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Type IIB moduli stabilization, inflation and waterfall fields

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In this paper, we present a string realization of the hybrid inflationary scenario within type IIB effective string theory constructions and a geometric configuration of intersecting D7 branes. A metastable de Sitter minimum is ensured by perturbative logarithmic corrections and D-term contributions from abelian factors associated with the D7 branes. The inflaton is identified with the internal volume modulus whereas possible waterfall fields correspond to excitations of open strings attached to the magnetized D7 branes. By incorporating contributions of these fields in the scalar potential, inflation stops and the metastable vacuum settles to a minimum with the observed tunable value of the cosmological constant.

Keywords: D-branes; superstring effective models; superstring vacua; inflation.

1. Introduction

At present, String Theory formulated in 10 or 11 dimensions appears to be the only promising candidate for a consistent quantum theory of the four known fundamental forces and their interactions. Compactification of the higher dimensional theory to four space–time dimensions entails an immense number of string vacua dubbed as the string landscape. Numerous Effective Quantum Field Theories, on the other hand, have been built to describe the low-energy physics and make cosmological predictions.

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Amongst their most important features, such theories should posses a tiny cosmological constant $\Lambda \approx 10^{-120}\, M_{\rm Planck}^4$ in order to account for the dark energy suggested by cosmological observations. The simplest way to realize the dark energy scenario is to introduce a scalar field ϕ with a potential $V(\phi)$, which displays a minimum value equal to the cosmological constant $V_{\rm min}(\phi_0) = \Lambda$, at some suitable point ϕ_0. There is a significant ongoing debate, however, on whether the string landscape contains any de Sitter vacua which comply with the prediction of positive Λ. Recent Swampland conjectures,[1] in particular, suggest that the first and second derivatives of $V(\phi)$ must satisfy the inequalities $|\nabla V|/V \geq c$ or $\min(\nabla_i \nabla_j V) \leq -c'$ (in Planck units) where c, c' are positive constants of order one. If these inequalities are true, some apparently consistent (anomaly free) theories in four dimensions do not have an ultraviolet completion and cannot be derived from string theory. In other words, they belong to the Swampland.[a] Putting it differently, starting from a successful Effective Field Theory weakly coupled to gravity which describes adequately the known physics phenomena, we cannot always embed it in the string theory landscape.

The above considerations have far reaching consequences both in cosmology and particle physics.[5] Here, we mention a few implications on otherwise very successful cosmological scenarios. For example, it is rather obvious that the Swampland criteria summarized in the aforementioned inequalities contradict the assumption that the cosmological constant can account for the dark energy of the universe. Furthermore, slow-roll inflation is inconsistent with these criteria. Instead, there are suggestions[5] that quintessence models where the cosmological constant varies over time satisfy current observational constraints. If this scenario prevails, the present acceleration phase eventually will terminate whereas the expansion of the universe will come to an end in the distant future.

The ensuing years since their formulation, Swampland conjectures have faced increased scrutiny. Most of the criticism focused on the assumed heuristic arguments, and the neglected role of string quantum corrections. Indeed, the latter are anticipated to be essential for the final form of the effective scalar potential in the resulting field theory model after compactification. This presentation will focus on investigations of de Sitter vacua and the realization of inflation in type IIB superstring theory. These investigations will take place assuming a geometric configuration of intersecting D-brane stacks with magnetic fluxes.[6] At the same time, we will consider the effects of a new four-dimensional Einstein–Hilbert term (localized in the internal space) which is generated from higher derivative terms in the 10-dimensional string effective action.[7,8] This set up induces logarithmic corrections to the scalar potential via loop effects.[9] Minimization of the whole scalar potential of the theory fixes the internal volume Kähler modulus, $\mathcal{V}$, whereas the ratios of the world-volumes along the three $D7$-brane stacks are fixed by virtue of D-term contributions and their parameters depending on the quantized magnetic fluxes. In addition, slow-roll inflation can be realized considering the (canonically normalized) inflaton field to be

[a] For reviews and further references, see Refs. 2–4.

proportional to the logarithm of the internal volume $\mathcal{V}$. Furthermore, the open string spectrum associated with the $D7$ brane stacks plays a significant role. One can fix magnetic fluxes and brane separations so that charged open string states have positive squared-masses, except for one of them which becomes tachyonic when $\mathcal{V}$ becomes less than some critical value. It turns out that this state can be identified with a waterfall field which can be used to stop the inflationary phase and deepen the vacuum. A generalization of this scenario with several waterfall fields shows that the model can accommodate the present dark energy.

2. Type IIB Moduli Stabilization

We briefly introduce the basic geometric set up and the moduli field content. We consider a six-dimensional compactification on a Calabi–Yau (CY) threefold within a type IIB framework in the presence of quantized 3-form fluxes. Deformations of the compactification correspond to massless scalars which do not acquire tree-level potential and do not affect the four-dimensional action. Such scalars are the dilaton field Φ, the Kähler moduli $\mathcal{T}_i$, the complex structure (CS) ones z_a and moduli corresponding to brane deformations. We further introduce a two index antisymmetric tensor denoted with $B_{\mu\nu}$ (the Kalb–Ramond field) and the p-form potentials C_p, $p = 0, 2, 4$. The C_0 potential and the dilaton field, define the usual axion–dilaton combination $S = C_0 + ie^{-\Phi} \to C_0 + \frac{i}{g_s}$ where g_s is the string coupling. At the effective theory level, there are two basic ingredients: the superpotential of the moduli fields and the Kähler potential.

