
 208 by

$$I_s = \int_{w_{\min}}^{w_h} i_s(w) f(w) dw \quad (32)$$

209 From Eq. (5), Eq. (31) and Eq. (32), the following is obtained

$$\begin{aligned} I_s &= -\frac{\epsilon_r \epsilon_o \zeta \rho_w g \Delta h}{\eta \tau} \frac{\rho_w g \Delta h}{L_o} (\beta D_f w_{\max}^{D_f}) \int_{w_{\min}}^{w_h} w^{1-D_f} dw \\ &= -\frac{\epsilon_r \epsilon_o \zeta \beta D_f}{\eta \tau} \frac{w_{\max}^{D_f}}{(2-D_f)} (w_h^{2-D_f} - w_{\min}^{2-D_f}) \frac{\rho_w g \Delta h}{L_o}. \end{aligned} \quad (33)$$

210 *Conduction current through the REV*

211 The streaming current is accounted for the streaming potential  $\Delta V$  that is built up across  
 212 the REV due to the water flow. In turn, an electric conduction current is generated in the  
 213 REV due to the electrical potential difference  $\Delta V$ . Namely, the conduction current in a  
 214 single fracture is given by Thanh et al. (2021)

$$i_c(w) = \left[ \sigma_w \frac{\beta w^2}{L_o \tau} + \Sigma_s \frac{2(1+\beta)w}{L_o \tau} \right] \Delta V. \quad (34)$$

215 The total electric conduction current through the REV under unsaturated conditions is  
 216 given by

$$\begin{aligned} I_c &= \int_{w_{\min}}^{w_h} i_c(w) f(w) dw \\ &= \frac{\beta D_f w_{\max}^{D_f} (w_h^{2-D_f} - w_{\min}^{2-D_f})}{(2-D_f) \tau} \left[ \sigma_w + \frac{2(1+\beta) \Sigma_s}{\beta} \frac{2-D_f}{1-D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right] \frac{\Delta V}{L_o}. \end{aligned} \quad (35)$$

217 *Streaming potential coupling coefficient*

218 Considering thermodynamic equilibrium, the following condition is satisfied

$$I_s + I_c = 0. \quad (36)$$

219 Consequently, the SPCC is given by

$$C_S = \frac{\Delta V}{\Delta P} = \frac{\Delta V}{\rho_w g \Delta h} = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[ \sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta} \frac{2-D_f}{1-D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right]}. \quad (37)$$

220 Invoking Eq. (12), the SPCC can be expressed in terms of  $S_e$  as

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[ \sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{\left[ S_e(1-\alpha^{2-D_f}) + \alpha^{2-D_f} \right]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1-\alpha^{2-D_f})} \right]}. \quad (38)$$

221 Eq. (38) is an expression for the SPCC of fractured media under unsaturated conditions. It  
 222 predicts that the SPCC is dependent of water properties ( $\sigma_w$ ,  $\epsilon_r$  and  $\eta$ ), physico-chemical  
 223 properties of the solid–water interface ( $\Sigma_s$  and  $\zeta$ ), microstructural parameters of fractured  
 224 media ( $D_f$ ,  $\phi$ ,  $\alpha$ ,  $\beta$ ,  $w_{\max}$ ) and saturation state ( $S_e$ ). When  $\Sigma_s = 0$ , Eq. (38) reduces to  
 225 the HS equation given by Eq. (2) that has been proposed for porous media rather than  
 226 fractured media regardless of  $S_e$ . Under saturated conditions  $S_e = 1$ , Eq. (38) simplifies to  
 227 that proposed by Thanh et al. (2021):

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[ \sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{1-\alpha^{1-D_f}}{1-\alpha^{2-D_f}} \right]}. \quad (39)$$

228 Combining Eq. (4), Eq. (30) and Eq. (38), one can infer an expression for the effective  
 229 excess charge density as following

$$\widehat{Q}_v = -\epsilon_r \epsilon_0 \zeta \frac{\phi S_e}{k \tau^2}. \quad (40)$$

