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# Dualities in G-Spaces May Underly Pre-Reflective Self-Consciousness

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## Context

- The Projective Consciousness model:
  - **Reinforcement learning** + **phenomenological** aspects of consciousness
  - Reproduce the experience of **space** in 'robots'  
*Consciousness involves a subjective perspective, characterized by viewpoint-structured organization, a sense of unity (holistic world), embodiment, and an internal representation of the world in perspective from a specific standpoint.*

## Some definitions

### Partially Observable Markov decision process + Group action

**Definition 1** (Markov Decision Process: Definition 1). A Markov Decision Process, is a collection  $\langle S, A, T, r \rangle$  where,

- $S$  is the set of configurations of the environment
- $A$  is the collection of actions of the agent
- $T : S \times A \rightarrow S$  is the transition probability; it captures the consequences of the action  $a \in A$  of the agent on the environment that changes from  $s_t$  to  $s_{t+1}$
- $r : S \times A \times S \rightarrow \mathbb{R}$ ; it is the reward function for an action  $a \in A$  and two states  $(s, s')$  thought of as  $s_t$  and  $s_{t+1}$ .

**Definition 2** (Partially Observable Markov Decision Process). A POMDP is defined as a tuple  $\langle S, A, T, r, O, Z \rangle$ , where  $\langle S, A, T, r \rangle$  is an MDP and,

- $O$  is the set of possible observations.
- $Z$  is the observation kernel,  $Z : S \times A \rightarrow O$ , which specifies the probability of observing a particular observation given the current state and action.
- $r$  is a reward function which domain is  $S \times A$ ;  $r : S \times A \rightarrow \mathbb{R}$ .

**Definition 3** (Group-structured space, G-space).  $S$  is a group-structured space for the group  $G$  when there is a map  $h : G \times S \rightarrow S$  denoted as  $h(g, s) = g \cdot s$  for  $g \in G$  and  $s \in S$ , such that,

1.  $(g \cdot g_1) \cdot s = g \cdot (g_1 \cdot s)$  for all  $g, g_1 \in G, s \in S$
2.  $e \cdot s = s$ , for all  $s \in S$

For a given group  $G$ , such space is called a G-space.

## Group-structured world model

Perspectives  $\rightsquigarrow$  Group  $G$   
World model  $\rightsquigarrow$  G-space

**Definition 4** (MDP and POMDP with group-structured state space). A MDP with a group-structured state space is a tuple  $\langle S, A, T, r, G \rangle$  where  $G$  is a group and  $\langle S, A, T, r \rangle$  is a MDP that satisfies the following properties:

- $S$  is a G-space
- $G$  is a subset of the set of actions  $A$ ,
- for all  $g \in G, T(s'|s, g) = 1[s' = g \cdot s]$

A POMDP with a group-structured state space is a tuple  $\langle S, A, T, r, O, Z, G \rangle$  where  $\langle S, A, T, r, G \rangle$  is a group-structured MDP (structured by  $G$ ) and  $\langle S, A, T, r, O, Z \rangle$  is a POMDP.

## Pre-reflective Self-consciousness

- Pre-reflective Self-consciousness (PRSC):
  - the property of consciousness to be conscious of itself as an intrinsic part of the moment-to-moment constitution of consciousness and not as the result of a secondary, reflective act of constitution of consciousness, taking as an object a previous state of subject-less consciousness
- Our focus is on:
  - accounting for how an agent could singularize itself in a pre-reflective manner, i. e. find a trace of its own presence and existence or be directly informed about its own existence through the representation of the environment it creates, as a result of intrinsic properties of its internal space of representation.

## PRSC in G-space?

- Simplest example:  $X$  is a homogeneous space,
  - G-space with  $X = \{g \cdot x_0 \mid g \in G\}$  for any  $x_0$  in  $X$ .
- In this case:
  - For any choice  $x_0 \in X, X \simeq G/H_{x_0}$  with  $H_{x_0} = \{g \mid g \cdot x_0 = x_0\}$ .
  - In other words, a choice of a reference point  $x_0$  in the state space is enough to *frame* the agent's actions in space through the map  $g \in G \mapsto g \cdot x_0 \in X$ .

## Dualities in G-spaces: PRSC?

A very simple instance of the **Yoneda Lemma**

**Definition 5.** The category associated to a group  $G$  is the one with one object  $*$  and morphisms  $[\ast, \ast] = G$  with composition the one defined on groups.

**Proposition 1.** A G-space is a functor from the category  $G$  to **Set**

**Proposition 2** (Yoneda Lemma for G-spaces). For a functor  $F$  from  $G$  to **Set**,

$$[h_*, F] \simeq FX \quad (1)$$

in a functorial manner.

In particular,  $\phi \in [h_*, F]$  is uniquely defined by  $\phi(id)$ , where  $id$  is the identity map  $id \in [\ast, \ast]$ ;  $\phi(id) \in X$  is simply a choice of point  $x_0$ .

## Limitation of G-spaces

- Not all G-space are good candidates for encoding the phenomenology of space and its content
    - Assume that a sensory modalities  $s \in \mathbb{R}^N$  is reconstructed into a point  $x \in X$  in the state space
    - for a perturbation  $\epsilon \in \mathbb{R}^N, s + \epsilon$  is represented as  $\tilde{x}$
    - in general  $\{g \cdot x \mid g \in G\}$  and  $\{g \cdot \tilde{x} \mid g \in G\}$  are different
    - ↪ **Instability!**
  - One possible solution:
    - Consider particular G-spaces : with more properties
    - E.g. a fiber bundle over a homogeneous space.
- Future direction, explore the relation between the duality of such spaces and PRSC.

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