To construct the superpotential, one introduces p-form field strengths $F_p = dC_{p-1}$, $H_3 = dB_2$ and defines $\mathbf{G_3} = F_3 - SH_3$. In terms of these, the fluxed induced superpotential $\mathcal{W}_0$ is given, at the classical level, by the well-known formula:[10]

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a), \tag{2.1}$$

where $\Omega(z_a)$ is a holomorphic 3-form. It turns out that the perturbative superpotential $\mathcal{W}_0$ is a holomorphic function which depends on the axion–dilaton modulus S, and the CS moduli z_a. Imposing the supersymmetric conditions, the moduli z_a, S can be stabilized. On the contrary, the Kähler moduli do not participate in the perturbative superpotential and thus remain completely undetermined at this stage.

The second ingredient is the Kähler potential which depends logarithmically on the various moduli fields through the following expression:

$$\mathcal{K}_0 = -2\ln(\mathcal{V}) - \ln\left(-i \int \Omega \wedge \bar{\Omega}\right), \tag{2.2}$$

where $\mathcal{V}$ is the volume of the 6D internal CY manifold $\mathcal{X}_6$, in string units. The effective potential is computed from (2.2) using the standard supergravity formula

$$V_{\text{eff}} = e^{\mathcal{K}}\left(\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2\right), \tag{2.3}$$

where $\mathcal{D}_I = \partial_I + \mathcal{K}_I$ is the Kähler covariant derivative. At the classical level, this potential vanishes identically due to its no-scale structure, and appropriate supersymmetric (flatness) conditions for the dilaton and the CS moduli. It is thus impossible to stabilize the Kähler moduli at this level. These moduli can be stabilized when quantum corrections breaking the no-scale structure of the Kähler potential are included.

Several ways to fix this problem have appeared over the last two decades. A first approach[11,12] was based on the inclusion of nonperturbative superpotential terms of the form $\mathcal{W}_{\rm np} \sim \sum_i A_i e^{-a_i \mathcal{T}_i}$. The coefficients A_i may depend on the complex structure moduli, and the exponential factors on the Kähler ones $\mathcal{T}_i$. The parameters a_i may arise form gaugino condensation on D-brane stacks and for the $SU(N)$ case, they are of the form $\frac{2\pi}{N}$. The above ingredients can stabilize the Kähler fields, however the potential acquires an anti-de Sitter (AdS) vacuum.[11] A possible solution to this problem[12] is to uplift the vacuum by taking into account contributions from $\overline{D3}$ branes. There are two issues regarding this solution. First, in order to obtain an AdS, minimum the coefficients $\mathcal{W}_0$, A_i and a_i require unnatural fine-tuning. Second, these contributions rely on nonperturbative effects which cannot be controlled at the full string level. Some improvements of the original models, however, have appeared using nilpotent chiral multiplets,[13] which lead to a new mechanism for uplifting the vacua in the string landscape.[14]

A different way to stabilize the moduli is based on Large Volume Scenario (LVS).[15] This proposal takes advantage of the leading α' corrections to the Kähler potential (together with the nonperturbative contributions) which ensure an AdS solution in the Large Volume Limit but avoid tuning $\mathcal{W}_0$ in (2.1) at extremely small values. Uplift to a de Sitter (dS) vacuum can be realized through D-terms.

Perturbative moduli-dependent corrections in weakly coupled string theory, on the other hand, are fully controllable and therefore more reliable. However, not all types of corrections are suitable for moduli stabilization. Ordinary perturbative expansions, either in α' or in powers of the weak string coupling g_s, fail to generate a (meta)stable dS minimum in a controllable way. This is the well-known Dine–Seiberg problem which we now describe in brief. When perturbative moduli-dependent quantum corrections are included in the Kähler potential they induce contributions to the scalar potential, $V(\tau_i)$ where τ_i are the imaginary parts of the Kähler moduli $\mathcal{T}_i$ and are associated with the internal volume. The validity of perturbation theory implies that such corrections should vanish for $\tau_i \to \infty$ implying also the vanishing of the scalar potential $V(\tau_i) \xrightarrow[\tau_i \to 0]{} 0$. If the zero at infinity is reached from negative values, then, for noncontrived scalar potentials $V(\tau_i)$, this implies an AdS minimum which is not acceptable. Thus, the vanishing of the potential at infinity should be approached from positive values. Again, for reasonable $V(\tau_i)$, this implies that there should be somewhere a maximum before a dS minimum is formed. These three shapes are plotted in Fig. 1. The potential on the right-hand side exhibits **local** minimum and maximum and its shape suggests that there should be **two** competing terms of different functional dependence on τ. While previously considered perturbative

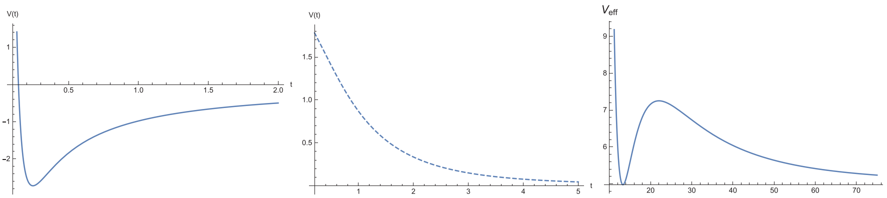

Fig. 1. Left figure: Vanishing of $V(\tau)$ from 0^- happens for potentials with an AdS minimum. Middle: Large τ behavior of $V(\tau)$ with power-law correction $\sim \frac{1}{\tau^n}$. The potential on the right-hand side exhibits **local** minimum and maximum.

corrections do not share this property at large volumes, a possible exception known from the field theory are logarithmic corrections similar to those in the Coleman–Weinberg mechanism.[16]

The above observation shows the way to overcome the difficulties in superstring constructions. We recall that string theory has a reach structure including non-perturbative objects such as D-branes which open up possibilities to construct realistic cosmological models. Another ingredient, of particular interest in this study, comes from high-order curvature terms in the 10-dimensional effective action. These elements are sufficient to generate loop corrections which induce new contributions to the Kähler potential $\mathcal{K}$, break its no scale invariance and stabilize the moduli. We will describe in short how perturbative logarithmic corrections are generated with the above constituents.