230 We remark that Guarracino and Jougnot (2022) proposed a model for  $\widehat{Q}_v$ , that is deduced  
 231 from their Eq. (14) and Eq. (23), as below

$$\widehat{Q}_v = -\epsilon_r \epsilon_0 \zeta \left[ 1 + \frac{1}{54} \left( \frac{e\zeta}{k_B T} \right)^2 \right] \frac{\phi S_e}{k}, \quad (41)$$

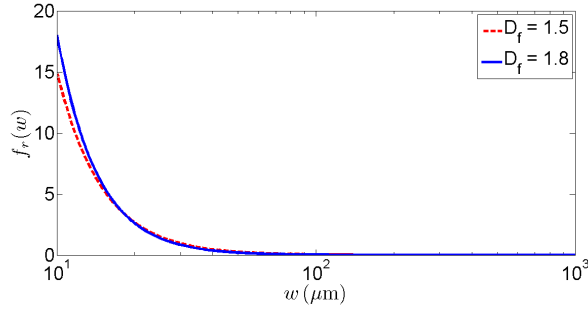


Figure 2: Probability density function associated with fracture size distribution in this work in the width range from  $w_{\min} = 10 \mu\text{m}$  to  $w_{\max} = 1000 \mu\text{m}$  ( $\alpha = 0.01$ ) for two values of  $D_f$  (1.5 and 1.8).

232 where  $e$  (C) the elementary charge,  $k_B$  (J/K) the Boltzman constant and  $T$  (K) is the  
 233 absolute temperature.

234 Under the Debye-Hückel approximation in which  $\left(\frac{e\zeta}{2k_B T}\right)^2 \ll 1$  (e.g., Pride, 1994), Eq.  
 235 (41) reduces to

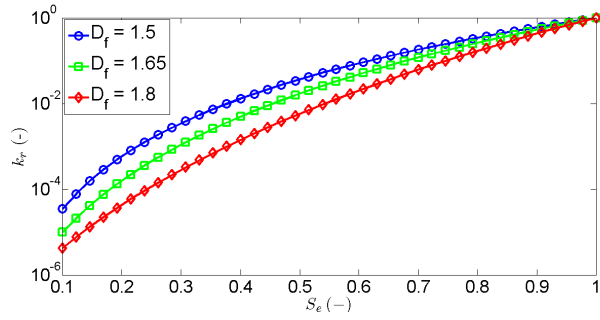
$$\hat{Q}_v = -\epsilon_r \epsilon_0 \zeta \frac{\phi S_e}{k}. \quad (42)$$

236 Obviously, our finding given by Eq. (40) is the same as that proposed by Guarracino and  
 237 Jougnot (2022) under the Debye-Hückel approximation given by Eq. (42). It is noted that  
 238 Guarracino and Jougnot (2022) did not consider  $\tau$  in their model (i.e.,  $\tau = 1$ ).

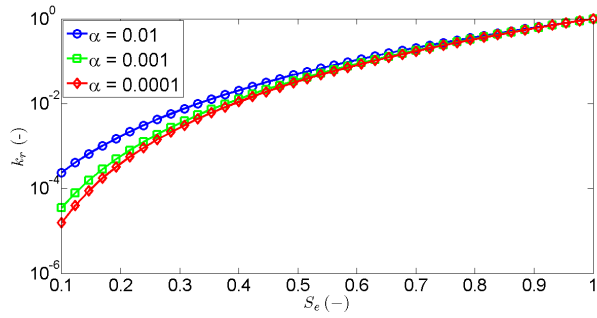
## RESULTS AND DISCUSSION

### 239 Sensitivity of the model

240 Figure 2 shows the representative probability density function of fractures predicted from  
 241 Eq. (7) in the width range from  $w_{\min} = 10 \mu\text{m}$  to  $w_{\max} = 1000 \mu\text{m}$  ( $\alpha = 0.01$ ) for two  
 242 different values of  $D_f$  (1.5 and 1.8). It is seen that: (i) the frequency distribution of fractures  
 243 becomes skewed toward smaller fracture width and (ii) there is a larger number of small  
 244 fractures for larger value of  $D_f$ .



