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1 **A unified model for the permeability, electrical conductivity**
2 **and streaming potential coupling coefficient in variably**
3 **saturated fractured media**

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10 **ABSTRACT**

11 We present a new unified model for the permeability, electrical conductivity, and streaming
12 potential coupling coefficient in variably saturated fractured media. For those, we con-
13 ceptualize the fractured medium as a partially saturated bundle of parallel capillary slits
14 with varying sizes. We assume that the fracture size distribution of the corresponding
15 medium follows a fractal scaling law, which allows us establish a pressure head-saturation
16 relationship based on the Laplace equation. We first describe the flow rate, the conduc-
17 tion current, and the electrokinetic streaming current within a single fracture. Then, we
18 upscale these properties at the scale of an equivalent fractured media partially saturated
19 in order to obtain the relative permeability, the electrical conductivity and the streaming
20 potential coupling coefficient. The newly proposed model explicitly depends on pore water
21 chemistry, interface properties, microstructural parameters of fractured media, and water
saturation. Model predictions are in good agreement with both experimental and simulated
data and with another model from the literature. The results of this work constitute a useful
framework to estimate hydraulic properties and monitor water flow in fractured media.

22 Keywords: Fractured media; Streaming potential; Electrical conductivity; Permeability;
23 Fractal

INTRODUCTION

24 The streaming potential (SP) is a contribution to the self-potential signal that is generated
25 by water flow in porous media. Due to the sensitivity of the SP method to subsurface water
26 flow, the SP technique has drawn an increasing attention to find or track underground
27 water in aquifers or reservoirs (e.g., Revil et al., 2012; Parsekian et al., 2015; Binley et al.,
28 2015). This technique has been used for identifying and monitoring subsurface water flow
29 (e.g., Jouniaux et al., 1999; Fagerlund and Heinson, 2003; Titov et al., 2005; Aizawa et al.,
30 2009), monitoring geothermal and volcanic areas (e.g., Corwin and Hoover, 1979; Finizola
31 et al., 2004; Mauri et al., 2010; Soueid Ahmed et al., 2018; Grobbe and Barde-Cabusson,
32 2019), mapping areas influenced by a contaminant plume (e.g., Martinez-Pagan et al., 2010;
33 Naudet et al., 2003; Roy, 2022), monitoring water flow in the vadose zone (e.g., Doussan
34 et al., 2002; Jougnot et al., 2015; Hu et al., 2020) or eco-hydrology (e.g., Voytek et al.,
35 2019). The SP technique can be applied to estimate hydrogeological parameters of the
36 aquifer (e.g., Jardani et al., 2007; Straface et al., 2010; Revil and Jardani, 2013).

37 Fractured rocks are ubiquitous in the environment and they play a major role in a wide
38 range of geoscience issues, such as groundwater flow and contaminant transport (e.g., Neu-
39 man, 2005; Medici et al., 2019), hydraulic fracturing (e.g., Osiptsov, 2017; Peshcherenko
40 et al., 2022), storage of CO₂ and nuclear waste (e.g., Bodvarsson et al., 1999; Wang and
41 Hudson, 2015; Ren et al., 2017), geothermal production (e.g., Murphy et al., 1981; Patter-
42 son et al., 2020). Geophysical methods offer a variety of tools to obtain information on
43 subsurface structure and physical properties of fractured rocks. Examples of those methods
44 include the electrical conductivity imaging (e.g., Stesky, 1986; Shen et al., 2009; Roubinet
45 and Irving, 2014), seismic technique (e.g., Herwanger et al., 2004; Li, 1997; Clair et al.,
46 2015), or the self potential technique (e.g., Fagerlund and Heinson, 2003; Wishart et al.,
47 2006; Mainault et al., 2013). It is shown that numerical approaches are effective to char-
48 acterize fractured media (e.g., Roubinet and Irving, 2014; Roubinet et al., 2016; Demirel

49 et al., 2018; Haas et al., 2013; DesRoches et al., 2018; Jougnot et al., 2020). However, to
50 the best of our knowledge, there are only few analytical models for fractured rocks in the
51 literature. For example, Thanh et al. (2021) proposed a model for the electrical conductivity
52 and streaming potential coupling coefficient in fractured media under saturated conditions
53 using a capillary bundle model following the fractal scaling law. Guarracino and Jougnot
54 (2022) presented a model to predict the effective excess charge density for fully and partially
55 water saturated fractured media that are described by the fractal Sierpinski carpet.

56 The aim of this study is to develop a unified model for the permeability, electrical
57 conductivity, and streaming potential coupling coefficient in fractured media by extend-
58 ing the work proposed by Thanh et al. (2021) to partially saturated conditions. For this
59 purpose, we conceptualize a fractured medium as a partially saturated bundle of parallel
60 capillary slits following the fractal scaling law. This conceptualization allows us to de-
61 termine the capillary pressure-saturation relationship and later deduce expressions for the
62 electrical conductivity and permeability of fractured media under partially saturated con-
63 ditions. From the electrokinetic streaming current and conduction current within a single
64 slit, we obtain an upscaled expression for the streaming potential coupling coefficient. The
65 new obtained model explicitly depends on properties of fracture water, interface properties,
66 microstructural parameters of fractured media and water saturation. Model predictions are
67 then compared with experimental data, simulated data as well as another previous model
68 in the literature.

THEORETICAL BACKGROUND OF STREAMING POTENTIAL

69 The streaming current is caused by electrokinetic coupling, that is the drag of electrical
70 charge by water flow in porous media conceptualized as a bundle of cylindrical capillary
71 tubes or a bundle of capillary slits. This phenomenon is directly related to the presence
72 of an electric double layer (EDL) that exists at the solid-water interface of the tubes or
73 fractures (e.g., Overbeek, 1952; Hunter, 1981). This EDL contains an excess of charge in
74 water to compensate the charge deficit of the capillary inner surface. The EDL is composed
75 of the Stern and diffuse layers. The Stern layer only contains counter-ions, i.e., the ions