The low-energy expansion of the type IIB superstring action contains fourth-order terms in the Riemann curvature, R^4, which do not receive any perturbative corrections beyond one loop.[7,17,18] Upon compactification to our four-dimensional space–time M_4, these one-loop corrections induce a novel Einstein–Hilbert (EH) term $\mathcal{R}_{(4)}$. Its coefficient is proportional to the Euler characteristic χ, defined on $\mathcal{X}_6$ by

$$\chi = \frac{3}{4\pi^3}\int_{\mathcal{X}_6} R\wedge R\wedge R.$$

Observing that χ contains three powers of R, we deduce that the effective EH term $\mathcal{R}_{(4)}$ (originating from R^4) is only possible in four dimensions. Furthermore, such an EH term can be viewed as a vertex localized at certain points in the six-dimensional bulk where χ acquires nonzero values, emitting closed strings (gravitons). We thus study the case of three-graviton scattering involving two massless gravitons and a Kaluza–Klein (KK) excitation propagating towards a $D7$-brane stack. The sum over the KK modes corresponds to a propagation that takes place in a two-dimensional bulk space transverse to the $D7$ stack, see Fig. 2. Consequently, this process yields logarithmic contributions breaking the no-scale invariance of the Kähler potential.[6,9] Taking these logarithmic contributions into account the final effective action (obtained in the T^6/Z_N orbifold limit) contains[9]

$$S \ni \frac{1}{(2\pi)^3}\int_{M_4\times\mathcal{X}_6} e^{-2\Phi}\mathcal{R}_{(10)} + \frac{4\zeta(2)\chi}{(2\pi)^3}\int_{M_4}\left(1-\sum_{k=1,2,3} e^{2\Phi}T_k \log log\frac{R_\perp^k}{w}\right)\mathcal{R}_{(4)}. \quad (2.4)$$

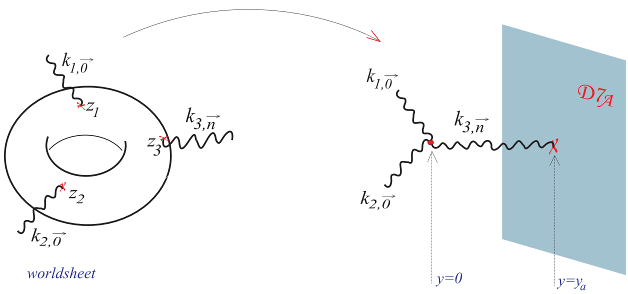

Fig. 2. Nonzero contribution from one-loop; 3-graviton scattering amplitude of two massless gravitons and 1 KK mode corresponding to a closed string propagation in two-dimensions towards a $D7$ brane.

Here T_k is the brane tension of the kth stack, $R_\perp^k$ the size of the two-dimensional space transverse to the $D7$-stack and w an "effective" localization width of the graviton vertex, given by $w = \ell_s/\sqrt{N}$ with $l_s = \sqrt{\alpha'}$ the fundamental string length.[8]

From the correction terms (2.4) in the 4D reduced action we can readily extract the corresponding induced terms in the Kähler potential. For simplicity, we assume the same tension for all three brane stacks, so that $T_k \equiv T = e^{-\Phi}T_0$, and for each Kähler modulus $\mathcal{T}_k$ we denote $\tau_k = \mathrm{Im}\mathcal{T}_k$. For $D7$-brane stacks with orthogonal co-volumes, the internal volume is simply $\mathcal{V} = \sqrt{\tau_1\tau_2\tau_3}$, and the Kähler potential takes the following form:

$$\mathcal{K} = -2\ln\left(\sqrt{\tau_1\tau_2\tau_3} + \xi + \gamma\ln(\tau_1\tau_2\tau_3)\right) \equiv -2\ln(\mathcal{V} + \xi + \gamma\ln\mathcal{V}). \tag{2.5}$$

Computations for the orbifold and smooth CY cases show that the parameters ξ and γ are given by[8,9]

$$\gamma \equiv -\frac{1}{2}g_s T_0 \xi, \quad \text{with} \quad \xi = -\frac{\chi}{4} \times \begin{cases} \dfrac{\pi^2}{3} g_s^2 & \text{for orbifolds,} \\ \zeta(3) & \text{for smooth CY.} \end{cases} \tag{2.6}$$

In (2.6) tree-level contributions for the orbifold case have not been included, since the $\zeta(3)\chi$ correction to the EH term vanishes.[7,8] The identity $\zeta(2) = \frac{\pi^2}{8}$ has also been used in the orbifold action (2.4).

3. Inflationary Phase

From (2.3), we can readily compute the F-part of the scalar potential V_F. To this end, we assume that all complex structure moduli are stabilized and the fluxed induced superpotential $\mathcal{W}_0$ can be taken as a constant, while for convenience we introduce the new parameter $\mu = e^{\frac{\xi}{2\gamma}}$. The exact expression for V_F can thus be written as

$$V_F = -\frac{3\gamma\mathcal{W}_0^2}{\kappa^4}\frac{2(\gamma + 2\mathcal{V}) + (4\gamma - \mathcal{V})\ln(\mu\mathcal{V})}{(\mathcal{V} + 2\gamma\ln(\mu\mathcal{V}))^2(6\gamma^2 + \mathcal{V}^2 + 8\gamma\mathcal{V} + \gamma(4\gamma - \mathcal{V})\ln(\mu\mathcal{V}))}, \tag{3.1}$$

where $\kappa = \sqrt{8\pi G_N}$ is the reduced Planck length. In the large volume limit, V_F takes the simplified form

$$V_F = \frac{3\mathcal{W}_0^2}{2\kappa^4\mathcal{V}^3}(\xi + 2\gamma(\ln\mathcal{V} - 4)) + \cdots \tag{3.2}$$

By virtue of the logarithmic term the potential (3.2) acquires a global minimum, although this is an anti-de Sitter vacuum. Yet, a D-part contribution to the scalar potential comes from the existence of universal $U(1)$ factors associated with the three $D7$-brane stacks. In the large world-volume limit, this contribution takes the following form:

$$V_D = \frac{d_1}{\kappa^4\tau_1^3} + \frac{d_2}{\kappa^4\tau_2^3} + \frac{d_3}{\kappa^4\tau_3^3} + \cdots \tag{3.3}$$

where the d_i for $i = 1, 2, 3$ are model-dependent constants related to $U(1)$ Fayet–Iliopoulos (FI) terms.