(a)



(b)

Figure 3: Variation of the  $k_r$  with  $S_e$  predicted from Eq. (22): (a) for three representative values of  $D_f$  (1.5, 1.65 and 1.8) at a given representative value of  $\alpha=0.001$ , (b) for three representative values of  $\alpha$  (0.01, 0.001 and 0.0001) at a given representative value of  $D_f = 1.5$ .

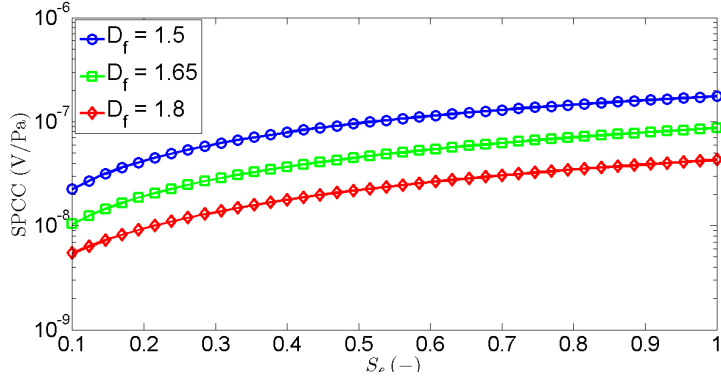


Figure 4: Dependence of the SPCC in magnitude on the effective water saturation for different values of  $D_f$  (1.6, 1.65 and 1.8) predicted from Eq. (38)

245 From Eq. (22), we can predict the variation of  $k_r$  with  $S_e$  for fractured media. For  
 246 example, Fig. 3 shows the  $S_e$ - $k_r$  relationship for: (a) three representative values of  $D_f$  (1.5,  
 247 1.65 and 1.8) at a given representative value of  $\alpha=0.001$ , (b) three representative values of  
 248  $\alpha$  (0.01, 0.001 and 0.0001) at a given representative value of  $D_f = 1.5$ . It is seen that  $k_r$  is  
 249 sensitive to parameters of  $S_e$ ,  $D_f$  and  $\alpha$ . At given values of  $S_e$  and  $\alpha$ ,  $k_r$  decreases with an  
 250 increase of  $D_f$ . The reason is that when  $D_f$  increases, the number of fractures in the REV  
 251 with small widths increases as indicated in Fig. 2. Therefore, at the same water saturation,  
 252  $w_h$  decreases and the total flow rate through the REV becomes smaller. Consequently,  
 253  $k_r$  decreases with increasing  $D_f$ . It is also predicted that at given values of  $S_e$  and  $D_f$ ,  
 254  $k_r$  decreases with a decrease of  $\alpha$ . The reason is that when  $\alpha$  decreases, there is a larger  
 255 fraction of fractures with smaller widths due to the property of the fractal distribution. As  
 256 a result,  $k_r$  decreases.

257 Figure 4 shows the variation of the SPCC in magnitude with  $S_e$  for different values of  
 258  $D_f$  (1.6, 1.65 and 1.8) predicted from Eq. (38). Representative parameters used in Eq.  
 259 (38) are:  $\sigma_w = 0.02$  S/m,  $\Sigma_s = 10^{-9}$  S,  $\zeta = -0.030$  V,  $w_{\max} = 200 \cdot 10^{-6}$  m and  $\beta = 0.01$ .  
 260 It is shown that the SPCC in magnitude increases with increasing  $S_e$ . This prediction is in  
 261 good agreement with those observed in published work but for porous media such as sand  
 262 columns, dolomite cores or limestone cores (e.g., Guichet et al., 2003; Revil and Cerepi,

