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1 Online learning of the transmission matrix of 2 dynamic scattering media

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8 **Abstract:** Thanks to the latest advancements in wavefront shaping, optical methods have
9 proven crucial to achieve imaging and control light in multiply scattering media, like biological
10 tissues. However, the stability times of living biological specimens often prevent such methods
11 from gaining insights into relevant functioning mechanisms in cellular and organ systems.
12 Here we present a recursive and online optimization routine, borrowed from time series
13 analysis, to optimally track the transmission matrix of dynamic scattering media over arbitrarily
14 long timescales. While preserving the advantages of both optimization-based routines and
15 transmission-matrix measurements, it operates in a memory-efficient manner. Because it can
16 be readily implemented in existing wavefront shaping setups, featuring amplitude and/or phase
17 modulation and phase-resolved or intensity-only acquisition, it paves the way for efficient optical
18 investigations of living biological specimens.

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20 1. Introduction

21 Optical methods are an irreplaceable tool to investigate biological media. They deliver images at
22 numerous contrast mechanisms [1], and can activate injected biomolecules [2] and fluorescent
23 markers [3]. However, precisely delivering light in space and time through biological tissues is
24 not straightforward, as photons get multiply scattered by heterogeneities of tissues, limiting their
25 penetration depth [4].

26 Another current challenge lies in tracking the scattering behaviour of living specimens, with
27 decorrelation times up to only a few ms [5]. This proves crucial to understand the functioning
28 mechanisms of cells and organisms, which requires their observation at extremely different
29 timescales, from nanoseconds (at a molecular level) to minutes (for organ systems) [6]. The need
30 for fast data acquisitions results, in turn, in measurements with inherently low signal-to-noise
31 ratios (SNRs), and requires solving long and multidimensional time series [7], whose prohibitive
32 size can make their evaluation problematic.

33 Wavefront shaping techniques have established themselves as the tools of choice to guide
34 light in scattering media [8]. The transmission of arbitrary fields [9], point-spread-function
35 (PSF) engineering [10], imaging [11], as well as tuning energy transmission through scattering
36 media [12], become all accessible if the transmission or reflection matrix of the medium is
37 measured [8, 13]. In what follows, we will generically refer to transmission and reflection matrices
38 as 'transfer matrices'. Conventional methods to retrieve the transfer matrix yield sub-optimal
39 solutions in noisy environments [8]. Those optimization routines which can compensate for noise
40 in the transfer matrix [14] require storing in memory the whole history of past measurements,
41 making them unsuited with long streams of data.

42 Iterative, optimization-based, sequential algorithms to focus through scattering media yield an
43 increase in the focus intensity already at their early iterations, which makes them the preferred
44 option on dynamic media. Importantly, they are cast as *recursive* procedures, *i.e.*, computing
45 the new estimate of the solution only requires the previous estimate and the new data point.

46 Unfortunately, their stochastic nature makes optimization over a set of output modes less
 47 reliable and the transmission of arbitrary fields prohibitive. Moreover, these procedures rely on
 48 maximizing a given metric, limiting light control to one predefined task. Various implementations
 49 derived from genetic algorithms [15, 16] have shown better resilience to noise than sequential
 50 algorithms, however at the cost of a higher computational complexity and careful choice of
 51 several adjustable parameters.

52 In signal processing, communications and finance, where most datasets are multidimensional
 53 time series, the recursive least-squares (RLS) algorithm has played a central role for system
 54 identification and prediction [17–19]. It allows optimal learning of linear predictors in an
 55 online manner—predictors are updated every time a new piece of data is sequentially made
 56 available, however past data do not need to be stored in memory. Consequently, its computational
 57 complexity is independent of the length of the time series, so iterations can be run over and over,
 58 ideally at the same rate as data acquisition (real-time operation).

59 Here, we demonstrate that the RLS algorithm represents a valuable tool to optimally estimate
 60 the transfer matrix of dynamic scattering media online and recursively. The least-squares
 61 optimization ensures resilience to noise. The algorithm is provided with a tunable memory,
 62 such that the dynamics of the scattering medium is accounted for. By doing so only the most
 63 reliable data points, *i.e.*, those acquired within the stability time of the medium, are used
 64 during the optimization. We justify how the RLS model can fit a wide variety of dynamic
 65 mechanisms happening in scattering media. Its performance is showcased with both simulated
 66 and experimental results, tracking the transmission matrix and the time-gated reflection matrix at
 67 realistic noise levels and well-controlled stability times, upon translating the scattering medium
 68 across the incident beam. We further show how light optimization can be achieved with binary
 69 amplitude or phase modulation and with phase-resolved or intensity-only measurements. Based
 70 on its computational complexity, we discuss its feasibility for light control in living biological
 71 specimens at large fields of view. Its simple implementation and the low number of adjustable
 72 parameters (whose choice is motivated in the next sections) make our proposed method readily
 73 applicable in existing wavefront shaping setups.

74 2. Methods

75 The method bears similarities with conventional routines for the measurement of the transfer
 76 matrix, and its working principle is graphically summarized in Fig. 1(a). However, here we allow
 77 the transfer matrix $\mathbf{X}_t \in \mathbb{C}^{M \times N}$ of the scattering medium to be dynamic, where we have denoted
 78 the number of output and input degrees of freedom with M and N , respectively. At every time
 79 step t , while probing the medium with the input $\mathbf{a}_t \in \mathbb{C}^N$ and collecting the corresponding output
 80 $\mathbf{y}_t = \mathbf{X}_t \mathbf{a}_t \in \mathbb{C}^M$, we aim to solve the optimization problem $\hat{\mathbf{X}}_t = \arg \min_{\mathbf{X}_t} \mathcal{L}_t(\mathbf{X}_t)$, with

$$\mathcal{L}_t(\mathbf{X}_t) \equiv \sum_{\tau=1}^t \left(\lambda^{t-\tau} \|\mathbf{y}_\tau - \mathbf{X}_t \mathbf{a}_\tau\|^2 \right) + \delta \lambda^t \|\mathbf{X}_t\|_F^2, \quad (1)$$