For the subsequent discussion, it is useful to replace the dependence of the potential on Kähler moduli with the canonically normalized fields. We identify them with a logarithmic function of the volume and two perpendicular directions defined in terms of τ_i ratios. We also recall that we consider a simple setup with "orthogonal" $D7$-brane stacks, such that $\mathcal{V} = \sqrt{\tau_1\tau_2\tau_3}$. The new basis then reads

$$\phi = \sqrt{\frac{2}{3}}\ln(\mathcal{V}), \tag{3.4}$$

$$u = \frac{1}{2}\log\left(\frac{\tau_1}{\tau_2}\right), \tag{3.5}$$

$$v = \frac{\sqrt{3}}{6}\log\left(\frac{\tau_1}{\tau_3}\frac{\tau_2}{\tau_3}\right). \tag{3.6}$$

In terms of these, the total scalar potential $V_{\text{eff}} = V_F + V_D$ in the large volume limit is

$$\begin{aligned} V_{\text{eff}} \approx\ & \frac{3\mathcal{W}_0^2}{2\kappa^4} e^{-3\sqrt{\frac{3}{2}}\phi}(\gamma(\sqrt{6}\phi - 4) + \xi) \\ & + \frac{e^{-\sqrt{6}\phi}}{\kappa^4}(d_1 e^{-\sqrt{3}v - 3u} + d_2 e^{-\sqrt{3}v + 3u} + d_3 e^{2\sqrt{3}v}). \end{aligned} \tag{3.7}$$

In the inflationary scenario that we will discuss shortly, the field ϕ defined in Eq. (3.4) will play the role of the inflaton. In order to examine its evolution during the inflation era, we need first to stabilize the three moduli $u, v, \mathcal{V} = e^{\sqrt{\frac{2}{3}}\phi}$ and derive the constraints in order to ensure a dS vacuum. We first minimize V_{eff} with respect to the two transverse fields u, v, and find their values at the minimum:

$$u_0 = \frac{1}{6}\ln\left(\frac{d_1}{d_2}\right), \quad v_0 = \frac{1}{6\sqrt{3}}\ln\left(\frac{d_1 d_2}{d_3^2}\right). \tag{3.8}$$

Substituting back into (3.7) we obtain the simple expression

$$V(\phi) \simeq -\frac{C}{\kappa^4} e^{-3\sqrt{\frac{3}{2}}\phi}\left(\sqrt{\frac{3}{2}}\phi - 4 + q + \frac{3}{2}\sigma e^{\sqrt{\frac{3}{2}}\phi}\right), \tag{3.9}$$

where we have defined

$$C \equiv -3\mathcal{W}_0{}^2\gamma > 0, \quad d \equiv 3(d_1 d_2 d_3)^{\frac{1}{3}}, \quad q \equiv \frac{\xi}{2\gamma}, \quad \sigma \equiv \frac{2d}{9\mathcal{W}_0{}^2\gamma}. \tag{3.10}$$

A few comments are in order. First, in order to ensure a dS vacuum, the parameter γ must be negative, hence the coefficient C is positive. Moreover, the parameter d, related to the D-term part of the potential, is always positive. Furthermore, increasing of the value of the parameter q shifts the local extrema towards larger volumes. Finally, σ is the only free parameter of the model. It acquires negative values, hence the total coefficient of the last term is positive and is expected to uplift the minimum of the potential to positive values.

To study inflation and compute the slow-roll parameters we need to determine the extrema of the potential with respect to the inflaton field ϕ.[19] Thus, we take the first and second derivatives of the potential with respect to ϕ and obtain

$$V'(\phi) = 3\sqrt{\frac{3}{2}}\frac{C}{\kappa^4} e^{-3\sqrt{\frac{3}{2}}\phi}\left(\sqrt{\frac{3}{2}}\phi + q - \frac{13}{3} + \sigma e^{\sqrt{\frac{3}{2}}\phi}\right), \tag{3.11}$$

$$V''(\phi) = -\frac{27}{2}\frac{C}{\kappa^4} e^{-3\sqrt{\frac{3}{2}}\phi}\left(\sqrt{\frac{3}{2}}\phi + q - \frac{14}{3} + \frac{2}{3}\sigma e^{\sqrt{\frac{3}{2}}\phi}\right). \tag{3.12}$$

Requiring the vanishing of the first derivative, $V'(\phi) = 0$, we obtain two solutions which are expressed in terms of the two branches W_0 and W_{-1} of the Lambert W function (product logarithm):

$$\phi_- = -\sqrt{\frac{2}{3}}\left(q - \frac{13}{3} + W_0(-e^{-x-1})\right), \tag{3.13}$$

$$\phi_+ = -\sqrt{\frac{2}{3}}\left(q - \frac{13}{3} + W_{-1}(-e^{-x-1})\right). \tag{3.14}$$

The new parameter x introduced in the above solutions is defined by

$$x \equiv q - \frac{16}{3} - \log(-\sigma) \leftrightarrow \sigma = -e^{q-\frac{16}{3}-x}. \tag{3.15}$$

while ϕ_- is the local minimum and ϕ_+ the local maximum. Large volumes can be achieved at weak coupling for $q < 0$, implying a negative Euler number $\chi < 0$, see Eqs. (2.6) and (3.10).