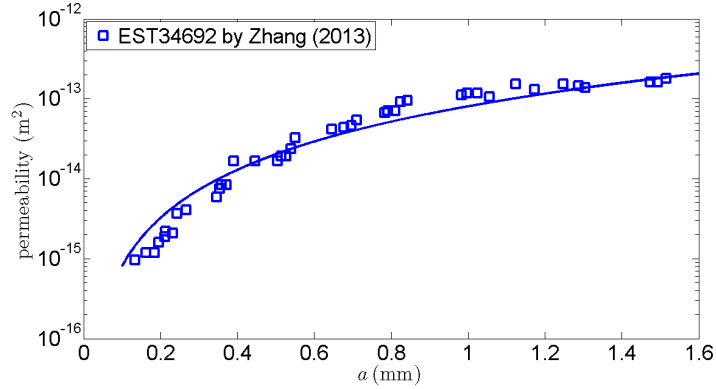


Figure 5: Variation of the  $k_s$  of a single fracture as a function of aperture  $a$  obtained from Zhang (2013) for the sample EST34692 and prediction from Eq. (25) for a fractured medium with  $D_f = 1.8$ ,  $\tau = 1.2$ ,  $\phi = 0.025$  and  $a_{\max} = a/40$ .

263 2004; Vinogradov and Jackson, 2011). Additionally, it is also seen that the SPCC decreases  
 264 with increasing  $D_f$  for the same value of  $S_e$ . The reason is that when  $D_f$  increases, the  
 265 number of fractures characterized by relatively small widths increases as shown by Fig. 2.  
 266 Hence, the surface conductivity of fractured media increases and the SPCC in magnitude  
 267 decreases. It is remarked that the surface conductivity of fractured media is dominated by  
 268 the contribution from the smaller width fractures for given values of  $\sigma_w$  and  $\Sigma_s$ .

## 269 Comparison with published data

270 There are not many published experimental data of single fracture permeability measure-  
 271 ment in the literature. Nevertheless, Zhang (2013) presents gas permeability measurement  
 272 on a Callovo-Oxfordian (COx) clayrock sample which exhibits a single fracture. Non frac-  
 273 tured COx clay rocks are known for their very low permeability (typically 10 nd at saturation  
 274 as indicated by Jougnot et al. (2010)). Hence, the measured permeability of a fractured  
 275 sample is largely due to the fracture itself. Fig. 5 shows the evolution of the sample per-  
 276 meability as a function of the aperture  $a$  of the fracture. The variation of the fractured  
 277 medium  $k_s$  under saturated conditions with  $a_{\max}$  is predicted from Eq. (25) as shown by  
 278 the solid line in Fig. 5. The model can describe the behavior of the permeability well with

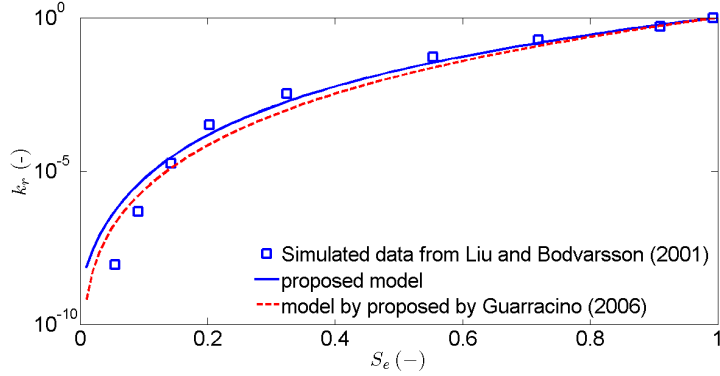


Figure 6: Variation of  $k_r$  with  $S_e$  simulated by Liu and Bodvarsson (2001) for two dimensional fracture networks (symbols) and corresponding predictions from Eq. (22) and Eq. (23).

279  $D_f = 1.8$ ,  $\tau = 1.2$ ,  $\phi = 0.025$ ,  $a_{\max} = a/40$ . It should be remarked that the fitting param-  
 280 eters ( $D_f$ ,  $\tau$ ,  $\phi$ ) are optimized using a function "fmincon" in Matlab to seek a minimum  
 281 root-mean-square error between the experimental data and predicted values.

282 Figure 6 shows the variation of  $k_r$  with  $S_e$  simulated by Liu and Bodvarsson (2001) for  
 283 two dimensional fracture networks (symbols). This observation can be predicted by Eq.  
 284 (22) with optimized parameters  $D_f = 1.6$  and  $\alpha = 2.5 \times 10^{-4}$  (solid line). As pointed out  
 285 by Guarracino (2006), the simulated data in Fig. 6 can also be reproduced by Eq. (23)  
 286 with  $D_f = 1.5$  and  $\alpha = 0.01$  (dashed line). It is seen that the proposed model is in very  
 287 good agreement with simulated data and the model proposed by Guarracino (2006) (The  
 288 root mean square deviation (RMSD) of the proposed model and that of Guarracino (2006)  
 289 are calculated to be 0.0241 and 0.0254, respectively).