81 and where $\|\cdot\|$ and $\|\cdot\|_F$ denote the L^2 -norm of a vector and the Frobenius norm of a matrix,
 82 respectively. Although for sake of generality the inputs and the outputs are assumed to be complex,
 83 we will also report an implementation where they are real, meaning that only the amplitude of
 84 the input beam is modulated and the intensity of the output fields is measured. Equation (1)
 85 is a linear least-squares loss function, featuring Tikhonov regularization via the regularization
 86 constant δ . Note, however, that each *data-fidelity term* $\|\mathbf{y}_\tau - \mathbf{X}_t \mathbf{a}_\tau\|^2$ is exponentially weighted
 87 in time, such that the old pieces of data (corresponding to $\tau \ll t$) are less relevant than the
 88 most recent ones in the current estimation of the transfer matrix at time t . In other words, the
 89 *forgetting factor* $\lambda \leq 1$ endows the algorithm with a memory, which allows it to cope with
 90 dynamic transfer matrices—at every time step t , the optimization problem is solved anew, using

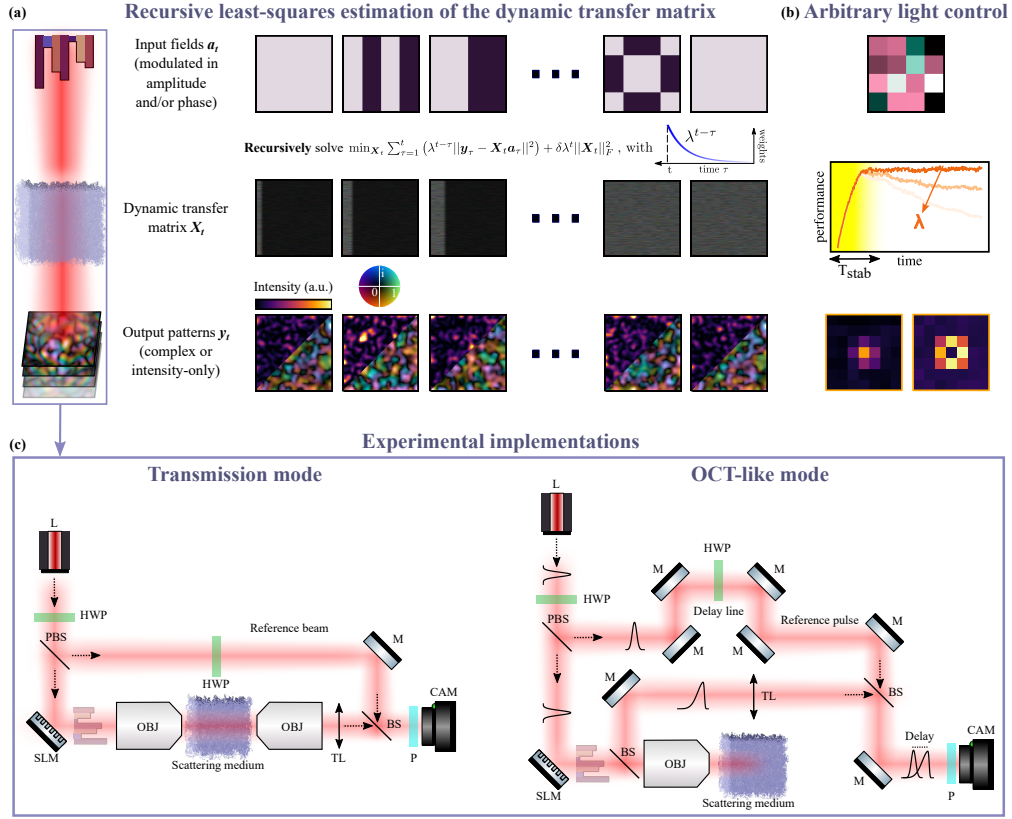


Fig. 1. Graphical summary of the RLS estimation technique and experimental implementations. (a) A sequence of input fields, modulated in amplitude and/or in phase (here Hadamard modulation patterns are shown), interacts with a dynamic scattering medium with unknown transfer matrix. Each (input, output) pair is used to update recursively the estimation of the dynamic transfer matrix, minimizing a regularized linear least-squares loss function, where each data-fidelity term is weighted via the coefficient $\lambda^{t-\tau}$. (b) At every time step, upon optimizing the coefficient λ , the current estimate of the transfer matrix can be used to achieve arbitrary light control through the scattering medium (here, a focus and a donut-shaped beam are displayed). (c) We demonstrated our method with a setup in transmission mode, for the retrieval of the transmission matrix (left), and with an OCT setup, for the retrieval of the time-gated reflection matrix (right). L: laser source; HWP: half-wave plate; (P)BS: (polarizing) beam-splitter; SLM: liquid-crystal-based spatial light modulator; OBJ: objective lens; TL: tube lens; P: polarizer; M: mirror; CAM: camera.

91 the whole history of past data, where more contribution is given to newest data. Evidently, in the
 92 case of a static scattering medium, all measurements can be equally trusted, thus Eq. (1) reduces
 93 to a typical regularized linear least-squares problem upon setting $\lambda = 1$. Once λ and δ are fixed,
 94 the least-squares problem has a unique solution, provided the inputs are linearly independent,
 95 which is the case in conventional transfer-matrix measurements, where the inputs are drawn from
 96 the Hadamard basis of order N .

97 The choice of exponential weights for Eq. (1) is motivated by the physics of our problem.
 98 We aim to follow the evolution of the transfer matrix of dynamic scattering media, subjected to
 99 uncorrelated variations, whereby the total transferred power fraction is constant in time. These

100 conditions apply in a wide variety of dynamic mechanisms in scattering media investigated
 101 with visible and near-infrared light, *e.g.* whenever their inner scatterers move due to functional
 102 changes [5, 20], or even when the sample drifts away from its initial position, suggesting that our
 103 method can also be used as an online calibration tool of imaging systems. In all these situations,
 104 the transfer matrix can indeed be described by the time series [21],

$$X_t = \frac{\sigma_X}{\sqrt{\sigma_X^2 + \sigma_P^2}} (X_{t-1} + P_t), \quad (2)$$

105 where we assume that both the transfer matrix and the perturbation matrix P_t are random
 106 variables independently drawn from complex Gaussian distributions with zero mean and constant
 107 variance σ_X^2 and σ_P^2 , respectively [22]. Equation (2) denotes an autoregressive model of order 1,
 108 AR(1), whose autocovariance is proportional to $(\sigma_X / \sqrt{\sigma_X^2 + \sigma_P^2})^t$, justifying our exponentially
 109 weighted model of Eq. (1). When focusing through dynamic scattering media following Eq. (2),
 110 the stability time of the enhancement is proportional to σ_P^{-2} [21]. This means that the optimal
 111 weight λ should follow the same dependence, thus in principle requiring the knowledge of the
 112 rate of change of the scattering medium. A strategy for automatically tuning the forgetting factor
 113 will be discussed in section 4.