76 with opposite sign to the charged surface. The ions in the Stern layer can be considered
77 as immobile due to strong electrostatic attraction. The diffuse layer contains both the
78 counter-ions and co-ions, i.e., the ions with same sign as the charged surface. The ions in
79 the diffuse layer are free to move but with a net excess of charge (e.g., Hunter, 1981; Jougnot
80 et al., 2020). The interface between the Stern layer and the diffuse layer corresponds to the
81 shear plane or slipping plane that separates the stationary fluid and the moving fluid. The
82 electrical potential at this plane is called the zeta potential ζ (V) that mostly depends on
83 mineral composition of porous media, ionic strength, temperature and pH of water (e.g.,
84 Hunter, 1981; Vinogradov et al., 2022b). The generated streaming current is, in turn,
85 balanced out by an electrical conduction current in the opposite direction, leading to a so-
86 called streaming potential. At the steady state condition, the streaming potential coupling
87 coefficient (SPCC) is defined as (e.g., Smoluchowski, 1903; Morgan et al., 1989):

$$C_S = \frac{\Delta V}{\Delta P}, \quad (1)$$

88 where ΔV (V) and ΔP (Pa) are the measured streaming potential and the imposed pres-
89 sure difference across a probed medium, respectively. There have been two approaches to
90 determine the SPCC at saturated conditions in the literature. For the first approach, the
91 classical one, the SPCC that is expressed in terms of the zeta potential ζ is given by (e.g.,
92 Smoluchowski, 1903)

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \sigma_w}, \quad (2)$$

93 where ϵ_r (no units) is the relative permittivity, ϵ_0 (F/m) is the dielectric permittivity in
94 vacuum, η (Pa s) is the dynamic viscosity and σ_w (S/m) is the electrical conductivity of
95 water. Eq. (2) is called the Helmholtz-Smoluchoski (HS) equation. Note that the surface
96 electrical conductivity σ_s is not considered in Eq. (2). If σ_s is taken into consideration,
97 Eq. (2) can be replaced by the following equation (e.g., Hunter, 1981; Ishido and Mizutani,
98 1981)

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta(\sigma_w + 2\frac{\Sigma_s}{\Lambda})}, \quad (3)$$

99 where Σ_s (S) is the specific surface conductance and Λ (m) is a characteristic length scale
 100 of porous media (Johnson et al., 1986). For the second approach, the SPCC is expressed in
 101 terms of the effective excess charge density \widehat{Q}_v (C/m³) dragged by water (e.g., Kormiltsev
 102 et al., 1998; Revil and Leroy, 2004; Jougnot et al., 2020)

$$C_S = -\frac{k\widehat{Q}_v}{\eta\sigma}, \quad (4)$$

103 where k (m²) and σ (S/m) are the permeability and electrical conductivity of fully saturated
 104 porous media, respectively.

105 One can note that Thanh et al. (2021) developed a model for the SPCC using the zeta
 106 potential ζ for fully saturated fractured media, whereas Guarracino and Jougnot (2022)
 107 proposed a model to predict the \widehat{Q}_v , for fully and partially water saturated fractured media,
 108 that also permits to determine the SPCC.

THEORETICAL DEVELOPMENT

109 Description of fractured media

110 Fractures in geological media exist over a wide range of scales, from microns to thousands of
 111 kilometers, and fractal patterns for fractured rocks have been reported in published works
 112 (e.g., Okubo and Aki, 1987; Bonnet et al., 2001; Kruhl, 2013). To derive the SPCC in
 113 fractured media, we regard the geometrical description reported in the literature for frac-
 114 tured media which are assumed to be made up of the fractures and the surrounding matrix
 115 (e.g., Tyler and Wheatcraft, 1990; Miao et al., 2015; Roubinet et al., 2016; Guarracino and
 116 Jougnot, 2022). The matrix permeability is usually much smaller than that of the fractures
 117 and thus the matrix can be considered as impermeable and no fluid exchange through the
 118 fracture walls. Note that, for consideration of fluid transfer from the matrix to the fractures,
 119 we refer readers to the work reported by Miao et al. (2019), for example. The representative
 120 elementary volume (REV) is assumed to be a cuboid of length of L_o (m) and cross-section
 121 area A (m²) as shown in Fig. 1. We conceptualize the fractures of the REV as a bunch

122 of parallel tortuous slits of varying aperture a (m) and width w (m) following the fractal
 123 scaling law (e.g., Tyler and Wheatcraft, 1990; Miao et al., 2015, 2019):

$$f(w) = D_f w_{\max}^{D_f} w^{-D_f-1}, \quad w_{\min} \leq w \leq w_{\max}, \quad (5)$$

124 where D_f (no units) is the fractal dimension that is between 1 and 2 in two-dimensional
 125 spaces and it can be determined by a box-counting method (e.g., Miao et al., 2015, 2019),
 126 w_{\min} (m) and w_{\max} (m) are the smallest and largest fracture widths in the REV, respectively,
 127 representing the lower and upper bounds of the fractal distribution. Therefore, the number
 128 of fractures whose widths in the range from w to $w+dw$ is given by $f(w)dw$ (e.g., Majumdar
 129 and Bhushan, 1990; Miao et al., 2015). The total number of fractures, from w_{\min} to w_{\max} ,
 130 is given by

$$N_t = \int_{w_{\min}}^{w_{\max}} f(w)dw \approx \left(\frac{w_{\max}}{w_{\min}} \right)^{D_f}. \quad (6)$$

131 Dividing Eq. (5) by Eq. (6), one is able to obtain the probability density function $f_r(w)$

$$f_r(w) = D_f w_{\min}^{D_f} w^{-D_f-1}. \quad (7)$$

132 It is shown that the aperture a is normally related to the width w by a linear scaling law
 133 (e.g., Torabi and Berg, 2011; Miao et al., 2015):

$$a = \beta w, \quad (8)$$

134 where β (unitless) is the proportionality coefficient called the fracture aspect ratio.

135 **Hydraulic properties**

136 The porosity of the REV is defined as

$$\begin{aligned} \phi &= \frac{V_p}{V_t} = \frac{\int_{w_{\min}}^{w_{\max}} (aw)(L_\tau)f(w)dw}{L_o A} = \frac{\beta \tau D_f w_{\max}^{D_f}}{A} \int_{w_{\min}}^{w_{\max}} w^{1-D_f} dw \\ &= \frac{\beta \tau D_f w_{\max}^2}{A(2-D_f)} (1 - \alpha^{2-D_f}), \end{aligned} \quad (9)$$

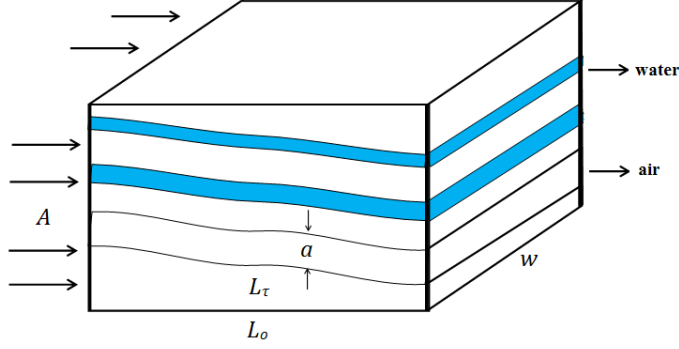


Figure 1: Schematic view of a fractured medium conceptualized as of a bunch of parallel fractures.

