



HAL
open science

Dualities in G-Spaces May Underly Pre-Reflective Self-Consciousness

Grégoire Sergeant-Perthuis, Nils Ruet, Tonglin Yan, Kenneth Williford, David Rudrauf

► **To cite this version:**

Grégoire Sergeant-Perthuis, Nils Ruet, Tonglin Yan, Kenneth Williford, David Rudrauf. Dualities in G-Spaces May Underly Pre-Reflective Self-Consciousness. 27th annual meeting of the Association for the Scientific Study of Consciousness (ASSC 27), Jul 2024, Tokyo, Japan. 13, pp.365 - 396, 2022, 10.1007/s13164-022-00638-w . hal-04636572

HAL Id: hal-04636572

<https://hal.sorbonne-universite.fr/hal-04636572>

Submitted on 5 Jul 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

Dualities in G-Spaces May Underly Pre-Reflective Self-Consciousness

Grégoire Sergeant-Perthuis¹, Nils Ruet², Tonglin Yan², Kenneth Williford³, David Rudrauf²

¹: LCQB, Sorbonne Université, ²: CIAMS, Université Paris-Saclay, ³: Department of Philosophy & Humanities The University of Texas at Arlington.

Context

- The Projective Consciousness model:
 - **Reinforcement learning** + **phenomenological** aspects of consciousness
 - Reproduce the experience of **space** in 'robots'
 - Consciousness involves a subjective perspective, characterized by viewpoint-structured organization, a sense of unity (holistic world), embodiment, and an internal representation of the world in perspective from a specific standpoint.

Some definitions

Partially Observable Markov decision process + Group action

Definition 1 (Markov Decision Process: Definition 1). A Markov Decision Process, is a collection $\langle S, A, T, r \rangle$ where,

- S is the set of configurations of the environment
- A is the collection of actions of the agent
- $T : S \times A \rightarrow S$ is the transition probability; it captures the consequences of the action $a \in A$ of the agent on the environment that changes from s_t to s_{t+1}
- $r : S \times A \times S \rightarrow \mathbb{R}$; it is the reward function for an action $a \in A$ and two states (s, s') thought of as s_t and s_{t+1} .

Definition 2 (Partially Observable Markov Decision Process). A POMDP is defined as a tuple $\langle S, A, T, r, O, Z \rangle$, where $\langle S, A, T, r \rangle$ is an MDP and,

- O is the set of possible observations.
- Z is the observation kernel, $Z : S \times A \rightarrow O$, which specifies the probability of observing a particular observation given the current state and action.
- r is a reward function which domain is $S \times A$; $r : S \times A \rightarrow \mathbb{R}$.

Definition 3 (Group-structured space, G-space). S is a group-structured space for the group G when there is a map $h : G \times S \rightarrow S$ denoted as $h(g, s) = g \cdot s$ for $g \in G$ and $s \in S$, such that,

1. $(g \cdot g_1) \cdot s = g \cdot (g_1 \cdot s)$ for all $g, g_1 \in G, s \in S$
2. $e \cdot s = s$, for all $s \in S$

For a given group G , such space is called a G-space.

Group-structured world model

Perspectives \rightsquigarrow Group G
World model \rightsquigarrow G-space

Definition 4 (MDP and POMDP with group-structured state space). A MDP with a group-structured state space is a tuple $\langle S, A, T, r, G \rangle$ where G is a group and $\langle S, A, T, r \rangle$ is a MDP that satisfies the following properties:

- S is a G-space
- G is a subset of the set of actions A ,
- for all $g \in G, T(s'|s, g) = 1[s' = g \cdot s]$

A POMDP with a group-structured state space is a tuple $\langle S, A, T, r, O, Z, G \rangle$ where $\langle S, A, T, r, G \rangle$ is a group-structured MDP (structured by G) and $\langle S, A, T, r, O, Z \rangle$ is a POMDP.

Pre-reflective Self-consciousness

- Pre-reflective Self-consciousness (PRSC):
 - the property of consciousness to be conscious of itself as an intrinsic part of the moment-to-moment constitution of consciousness and not as the result of a secondary, reflective act of constitution of consciousness, taking as an object a previous state of subject-less consciousness
- Our focus is on:
 - accounting for how an agent could singularize itself in a pre-reflective manner, i. e. find a trace of its own presence and existence or be directly informed about its own existence through the representation of the environment it creates, as a result of intrinsic properties of its internal space of representation.

PRSC in G-space?

- Simplest example: X is a homogeneous space,
 - G-space with $X = \{g \cdot x_0 \mid g \in G\}$ for any x_0 in X .
- In this case:
 - For any choice $x_0 \in X, X \simeq G/H_{x_0}$ with $H_{x_0} = \{g \mid g \cdot x_0 = x_0\}$.
 - In other words, a choice of a reference point x_0 in the state space is enough to *frame* the agent's actions in space through the map $g \in G \mapsto g \cdot x_0 \in X$.

Dualities in G-spaces: PRSC?

A very simple instance of the **Yoneda Lemma**

Definition 5. The category associated to a group G is the one with one object $*$ and morphisms $[\ast, \ast] = G$ with composition the one defined on groups.

Proposition 1. A G-space is a functor from the category G to **Set**

Proposition 2 (Yoneda Lemma for G-spaces). For a functor F from G to **Set**,

$$[h_*, F] \simeq FX \quad (1)$$

in a functorial manner.

In particular, $\phi \in [h_*, F]$ is uniquely defined by $\phi(id)$, where id is the identity map $id \in [\ast, \ast]$; $\phi(id) \in X$ is simply a choice of point x_0 .

Limitation of G-spaces

- Not all G-space are good candidates for encoding the phenomenology of space and its content
 - Assume that a sensory modalities $s \in \mathbb{R}^N$ is reconstructed into a point $x \in X$ in the state space
 - for a perturbation $\epsilon \in \mathbb{R}^N, s + \epsilon$ is represented as \tilde{x}
 - in general $\{g \cdot x \mid g \in G\}$ and $\{g \cdot \tilde{x} \mid g \in G\}$ are different
 - ↪ **Instability!**
 - One possible solution:
 - Consider particular G-spaces : with more properties
 - E.g. a fiber bundle over a homogeneous space.
- Future direction, explore the relation between the duality of such spaces and PRSC.

References

- [1] K. Williford, D. Bennequin, K. Friston, and D. Rudrauf, "The projective consciousness model and phenomenal selfhood," *Frontiers in Psychology*, 2018.
- [2] D. Rudrauf, D. Bennequin, and K. Williford, "The Moon illusion explained by the projective consciousness model," *Journal of Theoretical Biology*, 2020.
- [3] G. Sergeant-Perthuis, N. Ruet, D. Rudrauf, D. Ognibene, and Y. Tisserand, "Influence of the geometry of the feature space on curiosity based exploration," in *NeurIPS 2023 workshop: Information-Theoretic Principles in Cognitive Systems*, 2023. [Online]. Available: <https://openreview.net/forum?id=5r2mVydu4r>.
- [4] K. Williford, D. Bennequin, and D. Rudrauf, "Pre-Reflective Self-Consciousness & Projective Geometry," *Review of Philosophy and Psychology*, vol. 13, no. 2, pp. 365–396, Jun. 2022, ISSN: 1878-5166. DOI: 10.1007/s13164-022-00638-w. [Online]. Available: <https://doi.org/10.1007/s13164-022-00638-w>.