114 Crucially, minimizing the loss function of Eq. (1) does not require storing the whole history of
 115 past data. This becomes apparent if we recall that the linear least-squares estimate of X_t , \hat{X}_t ,
 116 satisfies the normal equations,

$$C_t \hat{X}_t^H = K_t, \quad (3)$$

117 with the covariance matrix of inputs and the cross-covariance matrix at time t respectively defined
 118 as,

$$C_t \equiv \sum_{\tau=1}^t \left(\lambda^{t-\tau} \mathbf{a}_\tau \mathbf{a}_\tau^H \right) + \delta \lambda^t \mathbf{I}_N \in \mathbb{C}^{N \times N} \quad (4a)$$

119

$$\mathbf{K}_t \equiv \sum_{\tau=1}^t \lambda^{t-\tau} \mathbf{a}_\tau \mathbf{y}_\tau^H \in \mathbb{C}^{N \times M}, \quad (4b)$$

120 with \mathbf{I}_N denoting the identity matrix of order N and the superscript H standing for Hermitian
 121 transposition. The quantities calculated in Eqs. (4) can be both estimated recursively, as follows:

$$C_t = \lambda C_{t-1} + \mathbf{a}_t \mathbf{a}_t^H \quad (5a)$$

122

$$\mathbf{K}_t = \lambda \mathbf{K}_{t-1} + \mathbf{a}_t \mathbf{y}_t^H. \quad (5b)$$

123

124 Equations (5) mean the loss defined in Eq. (1) can be minimized from the new piece of data
 125 $(\mathbf{a}_t, \mathbf{y}_t)$ and the previous estimates of the covariance and cross-covariance matrices, whose sizes
 126 are independent of the amount of past data. It becomes now clear how the RLS algorithm
 127 combines the benefits of transfer-matrix-based and optimization approaches. Using a recursive
 128 procedure, a typical asset of, *e.g.*, the continuous sequential algorithm (CSA), the partitioning
 129 algorithm [21], or more computationally intense genetic algorithms [15], the full X_t is estimated
 130 *in parallel* at all output pixels, thereby preserving all light-control capabilities allowed by the
 131 knowledge of the transfer matrix [10–12, 14] [Fig. 1(b)]. In principle, the transfer matrix could
 132 be obtained from Eq. (3) as $\hat{X}_t = \mathbf{K}_t^H (C_t^{-1})^H$. However, in what follows we will implement
 133 the *inverse QR-decomposition-based RLS* (abbreviated as inverse QRD-RLS) algorithm [23].
 134 Because it avoids matrix inversions and it always preserves the non-negativeness of the covariance
 135 matrix, it possesses higher numerical stability than directly inverting Eq. (3). Overall, it boils

136 down to performing a QR decomposition of a matrix constructed from the new data and the
 137 previous estimate of the square root of the inverse covariance matrix. This results in few lines of
 138 code which can be readily implemented in any programming language using standard libraries or
 139 built-in functions (see the box Algorithm 1 and the corresponding code available at Ref. [24]).
 140 As can be seen from Eq. (1) and Algorithm 1, the regularization constant δ is used to construct
 141 the initial estimate of the square root of the inverse correlation matrix, hence it mostly impacts
 142 the convergence speed at early iterations. In section 4, the choice of its value will be discussed.

Algorithm 1: Inverse QRD-RLS update

Initializations: $\hat{\mathbf{X}}_0 = \mathbf{0}$, $(\mathbf{C}_0^{-1})^{1/2} = \delta^{-1/2} \mathbf{I}_N$

Input: New input pattern \mathbf{a}_t , new output pattern \mathbf{y}_t , previous estimate of the transfer
 matrix $\hat{\mathbf{X}}_{t-1}$, previous estimate of the square root of the inverse covariance matrix
 $(\mathbf{C}_{t-1}^{-1})^{1/2}$, forgetting factor λ

/* Construction of the matrix \mathbf{U} */

1 $\mathbf{U} = \begin{bmatrix} 1 & \lambda^{-1/2} \mathbf{a}_t^H (\mathbf{C}_{t-1}^{-1})^{1/2} \\ \mathbf{0} & \lambda^{-1/2} (\mathbf{C}_{t-1}^{-1})^{1/2} \end{bmatrix}$

/* QR decomposition of \mathbf{U}^H */

2 $\mathbf{U}^H = \mathbf{Q} \mathbf{V}^H$

3 $\mathbf{V} = \begin{bmatrix} v_{11} & \mathbf{0}^H \\ v_{21} & (\mathbf{C}_t^{-1})^{1/2} \end{bmatrix}$

/* Update of the transfer matrix */

4 $\hat{\mathbf{X}}_t = \hat{\mathbf{X}}_{t-1} + (\mathbf{y}_t - \hat{\mathbf{X}}_{t-1} \mathbf{a}_t) v_{21}^H v_{11}^{-1}$

5 **return** $\hat{\mathbf{X}}_t$ and $(\mathbf{C}_t^{-1})^{1/2}$

143 **3. Experiments**

144 Figure 1(c) shows the sketches of the experimental implementations used to demonstrate our
 145 method. Both are based on phase-shifting digital holography to retrieve the complex output fields
 146 \mathbf{y}_t after interacting with a multiply scattering medium. The medium is an opaque deposit of ZnO
 147 nanoparticles (size < 100 nm, relative transmittance ~ 0.15), whose thickness ($20 \mu\text{m}$) is 5 to
 148 7 transport mean free paths, ensuring full mixing of its optical modes at the output. The input
 149 fields are shaped via a reflective, phase-only and liquid-crystal-based spatial light modulator
 150 (SLM, Meadowlark Optics HSP512L-1064) and focused on the scattering medium with an
 151 objective with a numerical aperture of 0.4 (Olympus PLN20X). A region-of-interest containing
 152 ~ 80 speckle grains is imaged onto a CCD camera (Manta G-046B, Allied Vision) via a tube lens,
 153 yielding a pixel size of $0.2 \mu\text{m}$ at the CCD plane. Before impinging onto the SLM, part of the
 154 beam is redirected along a reference arm with a polarizing beam splitter (PBS), and subsequently
 155 recombined with the scattered beam through a beam splitter (BS). The relative power of the two
 156 beams, yielding the maximum interference contrast, is adjusted via two half-wave plates, one
 157 along the common path and one along the reference arm, while a polarizer in front of the camera
 158 filters out any potential residual ballistic component traveling along with the scattered beam.
 159 In the experiments in transmission [Fig. 1(c), left], the beam exiting the scattering medium
 160 is collected at a distance of ~ 1.5 mm, where a fully developed speckle pattern was observed,
 161 with another Olympus PLN20X 0.4 NA objective. The light source (MaiTai HP Ti:Sapphire
 162 laser, Spectra-Physics) is set to monochromatic operation mode at a wavelength of 808 nm. The
 163 experiments in reflection [Fig. 1(c), right] reproduce a typical optical coherence tomography

164 (OCT) setup, whereby ultrashort pulses (with a central wavelength of 808 nm and a duration of
165 100 fs) are sent through the scattering medium and the backscattered, elongated pulses are gated
166 at a time delay set by a delay line along the reference arm.