137 where V_p and V_t are the pore volume and total volume of the REV, respectively, L_o is the
 138 length of the REV, L_τ is the real length of the fracture, $\tau=L_\tau/L_o$ is the dimensionless
 139 hydraulic tortuosity of the fracture and $\alpha=w_{\min}/w_{\max}$. For the purpose of simplification,
 140 the fracture length is assumed to be unchanging with its width, hence τ is considered to be
 141 constant over the REV and therefore independent from the water saturation.

142 We suppose that the REV is initially filled by water and dewatered by the application
 143 of a pressure head h (m). For a capillary slit, the relationship between the fracture width
 144 w_h and the pressure head h is approximately given by (e.g., Bullard and Garboczi, 2009)

$$h = \frac{2T_s \cos\theta}{\rho_w g \beta w_h}, \quad (10)$$

145 where T_s (N/m) is the surface tension of water, θ ($^\circ$) is the contact angle, ρ_w (kg/m^3) is the
 146 water density and g (m/s^2) is the gravitational acceleration. A fracture is fully desaturated
 147 when its width w is greater than value w_h given by Eq. (10). We assume that each fracture
 148 is filled by either water or air. Therefore, in water-wet systems, water fills the fractures of
 149 the smallest widths, while air occupies fractures of the largest widths when all the fractures
 150 are simultaneously accessible. In other words, fractures with widths w comprised between
 151 w_{\min} and w_h will be occupied by water while those with widths comprised between w_h and
 152 w_{\max} will be filled by air under application of the pressure head h . The contribution of

153 water in the REV depends on the effective water saturation S_e (unitless), that is defined as

$$S_e = \frac{S_w - S_{wr}}{1 - S_{wr}}, \quad (11)$$

154 where S_w (unitless) is the water saturation and S_{wr} (unitless) is the residual water saturation
 155 that represents the water held as films on the fracture walls which can not be drained by
 156 the pressure head h or in non-connected fractures which cannot be taken into account in
 157 the present conceptual model.

158 Following a similar approach to what is reported in the literature (e.g., Guarracino,
 159 2006; Thanh et al., 2020; Guarracino and Jougnot, 2022), S_e is expressed in terms of w_h as
 160 follows:

$$S_e = \frac{\int_{w_{\min}}^{w_h} (aw)L_\tau f(w)dw}{\int_{w_{\min}}^{w_{\max}} (aw)L_\tau f(w)dw} = \frac{w_h^{2-D_f} - w_{\min}^{2-D_f}}{w_{\max}^{2-D_f} - w_{\min}^{2-D_f}}. \quad (12)$$

161 Combining Eq. (10) and Eq. (12), the capillary pressure curve for fractured media can be
 162 obtained as

$$S_e = \frac{h^{D_f-2} - h_{\max}^{D_f-2}}{h_{\min}^{D_f-2} - h_{\max}^{D_f-2}}, \quad h_{\min} \leq h \leq h_{\max}, \quad (13)$$

163 where $h_{\min} = \frac{2T_s \cos\theta}{\rho_w g \beta w_{\max}}$ and $h_{\max} = \frac{2T_s \cos\theta}{\rho_w g \beta w_{\min}}$.

164 Under laminar flow conditions, the average velocity in a single fracture of aperture a is
 165 given by (e.g., Chung, 2010)

$$\bar{v} = \frac{\rho_w g a^2 \Delta h}{12\eta\tau L_o}, \quad (14)$$

166 where Δh is the pressure head drop across the REV.

167 The flow rate in a single fracture follows the well-known cubic law as (e.g., Neuzil and Tracy,
 168 1981; Klimczak et al., 2010)

$$q = \bar{v} \cdot (aw) = \frac{\rho_w g a^3 w \Delta h}{12\eta\tau L_o}. \quad (15)$$

169 The total volumetric flow through the REV under unsaturated conditions is given by

$$q^{\text{REV}} = \int_{w_{\min}}^{w_h} qf(w)dw. \quad (16)$$

170 Combining Eq. (5), Eq. (8), Eq. (15) and Eq. (16), one obtains

$$\begin{aligned} q^{\text{REV}} &= \int_{w_{\min}}^{w_h} \frac{\rho_w g a^3 w}{12\eta\tau} \frac{\Delta h}{L_o} [D_f w_{\max}^{D_f} w^{-D_f-1} dw] \\ &= \frac{\rho_w g \beta^3 D_f}{12\eta\tau} w_{\max}^{D_f} \frac{w_h^{4-D_f} - w_{\min}^{4-D_f}}{4-D_f} \frac{\Delta h}{L_o}. \end{aligned} \quad (17)$$

171 Combining Eq. (12) and Eq. (17), one can express q^{REV} in terms of S_e as

$$q^{\text{REV}} = \frac{\rho_w g \beta^3 D_f}{12\eta\tau} w_{\max}^4 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{4-D_f} \frac{\Delta h}{L_o}. \quad (18)$$

172 Following Darcy's law for Newtonian fluid flow in fractured media, q^{REV} is given by

$$q^{\text{REV}} = \frac{kA}{\eta} \frac{\rho_w g \Delta h}{L_o}, \quad (19)$$

173 where k and A are the permeability and the cross sectional area of the REV.

174 Combining Eq. (18) and Eq. (19), we obtain an expression for k under unsaturated
175 conditions as

$$k(S_e) = \frac{\beta^3 D_f}{12\tau A} w_{\max}^4 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{4-D_f}. \quad (20)$$

176 Hence, the permeability under fully saturated conditions ($S_e = 1$) is given by

$$k_s = \frac{\beta^3 D_f}{12\tau A} w_{\max}^4 \frac{1 - \alpha^{4-D_f}}{4-D_f}. \quad (21)$$

177 The relative permeability of fractured media, that is defined as $k_r = k(S_e)/k_s$, is given by

$$k_r(S_e) = \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{1 - \alpha^{4-D_f}}. \quad (22)$$

178 It is remarked that using a classical fractal object known as the Sierpinski carpet for the
 179 fracture network in combination with the Burdine model, Guarracino (2006) obtained an
 180 expression for k_r as a function of S_e as follows:

$$k_r(S_e) = S_e^2 \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{4-D_f}{2-D_f}} - \alpha^{4-D_f}}{1 - \alpha^{4-D_f}}. \quad (23)$$

181 The only difference between our proposed model given by Eq. (22) and the one proposed
 182 by Guarracino (2006) given by Eq. (23) for k_r is a prefactor S_e^2 .