290 Similarly, Fig. 7 shows the variation of  $k_r$  with  $S_e$  for the fractured wellbore cement  
 291 measured by Rod et al. (2019) (symbols) and corresponding predictions from our model  
 292 and Guarracino (2006). The fitting parameters are optimized in the same way as previously  
 293 mentioned are  $D_f = 1.2$ ,  $\alpha = 0.001$  and  $D_f = 1.1$ ,  $\alpha = 0.01$  for Eq. (22) and Eq. (23),  
 294 respectively. One can see that our proposed model (RMSD = 0.0985) can provide a better  
 295 fit than Guarracino (2006) (RMSD = 0.1469). The main reason may be come from the

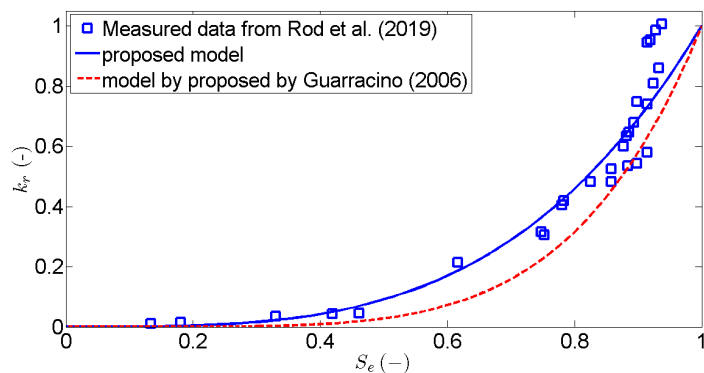


Figure 7: Variation of  $k_r$  with  $S_e$  for the fractured wellbore cement measured by Rod et al. (2019) (symbols) and corresponding predictions from Eq. (22) and Eq. (23).

296 difference in the prefactor  $S_e^2$  in Eq. (23) compared to Eq. (22). Therefore,  $k_r$  predicted  
 297 from Guarracino (2006) is lower than our model, especially at low values of  $S_e$ . Note that  
 298  $S_e$  is always less than 1.

299 To the best of our knowledge, there have been only few publications on SPCC mea-  
 300 surements of fractured media under saturated conditions (e.g., Moore and Glaser, 2007;  
 301 Vinogradov et al., 2022a) and no publications under unsaturated conditions. For example,  
 302 Fagerlund and Heinson (2003) measured the zeta potential of fractured rocks by crushing  
 303 rocks and packing obtained crushed material into a tube. Hence, the experimental data for  
 304 crushed material reported by Fagerlund and Heinson (2003) is not applicable for our model.  
 305 Vinogradov et al. (2022a) measured the SPCC for a fractured Lewisian gneiss sample which  
 306 was assumed to have a single fracture at different values of confining pressure and ionic  
 307 strengths. For example, the measured values for the SPCC at two different values of confin-  
 308 ing pressures (4 MPa and 7 MPa) for the ionic strength of NaCl of 0.7 M, that is denoted  
 309 by  $C_f$  were reported to be -1.21 mV/MPa and -1.23 mV/MPa, respectively. The pore elec-  
 310 trical conductivity  $\sigma_w$  can be estimated by the relationship  $\sigma_w = 10C_f = 7$  S/m (e.g., Sen  
 311 and Goode, 1992). For this high value of  $\sigma_w$ , the contribution of the surface conductivity  
 312 to the total effective conductivity can be neglected (e.g., Alkafeef and Alajmi, 2006; Thanh