Notably, most of the important quantities are expressed through simple analytical forms in terms of x. For example, the slow-roll parameter η depends only on x

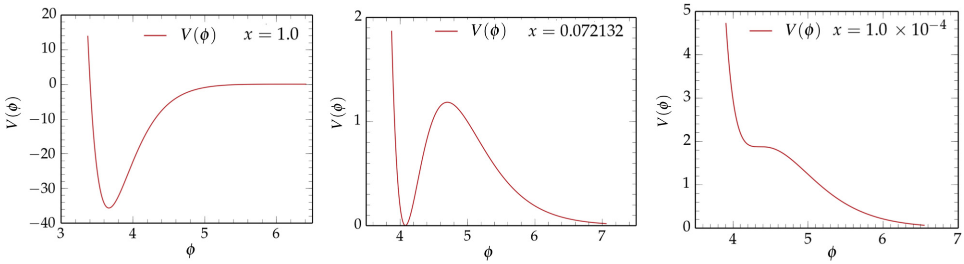

Fig. 3. Scalar potential $V(\phi)$ for different values of x giving an AdS, Minkowski or dS vacuum.

through the Lambert W function

$$\eta(\phi_{-/+}) = \frac{V''(\phi_{-/+})}{V(\phi_{-/+})} = -9\,\frac{1 + W_{0/-1}(-e^{-x-1})}{\frac{2}{3} + W_{0/-1}(-e^{-x-1})}. \tag{3.16}$$

Similarly, the distance between the two extrema is

$$\phi_+ - \phi_- = \sqrt{\frac{2}{3}}[W_0(-e^{-x-1}) - W_{-1}(-e^{-x-1})] > 0. \tag{3.17}$$

The parameter x thus clearly plays a significant role. For the critical value $x_c \simeq 0.072132$ the potential at the minimum vanishes, $V(\phi_-) = 0$, which corresponds to a Minkowski minimum. Below this critical value, in the region $0 < x < x_c$, the potential acquires a dS vacuum whereas for $x > x_c$ it displays an AdS minimum. For $x < 0$, the two branches of the Lambert function join and the potential loses its local extrema. The potential for the three regimes described above is depicted in Fig. 3.

Having determined the region of the parameter x which is consistent with dS minima, we are now ready to study cosmological implications and in particular inflationary observables. We first find that some well-known inflationary scenarios such as slow-roll inflation hilltop, cannot be realized in our restricted model. We can easily adjust the value of the slow-roll parameter η (which depends only on x) by varying $x \in (0, x_c)$, so that inflation starts near the maximum, and the modes exit horizon with the required value of the spectral index. It is found, however, that the slow-roll parameters ϵ, η remain much less than unity all the way down the slope, hence inflation does not stop, and as a result an unacceptably large number of e-folds is generated.

As we describe below, in order to study more general inflationary scenarios, we will scan the x parameter space. For each value of x, we can solve the evolution equation for the Hubble parameter and derive the relevant parameters to study the eventual inflationary stage. Before entering the details of such a procedure, we thus recall a few basic equations regarding the evolution of the expansion of the universe and the inflationary epoch assuming a single scalar field ϕ in the standard

Friedmann–Lemaître–Robertson–Walker (FLRW) background. The Friedmann equations for an expanding universe are

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + \kappa^2 V(\phi), \tag{3.18}$$

$$2\dot{H} = -\dot{\phi}^2, \tag{3.19}$$

where as usual $H(t) = \frac{\dot{a}}{a}$, represents the Hubble parameter. The equation of motion for the scalar field reads

$$\ddot{\phi} + 3H\dot{\phi} + \kappa^2 V'(\phi) = 0. \tag{3.20}$$

Changing variable through $\dot{H} = \frac{dH}{d\phi}\dot{\phi}$, Eq. (3.19) yields

$$\frac{dH}{d\phi} = H'(\phi) = -\frac{1}{2}\dot{\phi}. \tag{3.21}$$

Using (3.18) and expressing $\dot{\phi}$ as a function of H and V, we obtain the Hubble parameter evolution equation:

$$H'(\phi) = \mp\frac{1}{\sqrt{2}}\sqrt{3H^2(\phi) - \kappa^2 V(\phi)}. \tag{3.22}$$

The exact forms of the slow-roll parameters η, ϵ are[20]

$$\eta(\phi) = 2\frac{H''(\phi)}{H(\phi)}, \quad \epsilon(\phi) = -\frac{\dot{H}}{H^2} = 2\left(\frac{H'(\phi)}{H(\phi)}\right)^2, \tag{3.23}$$

while in the slow-roll limit they acquire the usual forms $\eta(\phi) \approx \frac{V''(\phi)}{V(\phi)}$, and $\epsilon(\phi) \approx \frac{1}{2}(\frac{V'(\phi)}{V(\phi)})^2$. From the first expression of ϵ in (3.23), we obtain

$$\frac{\ddot{a}}{aH^2} = 1 - \epsilon, \tag{3.24}$$

so that $\epsilon < 1$ is the natural criterium characterizing inflation, a phase with $\ddot{a} > 0$. Finally, the number of e-folds N is given by

$$N = \int_t^{t_{\text{end}}} H dt = \frac{1}{\sqrt{2}}\int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{\epsilon}}. \tag{3.25}$$

As mentioned above, one can investigate inflationary possibilities through a scan of the x parameter in the following way. The value of x determines the shape of the inflaton scalar potential $V(\phi)$, which enters the evolution equation (3.22) for the Hubble parameter. For a given value of x, solving this equation thus allows to compute the slow-roll parameters and number of e-folds, through Eqs. (3.23) and (3.25), and study the inflationary phase.