183 If one invokes Eq. (9), k_s can be expressed as

$$k_s = \frac{\beta^2 w_{\max}^2 \phi}{12\tau^2} \frac{1 - \alpha^{4-D_f}}{1 - \alpha^{2-D_f}} \frac{2 - D_f}{4 - D_f} = \frac{a_{\max}^2 \phi}{12\tau^2} \frac{1 - \alpha^{4-D_f}}{1 - \alpha^{2-D_f}} \frac{2 - D_f}{4 - D_f}. \quad (24)$$

184 For $w_{\max} \gg w_{\min}$ ($\alpha \rightarrow 0$) that is normally reported to be satisfied for the fractal fractured
 185 media (e.g., Guarracino, 2006; Miao et al., 2015, 2019; Thanh et al., 2021), Eq. (24) reduces
 186 to

$$k_s = \frac{\beta^2 w_{\max}^2 \phi}{12\tau^2} \frac{2 - D_f}{4 - D_f} = \frac{a_{\max}^2 \phi}{12\tau^2} \frac{2 - D_f}{4 - D_f}. \quad (25)$$

187 **Electrical conductivity**

188 Following Thanh et al. (2021), the total electrical conductivity in a single fracture with
 189 consideration of the surface conductivity is given by

$$\sigma_f(w) = \sigma_w \frac{\beta w^2}{A\tau} + \Sigma_s \frac{2(1 + \beta)w}{A\tau}. \quad (26)$$

190 Recall that σ_w (S/m) and Σ_s (S) are the electrical conductivity of water and specific surface
 191 conductance at the solid–water interface as previously mentioned.

192 The total electrical conductivity of considered porous media under unsaturated conditions
 193 is therefore obtained by

$$\sigma = \int_{w_{\min}}^{w_h} \sigma_f(w) f(w) dw. \quad (27)$$

194 Combining Eq. (5), Eq. (26) and Eq. (27) yields the following:

$$\begin{aligned}\sigma &= \frac{1}{A\tau} \left\{ \sigma_w \beta D_f w_{\max}^{D_f} \frac{w_h^{2-D_f} - w_{\min}^{2-D_f}}{2 - D_f} + 2\Sigma_s(1 + \beta) D_f w_{\max}^{D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{1 - D_f} \right\}. \\ &= \frac{\beta D_f w_{\max}^{D_f} (w_h^{2-D_f} - w_{\min}^{2-D_f})}{A(2 - D_f)\tau} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta} \frac{2 - D_f}{1 - D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right\}.\end{aligned}\quad (28)$$

195 Invoking Eq. (12), the σ under unsaturated conditions is expressed in terms of S_e as

$$\sigma = \frac{\beta D_f w_{\max}^2 S_e (1 - \alpha^{2-D_f})}{A(2 - D_f)\tau} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta w_{\max}} \frac{2 - D_f}{1 - D_f} \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1 - \alpha^{2-D_f})} \right\}.\quad (29)$$

196 Substituting A from Eq. (9) into Eq. (28), the σ is obtained as

$$\sigma = \frac{\phi S_e}{\tau^2} \left\{ \sigma_w + \frac{2(1 + \beta)\Sigma_s}{\beta w_{\max}} \frac{2 - D_f}{1 - D_f} \frac{[S_e(1 - \alpha^{2-D_f}) + \alpha^{2-D_f}]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1 - \alpha^{2-D_f})} \right\}.\quad (30)$$

197 Eq. (30) shows the dependence of σ under unsaturated conditions on microstructural
198 parameters of the fractured media (D_f , ϕ , α , β , w_{max} , τ), water electrical conductivity σ_w ,
199 specific surface conductance Σ_s , and effective water saturation S_e .

200 Streaming potential coupling coefficient

201 *Streaming current through the REV*

202 Under the thin EDL assumption in which the Debye length is small compared to fracture
203 widths and the Debye-Hückel approximation that is applicable for small values of ζ (i.e., 50
204 mV) for a binary symmetric 1:1 electrolyte, for example (e.g., Rice and Whitehead, 1965;
205 Pride, 1994), the electrokinetic streaming current in a single fracture due to transport of
206 excess charge in the EDL by water flow is given by (e.g., Thanh et al., 2021)

$$i_s(w) = -\frac{\epsilon_r \epsilon_o \zeta}{\eta} \frac{w a}{\tau} \frac{\rho_w g \Delta h}{L_o} = -\frac{\epsilon_r \epsilon_o \zeta}{\eta} \frac{\beta w^2}{\tau} \frac{\rho_w g \Delta h}{L_o}.\quad (31)$$

207 The total streaming current through the REV under unsaturated conditions is determined
 208 by

$$I_s = \int_{w_{\min}}^{w_h} i_s(w) f(w) dw \quad (32)$$

209 From Eq. (5), Eq. (31) and Eq. (32), the following is obtained

$$\begin{aligned} I_s &= -\frac{\epsilon_r \epsilon_o \zeta \rho_w g \Delta h}{\eta \tau} \frac{\rho_w g \Delta h}{L_o} (\beta D_f w_{\max}^{D_f}) \int_{w_{\min}}^{w_h} w^{1-D_f} dw \\ &= -\frac{\epsilon_r \epsilon_o \zeta \beta D_f}{\eta \tau} \frac{w_{\max}^{D_f}}{(2-D_f)} (w_h^{2-D_f} - w_{\min}^{2-D_f}) \frac{\rho_w g \Delta h}{L_o}. \end{aligned} \quad (33)$$

210 *Conduction current through the REV*

211 The streaming current is accounted for the streaming potential ΔV that is built up across
 212 the REV due to the water flow. In turn, an electric conduction current is generated in the
 213 REV due to the electrical potential difference ΔV . Namely, the conduction current in a
 214 single fracture is given by Thanh et al. (2021)

$$i_c(w) = \left[\sigma_w \frac{\beta w^2}{L_o \tau} + \Sigma_s \frac{2(1+\beta)w}{L_o \tau} \right] \Delta V. \quad (34)$$