167 Dynamics is introduced by transversally translating the scattering medium across the incident
168 beam, with independent and randomly distributed Gaussian steps, whose standard deviation
169 determines the stability time of the medium. More details on it will be provided in the next
170 section.

171 4. Results

172 Figure 2 summarizes the performance of the RLS algorithm for the online estimation of the
173 transmission matrix. The beam incident onto the SLM is modulated according to the Hadamard
174 patterns with $N = 64$ pixels. Every time an input \mathbf{a}_t is sent through the scattering medium and the
175 corresponding output field \mathbf{y}_t is measured, the inverse QRD-RLS update routine of Algorithm
176 1 is executed, yielding an estimate $\hat{\mathbf{X}}_t$ of the transfer matrix. Note, that this procedure can be
177 continuously repeated—after sending the N -th input, the first Hadamard vector or any other
178 known input pattern can be sent. As long as the scattering medium is static, probing it with the
179 same input multiple times corresponds to oversampling the unknown $N \times M$ coefficients of its
180 transfer matrix, thereby improving their estimation. It is indeed known that the covariance of the
181 estimated transfer matrix is inversely proportional to \mathbf{C}_t^{-1} , thus decreasing as t^{-1} [17]. Since the
182 true value \mathbf{X}_t is unknown, the quality of our reconstruction is evaluated via the intensity of a focus
183 produced behind the scattering medium. We report the intensity enhancement, relative to the
184 average intensity of a non-optimized speckle pattern [8]. The learning curve for a static scattering
185 medium, obtained from the RLS algorithm, is shown as an orange trace in Fig. 2(a). The temporal
186 axis is expressed in units of T_{TM} , which is defined as the time needed to update the estimation of
187 transfer matrix N times. In other words, a conventional transfer matrix experiment lasts T_{TM} .
188 Equivalently, a normalized time of 2 means the oversampling ratio is 2. To showcase the beneficial
189 effect of oversampling, the blue trace shows the performance of a conventional transfer-matrix
190 measurement, lasting until $t/T_{TM} = 1$, thus using N measurements. At times $t/T_{TM} \leq 1$, the
191 two approaches are equivalent—data are not oversampled. At later times, however, one can take
192 advantage of the whole history of past data to build an estimate more resilient against noise. Our
193 values of the enhancement, when compared to the number of input degrees of freedom N , are on a
194 par with previously reported measurements with no oversampling [13, 25, 26].

195 The same procedure is repeated with dynamic scattering media. By duly tuning the amplitude
196 of their movements, we achieve different stability times T_{stab} (also expressed in units of T_{TM}).
197 These are estimated as the time constant of an exponential function fitting the tails of the blue
198 traces. Note, that the scattering medium is dynamic for the whole duration of the experiments. At
199 oversampling ratios in the range 3-4, we increase the focus intensity by a factor between 1.5 and 2,
200 compared to the values after a conventional transfer-matrix approach. Upon decreasing T_{stab} , the
201 oversampling ratio decreases too, and the performances of the two approaches gradually match,
202 however the RLS estimation always operates in a memory-efficient manner. With dynamic media,
203 forgetting factors $\lambda < 1$ should be used. In our experiments featuring T_{stab} in the range 1-4,
204 we have chosen $1 - \lambda \approx 10^{-5}$, achieving a good compromise between tracking capability and
205 numerical stability. Interestingly, it has been shown that the optimal forgetting factor heavily
206 depends on the number of unknown parameters N which, fortunately, is under user control [27].
207 Furthermore, the structure of Algorithm 1 suggests that each inverse QRD-RLS iteration may
208 be run at a different value of λ , allowing the user to pick the one yielding the best performance
209 in an online manner, *i.e.*, with no need to restart the optimization anew and using the current
210 enhancement as a feedback to tune the next value of λ . Trivially, the optimal value for static media
211 is instead $\lambda = 1$. The best regularization constant δ depends on the SNR of the measurements. In
212 our experiments, the fact that the RLS algorithm is on a par with the conventional transfer-matrix

213 approach at $t/T_{TM} \leq 1$ and $T_{TM} < T_{stab}$ suggests that the selected regularization constant (here
 214 $\delta = 1$) is optimized for the best performance.

215 In order to test the suitability of the cost function in Eq. (1) and the model in Eq. (2) to our
 216 experimental settings, the experiments at the top row of Fig. 2 are reproduced with numerical
 217 simulations, following the AR(1) model of Eq. (2) [Figs. 2(d)-(f)]. We simulated a finite SNR by
 218 corrupting the outputs y_i , with average power σ_y^2 , with additive white complex Gaussian noise
 219 with variance σ_{noise}^2 and setting the simulated SNR as $\text{SNR}_{sim} \equiv \sigma_y^2/\sigma_{noise}^2$. In experiments, the
 220 noise was estimated from the standard deviation of the intensity enhancement at $t/T_{TM} > 1$ in
 221 Fig. 2(a), while the signal level was calculated as the average intensity of *non-optimized* speckle
 222 patterns, with the ratio between the two yielding the experimental SNR, SNR_{exp} . At comparable
 223 values of stability times and SNR_{exp} and SNR_{sim} [noted in Figs. 2(a) and (d), respectively,
 224 for a static medium], quantitative agreement is overall obtained. For example, if the SNR is
 225 increased by a factor of 2 doubling the number of phase-stepped images for field reconstruction,
 226 the experimental performance is the one plotted as an inset in Fig. 2(a). The same trend is
 retrieved by simulating measurements with halved σ_{noise}^2 [inset in Fig. 2(d)].