The above scan gave rise to a novel scenario where most of the e-folds are obtained near the minimum. In this scenario, the inflaton starts rolling down from a point close to the maximum towards the minimum of its potential with zero initial speed. If $\eta(\phi_+) < -0.02$, because at the inflection point the second derivative $V''(\phi)$ changes sign, the inflaton will pass through the point where $\eta(\phi_*) = -0.02$ before it crosses the inflection point. We can then choose the parameter x so that 60 e-folds are obtained from this point to minimum. Thus, in order to reproduce the observational data, the initial position of the inflaton has to be higher than the inflection point, where η is negative, so that $\eta = -0.02$ is taken at the horizon exit.

In order to realize this scenario, we have solved numerically the evolution equation (3.22) for various values of x, starting near the maximum with vanishing initial speed for the inflaton. The required number of e-folds, $N_* \simeq 60$ are achieved for $x \simeq 3.310^{-4}$ while the two extrema of the potential are found at $\phi_- = 4.334$ and $\phi_+ = 4.376$. The e-folds are computed from the horizon exit $\phi_* \simeq 4.354$ at which $\eta(\phi_*) = -0.02$, down to the minimum ϕ_-. Is it worth observing that the corresponding inflaton field displacement $\Delta\phi \simeq 0.02$, is much less than one in Planck units. Hence, it corresponds to small field inflation, and as such is compatible with the validity of the effective field theory. Finally, this model predicts an inflation scale $H_* \simeq 5 \times 10^{12}\,\mathrm{GeV}$ and a ratio of tensor to scalar perturbations $r \simeq 4 \times 10^{-4}$.

4. Waterfall Fields and Hybrid Inflation

Up to this point, we have explained how in the simple geometric set up of three $D7$-brane stacks we can ensure Kähler moduli stabilization in a dS vacuum and investigated the conditions to realize inflation. We found that logarithmic radiative corrections and brane magnetizations generate a scalar potential with a very shallow dS minimum, which can realize inflation with the required 60 e-folds collected near the minimum (as opposed -for example- to the case of hilltop scenario). However, the tight constraints imposed by the various requirements entail a metastable minimum with a cosmological constant much larger than the one observed today. A detailed consideration shows that this false vacuum of the so-obtained scalar potential is suggestive for a solution through hybrid inflation[21] where a waterfall field ends the inflation phase and settles to a lower (true) vacuum with the anticipated value of the cosmological constant. Such a waterfall field is realized by a scalar field with effective mass depending on the value of the inflaton. If this field becomes tachyonic under a certain critical value for the inflaton, it generates the waterfall direction of the scalar potential.

Within the present geometric configuration, potential waterfall field candidates are the various states associated with the excitations of open strings with endpoints attached to $D7$ brane stacks. The scalar components of these states may receive supersymmetric positive square masses from brane separation or Wilson lines, and nonsupersymmetric contributions due to the presence of the world-volume magnetic fields generating the D-terms required for moduli stabilization.

In the following, we briefly describe how these fields contribute to the materialisation of this scenario in the context of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. We assume a factorized 6-torus into three 2-tori $T^6 = T^2 \times T^2 \times T^2$ spanning the internal dimensions (45),

(67) and (89), respectively. The model under consideration consists of three $D7$ brane stacks, which we denote with $D7_1$, $D7_2$ and $D7_3$. Each of them spans four internal dimensions and is localized in the remaining two. This setup can be considered as dual to the configuration of the $D9$ and $D5$ branes as in the toroidal orbifold model described in the literature.[22,23] This is shown schematically in the following table where we impose T-duality along (45) dimensions.

	(45)	(67)	(89)
$D7_1$	$\cdot$	$\times$	$\times$
$D7_2$	$\times$	$\times$	$\cdot$
$D7_3$	$\times$	$\cdot$	$\times$

$\longrightarrow$

	(45)	(67)	(89)
$D9_1$	$\times$	$\times$	$\times$
$D5_2$	$\cdot$	$\times$	$\cdot$
$D5_3$	$\cdot$	$\cdot$	$\times$

We use a cross $\times$ to represent the $D7$ world-volume spanning the corresponding torus, and a dot $\cdot$ to indicate the transverse directions where the $D7$ brane is localised.

As motivated above, we can introduce magnetic fields $H_a^{(i)}$, on the a-th stack $D7_a$ and in the ith torus T_i^2. They are subject to the Dirac quantization condition $m_a^{(i)} \int H_a^{(i)} = 2\pi n_a^{(i)}$, leading to the magnetic field quantization $2\pi H_a^{(i)} \mathcal{A}_i = k_a^{(i)}$, where $4\pi^2 \mathcal{A}_i$ is the T_i^2 area. Here $m_a^{(i)}$, $n_a^{(i)}$ are the winding numbers and the flux quanta and we defined the ratio $k_a^{(i)} = n_a^{(i)}/m_a^{(i)} \in \mathbb{Q}$. The magnetic fields modify the world-sheet action by introducing boundary terms[24,25] and shift the modes of the charged oscillators by

$$\zeta_a^{(i)} = \frac{1}{\pi} \mathrm{Arctan}(2\pi\alpha' q_a H_a^{(i)}), \tag{4.1}$$

where $q_a = \pm 1, 0$ are the $U(1)$ charges of the open string endpoints.

The mass spectrum can be extracted, either from the field theory mass formula or from vacuum amplitudes, and one sees that when magnetic fields are introduced into the $D7$-brane configuration, tachyonic states may appear in the spectrum.[25,26] In general, one can eliminate them by introducing appropriate brane separations or Wilson lines.

To be concrete, we consider magnetic fields on each $D7$ stack, denoted by a circled cross $\otimes$ as the following table.