215 The total electric conduction current through the REV under unsaturated conditions is
 216 given by

$$\begin{aligned} I_c &= \int_{w_{\min}}^{w_h} i_c(w) f(w) dw \\ &= \frac{\beta D_f w_{\max}^{D_f} (w_h^{2-D_f} - w_{\min}^{2-D_f})}{(2-D_f) \tau} \left[\sigma_w + \frac{2(1+\beta) \Sigma_s}{\beta} \frac{2-D_f}{1-D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right] \frac{\Delta V}{L_o}. \end{aligned} \quad (35)$$

217 *Streaming potential coupling coefficient*

218 Considering thermodynamic equilibrium, the following condition is satisfied

$$I_s + I_c = 0. \quad (36)$$

219 Consequently, the SPCC is given by

$$C_S = \frac{\Delta V}{\Delta P} = \frac{\Delta V}{\rho_w g \Delta h} = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[\sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta} \frac{2-D_f}{1-D_f} \frac{w_h^{1-D_f} - w_{\min}^{1-D_f}}{w_h^{2-D_f} - w_{\min}^{2-D_f}} \right]}. \quad (37)$$

220 Invoking Eq. (12), the SPCC can be expressed in terms of S_e as

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[\sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{\left[S_e(1-\alpha^{2-D_f}) + \alpha^{2-D_f} \right]^{\frac{1-D_f}{2-D_f}} - \alpha^{1-D_f}}{S_e(1-\alpha^{2-D_f})} \right]}. \quad (38)$$

221 Eq. (38) is an expression for the SPCC of fractured media under unsaturated conditions. It
 222 predicts that the SPCC is dependent of water properties (σ_w , ϵ_r and η), physico-chemical
 223 properties of the solid–water interface (Σ_s and ζ), microstructural parameters of fractured
 224 media (D_f , ϕ , α , β , w_{\max}) and saturation state (S_e). When $\Sigma_s = 0$, Eq. (38) reduces to
 225 the HS equation given by Eq. (2) that has been proposed for porous media rather than
 226 fractured media regardless of S_e . Under saturated conditions $S_e = 1$, Eq. (38) simplifies to
 227 that proposed by Thanh et al. (2021):

$$C_S = \frac{\epsilon_r \epsilon_0 \zeta}{\eta \left[\sigma_w + \frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{1-\alpha^{1-D_f}}{1-\alpha^{2-D_f}} \right]}. \quad (39)$$

228 Combining Eq. (4), Eq. (30) and Eq. (38), one can infer an expression for the effective
 229 excess charge density as following

$$\widehat{Q}_v = -\epsilon_r \epsilon_0 \zeta \frac{\phi S_e}{k \tau^2}. \quad (40)$$

230 We remark that Guarracino and Jougnot (2022) proposed a model for \widehat{Q}_v , that is deduced
 231 from their Eq. (14) and Eq. (23), as below

$$\widehat{Q}_v = -\epsilon_r \epsilon_0 \zeta \left[1 + \frac{1}{54} \left(\frac{e \zeta}{k_B T} \right)^2 \right] \frac{\phi S_e}{k}, \quad (41)$$

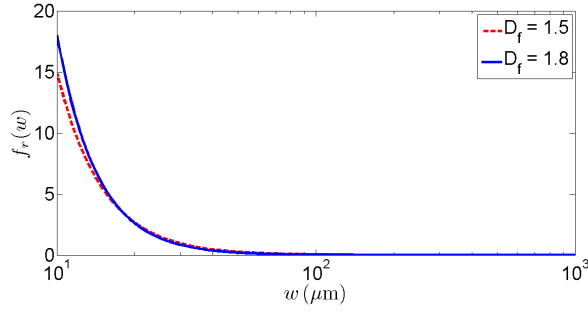


Figure 2: Probability density function associated with fracture size distribution in this work in the width range from $w_{\min} = 10 \mu\text{m}$ to $w_{\max} = 1000 \mu\text{m}$ ($\alpha = 0.01$) for two values of D_f (1.5 and 1.8).

232 where e (C) the elementary charge, k_B (J/K) the Boltzman constant and T (K) is the
 233 absolute temperature.

234 Under the Debye-Hückel approximation in which $\left(\frac{e\zeta}{2k_B T}\right)^2 \ll 1$ (e.g., Pride, 1994), Eq.
 235 (41) reduces to

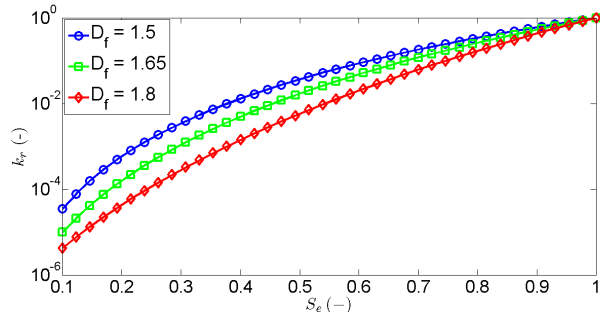
$$\hat{Q}_v = -\epsilon_r \epsilon_0 \zeta \frac{\phi S_e}{k}. \quad (42)$$

236 Obviously, our finding given by Eq. (40) is the same as that proposed by Guarracino and
 237 Jougnot (2022) under the Debye-Hückel approximation given by Eq. (42). It is noted that
 238 Guarracino and Jougnot (2022) did not consider τ in their model (i.e., $\tau = 1$).

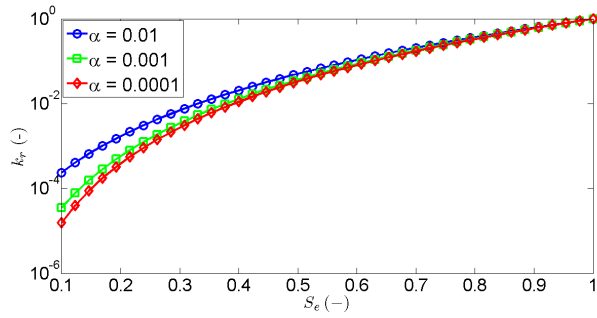
RESULTS AND DISCUSSION

239 Sensitivity of the model

240 Figure 2 shows the representative probability density function of fractures predicted from
 241 Eq. (7) in the width range from $w_{\min} = 10 \mu\text{m}$ to $w_{\max} = 1000 \mu\text{m}$ ($\alpha = 0.01$) for two
 242 different values of D_f (1.5 and 1.8). It is seen that: (i) the frequency distribution of fractures
 243 becomes skewed toward smaller fracture width and (ii) there is a larger number of small
 244 fractures for larger value of D_f .