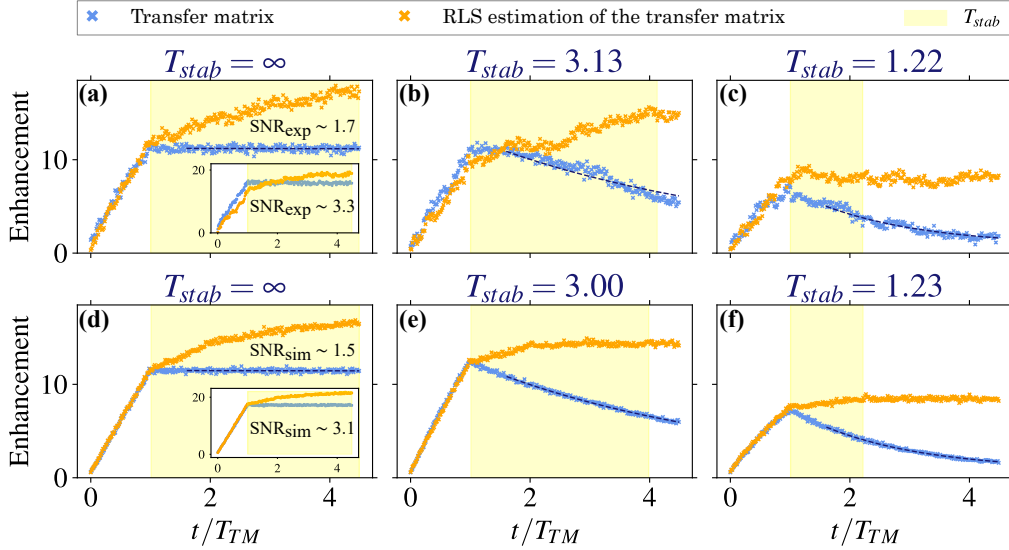


Fig. 2. Enhancement of the intensity at one output pixel produced through a scattering medium, as a function time. T_{TM} is the time to optimize over all the $N = 64$ degrees of freedom once. Blue points: conventional transfer-matrix measurement, lasting until $t/T_{TM} = 1$. Its estimate is held constant for later times, allowing the extraction of the stability time T_{stab} of the scattering medium via exponential fitting (dashed dark blue traces). Note that, when a finite T_{stab} is reported, the scattering medium is dynamic for the whole duration of the experiment. Orange points: RLS estimation of the transfer matrix. (a)-(c): experimental results, averaged over 9 realizations of a focus produced at the center of the camera field of view, upon measuring different regions of the scattering medium. (d)-(f): corresponding simulations at comparable T_{stab} . The insets in panels (a), (d) show the same results obtained after doubling the SNR, yielding an improvement of the enhancement by a factor of $\sim \sqrt{2}$. The SNR is noted in panels (a) and (d) for the case of a static medium.

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Analogous results, plotted in Fig. 3, are obtained with the non-invasive OCT setup on the right-hand side of Fig. 1(c), setting the time delay yielding the maximum average gated intensity. In this instance, the transfer matrix is the time-gated reflection matrix [28]. The two learning curves corresponding to the retrieval of the transfer matrix are compared to a conventional

232 optimization routine, which can recursively track the changes in the scattering medium, namely
 233 the CSA (in cyan). After blocking the beam along the reference arm, we implement a version of
 234 the CSA modulating half of the SLM pixels (corresponding to the +1 or -1 entries of the
 235 N Hadamard patterns) at each iteration, yielding the best interference contrast (thus bearing
 236 similarities to the partitioning algorithm too [21]). It displays comparable performances to a
 237 conventional transfer-matrix measurement with a static medium [although convergence is reached
 238 later, owing to its stochastic nature, Fig. 3(a)], and it shows solid tracking capabilities in dynamic
 239 environments [Figs. 3(b)-(c)]. Still, the resilience to noise of the inverse QRD-RLS algorithm
 240 makes it the preferred choice in this setting too, achieving an intensity twice as high as the one
 241 obtained with the CSA. The bottom row of Fig. 3 shows the corresponding focal spots produced
 by each algorithm at the last time step.

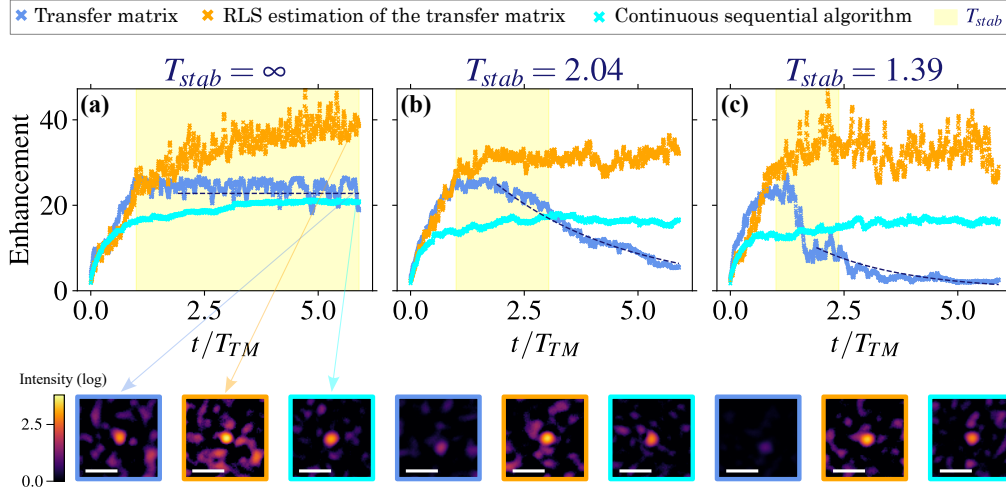


Fig. 3. Enhancement of the intensity at one output pixel produced in a scattering medium, as a function time. T_{TM} is the time to optimize over all the $N = 256$ degrees of freedom once. Blue points: conventional transfer-matrix measurement, lasting until $t/T_{TM} = 1$. Its estimate is held constant for later times, allowing the extraction of the stability time T_{stab} of the scattering medium via exponential fitting (dashed dark blue traces). When a finite T_{stab} is reported, the scattering medium is dynamic for the whole duration of the experiment. Orange points: RLS estimation of the transfer matrix. Cyan points: optimization via the continuous sequential algorithm (CSA). (a)-(c): experimental tracking performance, averaged over 8 realizations of foci produced across the full camera field of view, whence the higher variability than in Fig. 2. The bottom row displays typical images of the focus at the last time step, for each algorithm and each value of T_{stab} (in a common logarithmic scale to ease visibility and comparison). All the images show one realization only, and not an average of the 8 realizations. Note that the foci produced by the transfer matrix and RLS algorithms stand on a higher speckle background than those obtained from the CSA because of the superposition with the static reference field needed for interferometric phase estimation. We have accordingly accounted for this aspect when estimating the enhancements reported in the top row. The length of the white scale bars is $5 \mu\text{m}$. The data underlying this figure are available in Dataset 1 at Ref. [29].