	(45)	(67)	(89)
$D7_1$	$\cdot$	$\otimes$	$\times$
$D7_2$	$\times$	$\cdot$	$\otimes$
$D7_3$	$\otimes$	$\times$	$\cdot$

Three different kinds of states appear. The first two describe strings with both endpoints on the "same" stack $D7_i$-$D7_i$ which are either neutral (attached to the same brane, hence with opposite endpoints charges) or doubly charged (stretching between the brane and its orientifold image). The last ones are mixed states $D7_i$-$D7_j$, with $i \neq j$. Due to the presence of magnetic fields, the massless states of the original orbifold model are modified. The masses of the $D7_i$-$D7_i$ doubly charged states read $\alpha' m^2 = -2|\zeta_i^{(j)}|$ whereas those of the $D7_i$-$D7_j$ states are of the form $(|\zeta_2^{(3)}| - |\zeta_1^{(2)}|)$, $(|\zeta_1^{(2)}| - |\zeta_3^{(1)}|)$ and $(|\zeta_3^{(1)}| - |\zeta_2^{(3)}|)$.

Observing the above mass formulae, it can be deduced that tachyonic states indeed appear in the spectrum.[25,26] The only way to eliminate all three potential tachyons along the $D7$-brane intersections ($D7_i$-$D7_j$ mixed states) is to choose $|\zeta_1^{(2)}| = |\zeta_2^{(3)}| = |\zeta_3^{(1)}|$. On the other hand, in order to uplift the tachyons on the $D7_i$-$D7_i$ sectors, we can introduce distance separations between branes and their images in the direction orthogonal to their world-volume, or Wilson lines. That is constant background gauge fields on unmagnetized world-volume tori. In the table below, we present a configuration keeping only one potential tachyonic state that can play the role of the waterfall field:[b]

	(45)	(67)	(89)
$D7_1$	$\cdot$	$\otimes$	$\times$
$D7_2$	$\times$	$\cdot$	$\otimes$
$D7_3$	$\otimes$	$\times$	$\cdot$

$\longrightarrow$

	(45)	(67)	(89)
$D7_1$	$\cdot$	$\otimes$	$\times_{A_1}$
$D7_2$	$\times$	$\cdot_{\pm x_2}$	$\otimes$
$D7_3$	$\otimes$	$\times_{A_3}$	$\cdot$

We introduce discrete Wilson lines along the third torus T_3^2 for the $D7_1$ stack and along the second torus T_2^2 for the $D7_3$ stack, while we separate the $D7_2$ stack from its orientifold image in its transverse directions. Next, we denote the $\mathcal{A}_i$ tori areas $(i = 1, 2, 3)$ as power fractions of the total volume $\mathcal{A}_i \equiv \alpha' r_i \mathcal{V}^{1/3}$, with $r_1 r_2 r_3 = 1$ and U_i the corresponding complex structure moduli. Then, the masses for the doubly charged states in the three brane stacks are found to be[19]

$$\alpha' m_{11}^2 \approx -\frac{2|k_1^{(2)}|}{\pi r_2 \mathcal{V}^{1/3}} + \frac{a_1^2}{r_3 \mathcal{V}^{1/3}}, \tag{4.2}$$

$$\alpha' m_{22}^2 \approx -\frac{2|k_2^{(3)}|}{\pi r_3 \mathcal{V}^{1/3}} + y_2 r_2 \mathcal{V}^{1/3}, \tag{4.3}$$

$$\alpha' m_{33}^2 \approx -\frac{2|k_3^{(1)}|}{\pi r_1 \mathcal{V}^{1/3}} + \frac{a_3^2}{r_2 \mathcal{V}^{1/3}}, \tag{4.4}$$

where a_1, a_3 and y_2 are functions of the complex structure moduli U_i defined in footnote b. By choosing appropriately a_1, a_3 with respect to the values of the magnetic fluxes $|k_1^{(2)}|$ and $|k_3^{(1)}|$, one can eliminate the $D7_1$-$D7_1$ and $D7_3$-$D7_3$ tachyons. For $a_i = 1/2$, typical for $\mathbb{Z}_2$ orbifolds, this requires flux numbers smaller than wrapping numbers. On the other hand, the $D7_2$-$D7_2$ state becomes tachyonic at and below a

[b] The following definitions are introduced: the discrete Wilson lines in the dual lattice are expressed as $A_k = a_{kx}\mathbf{R}_k^{*x} + a_{ky}\mathbf{R}_k^{*y}$, with $a_{kx}, a_{ky} \in \mathbb{Q}$. The $D7_k$ brane position x_k as $x_k \equiv x_k^x \mathbf{R}_{kx} + x_k^y \mathbf{R}_{ky}$ with $x_k^x, x_k^y \in \mathbb{Q}$, while $\mathbf{R}_{ik} \cdot \mathbf{R}_i^{*l} = \delta_k^l$. For later use, we also define $y_k(U) = \frac{4|x_k^x - iU x_k^y|^2}{\mathrm{Re}(U)}$ and $a_k(U) = \frac{|a_{ky} + iU a_{kx}|^2}{\mathrm{Re}(U)}$.

critical value of the volume that can be chosen to be in the vicinity of the minimum of the potential, as required for the waterfall field, denoted by φ_- in the following.

We turn now to the scalar potential. The magnetic fields contribute through a D-term of the form

$$\begin{aligned} V_D &= \sum_a \frac{g^2_{U(1)_a}}{2}\left(\xi_a + \sum_n q^n_a|\varphi^n_a|^2\right)^2 + \cdots \\ &= \sum_{a=1,3} \frac{g^2_{U(1)_a}}{2}\xi^2_a + \frac{g^2_{U(1)_2}}{2}(\xi_2 + 2|\varphi_+|^2 - 2|\varphi_-|^2 + \cdots)^2 + \cdots , \end{aligned} \tag{4.5}$$

where in the second line contributions only from the tachyonic field and its charge conjugate are taken into account.