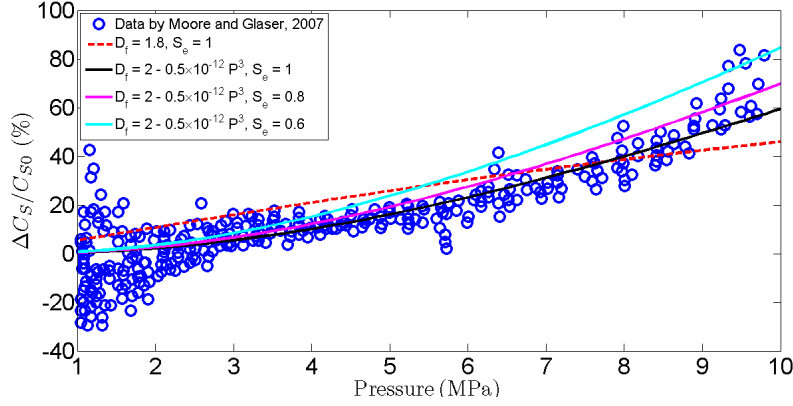


Figure 8: Variation of the relative SPCC difference with injection pressure  $\Delta C_S(P)/C_{S0}$ . The symbols are experimental data reported by Moore and Glaser (2007). The best fit of the proposed model to experimental data are displayed by lines.

313 and Sprik, 2016). Therefore, we can apply Eq.(39) without consideration of the term of  
 314  $\frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{1-\alpha^{1-D_f}}{1-\alpha^{2-D_f}}$  to determine the  $\zeta$ . The obtained values of the  $\zeta$  are -11.8 mV and  
 315 -12.0 mV for the confining pressures of 4 MPa and 7 MPa, respectively. This finding is in  
 316 good agreement with the result of Vinogradov et al. (2022a) where  $\zeta$  was reported to be  
 317 -10.52 mV and -10.69 mV, respectively.

318 Additionally, Moore and Glaser (2007) measured the relative SPCC difference as a  
 319 function of injection pressures for microcracked Sierra granite samples during hydraulic  
 320 fracturing in the laboratory as shown in Fig. 8 (symbols). It is remarked that the relative  
 321 SPCC difference is defined as  $\Delta C_S(P)/C_{S0}$ , where  $C_{S0}$  is the SPCC at zero pressure drop.  
 322 It is seen that there is a tendency of increase of the SPCC with pressure. The reason is  
 323 related to an increase of dilatancy of microcracks with increasing pressures, which causes an  
 324 increase in permeability and therefore in apertures. The increase of apertures of fractured  
 325 media results in a decrease of the surface electrical conductivity and hence the SPCC.  
 326 Moore and Glaser (2007) showed the variation of the permeability  $k_s$  with pressure drop  $P$   
 327 as follows:

$$k_s = 10^{-18} e^{2.5 \times 10^{-4} P}, \quad (43)$$

328 where  $k_s$  is in  $\text{m}^2$  and  $P$  is in  $\text{kPa}$ .

329 Combining Eq. (24) and Eq. (43), one can obtain  $w_{\max}$  as a function of  $P$  as following:

$$w_{\max} = \chi \sqrt{k_s} = \chi 10^{-9} e^{1.25 \times 10^{-4} P}, \quad (44)$$

330 where  $\chi = \frac{\tau}{\beta} \sqrt{\frac{12(1-\alpha^{2-D_f})(4-D_f)}{\phi(1-\alpha^{4-D_f})(2-D_f)}}$ .