242
 243 Because our proposed routine retrieves the coefficients of the transfer matrix at all output pixels
 244 simultaneously, its applications go beyond focusing. Figure 4 showcases two light-control tasks,
 245 through dynamic scattering media, enabled by the recursive and online estimation of the transfer
 246 matrix. The first one is maximal energy transmission, upon sending the leading singular vector

247 of the transfer matrix (top row) [12], and the second one consists in arbitrarily shaping a PSF
 248 (bottom row), here in a donut shape [10]. As expected, these trends replicate the performance on
 249 the focusing task of Fig. 2.

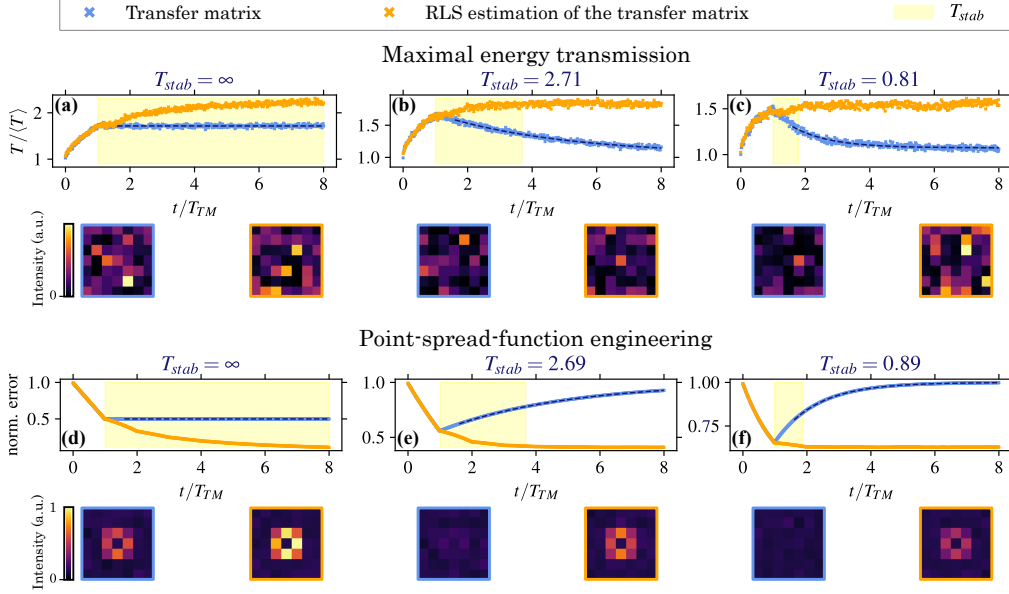


Fig. 4. Light control through dynamic scattering media goes beyond focusing. (a)-(c): Enhancement of the total transmittance across the full field of view through a scattering medium, as a function time and for different stability times T_{stab} . T_{TM} is the time to optimize over all the N degrees of freedom once. Blue points: conventional transfer-matrix measurement, lasting until $t/T_{TM} = 1$. Its estimate is held constant for later times, allowing the extraction of the stability time T_{stab} of the scattering medium via exponential fitting (dashed dark blue traces). When a finite T_{stab} is reported, the scattering medium is dynamic for the whole duration of the experiment. Orange points: RLS estimation of the transfer matrix. Here we used $N = 64$ and $M = 49$. (d)-(f): normalized error in the estimation of the transfer matrix of the scattering medium, used to project a donut-shaped beam at the output plane. The same color legend as in (a)-(c) applies. In each set of results, the bottom row displays typical camera images at the last time step. Simulated results.

250 The most important asset to characterize dynamic scattering media is the wavefront shaping
 251 device. Digital micromirror devices (DMD) offer a valuable alternative to liquid-crystal-based
 252 arrays and microelectromechanical systems (MEMS) modulators in terms of cost (~ 1 kUSD),
 253 pixel count ($> 10^5$) and operating frequencies (> 10 kHz). Despite their binary amplitude
 254 modulation, several strategies have been devised to enable light control through scattering
 255 media. Lee holography [30, 31] and superpixel-based related methods [32] achieve phase and
 256 amplitude control, at the expense of a more involved setup and a relatively low light efficiency,
 257 as they rely on an analog spatial filter. Aiming for a simple and non-invasive implementation
 258 suitable for real-life applications, Bayesian algorithms have been proposed to solve the phase
 259 retrieval problem $\mathbf{y}_t = |\mathbf{X}_t \mathbf{a}_t|^2$ (with $|\cdot|$ denoting the element-wise modulus operation), *i.e.*,
 260 recover the transfer matrix from intensity-only measurements and binary amplitude modulation
 261 of the inputs, thus transferring the hardware complexity to the software. Examples include
 262 the phase retrieval Variational Bayes Expectation-Maximization (prVBEM) [33, 34] algorithm,
 263 the phase retrieval Swept Approximate Message Passing (prSAMP) [35] algorithms, and their

264 corresponding compressive version, named phase retrieval Generalized AMP (prGAMP) [36].
 265 However, their complexities are of order $\mathcal{O}(t^2)$ per iteration, preventing their application to
 266 real-time online learning of long ($t \gg N$) and multidimensional time series.

267 In what follows, we show how to implement the RLS estimation technique using non-invasive,
 268 intensity-only measurements and binary amplitude modulation of the inputs. When performing
 269 wavefront shaping experiments with a DMD, light control is restricted to opening or blocking the
 270 modes of the scattering medium, so to achieve the desired output patterns. Hence, the knowledge
 271 of the complex-valued transfer matrix is of limited use. We now build on the contribution by Tao
 272 and colleagues [37]. They regard each binary input $\mathbf{a}_t \in \{0, 1\}^N$ as the sum of the first Hadamard
 273 vector $\mathbf{h}_1 = \{1\}^N$, referred to as “reference”, with any other Hadamard vector $\mathbf{h}_t \in \{+1, -1\}^N$,
 274 namely $\mathbf{a}_t = (\mathbf{h}_1 + \mathbf{h}_t)/2$. In a similar fashion to inline digital holography, in the output pixels
 275 where the reference intensity is larger than the response to an average input, the phase retrieval
 276 equation can be linearized. They derive the following linear approximation,

$$\frac{1}{2} |\mathbf{X}_t \mathbf{h}_1| \circ \left(|\mathbf{X}_t \mathbf{a}_t|^2 \oslash |\mathbf{X}_t \mathbf{h}_1|^2 - \mathbf{1} \right) \approx \text{Re}\{\mathbf{X}_t\} \mathbf{h}_t, \quad (6)$$

