Dualities in G-Spaces May Underly Pre-Reflective Self-Consciousness

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Context

- The Projective Consciousness model:

→ **Reinforcement learning** + **phenomenological** aspects of consciousness

→ Reproduce the experience of ***space*** in 'robots'

Consciousness involves a subjective perspective, characterized by viewpoint-structured organization, a sense of unity (holistic world), embodiment, and an internal representation of the world in perspective from a specific standpoint.

Some definitions

Partially Observable Markov decision process + Group action

Definition 1 (Markov Decision Process: Definition 1)**.** *A Markov Decision Process, is a collection $\langle S, A, T, r\rangle$ where,*

- *S is the set of configurations of the environment*
- *A is the collection of actions of the agent*
- *$T : S \times A \to S$ is the transition probability; it captures the consequences of the action $a \in A$ of the agent on the environment that changes from s_t to s_{t+1}*
- *$r : S \times A \times S \to \mathbb{R}$; it is the reward function for an action $a \in A$ and two states (s, s') thought of as s_t and s_{t+1}.*

Definition 2 (Partially Observable Markov Decision Process)**.** *A POMDP is defined as a tuple $\langle S, A, T, r, O, Z\rangle$, where $\langle S, A, T, r\rangle$ is an MDP and,*

- *O is the set of possible observations.*
- *Z is the observation kernel, $Z : S \times A \to O$, which specifies the probability of observing a particular observation given the current state and action.*
- *r is a reward function which domain is $S \times A$; $r : S \times A \to \mathbb{R}$.*

Definition 3 (Group-structured space, G-space)**.** *S is a group-structured space for the group G when there is a map $h : G \times S \to S$ denoted as $h(g, s) = g.s$ for $g \in G$ and $s \in S$, such that,*

1. $(g.g_1).s = g.(g_1.s)$ for all $g, g_1 \in G$, $s \in S$

2. $e.s = s$, for all $s \in S$

For a given group G, such space is called a G-space.

Group-structured world model

Perspectives $\rightsquigarrow$ Group G

World model $\rightsquigarrow$ G-space

Definition 4 (MDP and POMDP with group-structured state space)**.** *A MDP with a group-structured state space is a tuple $\langle S, A, T, r, G\rangle$ where G is a group and $\langle S, A, T, r\rangle$ is a MDP that satisfies the following properties:*

- *S is a $G-$space*
- *G is a subset of the set of actions A,*
- *for all $g \in G$, $T(s'|s, g) = 1[s' = g.s]$*

A POMDP with a group-structured state space is a tuple $\langle S, A, T, r, O, Z, G\rangle$ where $\langle S, A, T, r, G\rangle$ is a group-structured MDP (structured by G) and $\langle S, A, T, r, O, Z\rangle$ is a POMDP.

Pre-reflective Self-consciousness

- Pre-reflective Self-consciousness (PRSC):

→ the property of consciousness to be conscious of itself as an intrinsic part of the moment-to-moment constitution of consciousness and not as the result of a secondary, reflective act of constitution of consciousness, taking as an object a previous state of subjectless consciousness

- Our focus is on:

→ accounting for how an agent could singularize itself in a pre-reflective manner, i. e. find a trace of its own presence and existence or be directly informed about its own existence through the representation of the environment it creates, as a result of intrinsic properties of its internal space of representation.

PRSC in G-space?

- Simplest example: X is a homogeneous space,

→ G-space with $X = \{g \cdot x_0 \mid g \in G\}$ for any x_0 in X.

- In this case:

→ For any *choice* $x_0 \in X$, $X \simeq G/H_{x_0}$ with $H_{x_0} = \{g \mid g \cdot x_0 = x_0\}$.

→ In other words, a choice of a reference point x_0 in the state space is enough to *frame* the agent's actions in space through the map $g \in G \mapsto g \cdot x_0 \in X$.

Dualities in G-spaces: PRSC?

A very simple instance of the **Yoneda Lemma**

Definition 5. *The category associated to a group G is the one with one object $*$ and morphisms $[*, *] = G$ with composition the one defined on groups.*

Proposition 1. *A G-space is a functor from the category G to **Set***

Proposition 2 (Yoneda Lemma for G-spaces)**.** *For a functor F from G to **Set**,*

$$[h_*, F] \simeq FX \tag{1}$$

in a functorial manner.

In particular, $\phi \in [h_*, F]$ is uniquely defined by $\phi(id)$, where id is the identity map $id \in [*, *]$; $\phi(id) \in X$ is simply a choice of point x_0.

Limitation of G-spaces

- Not all G-space are good candidates for encoding the phenomenology of space and its content

→ Assume that a sensory modalities $s \in \mathbb{R}^N$ is reconstructed into a point $x \in X$ in the state space

→ for a perturbation $\epsilon \in \mathbb{R}^N$, $s + \epsilon$ is represented as $\tilde{x}$

→ in general $\{g.x | g \in G\}$ and $\{g.\tilde{x} | g \in G\}$ are different

↪ ***Instability!***

- One possible solution:

→ Consider particular G-spaces : with more properties

→ E.g. a fiber bundle over a homogeneous space.

Future direction, explore the relation between the duality of such spaces and PRSC.

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