(a)



(b)

Figure 3: Variation of the k_r with S_e predicted from Eq. (22): (a) for three representative values of D_f (1.5, 1.65 and 1.8) at a given representative value of $\alpha=0.001$, (b) for three representative values of α (0.01, 0.001 and 0.0001) at a given representative value of $D_f = 1.5$.

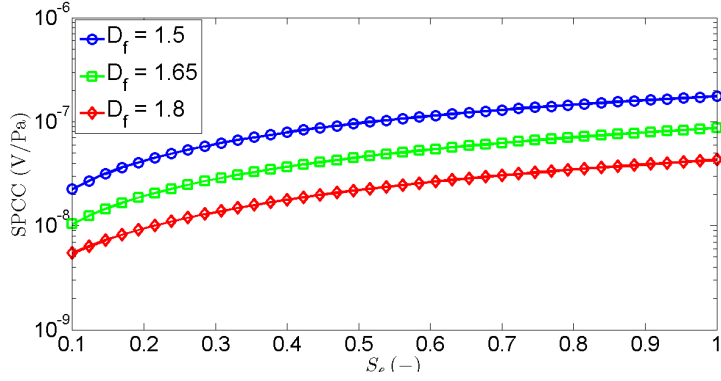


Figure 4: Dependence of the SPCC in magnitude on the effective water saturation for different values of D_f (1.6, 1.65 and 1.8) predicted from Eq. (38)

245 From Eq. (22), we can predict the variation of k_r with S_e for fractured media. For
 246 example, Fig. 3 shows the S_e - k_r relationship for: (a) three representative values of D_f (1.5,
 247 1.65 and 1.8) at a given representative value of $\alpha=0.001$, (b) three representative values of
 248 α (0.01, 0.001 and 0.0001) at a given representative value of $D_f = 1.5$. It is seen that k_r is
 249 sensitive to parameters of S_e , D_f and α . At given values of S_e and α , k_r decreases with an
 250 increase of D_f . The reason is that when D_f increases, the number of fractures in the REV
 251 with small widths increases as indicated in Fig. 2. Therefore, at the same water saturation,
 252 w_h decreases and the total flow rate through the REV becomes smaller. Consequently,
 253 k_r decreases with increasing D_f . It is also predicted that at given values of S_e and D_f ,
 254 k_r decreases with a decrease of α . The reason is that when α decreases, there is a larger
 255 fraction of fractures with smaller widths due to the property of the fractal distribution. As
 256 a result, k_r decreases.

257 Figure 4 shows the variation of the SPCC in magnitude with S_e for different values of
 258 D_f (1.6, 1.65 and 1.8) predicted from Eq. (38). Representative parameters used in Eq.
 259 (38) are: $\sigma_w = 0.02$ S/m, $\Sigma_s = 10^{-9}$ S, $\zeta = -0.030$ V, $w_{\max} = 200 \cdot 10^{-6}$ m and $\beta = 0.01$.
 260 It is shown that the SPCC in magnitude increases with increasing S_e . This prediction is in
 261 good agreement with those observed in published work but for porous media such as sand
 262 columns, dolomite cores or limestone cores (e.g., Guichet et al., 2003; Revil and Cerepi,

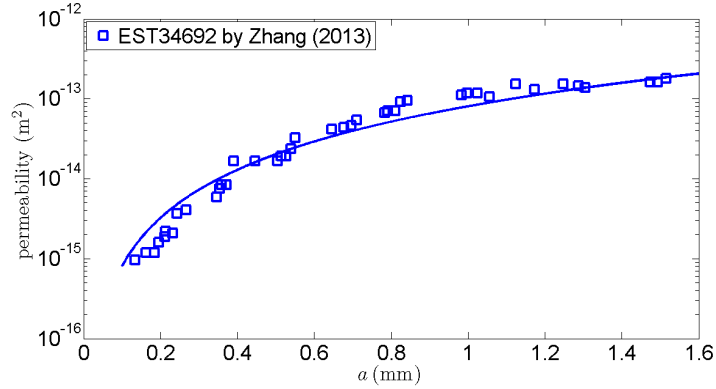


Figure 5: Variation of the k_s of a single fracture as a function of aperture a obtained from Zhang (2013) for the sample EST34692 and prediction from Eq. (25) for a fractured medium with $D_f = 1.8$, $\tau = 1.2$, $\phi = 0.025$ and $a_{\max} = a/40$.

263 2004; Vinogradov and Jackson, 2011). Additionally, it is also seen that the SPCC decreases
 264 with increasing D_f for the same value of S_e . The reason is that when D_f increases, the
 265 number of fractures characterized by relatively small widths increases as shown by Fig. 2.
 266 Hence, the surface conductivity of fractured media increases and the SPCC in magnitude
 267 decreases. It is remarked that the surface conductivity of fractured media is dominated by
 268 the contribution from the smaller width fractures for given values of σ_w and Σ_s .

269 Comparison with published data

270 There are not many published experimental data of single fracture permeability measure-
 271 ment in the literature. Nevertheless, Zhang (2013) presents gas permeability measurement
 272 on a Callovo-Oxfordian (COx) clayrock sample which exhibits a single fracture. Non frac-
 273 tured COx clay rocks are known for their very low permeability (typically 10 nd at saturation
 274 as indicated by Jougnot et al. (2010)). Hence, the measured permeability of a fractured
 275 sample is largely due to the fracture itself. Fig. 5 shows the evolution of the sample per-
 276 meability as a function of the aperture a of the fracture. The variation of the fractured
 277 medium k_s under saturated conditions with a_{\max} is predicted from Eq. (25) as shown by
 278 the solid line in Fig. 5. The model can describe the behavior of the permeability well with

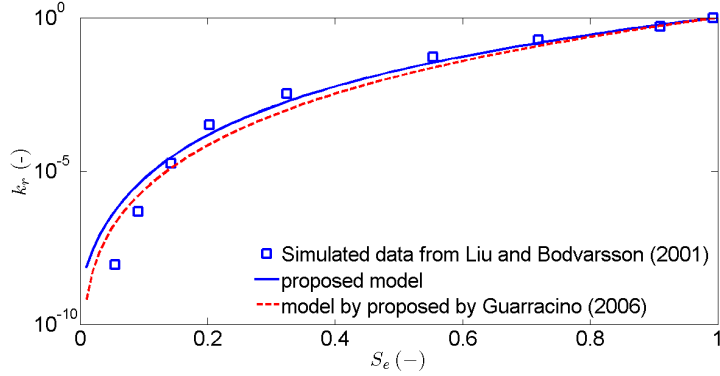


Figure 6: Variation of k_r with S_e simulated by Liu and Bodvarsson (2001) for two dimensional fracture networks (symbols) and corresponding predictions from Eq. (22) and Eq. (23).