277 where \circ and \oslash denote element-wise vector multiplication and division, respectively, $\mathbf{1} \equiv \{1\}^M$
 278 and $\text{Re}\{\cdot\}$ stands for real part. Note, that the condition for a proper linearization is met, assuming
 279 the output pixels are independent, with a probability

$$\mathcal{P}(I > \langle I \rangle) = \int_{\langle I \rangle}^{\infty} p(I) dI = e^{-1} \approx 40\%, \quad (7)$$

280 where we have used the probability distribution of the speckle intensity, $p(I) \equiv \exp(-I/\langle I \rangle) / \langle I \rangle$
 281 [38]. As all the terms in its left-hand side $\tilde{\mathbf{y}}_t$ are known, we can recursively solve Eq. (6)
 282 for $\text{Re}\{\mathbf{X}_t\}$, minimizing a loss function like the one in Eq. (1), and interpreting \mathbf{h}_t and $\tilde{\mathbf{y}}_t$ as
 283 real inputs and outputs, respectively. The real (or, equivalently, imaginary) part of the transfer
 284 matrix is all is needed to focus at any output pixel where the linear approximation holds. The
 285 corresponding results in Fig. 5 indeed show the same trend as in Figs. 2, 3 and 4. Here,
 286 the enhancement is expressed relative to the maximum enhancement achievable with binary
 287 amplitude modulation $\approx 1 + (N/2 - 1)/\pi$ [39]. Feedback-based routines, like the binary version
 288 of the CSA [39] (plotted in cyan in Fig. 5), are highly impacted by experimental noise, as they
 289 rely on one single output value. In contrast, exploiting the past data allows us to provide solutions
 290 more resilient to noise.

291 To gain more insight into the performance of our experimental system, its throughput is
 292 estimated with the parameters from [13], namely $M = 256$ and 4 phase-shifted intensity images
 293 to evaluate each output field. In Fig. 6 we plot, as a function of the number of input modes N ,
 294 the time to update the optimal focusing pattern from one new piece of data, therefore comprising
 295 one (complex) output measurement, the update of the transfer matrix and the computation
 296 of the optimal input pattern. For a sufficiently low number of input modes ($N \leq 256$ in our
 297 implementation), the bottleneck is set by the refresh rate of the SLM—we indeed recover
 298 a baseline at ~ 50 ms, which is consistent with the response time $\gtrsim 10$ ms reported by the
 299 manufacturer. With increasing values of N , the update of the transfer matrix and the computation
 300 of the optimal pattern take a non-negligible time at each iteration, hence an onset at $N \sim 256$ is
 301 observed. In a conventional transfer-matrix measurement (blue line and data points) performed
 302 with Hadamard inputs, an additional $\mathcal{O}(N^2)$ is required to bring the optimal focusing pattern
 303 from the Hadamard to the canonical basis. The inverse QRD-RLS estimation technique (orange
 304 line and data points), based on Algorithm 1, would run with a $\mathcal{O}(N^3)$ complexity, as it involves
 305 a QR decomposition [41], but we retrieve a lower power dependence (~ 2.6) owing to the low
 306 number of data points above the onset. We should, however, recall that Algorithm 1 has been
 307 implemented to enjoy superior numerical stability. A typical RLS algorithm propagating the

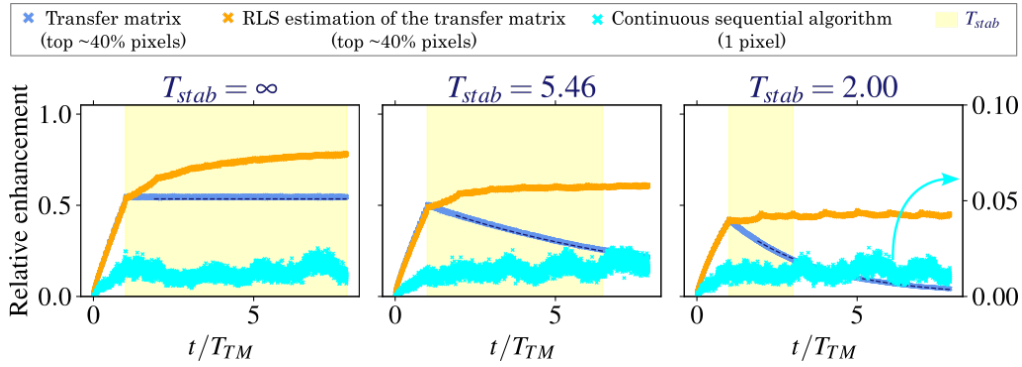


Fig. 5. Enhancement of the intensity produced at one output pixel through a scattering medium, as a function time and for different stability times T_{stab} . By using the linear approximation of Eq. (6) (valid across $\sim 40\%$ of the output pixels), we achieve wavefront shaping from intensity-only images. T_{TM} is the time to optimize over all the N degrees of freedom once. Blue points: conventional transfer-matrix measurement, lasting until $t/T_{TM} = 1$. Its estimate is held constant for later times, allowing the extraction of the stability time T_{stab} of the scattering medium via exponential fitting (dashed dark blue traces). When a finite T_{stab} is reported, the scattering medium is dynamic for the whole duration of the experiment. Orange points: RLS estimation of the transfer matrix. Cyan points: continuous sequential algorithm (CSA), plotted on a different scale on the right-hand vertical axis. To reproduce noisy measurements with suboptimal detector performance, all simulated intensities I were corrupted with additive Gaussian noise with a standard deviation of $20\sqrt{I}$ [40]. Simulated results.

308 inverse covariance matrix instead of its square root would require N^2 operations, thus matching
 309 the performance of a conventional transfer-matrix measurement. Owing to its updating routine,
 310 the iteration time of the CSA (cyan line and data points) is not impacted by the number of input
 311 modes, however its performance is limited in dynamic and noisy environments as shown above.
 312 As a final remark we stress that online optimization is run on the CPU of an Intel Core i7-6700
 313 processor with 4 cores, a clock speed of 3.4 GHz and 16 GB RAM, thus yielding the onset
 314 at $N \sim 256$. Therefore our experiments optimize over $N \leq 256$ modes. Such figures do not
 315 represent a bottleneck for real-time and online wavefront shaping at high enhancements. The
 316 number N can be definitely increased on a high-performance computing platform, for example
 317 implementing the RLS algorithm on a FPGA [42] (as was already done in [25] with a conventional
 318 continuous optimization algorithm for focusing through dynamic scattering media), or on a
 319 GPU [43].