We have also explained that the tachyonic scalar, coming from strings stretching between the $D7_2$ brane stack and its image, may receive a positive mass contribution due to the brane position. In the effective field theory, this contribution is described by a trilinear superpotential obtained by an appropriate $N = 1$ truncation of an $N = 4$ supersymmetric theory. The physical mass for the canonically normalized fields can be computed from the physical Yukawa couplings, derived from the supergravity action, and can be expressed as[27] $\mathcal{W}_{\rm tach} = Y_{ijk}\,\varphi_i\varphi_j\varphi_k$, where Y_{ijk} are Yukawa coefficients expressed in terms of the Kähler metrics of related matter fields. Their volume dependence can be worked out and the final form of the coupling is

$$\mathcal{W}_{\rm tach} = g_s^{1/2}\kappa^3\sqrt{\frac{\mathcal{A}_2}{\alpha'\mathcal{V}}}\varphi_2\varphi_+\varphi_-, \tag{4.6}$$

which induces a scalar potential F-part of the form $V_F \ni m^2_{x_2}(|\varphi_+|^2 + |\varphi_-|^2)$ with $m^2_{x_2} = y_2(g^2_s/\kappa^2\mathcal{V})\mathcal{A}_2/\alpha'$. In addition to this mass-squared terms, the F-term scalar potential also contains quartic terms. They can be worked out and the leading term in the scalar potential for the tachyonic scalar is found to be of the form $V_F \ni \kappa^2 m^2_{x_2}|\varphi_-|^4$.

The effective scalar potential includes the D-term and F-term contributions and its final form is achieved after the minimization procedure whose details can be found in Ref. 19. Neglecting, in particular, the massive φ_+ field, the scalar potential receives the simplified form

$$V(\mathcal{V},\varphi_-) = \frac{C}{\kappa^4}\left(-\frac{\ln\mathcal{V} - 4 + q}{\mathcal{V}^3} - \frac{3\sigma}{2\mathcal{V}^2}\right) + \frac{1}{2}m^2_Y(\mathcal{V})|\varphi_-|^2 + \frac{\lambda(\mathcal{V})}{4}|\varphi_-|^4, \tag{4.7}$$

where the explicit forms of the volume-dependent mass m^2_Y and quartic coupling λ are given in terms of integers representing magnetic fluxes[19] and other string parameters. The final dependence of $V(\mathcal{V},\varphi_-)$ on the two fields has been written in the form of the hybrid scenario[21] scalar potential. In this form, it is even clearer that the role of the waterfall field is played by the scalar field φ_- associated with the state stretching between the $D7_2$ brane and its orientifold image. Its mass squared m^2_Y depends on the internal volume $\mathcal{V}$, directly related to the inflaton, and turns negative when the internal volume acquires a critical value. A waterfall direction is thus generated, as in the hybrid scenario. This mechanism leads to a new lower minimum. It has been found[19] that when only a single tachyon is involved, the amount of reduction falls short to explain the observed value of dark energy of our universe. This situation can be remedied within our model by introducing more tachyons, coming from the two other $D7$-brane stacks and from a fourth magnetized stack, parallel to one of the initial stacks. These additional tachyons contribute negatively to the scalar potential and are sufficient to achieve the present value of the cosmological constant. Apart from (or instead of) these contributions, one should of course expect new physics at low energies, leading to other phase transitions that affect the scalar potential. Hence, the precise tuning of the vacuum energy within our high-energy model should be regarded as a proof of principle.

5. Conclusions

In this presentation, we have discussed aspects of perturbative corrections in the weak string coupling regime and large volume compactifications within the framework of type IIB string theory. We have considered a geometric configuration of intersecting $D7$-brane stacks and investigated the role of logarithmic corrections which are present by virtue of local tadpoles induced by localized gravity kinetic terms. Such terms are generated from the dimensional reduction of the R^4 terms in the effective 10-dimensional action and arise only in four space–time dimensions. We have shown that in this string theory context, metastable de Sitter vacua can be ensured together with Kähler moduli stabilization.

Subsequently, we have examined the possibility of realizing the mechanism of cosmological inflation. We have shown that the inflationary scenario can be naturally implemented when the internal volume modulus is considered to be the inflaton field. The effective scalar potential contains only a single free parameter, whose value is fixed in order to meet the inflationary conditions and in particular the requirement of 60 e-folds which, in our construction, are collected near the minimum, while the horizon exit occurs near the infection point. These requirements, however, lead to a very shallow potential with its minimum much larger than the known value of the cosmological constant.

To resolve this discrepancy, we have suggested that a string version of the hybrid inflationary scenario could be realized where possible waterfall fields could be identified with some of the charged string states stretching between the branes and their orientifold images. In the effective theory, the (volume dependent) masses squared of such excitations consist of positive contributions from brane separations and possible negative ones when world-volume magnetic fields are turned on. With suitable conditions on various quantities such as magnetic fluxes and geometric characteristics, tachyonic states may appear. For illustrative purposes, we have presented a simple scenario where a tachyonic field arises, with its mass squared turning negative as soon as the internal volume acquires a critical value. This is exactly what is required for a waterfall field. More specifically, in the effective field theory, states of the kind described above induce specific contributions to the F- and D-terms of the effective potential. When these contributions are included in the total scalar potential,[19] the tachyonic field can indeed play the role of the waterfall field, providing in this way an explicit string realization of the hybrid inflationary scenario. Finally, we have discussed the role of multiple tachyonic fields in order to obtain the present value of the cosmological constant. Remarkably, the present construction offers an explicit counter-example to de Sitter Swampland conjecture.

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