279 $D_f = 1.8$, $\tau = 1.2$, $\phi = 0.025$, $a_{\max} = a/40$. It should be remarked that the fitting param-
 280 eters (D_f , τ , ϕ) are optimized using a function "fmincon" in Matlab to seek a minimum
 281 root-mean-square error between the experimental data and predicted values.

282 Figure 6 shows the variation of k_r with S_e simulated by Liu and Bodvarsson (2001) for
 283 two dimensional fracture networks (symbols). This observation can be predicted by Eq.
 284 (22) with optimized parameters $D_f = 1.6$ and $\alpha = 2.5 \times 10^{-4}$ (solid line). As pointed out
 285 by Guarracino (2006), the simulated data in Fig. 6 can also be reproduced by Eq. (23)
 286 with $D_f = 1.5$ and $\alpha = 0.01$ (dashed line). It is seen that the proposed model is in very
 287 good agreement with simulated data and the model proposed by Guarracino (2006) (The
 288 root mean square deviation (RMSD) of the proposed model and that of Guarracino (2006)
 289 are calculated to be 0.0241 and 0.0254, respectively).

290 Similarly, Fig. 7 shows the variation of k_r with S_e for the fractured wellbore cement
 291 measured by Rod et al. (2019) (symbols) and corresponding predictions from our model
 292 and Guarracino (2006). The fitting parameters are optimized in the same way as previously
 293 mentioned are $D_f = 1.2$, $\alpha = 0.001$ and $D_f = 1.1$, $\alpha = 0.01$ for Eq. (22) and Eq. (23),
 294 respectively. One can see that our proposed model (RMSD = 0.0985) can provide a better
 295 fit than Guarracino (2006) (RMSD = 0.1469). The main reason may be come from the

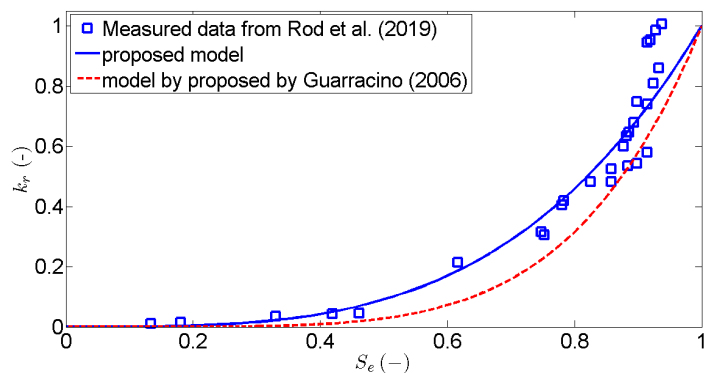


Figure 7: Variation of k_r with S_e for the fractured wellbore cement measured by Rod et al. (2019) (symbols) and corresponding predictions from Eq. (22) and Eq. (23).

296 difference in the prefactor S_e^2 in Eq. (23) compared to Eq. (22). Therefore, k_r predicted
 297 from Guarracino (2006) is lower than our model, especially at low values of S_e . Note that
 298 S_e is always less than 1.

299 To the best of our knowledge, there have been only few publications on SPCC mea-
 300 surements of fractured media under saturated conditions (e.g., Moore and Glaser, 2007;
 301 Vinogradov et al., 2022a) and no publications under unsaturated conditions. For example,
 302 Fagerlund and Heinson (2003) measured the zeta potential of fractured rocks by crushing
 303 rocks and packing obtained crushed material into a tube. Hence, the experimental data for
 304 crushed material reported by Fagerlund and Heinson (2003) is not applicable for our model.
 305 Vinogradov et al. (2022a) measured the SPCC for a fractured Lewisian gneiss sample which
 306 was assumed to have a single fracture at different values of confining pressure and ionic
 307 strengths. For example, the measured values for the SPCC at two different values of confin-
 308 ing pressures (4 MPa and 7 MPa) for the ionic strength of NaCl of 0.7 M, that is denoted
 309 by C_f were reported to be -1.21 mV/MPa and -1.23 mV/MPa, respectively. The pore elec-
 310 trical conductivity σ_w can be estimated by the relationship $\sigma_w = 10C_f = 7$ S/m (e.g., Sen
 311 and Goode, 1992). For this high value of σ_w , the contribution of the surface conductivity
 312 to the total effective conductivity can be neglected (e.g., Alkafeef and Alajmi, 2006; Thanh

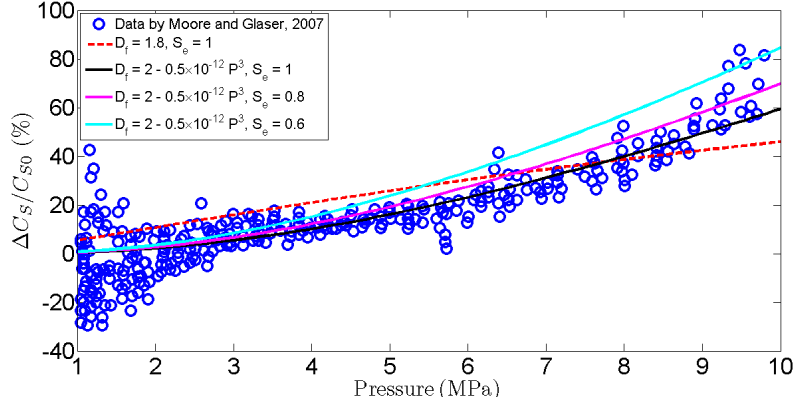


Figure 8: Variation of the relative SPCC difference with injection pressure $\Delta C_S(P)/C_{S0}$. The symbols are experimental data reported by Moore and Glaser (2007). The best fit of the proposed model to experimental data are displayed by lines.