320 5. Outlook

321 We have presented a recursive and online optimization procedure for the estimation of the transfer
 322 matrix of dynamic scattering media, combining the benefits of optimization-based routines and
 323 transfer-matrix measurements in wavefront shaping. Experimental and numerical demonstrations
 324 have been provided on conventional wavefront shaping setups and for different light-control tasks,
 325 noise levels and stability times. Its most intriguing feature is the possibility to optimize multi-
 326 and high-dimensional transfer matrices, without the need to store the history of past data in
 327 memory. Therefore, we foresee our method to turn out pivotal whenever the scattering behaviour
 328 of living biological specimens has to be tracked at various timescales.

329 In our proof-of-principle experiments, all optical modes change with the same rate, therefore
 330 they share the same oversampling ratio. However, when imaging large fields of view ($\sim 10^4 \mu\text{m}^2$)

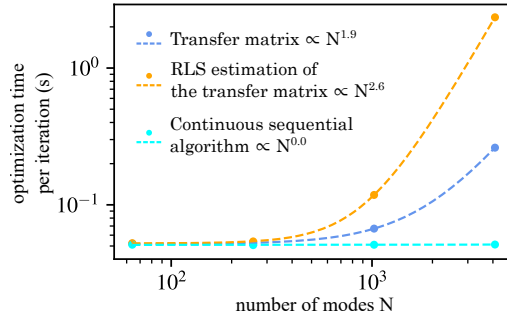


Fig. 6. Time required to update the optimal focusing pattern from one new piece of data, estimated in the experimental setup of Fig. 1(c) right, as a function of the number of input modes. Four phase-stepped intensity images are combined to estimate each output field across a field of view of $M = 256$ pixels. Blue points and line: conventional transfer-matrix measurement; orange: RLS estimation of the transfer matrix; cyan: continuous sequential algorithm (CSA). Each point is an average of 4096 measurements, such that the standard deviation of the mean is always within the marker size.

331 in biological media, timescales differing by factors as large as 100 are accessible. For example,
 332 the modes induced by blood flowing decorrelate in less than 10 ms (>100 Hz), while breathing
 333 modes can last as long as 800 ms (1.25 Hz) in mice [5]. As a result of that, the slowest modes
 334 enjoy an oversampling ratio close to 100. This means that, compared to an offline least-square
 335 estimation of the transfer matrix, a factor of 100 is saved in memory, which can be ultimately used
 336 to enlarge the field of view by 2 orders of magnitude. Using the latest MEMS modulators, $N =$
 337 600 modes can be optimized at a rate of 60 kHz in 10 ms, thus allowing the transfer matrix to be
 338 estimated at $M \sim 1.6 \cdot 10^6$ output pixels in parallel, assuming 16 GB RAM and double-precision
 339 floating-point format (16 B per complex matrix element). This is illustrated in Fig. 7(a), where
 340 the feasibility region for offline least-squares is shaded in blue and depends on the oversampling
 341 ratio. On the other hand, using the RLS estimation means oversampling does not play a role, so
 342 its feasibility region is much larger (orange shaded area). If we also consider that, in ultrafast
 343 wavefront shaping systems like [25], the SNR approaches 1, at an oversampling ratio of 100 the
 344 RLS estimation of the transfer matrix yields an improvement of the focus intensity by a factor of
 345 2, compared to a conventional transfer-matrix measurement with no oversampling [Fig. 7(b)].
 346 Focusing deep inside scattering media, at locations characterized by a specific stability time,
 347 may become a reality, thanks to the recent advancements in optimal light control, exploiting the
 348 knowledge of the transfer matrix measured at different times [44,45].

349 Besides sharing the same stability times, all the optical modes considered here are also
 350 unpredictable, as they feature random and independent increments according to Eq. (2). Should
 351 one possess prior knowledge on the medium dynamics, the RLS estimation may even be employed
 352 to predict future scattering behaviours as well as informing the user on the next most informative
 353 inputs to optimize information retrieval [46]. For example, breathing modes and heartbeat are
 354 known to induce revivals of correlations [5]. Such behaviour was neither reproduced in our
 355 experiments, nor accounted for in our model. In this context, the investigation of dynamic
 356 biological tissues would benefit from an implementation of the RLS algorithm dealing with
 357 quasi-periodic measurements.

358 We would finally like to remind that the effectiveness of the RLS algorithm is enabled
 359 by the linear relationship between the input and output patterns. Linearity is guaranteed by
 360 light-matter interaction via elastic scattering and thanks to our measurement scheme, allowing
 361 quantitative phase estimation of the output fields. When a non-linear transfer function was
 362 involved (Fig. 5), a linear approximation was made, at the cost of reduced performance. Towards

363 the recursive optimization of non-linear functions, a kernel version of the RLS algorithm has been
 364 proposed [47, 48]. It relies on performing linear regressions in a higher ($> N$) dimensional feature
 365 space, approximating the non-linear function. Its implementation, although more complicated
 366 than its linear counterpart, would be worth investigating, as it would unlock online learning of
 367 the transfer matrix of dynamic scattering media for a wide variety of contrast mechanisms, from
 fluorescence to non-linear coherent scattering.

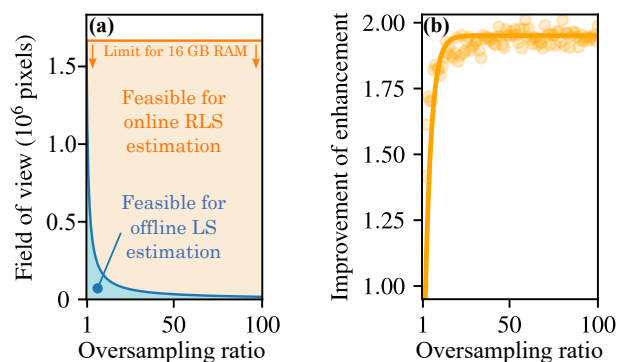


Fig. 7. (a) Feasibility regions of the offline least-squares algorithm (blue shaded area) and of the RLS algorithm (orange shaded area), on the plane spanned by the oversampling ratio and the field of view. (b) Evolution of the enhancement as a function of the oversampling ratio (relative to the value at oversampling = 1) at SNR = 1. Points: simulations; line: exponential fit.

368

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375 **Data Availability Statement.** Reconstruction and simulation codes underlying the results presented in
 376 this paper are available in Ref. [24]. The data underlying Fig. 3 are available in Dataset 1 at Ref. [29].

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