331 From Eq. (39) and Eq. (44), one can predict the  $\Delta C_S/C_{S0}$  as a function of  $P$  for  
 332 the microcracked samples reported by Moore and Glaser (2007) as shown by the solid line  
 333 in Fig. 8. Input parameters for the prediction are shown in Table 1 where superscripts \*  
 334 stands for values measured by Moore and Glaser (2007) and superscripts + stands for fitting  
 335 parameters. Namely,  $\sigma_w$ ,  $\phi$ , and  $\zeta$  are reported to be 0.015 S/m, 0.009 (unitless) and -34  
 336 mV, respectively. Due to constraints associated with a large number of model parameters  
 337 for  $\Delta C_S/C_{S0}$ , we search for those parameters that provide a relatively good fit by a trial-  
 338 and-error method. The obtained values for  $\tau$ ,  $\Sigma_s$ ,  $D_f$ ,  $\beta$ , and  $\alpha$  are 2 (unitless),  $0.17 \times 10^{-10}$   
 339 S, 1.8 (unitless), 0.01 (unitless), and 0.001 (unitless), respectively. It is noted that the found  
 340 fitting parameters are in the ranges normally reported in literature for fractured rocks. For  
 341 example,  $\tau$  was reported between 1.1 and 30 (e.g., Wang et al., 2022; Violay et al., 2010;  
 342 Roubinet et al., 2018). Using the box-counting technique,  $D_f$  was found between 1.2 and  
 343 1.85 (e.g., Walsh and Watterson, 1993; Roy et al., 2007). The surface conductance  $\Sigma_s$  was  
 344 reported in the range from  $0.1 \times 10^{-9}$  S to  $4 \times 10^{-9}$  S for silica surface in contact with NaCl  
 345 electrolytes (e.g., Thanh et al., 2019; Revil and Glover, 1998). Ghanbarian et al. (2019)  
 346 found  $\beta$  in the range from 0.001 to 0.1 for tensile fractures in the Krafla fissure swarm of  
 347 northeast Iceland. Additionally, Miao et al. (2015) also used  $\beta = 0.002$  for fitting their  
 348 model with simulated data. Values of  $\alpha$  were inferred between 0.0001 and 0.01 for fractured  
 349 carbonate core rock samples (e.g., Erol et al., 2017). Fig. 8 shows that the proposed model  
 350 can produce the key behavior of experimental data. We remark that the model can provide  
 351 a better fit if one takes into account the variation of  $D_f$  with  $P$  (Guarracino and Jougnot,  
 352 2022). For example, the relationship  $D(P) = 2 - 0.5 \times 10^{-12} P^3$  provides a better fit to the  
 353 experimental data as shown in Fig. 8. We also illustrate variations of  $\Delta C_S/C_{S0}$  with  $P$

Table 1: Input parameters for the microcracked samples reported by Moore and Glaser (2007). Note that superscripts \* stands for values measured by Moore and Glaser (2007) and superscripts + stands for fitting parameters

Parameter	Value	Units
$\sigma_w^*$	0.015	S/m
$\phi^*$	0.009	unitless
$\tau^+$	2	unitless
$\zeta^*$	-34	mV
$\Sigma_s^+$	$0.17 \times 10^{-10}$	S
$D_f^+$	1.8	unitless
$\beta^+$	0.01	unitless
$\alpha^+$	0.001	unitless

354 predicted from the proposed model for other effective saturations  $S_e$  (0.8 and 0.6) as shown  
 355 by the colored solid lines in Fig. 8.

## CONCLUSIONS

356 We present a new unified model for the permeability, electrical conductivity, and streaming  
 357 potential coupling coefficient in variably saturated fractured media. For those, we con-  
 358 ceptualize the fractured medium as a bundle of parallel capillary fractures or slits with  
 359 varying sizes that is partially saturated. We assume that the fracture size distribution of  
 360 the corresponding medium follows the fractal scaling law, therefore allowing us to determine  
 361 the pressure head-water saturation relationship. From the flow rate, conduction current,  
 362 and electrokinetic streaming current within a single saturated fracture, we can upscale ex-  
 363 pressions for the permeability, relative permeability, electrical conductivity, and streaming  
 364 potential coupling coefficient for fractured media under partially saturated conditions at  
 365 the REV scale. This new unified model explicitly depends on properties of fracture water  
 366 ( $\sigma_w$ ,  $\epsilon_r$  and  $\eta$ ), interface properties ( $\Sigma_s$  and  $\zeta$ ), microstructural parameters of fractured  
 367 media ( $D_f$ ,  $\phi$ ,  $\alpha$ ,  $\beta$ ,  $w_{\max}$ ) and saturation state ( $S_e$ ). Model predictions are in good with  
 368 experimental data, simulated data as well as another previous model in the literature. This  
 369 newly proposed model constitutes a practical framework to estimate hydraulic properties  
 370 and monitor water flow in fractured media based on self potential measurements and pos-

371 sibly monitor fracking processes.

## DATA AVAILABILITY STATEMENT

372 The data underlying this article will be shared on reasonable request to the corresponding  
373 author.

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