313 and Sprik, 2016). Therefore, we can apply Eq.(39) without consideration of the term of
 314 $\frac{2(1+\beta)\Sigma_s}{\beta w_{\max}} \frac{2-D_f}{1-D_f} \frac{1-\alpha^{1-D_f}}{1-\alpha^{2-D_f}}$ to determine the ζ . The obtained values of the ζ are -11.8 mV and
 315 -12.0 mV for the confining pressures of 4 MPa and 7 MPa, respectively. This finding is in
 316 good agreement with the result of Vinogradov et al. (2022a) where ζ was reported to be
 317 -10.52 mV and -10.69 mV, respectively.

318 Additionally, Moore and Glaser (2007) measured the relative SPCC difference as a
 319 function of injection pressures for microcracked Sierra granite samples during hydraulic
 320 fracturing in the laboratory as shown in Fig. 8 (symbols). It is remarked that the relative
 321 SPCC difference is defined as $\Delta C_S(P)/C_{S0}$, where C_{S0} is the SPCC at zero pressure drop.
 322 It is seen that there is a tendency of increase of the SPCC with pressure. The reason is
 323 related to an increase of dilatancy of microcracks with increasing pressures, which causes an
 324 increase in permeability and therefore in apertures. The increase of apertures of fractured
 325 media results in a decrease of the surface electrical conductivity and hence the SPCC.
 326 Moore and Glaser (2007) showed the variation of the permeability k_s with pressure drop P
 327 as follows:

$$k_s = 10^{-18} e^{2.5 \times 10^{-4} P}, \quad (43)$$

328 where k_s is in m^2 and P is in kPa.

329 Combining Eq. (24) and Eq. (43), one can obtain w_{\max} as a function of P as following:

$$w_{\max} = \chi \sqrt{k_s} = \chi 10^{-9} e^{1.25 \times 10^{-4} P}, \quad (44)$$

330 where $\chi = \frac{\tau}{\beta} \sqrt{\frac{12(1-\alpha^{2-D_f})(4-D_f)}{\phi(1-\alpha^{4-D_f})(2-D_f)}}$.

331 From Eq. (39) and Eq. (44), one can predict the $\Delta C_S/C_{S0}$ as a function of P for
 332 the microcracked samples reported by Moore and Glaser (2007) as shown by the solid line
 333 in Fig. 8. Input parameters for the prediction are shown in Table 1 where superscripts *
 334 stands for values measured by Moore and Glaser (2007) and superscripts + stands for fitting
 335 parameters. Namely, σ_w , ϕ , and ζ are reported to be 0.015 S/m, 0.009 (unitless) and -34
 336 mV, respectively. Due to constraints associated with a large number of model parameters
 337 for $\Delta C_S/C_{S0}$, we search for those parameters that provide a relatively good fit by a trial-
 338 and-error method. The obtained values for τ , Σ_s , D_f , β , and α are 2 (unitless), 0.17×10^{-10}
 339 S, 1.8 (unitless), 0.01 (unitless), and 0.001 (unitless), respectively. it is noted that the found
 340 fitting parameters are in the ranges normally reported in literature for fractured rocks. For
 341 example, τ was reported between 1.1 and 30 (e.g., Wang et al., 2022; Violay et al., 2010;
 342 Roubinet et al., 2018). Using the box-counting technique, D_f was found between 1.2 and
 343 1.85 (e.g., Walsh and Watterson, 1993; Roy et al., 2007). The surface conductance Σ_s was
 344 reported in the range from 0.1×10^{-9} S to 4×10^{-9} S for silica surface in contact with NaCl
 345 electrolytes (e.g., Thanh et al., 2019; Revil and Glover, 1998). Ghanbarian et al. (2019)
 346 found β in the range from 0.001 to 0.1 for tensile fractures in the Krafla fissure swarm of
 347 northeast Iceland. Additionally, Miao et al. (2015) also used $\beta = 0.002$ for fitting their
 348 model with simulated data. Values of α were inferred between 0.0001 and 0.01 for fractured
 349 carbonate core rock samples (e.g., Erol et al., 2017). Fig. 8 shows that the proposed model
 350 can produce the key behavior of experimental data. We remark that the model can provide
 351 a better fit if one takes into account the variation of D_f with P (Guarracino and Jougnot,
 352 2022). For example, the relationship $D(P) = 2 - 0.5 \times 10^{-12} P^3$ provides a better fit to the
 353 experimental data as shown in Fig. 8. We also illustrate variations of $\Delta C_S/C_{S0}$ with P

Table 1: Input parameters for the microcracked samples reported by Moore and Glaser (2007). Note that superscripts * stands for values measured by Moore and Glaser (2007) and superscripts + stands for fitting parameters

Parameter	Value	Units
σ_w^*	0.015	S/m
ϕ^*	0.009	unitless
τ^+	2	unitless
ζ^*	-34	mV
Σ_s^+	0.17×10^{-10}	S
D_f^+	1.8	unitless
β^+	0.01	unitless
α^+	0.001	unitless

354 predicted from the proposed model for other effective saturations S_e (0.8 and 0.6) as shown
 355 by the colored solid lines in Fig. 8.

CONCLUSIONS

356 We present a new unified model for the permeability, electrical conductivity, and streaming
 357 potential coupling coefficient in variably saturated fractured media. For those, we con-
 358 ceptualize the fractured medium as a bundle of parallel capillary fractures or slits with
 359 varying sizes that is partially saturated. We assume that the fracture size distribution of
 360 the corresponding medium follows the fractal scaling law, therefore allowing us to determine
 361 the pressure head-water saturation relationship. From the flow rate, conduction current,
 362 and electrokinetic streaming current within a single saturated fracture, we can upscale ex-
 363 pressions for the permeability, relative permeability, electrical conductivity, and streaming
 364 potential coupling coefficient for fractured media under partially saturated conditions at
 365 the REV scale. This new unified model explicitly depends on properties of fracture water
 366 (σ_w , ϵ_r and η), interface properties (Σ_s and ζ), microstructural parameters of fractured
 367 media (D_f , ϕ , α , β , w_{\max}) and saturation state (S_e). Model predictions are in good with
 368 experimental data, simulated data as well as another previous model in the literature. This
 369 newly proposed model constitutes a practical framework to estimate hydraulic properties
 370 and monitor water flow in fractured media based on self potential measurements and pos-

371 sibly monitor fracking processes.

DATA AVAILABILITY STATEMENT

372 The data underlying this article will be shared on reasonable request to the corresponding
373 author.

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